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REAL TIME LOSS DETECTION FOR SNM IN PROCESS*

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Abstract

In this paper we discuss the basis of a design for real time special nuclear material (SNM) loss detectors. The design utilizes process measurements and signal processing techniques to produce a timely estimate of material loss. A state estimator is employed as the primary signal processing algorithm. Material loss is indicated by changes in the states or process innovations (residuals). The design philosophy is discussed in the context of these changes.

I. Introduction

The need to account for and safeguard special nuclear materials (SNM) in the fabrication and reprocessing of nuclear fuels is apparent. Most techniques currently employed in obtaining the physical inventory of these SNM are off-line. If timely accounting is to be used to detect material loss, then more rapid and accurate inventory techniques are required. Some recent work in this area has proposed the use of on-line techniques which appear promising.[1,2,3] Part of our research program for the NRC has been to investigate the capability of on-line material estimator-detectors to perform prescribed inventory and loss detection tasks.

In this paper we are primarily concerned with the structural analysis of a process monitor as a device to detect material loss. We define a process monitor to be a device which (1) makes measurements on physical parameters, (2) estimates process states (material accounting), and (3) makes a decision with regard to material loss (diversion detection). We evaluated an optimal estimator-detector scheme, henceforth termed diversion detector, analogous to devices employed in current aerospace system failure detection systems.[4] The emphasis of this paper is on the signal processing prior to actual detection (see Figure 1). We discuss in Section II a particular philosophy of diversion detection - one which requires minimal modeling of diversion acts. The theoretical analysis of the design philosophy is discussed in Section III. Analysis of the design algorithms along with the corresponding performance evaluation is discussed in Section IV for a simulated reprocessing plant unit.

II. Diversion Detection Concepts

In this section we present (conceptually) the design of on-line, real-time diversion detectors. In the next section we discuss the theoretical justification for these concepts. Our basic objective is to design a robust device capable of providing accurate estimates of SNM in process and timely detection of material losses. By robust we mean a device that can detect diversion in a wide range of scenarios.

Suppose that a process has measurements $\{z_k\}$ contaminated with noise and is given by

$$z_k = h(x_k) + v_k \tag{1}$$

where x_k, z_k are the n-state and p-measurement vectors, $h(\cdot)$ is a p-vector function and v_k is a zero mean Gaussian vector with covariance R_k representing the measurement uncertainties.

The state x_k is related to the mass of SNM (e.g., concentration) in process. Changes in the state from normal or expected levels can be used to infer abnormal process conditions. These conditions can be interpreted either as an upset, which is important for control purposes, or as a potential theft of material.

Estimates of the state can be accomplished in many different ways depending on the accuracy and precision required. A state estimator is a computer algorithm which may incorporate: (i) knowledge of the chemical process phenomonology; (ii) knowledge of the measurement system; (iii) knowledge of measurement uncertainties in the form of mathematical models to produce an estimate of the quantity of SNM in process.

Estimates are calculated in a variety of ways. For example, there are process model-based estimators (Kalman filters[5]), statistical model-based estimators (Box-Jenkins filters[6]), statistic-based estimators (Covariance filters[7]), or even optimization-based estimators (Gradient filters[8]). In any case, most state estimators can be placed in a recursive form with the various subtleties emerging in the

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calculation of the current estimate ($\hat{x}_{old}$). The standard technique employed is based on updating the current estimate as a new piece of measurement data becomes available. The state estimates generally take the recurrence form

$$\hat{x}_{new} = \hat{x}_{old} + K_k \epsilon_{new} \tag{2}$$

where

$$\epsilon_{new} = z_k - \hat{z}_{old} = z_k - h(\hat{x}_{old}) \tag{3}$$

Here we see that the new state estimate is obtained by correcting the old estimate by a K-weighted amount. The term ϵ_{new} is the new information or innovations[5], i.e., it is the difference between the actual measurement and the predicted measurement ($\hat{z}_{old}$) based on our old state estimate. The computation of the weight K depends on the error criterion used (e.g., mean-squared, absolute, etc.)[7]

Equations (2) and (3) are the quantities of prime concern from a safeguards viewpoint. The corrected state ($\hat{x}_{new}$) is an estimate of the amount of SNM in the unit process under investigation (material accounting). Under normal conditions, an estimator is tuned such that the innovations sequence is a zero mean, white (independent) process and $\hat{x}_{new}$ is an accurate estimate of SNM in process. No attempt is made to model adversary diversion scenarios because the effect of a diversion will result in a model mismatch. This mismatch is reflected by variations in the innovations statistics, i.e., they become biased and correlated. Thus, a state estimator can provide us with two variables of interest - the state estimate for use in material accounting and detection, and the innovations for indicating the degree of process and measurement models mismatch. The latter can be used in an innovations-based detection operation.

State-based detectors rely primarily on the tracking capability of the filter while innovations-based detectors rely on model mismatching information provided in the statistics of the innovation sequence. The inverse relationships between state and innovations responses to a material loss (Δx) (i.e., $x \to x + \Delta x$) for an estimator tuned to different bandwidths or time constants, τ, is depicted in Figure 2. These features can be exploited for safeguards purposes. Without using the innovations information, a state-based estimator must trade off estimation tracking accuracy with response time. Figure 2a shows that low variance estimates (e.g., τ_1 or τ_2) have an inherent lag time before they begin to track a diversion. By using the innovations to detect diversion, an estimator can be tuned to yield improved steady state estimates (normal operation) for material accounting purposes. Note the drastic change in the innovations sequence in Figure 2b for τ_1 and for a large Δx change. In this case the estimator is insensitive to Δx changes and therefore the innovations indicate the mismatch or lack of tracking.

Material diversion detection is based on simple hypothesis testing techniques which result in a likelihood ratio test for a sufficient statistic. A simple threshold detector which senses changes in the estimator output $\hat{x}$ relative to a reference value can be implemented using the priori knowledge of the estimator error covariance while innovations-based detectors require further computation. In the next section we quantify these concepts.

III. Diversion Detection Theory

In this section we develop the theoretical background necessary to quantify the concepts of the previous section. We use a process model-based estimator---the Kalman estimator[5] as the primary signal processor in the diversion detector design. The robustness of the Kalman estimator led us to this choice.

Assume that a process upset or diversion of SNM is represented by a deviation of the process state (see Figure 3) from the normal process trajectory, i.e.,

$$x^d := x - \Delta x^\dagger \tag{4}$$

where

x^d is the process state trajectory during upset or diversion.

x is the process state trajectory during normal operation.

Δx is the deviation of the process state from the normal trajectory.

We examine the effect of the deviation Δx on the state estimator subsequently, but first recall that from the linearity property[5] of conditional expectations we obtain

$$\hat{x}^d = \hat{x} - \widehat{\Delta x} \tag{5}$$

Therefore we can decompose the state estimate $\hat{x}^d$ into two parts: the estimate during normal process operation and the deviation from normal during upset or diversion. Let us examine the state estimator $\hat{x}^d$ more closely.

$$\begin{aligned} \hat{x}^d_{new} &= \hat{x}^d_{old} + K\epsilon^d_{new} \\ \epsilon^d_{new} &= z^d - h\hat{x}_{old} \end{aligned} \tag{6}$$

(Note that a process model is used to operate $\hat{x}_{OLD}$[5]).

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†Time subscripts are suppressed throughout; therefore $x \to x_k$.

Substituting (5) into (6) and rearranging we see that

$$\hat{x}^d_{new} = \underbrace{\hat{x}_{new}}_{\text{Normal}} - \underbrace{(\Delta\hat{x}_{old} + K\Delta\epsilon_{new})}_{\text{Deviation}} \qquad (7)$$

where

$$\Delta\epsilon_{new} = \Delta z - h\widehat{\Delta x}_{old}$$

The performance of the state estimator is indicated by two quantities: estimation error ($\tilde{x} := x_{TRUE} - \hat{x}$)† and innovations. The tracking error indicates how accurately the process state is estimated, while the innovations indicate the model adequacy. For the decomposed estimator of (7) the estimation error is

$$\tilde{x}^d = \underbrace{\tilde{x}}_{\text{Normal}} - \underbrace{\widetilde{\Delta x}}_{\text{Deviation}} \qquad (8)$$

and the innovations are

$$\epsilon^d_{new} = \underbrace{\epsilon_{new}}_{\text{Normal}} - \underbrace{\Delta\epsilon_{new}}_{\text{Deviation}} \qquad (9)$$

This decomposition will help us analyze the estimator performance for the three cases presented in Figure 2:

(i) τ_3-Estimator-- Tracks†† State During Upset or Diversion
(ii) τ_1-Estimator-- Does Not Track the State During Upset or Diversion
(iii) τ_2-Estimator-- Partially Tracks the State During Upset or Diversion

To quantify how well the state estimator of (7) tracks, we examine the statistics of the estimation error and innovations for each case.

Case (i): τ_3-Estimator

The τ_3-Estimator tracks the diversion accurately within the diversion window ($\hat{x}^d = \hat{x} - \widehat{\Delta x}$; $\tilde{x}^d = \tilde{x} - \widetilde{\Delta x}$; $\epsilon^d = \epsilon - \Delta\epsilon$). Under this condition $\tilde{x}^d$, ϵ^d are zero mean with respective covariances $\tilde{\pi}^d$ and R^d_ϵ [5].

Case (ii): τ_1-Estimator

The τ_1-Estimator is not robust enough to track deviations from the normal state trajectory. ($x^d = \hat{x}$; $\tilde{x}^d = \tilde{x}$; $\epsilon^d = \epsilon$). The estimation error for this case is given by

$$\tilde{x}^* = \tilde{x} - \Delta x \qquad (10)$$

The mean tracking error is

$$E\tilde{x}^* = E\tilde{x} - E\Delta x = -E\Delta x =: -\overline{\Delta x}$$

and

$$\mathrm{Cov}(\tilde{x}^*) = E(\tilde{x} - \widetilde{\Delta x})(\tilde{x} - \Delta x)^T - \overline{\Delta x}\,\overline{\Delta x}^T =: \widetilde{\Delta\pi} - \overline{\Delta x}\,\overline{\Delta x}^T \dagger$$

The innovations sequence for this case is given by

$$\overset{*}{\epsilon}_{new} = \epsilon_{new} - \Delta z \qquad (11)$$

with the statistics

$$E\overset{*}{\epsilon}_{new} = E\epsilon_{new} - E\Delta z = -\overline{\Delta z}$$

and

$$\mathrm{Cov}(\overset{*}{\epsilon}_{new}) = E(\epsilon_{new} - \Delta z)(\epsilon_{new} - \Delta z)^T - \overline{\Delta z}\,\overline{\Delta z}^{T\dagger\dagger} =: \Delta R_\epsilon - \overline{\Delta z}\,\overline{\Delta z}^T$$

Finally, we consider the case of partial tracking where we assume that

$$x^\theta := x - \theta\Delta x + (\theta - I)\Delta x \qquad (12)$$

where $\theta \in R^{nxn}$ and θ = diagonal (θ_i).

Case (iii): τ_2-Estimator

The τ_2-Estimator is sensitive enough to respond to deviations, but not robust enough to accurately track ($\hat{x}^d = \hat{x} + (\theta - I)\widehat{\Delta x}$; $\tilde{x}^d = \tilde{x} + (\theta - I)\widetilde{\Delta x}$; $\epsilon^d = \epsilon - \Delta\epsilon^{(\theta-I)}$). The estimation error for this case is

$$\tilde{x}^\theta = \tilde{x} + \widetilde{\Delta x}^{(\theta-I)} - \theta\Delta x \qquad (13)$$

where

$$\widetilde{\Delta x}^{(\theta-I)} := (\theta - I)(\Delta x - \widehat{\Delta x})$$

The mean tracking error is given by

$$E\tilde{x}^\theta = E\tilde{x} + (\theta - I)\,E\widetilde{\Delta x}^{(\theta-I)} - \theta E\Delta x = -\theta\overline{\Delta x}$$

† The estimation error cannot be calculated in practice unless the true trajectory x_{TRUE} is known.

†† By tracking we mean the estimator accurately estimates the state within a reasonable time period called the diversion window.

† $\widetilde{\Delta\pi}$ contains cross terms since x and Δx are correlated.

†† The innovations are no longer white since they are time correlated due to the x and Δx cross terms.

and the

$$\mathrm{Cov}(\tilde{x}^\theta) = E(\tilde{x} - \widetilde{\Delta x}^{(\theta-I)})(\tilde{x} - \widetilde{\Delta x}^{(\theta-I)})^T - \theta\overline{\Delta x}\;\overline{\Delta x}^T\theta^T =: \widetilde{\Delta\pi}^{(\theta-I)} - \theta\overline{\Delta x}\;\overline{\Delta x}^T\theta^T$$

Similarly, the innovations are given by

$$\epsilon^\theta_{new} = \epsilon_{new} + \Delta\epsilon^{(\theta-I)}_{new} - \Delta z^\theta \tag{14}$$

where

$$\Delta\epsilon^{(\theta-I)} = H(\theta - I)\,\widetilde{\Delta x}_{old}$$

$$\Delta z^\theta := H\theta\Delta x$$

The statistics are

$$E\epsilon^\theta_{new} = E\epsilon_{new} + E\Delta\epsilon^{(\theta-I)}_{new} - E\Delta z^\theta = -\overline{\Delta z}^\theta$$

and

$$\mathrm{Cov}(\epsilon^\theta_{new}) = E(\epsilon_{new} + \Delta\epsilon^{(\theta-I)}_{new})(\epsilon_{new} + \Delta\epsilon^{(\theta-I)}_{new})^T - \overline{\Delta z}^\theta\;\overline{\Delta z}^{\theta^T} =: \Delta R^\theta_\epsilon - \overline{\Delta z}^\theta\;\overline{\Delta z}^{\theta^T}$$

We summarize these results in Table I.

Another approach to this problem exists by modeling the diversion of SNM as a change in model parameters. In this case, one can show that the innovations sequence also becomes non-zero mean and correlated. In either case, we see that parameter changes and tracking inaccuracies both result in model mismatch indicated by statistical changes in the innovations. In the next section we explore these concepts further by considering a simulated diversion of SNM from a plutonium nitrate storage tank.

IV. An Application

In this section we analyze the performance of the signal processing algorithm on noisy data simulating typical operation of a plutonium nitrate storage tank. We consider the three cases of estimator performance discussed in the previous section.

A linear, dynamic and stochastic model was developed for $Pu(NO_3)_4$ solution which was assumed to consist of three molar HNO_3, Pu and H_2O. The dynamics of the solution arise due to radiolysis effects, through evaporation of H_2O and HNO_3 and by adversary diversion activities.[1,9] See Figure 4. Uncertainties in these effects are represented as process noise. The process model relating solution mass-$M_s(t)$ and density -$\rho(t)$ while neglecting the secondary effects of radiolysis is given by

$$\dot{M}_s = -\lambda_H + w_1 \qquad \dot{\rho} = w_2 \tag{15}$$

where

λ_H is the H_2O, HNO_3 evaporation coefficient,

and

$$w \sim N(0,Q)$$

The measurement system used in the tank is a standard air bubbler instrument which measures differential pressures proportional to density and height. The measurement model can be developed from the pressure drops (ΔP_A, ΔP_B) as

$$z_k = \begin{bmatrix} g/\beta & -(\alpha/\beta)g \\ 0 & g'H \end{bmatrix} \begin{bmatrix} M_s(t_k) \\ \rho_k \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix} \tag{16}$$

where

- g is the gravity constant (m/sec^2);
- β is the tank cross-sectional area (m^2);
- α is the tank heel (m);
- H is the distance between density bubbler tubes (m);
- v_k is a random vector modeling instrument uncertainty (N/m^2) and is distributed $N(0,R_k)$;

A linear Kalman filter algorithm[5] was used to give least-squares estimates of the process model variables based on linear combinations of the measurement data. Even though $M_s(t)$ is not measured, it can be reconstructed from the measurements by the estimator.

Simulated data are used to illustrate the performance of the filter in estimating $M_s(t)$ and $\rho(t)$. The estimator input was simulated (corresponding to Δz = 300 $Kg/msec^2$) measurements (see Table III for parameters) of density $\rho(t)$ with a diversion of 10 kg of solution mass. In this example the storage tank capacity was 170 kg of Pu or 983 kg of $Pu(NO_3)_4$. Large diversion signal levels are used to illustrate the concepts.

We consider the three cases and examine the resulting estimation error and innovations.

<u>Case (i)</u>: τ_3-Estimator

In this case we obtain the estimate $\hat{x}^d$ of (7). Recall that an estimator is considered <u>tracking</u> when 95% of the error samples ($\tilde{x}^d$) reside within the 2σ confidence limits in the diversion window. As shown in Figure 5, the state estimate tracks the diversion, the estimation error is within the 2σ limits (Figure 6), and the innovations are indeed zero mean and white† (Figure 7). The $E\tilde{x}_1 \approx 0$, $E\epsilon_1 \approx 0$ and the covariances match those predicted by the estimator.

†The statistics of the innovations were calculated separately and a whiteness test performed; however, this property can be seen from the figure also in the example.

Case (ii): τ_1-Estimator

The estimator does not respond within the diversion window. In this case the error is Δx and all samples after the start of diversion lie outside the confidence limits. The statistics $E\tilde{x}_1 = -\overline{\Delta x}_1 \approx -10$ kg, $E\epsilon_1 = -\overline{\Delta z}_1 \approx -300$ kg/msec^2, and the innovations were no longer white. This case was not simulated but would result in plots similar to τ_1 of Figure 2.

Case (iii): τ_2-Estimator

The estimator partially responds to the diversion. The estimator underestimates the diversion by ~ 5 kg as shown in Figure 8 and the corresponding estimation error in Figure 9 ($E\tilde{x}_1 = -\theta\overline{\Delta x}_1 \approx -7.5$ kg). The innovations are shown in Figure 10. We see that they are biased ($E\epsilon_1 = -\overline{\Delta z}_1\theta \approx -115$ kg/msec^2) and correlated (from whiteness test). We summarize these results in Table III. The results appear quite reasonable and correspond to those predicted by the theory. Although we did not discuss the design of the decision algorithm (refer to Figure 1), previous work has reported these results[3] for a given detection algorithm. The signal processing algorithm is the crucial component of a diversion detector because it must improve the signal-to-noise ratio and thereby increase the detection sensitivity.

This analysis quantifies the tradeoff between estimation accuracy a d response time to changes in the process var ables. In any "measurement" scheme there is usually this tradeoff. This completes the application to a plutonium nitrate storage tank.

V. Summary

This paper has analyzed the performance of a model-based signal processing algorithm as an integral component of diversion detector design. We developed the analysis conceptually, theoretically, and through an example.

The concepts of model-based estimators imply that either the state or innovation signals provide different diversion information which can be used for detection purposes. The estimator was examined in three cases: (i) tracking; (ii) not tracking; and (iii) partially tracking. Theory shows that these cases can be reasonably quantified and an application to a plutonium nitrate storage tank was shown.

Acknowledgments

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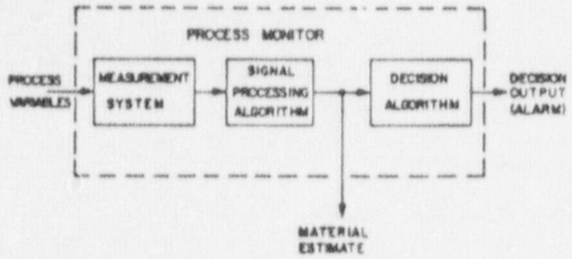

FIGURE 1 - PROCESS MONITOR COMPONENTS

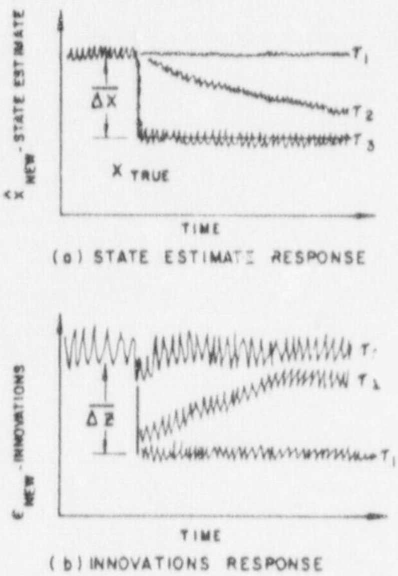

FIGURE 2 - STATE AND INNOVATIONS RESPONSE TO MODEL MISMATCH ($\tau_1 > \tau_2 > \tau_3$)

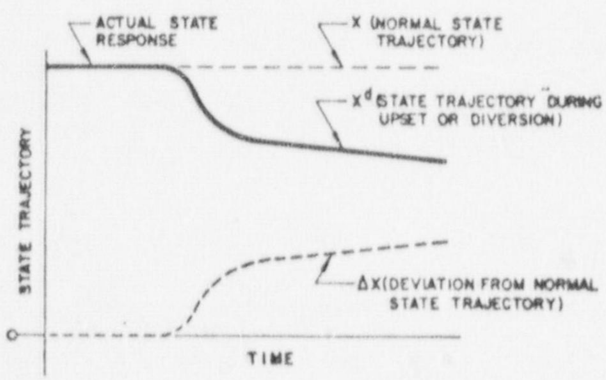

FIGURE 3 - DECOMPOSITION OF STATE TRAJECTORY DURING UPSET OR DIVERSION ($X^d = X - \Delta X$)

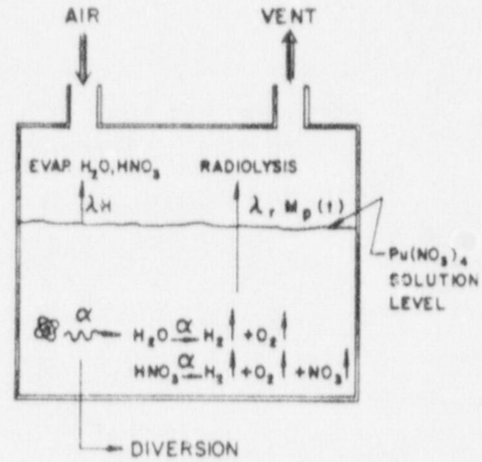

FIGURE 4 - DIAGRAM OF $Pu(NO_3)_4$ STORAGE TANK ILLUSTRATING SOLUTION DYNAMICS

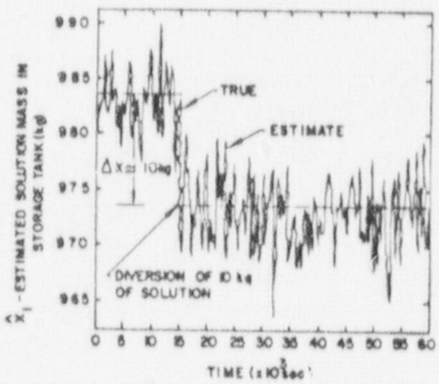

FIGURE 5 - STATE RESPONSE FOR τ_3 ESTIMATOR

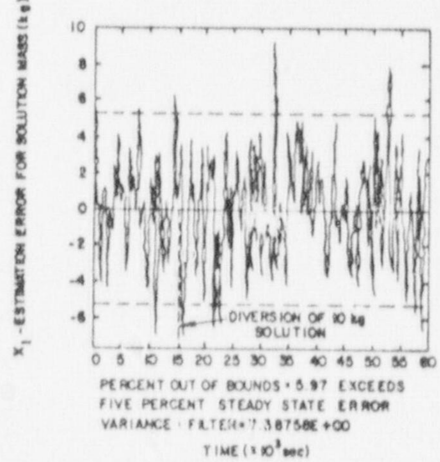

FIGURE 6 - ESTIMATION ERROR FOR τ_3 ESTIMATOR

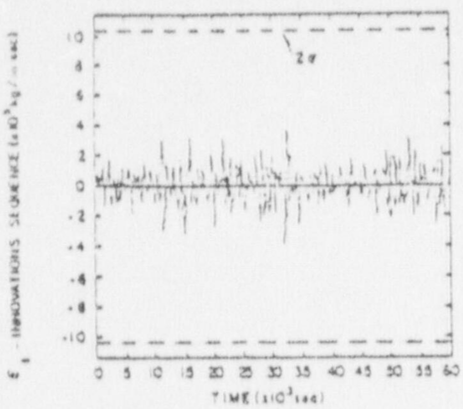

FIGURE 7 - INNOVATIONS RESPONSE FOR τ_1-ESTIMATOR

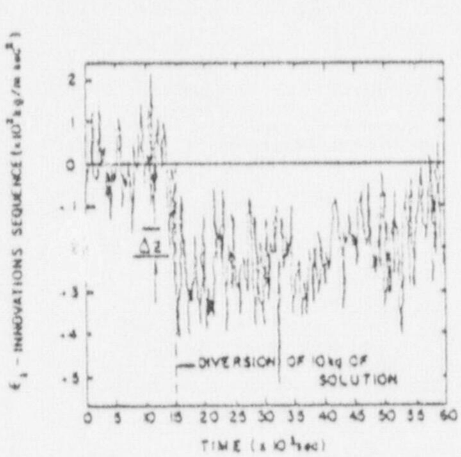

FIGURE 10 - INNOVATIONS RESPONSE FOR τ_1-ESTIMATOR

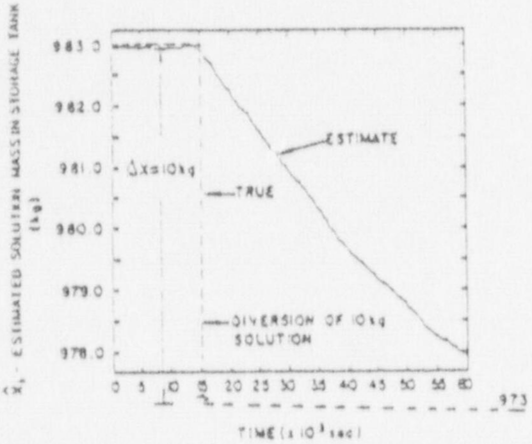

FIGURE 8 - STATE RESPONSE FOR τ_1 - ESTIMATOR

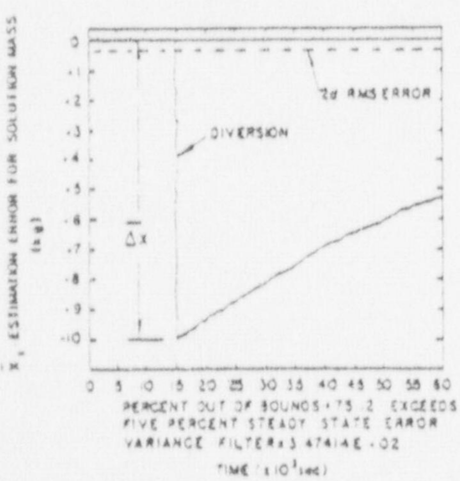

FIGURE 9 - ESTIMATION ERROR FOR τ_1 - ESTIMATOR

TABLE I: SUMMARY OF ESTIMATOR PERFORMANCE

STATISTIC	τ_1-ESTIMATOR	τ_2-ESTIMATOR	τ_3-ESTIMATOR
$E\tilde{x}$	$-\overline{\Delta x}$	$-\theta\,\overline{\Delta x}$	0
$Cov(\tilde{x})$	$\Delta\tilde{\pi} - \overline{\Delta x}\,\overline{\Delta x}^T$	$\Delta\tilde{\pi}^{(\theta-1)} - \theta\overline{\Delta x}\,\overline{\Delta x}^T\theta^T$	$\tilde{\pi}^d$
$E\epsilon$	$-\overline{\Delta z}$	$-\overline{\Delta z}^\theta$	0
$Cov(\epsilon)$	$\Delta R_\epsilon - \overline{\Delta z}\,\overline{\Delta z}^T$	$\Delta R_\epsilon\theta - \overline{\Delta z}^\theta\overline{z}^{\theta T}$	R_ϵ^d

TABLE II: STORAGE TANK SIMULATION PARAMETERS

λ_H = 5.79 kg soln/day

H = 2.54 m

α = .0209 m^3

β = .328 m^2

r_{11} = 6716 kg/m sec^2

r_{22} = 87 kg/m sec^2

TABLE III $Pu(NO_3)_4$ STORAGE TANK EXAMPLE ESTIMATOR PERFORMANCE

STATISTIC	Υ_1-ESTIMATOR	Υ_2-ESTIMATOR	Υ_3-ESTIMATOR
$E\tilde{X}_1$	-10 kg	-7.8 kg*	0
$Cov(\tilde{X}_1)$	–	3.8×10^{-4} (kg^2)	7.4 (kg^2)
$E\epsilon_1$	-300 kg/m-sec^2	-115 kg/m-sec^2	0
$Cov(\epsilon_1)$	–	$6.8\times10^{[illegible]}$ (kg/m-sec^2)^2	$2.8\times10^{[illegible]}$ (kg/m-sec^2)^2

* AVERAGED OVER THE DIVERSION WINDOW

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