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SYSTEMS RESEARCH

SYSTEMS ANALYSIS-EXPERIMENT SPECIFICATION

REVIEW OF SAFETY RELIEF-VALVE ANALYTICAL MODELS

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ABSTRACT

The General Electric safety relief-valve analytical models were reviewed, and the methods and applicability of the analytical models were analyzed. Concerns regarding these models have been addressed, and recommendations for future modification and development have been suggested.

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SUMMARY

General Electric has provided documents describing an analysis of the safety-relief valve discharge phenomena and the corresponding induced loads on the boundaries of the pressure-suppression pool. The INEL, responding to a request by the Nuclear Regulatory Commission, has reviewed these documents to analyze the methods and applicability of the analytical models. The attached report has been prepared to provide a succinct summary of the present INEL position regarding these models. Included within this report are a summary of all concerns, including appendixed details when appropriate, and recommendations for future development.

It is believed that the basic modeling approach adopted by General Electric is a viable one; however, concerns do exist regarding details of the representation of various processes involved in the safety-relief valve discharge phenomena. It is felt that these concerns must be addressed before the analytical model is considered complete; on-going Monticello experimental efforts are expected to provide the data required to complete and verify the model. The major areas of concern specifically addressed in this report relate to the bubble formation process and associated formation efficiency, the determination of initial conditions for bubble formation and oscillation, and the superposition of pressures from multiple bubbles. All of these concerns are discussed, and suggestions are made which would, it is felt, contribute to the usefulness of the safety-relief valve analytical models.

I. INTRODUCTION

The General Electric documents(1,2,3,4) describe an analysis of the safety-relief valve discharge phenomena and the corresponding induced loads on the boundaries of the pressure-suppression pool. The present report presents a summary of the INEL review of the analytical models used by General Electric for the safety-relief valve discharge phenomena. The review has included: 1) a study and critique of the assumptions made for each model, 2) a dimensional analysis of every equation and 3) a study and critique of the correctness and applicability of each equation used in the models. This report is being issued to carefully summarize the present INEL position on all material reviewed to date concerning the safety-relief valve model.

The General Electric models described in References (1), (2) and (3) are relatively simplistic models of a very complex process. It is felt, however, that they represent a reasonable and logical method of approach since they rely sufficiently, in most cases, on a basic principles representation of the processes involved; and, in addition, applicable test data are utilized to calibrate and verify results of the models. There are, however, some areas where there is not sufficient applicable test data and where the analytical representations are not sufficient to scale this data to other conditions.

In this review and summary, three major features of the analytical models which appear to require clarification or further analysis, and which, it is felt, have the potential to compromise the overall value of the General Electric models have been identified. These areas can be

described as:

1. The bubble formation process and the associated efficiency of formation.
2. The determination of the initial conditions for bubble formation and oscillation.
3. The superposition of pressures from multiple bubbles.

In addition to the three areas concerned with model development, the INEL has questions and suggestions regarding the comparison of theory and experiment as presented in NEDE-21062-P.[4]

The present INEL position review report presents a concise summary and discussion of all areas of concern. Within the body of this report, detail is limited to that required for clarity. Additional detail appears, when appropriate, as appendices.

II. SUMMARY OF PRESENT CONCERNS

Although reviews of reports documenting the General Electric safety-relief valve discharge analytical models are basically in agreement with the modeling approach adopted, concerns do exist regarding the details of representations of various processes involved in the safety relief-valve discharge phenomena. The present Monticello experimental tests are expected to provide the data required to fully address these concerns.

This section presents a summary and review of concerns expressed by several different reviewers during the course of the INEL analysis of the analytical models. Care has been taken to consolidate and review all areas of concern in light of all available General Electric

information. What follows in a succinct listing and discussion of all present concerns regarding the General Electric safety-relief valve analytical models:

1. The efficiency of bubble formation, η, has been defined in Equation 24 of NEDE-20942-P but is apparently not included in Equation 25 of this same document. Instead, the η appearing in Equation 25 is an empirically determined constant. The INEL suggests the development of an independent correlation for η in order to allow use of the models as a predictive tool; and, in addition, it is felt that further development and application of an analytical expression for η would add both simplification and completeness to the existing models.

Due to the importance of the bubble formation efficiency, Section III of this report presents a more detailed discussion of this concern.

2. It is felt that the assumption and logic behind the equation (Eq. 40, NEDE-20942-1P) derived to account and correct for the deviation from the assumption of infinitesimaly small bubbles should be re-analyzed. The derivation is confusing and contains apparent inconsistencies of notation. Two of the concerns were: the equation was not derived to account for multiple bubbles, and the equation was not applied to a three-dimensional grid. However, recent discussions with General Electric have indicated that both of the above concerns were recognized and addressed.

3. Some of the assumptions pertaining to the initial conditions for the bubble dynamics modeling are considered to be in need of clarification and further study. Specifically, the assumption that both the vent-pipe exit and the ram's head are in a choked condition is questioned. In addition, the initial conditions for bubble oscillation listed in Section 3.23 of NEDE-20942-P should be reconsidered in view of the fact that the initial jet penetration distance and position of bubble formation are not addressed in the model. The importance of the assumptions made when determining the initial conditions for bubble formation and oscillation have led to further discussion of this issue in Section III of the present report.

4. There exist concerns about the derivation and application of the equation for multiple bubble superposition (Eq. 39, NEDE-20942-P). A detailed discussion of these concerns, which are considered of major importance, appears also in Section III.

5. The analytical models described in NEDE-20942-P are not capable of predicting maximum and minimum pressures occurring prior to the end of bubble formation. Therefore, confidence should exist that these maximum and minumum pressures will be smaller than the pressures occurring after the completion of bubble formation.

6. It is felt that the rising acceleration of the bubble due to buoyancy and drag is 2g, rather than one g as stated in Section 3.4 of NEDE-20942-P. In addition, the assumption that the effect of bubble elevation on pressure is negligible is considered to be in need of review. Buoyancy effects are discussed in Appendix B.

7. The comparisons between pressure as a function of time as predicted by the models, and the trends indicated by data, do not appear to agree. Specifically, the first derivative of pressure with respect to time appears to be discontinuous at the peaks for the model, while the test data shows smooth peaks. Also, the model predicts that the second derivative of pressure with respect to time is always greater than zero while the test data indicates a definite point of inflection. The behavior of these derivatives is considered a major concern and their further discussion follows in Section III of this report.
8. Many of the non-conservative predictions presented in the comparison of safety-relief valve model predictions with test data (NEDE-21062-P) appear to be a result of a particular choice for the value for bubble formation efficiency and the method chosen to account for multiple bubble effects. The ultimate judgement of an analytical model depends upon the comparison of the model predictions to test data. Due to the importance of these comparisons, more discussion appears in Section III of this report.
9. Inconsistencies of notation and apparent typographical errors contribute to a lack of clarity throughout the reports documenting the General Electric safety-relief valve analytical models. Although minor, correction of the typographical errors together with more concern for consistency of notation would add considerably to the readability and overall value of the reports.

III. REVIEW OF AREAS OF PRIMARY CONCERN

The topics discussed in the previous section represent all current concerns regarding the General Electric analytical models. As indicated, some of these concerns are considered minor and are not felt to seriously compromise the overall value of the safety-relief valve discharge analytical models. However, those topics felt to be of more importance require further consideration, and what follows represents a more detailed discussion of these major concerns.

1. Bubble Formation Efficiency

The bubble-formation efficiency, η, has been defined in NEDE-20942-P (Page 15) as that fraction of the energy leaving the ram's head which is received and kept by the bubble. A consistent mathematical expression for η is given but is not incorporated into the formulation of the model. In the parametric study of η found in NEDE-21062-P, η was varied between 0.1 and 1 with apparent disregard for the definition. Lacking additional clarification, one must assume that the word energy in the definition of η refers to total energy. Using the present definition for η and appropriate reservoir conditions, it is apparent that in order for $\eta = .1$ to be physically possible, one must not refer to total energy but rather the energy difference between the thermodynamic state of the air as it leaves the ram's head and the state of the air if it were brought into thermodynamic equilibrium with the water in the reservoir.

As η is presently incorporated into the model, the irreversibilities associated with heat transfer, friction, and shocks are all lumped into the efficiency parameter. No effort has been made to include

an analysis of the relative magnitude of these losses or to include these losses as separate terms. A recent conversation with General Electric and the NRC indicated that, without justification, irreversible flow losses are ignored when analyzing bubble formation efficiency; and, instead, major energy losses associated with the flow are attributed only to heat transfer. If the present modeling approach is retained, an analysis should be presented to validate the present assumption that irreversible pressure losses are small compared to heat transfer losses.

2. Bubble Initial Conditions

In an effort to improve the agreement of pressure distribution predictions with data, the initial position of the bubble was located four feet from the ram's head. The data is thus far too limited to assume that the four feet assumption is valid over anything but a limited range of geometries and flow conditions. A more satisfactory solution might be to include a model of jet penetration taken from the available literature to predict the initial diameter and position of bubble formation and to correlate this simple model with existing data.

The initial bubble pressure is assumed in NEDE-20942-P, Page 16, to be equal to the air pressure at the ram's head exit the instant the water slug is cleared. An accurate gas dynamics analysis is necessary before the exit pressure can be predicted. Without adequate justification, it is stated that the flow is choked simultaneously at five locations within the ram's head assembly. The incorrect statement that the flow is choked simultaneously at five locations results from an incorrect application of a one-dimensional momentum analysis (Equation 14, NEDO-10859). The particular problem is so inherently multi-dimensional

that a one-dimensional analysis is inadequate; in fact, for the one-dimensional case, the system of governing equations (if obtained properly) is indeterminate.[6] From the limited dimension and state information given, it appears that a sonic condition is not possible simultaneously at all five locations. However, choking is likely somewhere in the system, and it is also likely that shocks will occur. It is felt that shocks and the associated irreversibilities should be adequately dealt with. Presently, it is assumed that the losses and irreversibilities can be lumped into a single parameter η. The results of the present Monticello experimental effort must be analyzed before this assumption can be justified.

3. Behavior of Pressure Derivatives

It was observed, from the pressure-time curves, (NEDE-20942-P Figure 12 Page 34 and NEDE-10859 Figure 4-4 Page 4-8) that the trends of the model differ from the trends of the actual tests in two very important ways. First, for the model, dp/dt appears to be discontinuous at the peaks while the test data show smooth peaks. Second, the model results appear to be always concave upward ($d^2p/dt^2 > 0$) while the actual test results have a definite point of inflection. The bubble pressure is assumed to oscillate about an equilibrium value, and the pressure force changes direction as the equilibrium pressure is passed. The change in direction of the pressure force should coincide with an inflection point. The behavior described above may be attributable in part to the classic behavior of a spherical Rayleigh bubble without damping. The fact that damping is neglected and the bubble is assumed to be

spherical may contribute to the disagreement of predicted peak pressures and frequencies with data. The simple inclusion of damping might result in significant improvement.

4. Superposition of Pressures from Multiple Bubbles

The basic General Electric approach to the superposition of pressures due to multiple bubbles is considered appropriate. However, details of the derivation of the equation of superposition (Equation 39, NEDE-20942-P) should be reviewed.

In addition to the form of Equation 39, there appears to be some confusion as to whether G. E. assumes multiple bubbles to be in phase (Pg R1-1 of NEDE-20947-P states, "However in calculating loads . . ., it is assumed that all bubbles are in phase.") or oscillating at random as is stated in the final sentence on Pg. 28 of NEDE-20942-P: "It is reasonable to assume that the bubbles will oscillate at random, therefore their combined effect at any point will be . . ."

Recent conversations with General Electic and the NRC indicated that the random oscillation assumption is used when determining the effects of multiple bubbles, but due to the inherent randomness of bubble formation and oscillation, the assumption of a particular phase difference associated with many different bubbles might not have great significance. The only assumption which can be confidently assumed to be conservative is the assumption that all bubbles are simultaneously in phase. Little can be said about the superposition of many bubbles until sufficient data exists to allow comparisons

of predictions with average measured pressures rather than with single-test values as has been done in NEDE-21062-P.

In addition to the questions concerning the proper phase difference associated with multiple bubbles, there exists a question concerning the generalization of the equation for the superposition of two bubbles (Eq. C-6, NEDE-21062-P) to N-bubbles as has apparently been done in Equation 39 of NEDE-20942-P. A preliminary INEL derivation of the general equation appropriate to N-bubbles indicates that Equation C-6 is valid only for two bubbles. In Appendix A of this summary report, there is developed, following the approach used by General Electric, an expression appropriate to three bubbles. It is apparent that the resulting expression differs from Equation 39 of NEDE-20942-P. Included with the derivation in Appendix A is a quantitiative comparison for the three-bubble superposition predictions resulting from Equation 39 of the General Electric report and the three-bubble equation derived in Appendix A of the present report. The results indicate that while both equations agree when the assumption that all bubbles are in phase is made, the equation used by G. E. can be on the order of twenty percent too low for the three-bubble case.

In addition to the recommendation that the equation for bubble superposition (Equation 39, NEDE-20942-P) be reviewed, it is felt that the formulation and application of Equation 40 of NEDE-20942-1P should be reviewed. Presently, Equation 40 is sometimes used in lieu of Equation 39 to determine multiple bubble effects. It is claimed that the application of Equation 40 results in a shift to a "best-estimate" model as opposed to a highly conservative one. However, the model has been formulated

using the assumption that the bubbles are in-phase; and, in addition, there remain other concerns about details of the derivation. Finally, it has not been demonstrated that the use of Equation 40 provides the required margin of conservatism to the "best-estimate" application of the model.

5. Comparison of Model Prediction with Test Data

Tables 1-a and 1-b summarize the test correlations found in NEDE-21062-P. Table 1-a shows some of the details of individual tests and Table 1-b shows the totals for the different tests. The first line of Table 1-b shows the combined results of the Quad Cities[4] comparison using the RMS formulation to account for multi-bubble effects and using a bubble-formulation efficiency $\eta = 0.1$. There were 53 data points accounted for. Of the 53 points, 47% were conservative (model predictions were more extreme than test data) and 53% were nonconservative. In addition to the 53 recorded points there were 27 possible points which were not reported. It seems appropriate, and is suggested for completeness, that account be given for all missing data points. Line two of Table 1-b show the most conservative of the comparisons. These data for the Small Scale Test[4] comparison utilized the RMS formulation for multiple bubbles and an efficiency $\eta = 1.0$. For the test, 85% of the points were conservative and 15% were nonconservative. Request No. 1 of NEDE-20942-1P contains a modification which is called a correction. Line 3 of Table 1-b shows the result of incorporating this modification into the correlation. The fraction of conservative points drops from 85% conservative to a very poor 46% conservative, leaving 54% of the points nonconservative.

Figure	Sample	% Conservative	% NonConservative	Not Accounted for
2-8 Valve B n=.1	16	44	56	4
2-9 B + E n =.1	12	33	67	8
2-10 AB + E n =.1	13	54	46	7
2-11 ABCD+E n =.1	12	58	42	8
3-9 RMS Max	44	89	11	4
3-10 RMS Min	43	81	19	5
3-11 RMS Max + Correct	45	62	38	3
3-12 RMS Min + Correct	44	30	70	4
3-13 ≤ Max + Correct	45	84	16	3
3-14 ≤ Min + Correct	42	48	52	6

1-A. Individual Test Results

n = .1 RMS Q.C.	53	47	53	27
n = 1 S.S. RMS	87	85	15	9
n = 1 S.S. RMS + Correct	89	46	54	7
n = 1 S.S. ≤ + Correct	87	67	33	9

1-B. Combined Test Results

Figure	Sample	% Conservative	% NonConservative	Not Accounted for
2-8 Valve B n=.1	16	44	56	4
2-9 B + E n =.1	12	33	67	8
2-10 AB + E n =.1	13	54	46	7
2-11 ABCD+E n =.1	12	58	42	8
3-9 RMS Max	44	89	11	4
3-10 RMS Min	43	81	19	5
3-11 RMS Max + Correct	45	62	38	3
3-12 RMS Min + Correct	44	30	70	4
3-13 Σ Max + Correct	45	84	16	3
3-14 Σ Min + Correct	42	48	52	6

1-A. Individual Test Results

n = .1 RMS Q.C.	53	47	53	27
n = 1 S.S. RMS	87	85	15	9
n = 1 S.S. RMS + Correct	89	46	54	7
n = 1 S.S. Σ + Correct	87	67	33	9

1-B. Combined Test Results

Conversations with General Electric and the NRC have indicated that the correction, Eqt. 40, is not intended to result in conservative predictions but a "best-estimate" prediction instead. However, the fact that this correction results in fewer conservative points than non-conservative ones points out the importance of insuring that a margin of conservatism be maintained when the model is applied. Line 4 of Table 1-b shows the result of using an algebraic summation (rather than an RMS method of combining the effects of multiple bubbles) along with the modification. With the algebraic summation and the modification, 67% of the points are conservative and 33% are nonconservative.

Finally, it should be mentioned that conversation with General Electric and the NRC indicated that the bubble formation efficiency of 0.1 was used for the Quad Cities comparison in order to bring the predicted values of maximum positive pressure within the range of experiment. Although a value of 0.1 does result in best-fit for the maximum positive pressures, a bubble formation efficiency of approximately 0.4 is required before the predicted values of frequency and peak negative pressure agree with experiment. Rather than ignore this trend, it might be more realistic to choose an efficiency resulting in agreement with frequency and peak negative pressure, and then to analyze the deviation of predicted peak positive pressure from experiment. As mentioned earlier, the model predicts a discontinuity at the maximum peak pressure while the data indicate smooth peaks.

Presently, a bubble formation efficiency factor is chosen in an attempt to achieve best-fit agreement with positive peak pressures while disregarding the resulting lack of agreement with frequency and negative peak pressure. As pointed out above, a suitable choice of a bubble formation efficiency factor will result in agreement with both frequency and negative pressures, although positive peak pressures will then be over-predicted. Therefore, an alternative and perhaps more logical approach to the efficiency selection process would be to choose a bubble formation efficiency factor resulting in a best fit agreement with frequency and negative peak pressure and to then investigate further the positive peak pressure disagreement in light of, for example, pressure transducer response and the omission of damping terms in the analytical models.

IV. SUGGESTIONS FOR FUTURE DEVELOPMENT

Based upon the discussions of areas of concern presented in this report, an outline of programs which it is felt would add significantly to the value of the safety-relief valve models as a predictive tool will now be presented:

Bubble Formation Efficiency

The model proposed by G. E. in NEDE-20942-P is quite simple and has much merit; however, empirical correlations are still necessary. One approach to the analysis of the bubble formation process would be to determine the position of formation of the bubble, via a jet-penetration model, and then to examine associated losses. Water entrainment into the jet is considered both important and relatively easy to incorporate into the existing bubble dynamics. Entrainment provides both the momentum exchange which damps out the kinetic energy and the mixing mechanism

which reduces the temperature (and consequently reduces the internal energy) of the air within the bubble.

Superposition of Multiple Bubbles

It is suggested that the validity of applying Equation 30 of NEDE-20942-P to the superposition of N-bubbles be reviewed. Also, it recommended that a general equation corresponding to the superposition of N-bubbles be developed. A preliminary INEL analysis of the general equation for superposition of N-bubbles indicates that Equation 39 will result in non-conservative predictions for the oscillation of more than two bubbles. Appendix A of this report illustrates the application of the approach used by General Electric for three bubbles and compares the results of this analysis to the predictions obtained from Equation 39 of NEDE-20742-P. The results indicate that the suggestions presented here are appropriate.

Appendix C of this report presents a detailed summary of the above recommendations along with estimates of the time required to complete the program.

V. CONCLUSIONS AND RECOMMENDATIONS

This summary report has addressed the present INEL concerns regarding the adequacy and completeness of the General Electric safety-relief valve discharge analytical models. Section II presented a summary of the consolidation and review of all previous INEL reports, while Sections III and IV presented, respectively, a discussion of those concerns judged by the INEL to be of more major importance, and suggestions for future development.

In summary, it is concluded that the basic General Electric approach to the development of the analytical models is sound; however, it is also felt that the concerns expressed within this report should be addressed before the model is considered complete. Specifically, it is felt that those areas of concern labeled "major" and appearing in Section III of this report be reviewed and re-analyzed when necessary. These areas include the bubble formation efficiency, the conditions for bubble formation and oscillation, and the superposition of multiple bubbles. The present Monticello experimental effort is expected to provide the data required to fully investigate all present concerns.

Finally, it is recommended that the suggestions for future development listed in Section IV of this report be undertaken in order to add to the completeness and viability of the General Electric safety-relief valve analytical models.

Regardless of the assumptions made in a model formulation, the real test of a model occurs in the laboratory, and the real value of the model is determined by how well it predicts actual behavior. As

indicated in Section III-5 of this report, the comparisons of model predictions with data quite often illustrate a significant lack of agreement; therefore, the detailed concerns presented in the body of this report are felt to be warranted and are presented solely in the interest of contributing to the usefulness of the model.

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VI. REFERENCES

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5. L. M. Milne - Thompson, Theoretical Hydrodynamics, 5th ed. (The Macmillan Company, N. J., 1968).
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APPENDIX A

Superposition of Multiple Bubbles

The pressure differential at a point in the pressure suppression pool due to an oscillating bubble can be approximated by:

$$P_{.} = P_{max_1} \cos wt \qquad (A-1)$$

where t = time,

$w = 2\pi f$, and f is the frequency of bubble oscillation.

For this analysis, it is assumed that these bubbles are oscillating simultaneously with the same frequency but with a phase angle relative to each. Assuming bubbles two and three to be oscillating with phase angles α and β relative to the first, then the local pressure differentials due to each bubble may be written:

$$P_1 = P_{max_1} \cos wt$$

$$P_2 = P_{max_2} \cos (wt - \alpha)$$

$$P_3 = P_{max_3} \cos (wt - \beta)$$

Now assuming that the pressure differentials are additive, the total pressure due to these bubbles may be written:

$$P = P_1 + P_2 + P_3 = P_{max_1} \cos wt + P_{max_2} \cos (wt - \alpha) + P_{max_3} \cos (wt - \beta),$$

or, $$P = (P_1 + P_2 \cos \alpha + P_3 \cos \beta) \cos wt + (P_2 \sin\alpha + P_3 \sin\beta) \sin wt \qquad (A-2)$$

Now, the maximum pressure may be determined by allowing

$\frac{dP}{dt} = 0$ at $t = t^*$. This operation yields:

$$w(P_1 + P_2 \cos\alpha + P_3 \cos\beta) \sin wt^* =$$

$$w(P_2 \sin\alpha + P_3 \sin\beta) \cos wt^*,$$

or,

$$(P_1 + P_2 \cos\alpha + P_3 \cos\beta) \sin wt^* = (P_2 \sin\alpha + P_3 \sin\beta) \cos wt^* \quad \text{(A-3)}$$

and from (A-3),

$$\tan wt^* = \frac{\sin wt^*}{\cos wt^*} = \frac{P_2 \sin\alpha\ P_3 \sin\beta}{P_1 + P_2 \cos\alpha + P_3 \cos\beta} \quad \text{(A-4)}$$

From the tangent of wt*, both cos wt* and sin wt* can be determined:

$$\cos wt^* = \frac{P_1 + P_2 \cos\alpha + P_3 \cos\beta}{\sqrt{P_1^2 + P_2^2 + P_3^2 + 2P_1(P_2 \cos\alpha + P_3 \cos\beta) + 2P_2P_3(\cos\alpha \cos\beta + \sin\alpha \sin\beta)}} \quad \text{(A-5)}$$

$$\sin wt^* = \frac{P_2 \sin\alpha + P_3 \sin\beta}{\sqrt{P_1^2 + P_2^2 + P_3^2 + 2P_2P_3(\sin\alpha \sin\beta + \cos\alpha \cos\beta) + 2P_1(P_2 \cos\alpha + P_3 \cos\beta)}} \quad \text{(A-6)}$$

Now, the expressions for sin wt* and cos wt* may be substituted into Equation (A-2), and then two cases can be considered.

Initially, consider the case where all three bubbles are oscillating in phase, that is $\alpha = \beta = 0$. This results in

$$\cos wt^* = 1$$

$$\sin wt^* = 0$$

and, $$P_{max} = P_1 + P_2 + P_3 \qquad \text{(A-7)}$$

This result, A-5, agrees with the result predicted by Equation 39 of NEDE-21062-P.

Now, if it is assumed that all three bubbles are oscillating with an average phase angle of $\frac{\pi}{2}$, then from A-2, A-5, and A-6,

$$\cos wt^* = \frac{P_1}{P_1^2 + P_2^2 + P_3^2 + 2\, P_2P_3}$$

$$\sin wt^* = \frac{P_2 + P_3}{P_1^2 + P_2^2 + P_3^2 + 2P_2P_3}$$

and, $$P_{max} = P_1^2 + P_2^2 + P_3^2 + 2P_2P_3 \qquad \text{(A-8)}$$

This result does not agree with Equation 39 of NEDE-21062-P.

As an example of the differences resulting from the application to three bubbles of the expression derived above and the Equation 39 in NEDE-21062-P, the simple comparison below is presented.

It is assumed that three bubbles are oscillating simultaneously in the pool; each bubble has a maximum pressure associated with it of 20 PSI. Using the expression derived in this appendix for the combined effect of three bubbles, the maximum pressure for the in-phase and out-of-phase conditions can be determined:

1. In - phase: assume $\alpha = \beta = 0$

   $$P_{max} = P_1 + P_2 + P_3 = 60 \text{ PSI}$$

2. Out-of-phase: assume $\alpha = \beta = \frac{\pi}{2}$

   $$P_{max} = \sqrt{P_1^2 + P_2^2 + P_3^2 + 2P_2P_3} = 44.72 \text{ PSI}$$

Applying Eq. 39 of NEDE-21062-P results in the following:

1. In-phase - assume $\alpha = \beta = 0$

   $$P_{max} = P_1 + P_2 + P_3 = 60 \text{ PSI}$$

2. Out-of-phase - assume $\alpha = \beta = \frac{\pi}{2}$

   $$P_{max} = \sqrt{P_1^2 + P_2^2 + P_3^2} = 34.64 \text{ PSI}$$

Therefore, the INEL expression agrees with the results of Equation 39 of NEDE-21062-P when the bubbles are assumed to be oscillating in-phase. When an average phase angle of $\frac{\pi}{2}$ is assumed, then the result predicted by Equation 39 is 23% lower than the prediction resulting from the INEL three-bubble equation.

It should be noted that the INEL equation reduces to Equation 39, for the out-of-phase case, if P_3 is zero; that is, the two expressions are identical for two bubbles.

APPENDIX B

BUOYANCY ANALYSIS

The total acceleration of the bubble should be two g rather than one g as stated in Section 3.4 of NEDE-20942-P. This point can be seen by analyzing the forces acting upon a bubble rising in an infinite liquid as discussed in Reference 5. The forces acting upon the bubble are simply the weight of the bubble, the buoyancy, and the resistance. Together they result in

$$M \frac{du}{dt} = -Mg + M'g - \frac{M'}{2} \frac{du}{dt} \qquad \text{(B-1)}$$

which leads to

$$\left(M + \frac{M'}{2}\right) \frac{du}{dt} = (M' - M)g \qquad \text{(B-2)}$$

or,

$$\frac{du}{dt} = \frac{M'-M}{M+\frac{M'}{2}} \, g \qquad \text{(B-3)}$$

where,

M = the mass of the bubble

M' = the mass of liquid displaced by the bubble

$\frac{du}{dt}$ = acceleration of the bubble

g = acceleration due to gravity

Therefore, if the mass of the bubble is ignored, the acceleration of the bubble through the liquid is two g.

It should be noted that if the buoyant force alone is considered, as is apparently done in NEDE-20942-P, then the force is simply $M'g$ and the acceleration on the bubble is indeed one g. However, it is not proper to calculate the buoyant force considering only one-half the displaced mass as is done in NEDE-20942-P; the correct expression for buoyant force must be calculated using the total mass of liquid which the sphere could displace.

PROPRIETARY INFORMATION

APPENDIX C

This appendix presents a summary of all recommendations for further analyses which are felt necessary before the General Electric safety-relief valve analytical model is complete and capable of being used confidently as a predictive tool. In addition to an outline of suggested tasks, estimated time requirements for the completion of the tasks are also presented.

As a minimum effort required to more completely judge the value of the safety-relief valve analytical models, it is recommended that the NRC initiate, perhaps through an external consultant, a two-fold program. This program, designed to further investigate a critical element of the safety-relief valve models and to provide verification of the models through adequate comparison with experiment, would incorporate the following recommendations:

1. Further investigation of the bubble formation efficiency should be performed including order of magnitude analyses to determine the relative magnitudes of heat transfer, entrainment and irreversible pressure losses on bubble formation efficiency. This effort would isolate the areas of major importance in the development of an analytical model for bubble formation efficiency; in addition, the present assumption that irreversibilities associated with heat transfer, friction, and shocks may be lumped into one parameter would be tested. It is estimated that these investigations would require four to six man-months depending on the level of effort dictated by the initial results of the analyses.
2. Verification studies should be conducted. Data previously obtained together with data forthcoming from present or future experimental efforts should be utilized to test the adequacy of the safety-relief

valve analytical models. In addition, if as a result of the current series of experiments modifications or additions are made to the analytical models, then these modifications or additions should be reviewed and analyzed by the NRC or an external consultant.

In addition to the minimum program outlined above, and depending upon the results and conclusions of that program, there exist further recommendations that General Electric could advantageously pursue. These additional recommendations include:

1. Develop a simple model for jet penetration and entrainment to provide a method of determining the bubble initial position and size and the scaling relationship for this portion of the bubble formation efficiency. This is estimated to be a three man-month task.
2. If warranted by further analyses of the bubble formation process, develop a simple gas dynamics model of the valves, vent pipe and ram's head to determine the effects of shocks, etc, on the initial bubble pressure. This task would require between six and nine man-months depending on the importance and refinements required.
3. Develop a simple model to calculate the enthalpy change of the gas forming the bubble from heat transfer and entrainment considerations. This is approximately a six man-month task.
4. Modify the superposition formula developed in Appendix C of NEDE-21062-P to correctly account for N-bubbles oscillating with given average phase difference. This necessitates the availability of sufficient data to properly determine a reasonable phase angle between the bubbles. It is estimated that the task would require one man-month.
5. Carry out parametric studies to establish important parameters for future testing and to determine the validity of extrapolating the model

from one system to another. Again, this assumes the availability of sufficient data. Approximately four man-months would be required.