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Sequential Test Procedures for Detecting Protracted Materials Losses

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EXECUTIVE SUMMARY

This report presents statistical testing procedures that might be useful for a licensee to comply with Section 70.83(b)(2) of the Material Control and Accounting Reform Amendment proposed by the Nuclear Regulatory Commission. The procedures address the problem of detecting protracted materials losses of strategic special nuclear material (SSNM) from a single materials control unit (MCU). These test procedures, which were selected to be compatible with the requirements of the Reform Amendment and the accounting data generated by operating facilities, include modified versions of Page's, power-one, and CUSUM procedures.

Principal conclusions with respect to testing procedures that meet the Reform Amendment requirements for detecting protracted losses are as follows:

- No single test procedure is best for both criteria and all diversion scenarios.
- In comparison with the other tests considered, Page's test gives the most timely detection of loss.
- The power-one test is most sensitive for protracted losses early in the 60-day accounting period and Page's test is most sensitive for protracted losses near the end of the period.
- For the process model given in this report, as low as reasonably achievable amounts of material to be detected with 90% probability range from 4.6 to 26.8 kg of SSNM, depending on the loss scenario and the false-alarm probability.
- The CUSUM approach as presented here is not recommended for licensees because of the low detection probabilities.
- Page's, power-one, and CUSUM tests are user-oriented because they are compatible with facility accounting data and are programmable for moderate-sized computer systems.
- If holdup and process variation are not included in the inventory difference (ID) model but present in the process, then assuming steady-state conditions, false alarms can increase substantially.

The test procedures were evaluated on simulated accounting data from a MCU in a conversion/fabrication process. The process model included variations in IDs caused by measurement uncertainties, process variations, and materials holdup in process equipment. Thus, the simulated accounting data approximate the ID behavior to be expected at operating facilities that use the conversion/fabrication process.

Criteria for evaluating test performance include the probability of detecting materials loss and the expected time from the onset of loss until detection. Each test was evaluated against several protracted-loss scenarios that included uniform losses early in the 60-day accounting period, uniform losses late in the period, and equally spaced abrupt losses during the period.

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SEQUENTIAL TEST PROCEDURES FOR DETECTING PROTRACTED MATERIALS LOSSES

by

A. S. Goldman

ABSTRACT

Sequential tests are required for detecting protracted (trickle) losses of strategic special nuclear materials from a single materials control unit (MCU). We compared applicable tests including modified versions of Page's test and power-one procedures. We used simulated data from a MCU in a conversion/fabrication process that took into account process variations, materials holdup, and measurement uncertainties. Comparisons were made over a 60-day accounting period under different loss scenarios. Some important findings include

(1) No single test procedure is best for all diversion scenarios.
(2) Power-one procedures are best for protracted losses that occur early in the accounting period and Page's test is best for late loss occurrence.
(3) If holdup and process variations are not included in the Inventory Difference model but are present in the process, then assuming steady-state conditions, false-alarm probabilities can double.

I. INTRODUCTION

This report presents statistical testing procedures that might be useful for a licensee to comply with Section 70.83(b)(2) of the Material Control and Accounting (MC&A) Reform Amendments proposed by the Nuclear Regulatory Commission (NRC). The proposed rule concerns detecting recurring losses of strategic special nuclear materials (SSNM) from a single unit process area, also called materials control unit (MCU), over an extended time period. The unit process area is defined as an area within a facility

wherein transfers of material across an arbitrary boundary can be measured or otherwise estimated. In addition, periodic measurements or estimates of material inventories can be made within the area.

The rule states:

> For each unit process, at least every seven calendar days the licensee shall evaluate measurement data accumulated since the last cleanout of the unit process. This evaluation must be able to detect a recurring loss with 90 percent power of detection. The amount to be detected for each unit process must be as low as reasonably achievable, but need not be less than 50 grams of SSNM.

The "seven calendar days" or multiple time aspect of the rule refers to the periodic examination of measured or estimated inventory and transfer values to detect evidence of materials loss.

To comply with these rules, licensees must first compute a periodic inventory difference (ID) for each unit process. Each ID amount is assumed to have process variation present, and successive IDs for a given unit are assumed to be correlated.

Compliance also requires sequential testing of successive IDs for a significant loss. This report suggests a "best" sequential testing procedure based upon a comparison of available tests.

A. Inventory Difference (ID)

Detection of materials loss in the following context is based on an ID that is determined periodically for each unit process and defined for the Nth time period as

$$ID_N = I_N - I_{N+1} + T_N \quad , \tag{1}$$

where I_N and I_{N+1} are the estimated inventories of material within the area at the beginning and end of period N, and T_N is the estimated net transfer of material across the area boundary during period N. Each inventory (transfer) term in Eq. (1) may be the sum of several inventory (transfer) values.

Where possible, all of the terms in the ID equation should be based on measured values. However, where practical considerations (such as technical difficulty or minimizing either cost or process disruption) preclude direct measurement of material quantities, historical process data or experimental data may be used to estimate these quantities.[1] Failure to either measure or estimate each term in the ID equation leads to a nonzero mean valued ID. Detection of material loss becomes complicated because statistical test procedures cannot distinguish between a positive ID caused by material loss (for example, diversion) and one caused by an unmodeled process loss. Thus, the test procedures reported here assume that the mean of the reported ID is zero in the absence of loss.

B. Sequential Tests

Sequential testing procedures are suggested for use in compliance with Section 70.83(b)(2) of the MC&A Reform Amendment because they offer more timely detection of protracted materials losses in comparison with fixed-length tests such as comparing an ID with its "limit of error" (commonly defined as twice the ID standard deviation).[2] In fact, sequential procedures can detect materials loss with a smaller average number of observations on the ID sequence than a fixed-length test requires, while maintaining the same false-alarm and detection probabilities (see, for example, Ref. 3).

Sequential testing procedures usually are applied in situations where data are observed during successive time intervals until sufficient evidence exists to permit decision between two hypothesized models describing the statistical distribution generating the data. For materials accounting applications, frequently assumed hypotheses are (1) a no-loss model in which the IDs have mean zero and (2) a loss model in which some of the IDs have a positive mean.

For this report, the suggested sequential decision procedure consists of selecting a statistic (usually the cumulative sum of IDs), and then using this statistic for either accepting hypothesis (2) when the statistic exceeds a decision threshold or boundary or continuing to observe and test the ID sequence when the test statistic falls within a boundary. Thus, testing of the ID sequence continues until loss is detected and the no-loss

model is implicitly accepted until the statistic exceeds the decision boundary. The part of the MC&A Amendment considered in this report does not suggest any specific time period for evaluating data. However, we choose to be consistent with Section 70.83(c)(4), which requires restarting tests after 60-day intervals.

The test procedures considered in this report are designed to detect a positive increase in the expected value of the ID sequence. This "change point" formulation of the detection problem assumes a null or no-loss hypothesis

$$H_0: \theta_i = 0 \quad (i = 1, 2, \ldots) \ ,$$

where θ_i is the expected value (mean) of the ID in period i, and an alternative or loss hypothesis

$$H_1: \theta_i = 0 \quad (i = 1, 2, \ldots, k) \quad \text{and}$$

$$\theta_i > 0 \quad (i = k + 1, \ldots) \ ,$$

where k is an unknown integer. Neither the magnitudes of the losses in each period nor the point of change is known in this setting.

C. Performance Criteria

Ideally, we want to find a test procedure that gives highest detection probabilities for all loss scenarios. A measure of the performance of a test for a loss scenario is the "average number" of IDs observed following onset of loss in order to achieve detection. We want this number to be as small as possible while maintaining a false-alarm probability (FAP) equal to a specified value. This average number does not include test periods that go 60 days without detection. Because we are conditioning the average number on detection, we will use "conditional average run length" (CARL) as a test criterion.

D. Statistical Correlations

For sequential test procedures, an important characteristic of ID sequences is ID correlations arising from two sources. First, there is a negative serial correlation between successive IDs because the same inventory term appears in both with an opposite sign. Second, instrument calibration errors produce positive correlations among IDs. Although most sequential testing procedures have been developed for independent data sequences, we have chosen three tests that are applicable to correlated data. The data sequence is mathematically transformed into an independent sequence, and then test procedures that assume independent data are applied.

E. Decision Boundaries

In the restricted time period of 60 days, our only interest is in the significant loss of material; consequently, we use only one decision boundary. The sequential testing stops in the 60-day period only if the boundary is crossed. In general, the decision boundary for sequential tests depends upon both the specified acceptable FAP (probability of incorrect decisions when the no-loss hypothesis is true) and a specified desired goal quantity with a known detection probability (DP). Except for one case, we have arbitrarily set a FAP = 0.05. No goal quantity is specified in the rules; consequently, we have used the CARL criterion with a FAP = 0.05 to determine the decision boundary. The "goal quantity" can be determined at DP = 0.90 from a computed DP curve. Setting the boundary usually involves a tradeoff between FAP and timeliness of detection. For example, tests with a boundary adjusted for a low FAP will require more observations to detect a protracted loss than the same test with a boundary adjusted for a higher FAP.

Decision boundaries (acceptance and rejection regions) are generally given in the statistical literature for independent data sequences and a testing framework that allows data to be taken until the test terminates. In materials accounting applications, we are constrained by finite testing periods and correlated data. In this situation analytical results are intractable. As a result, we obtain a rejection boundary computed from simulated data.

Under conditions of no loss, ID sequences with the appropriate correlation structure are generated and the decision boundary is chosen to attain

a FAP = 0.05. By accepting a higher FAP, we could have improved the detection probability of the tests at the expense of increased process disruption for alarm resolution.

F. ID Variability

Decisions about materials loss require statistical testing procedures because of the inherent uncertainties. These uncertainties can cause a nonzero ID when no material is missing. Although statistical tests allow one to estimate the probability of an incorrect decision, unmodeled sources of uncertainty in the ID can lead to a higher frequency of incorrect decisions than the statistical theory would predict based on modeled sources alone. For example, an underestimated variability of an ID sequence could lead to an unexpectedly high rate of false alarms.

For materials accounting, the principal sources of ID variability are measurement errors, process variability, and unmodeled terms in the ID equation, particularly material held up in process equipment that is inaccessible for measurement. In this study, we have incorporated each of these sources of uncertainty into our simulation of the ID sequence. Random and systematic errors for process measurements are based on actual instrument performance. Process variations such as changes in the quantity of material transferred from a process vessel are based on analysis of process data. Estimates of material holdup in process equipment are derived from experimental results. By incorporating these sources of uncertainty, we can generate simulated data that are representative of ID data for an operating facility. Thus, conclusions about statistical test performances should be valid for operating facilities.

II. EXAMPLE MATERIALS CONTROL UNIT (MCU)

The ID data used in this study are based on simulated measurement data from a MCU in a UF_6-to-U_3O_8 conversion process.[4] This MCU consists of the following process steps: (1) preheating of UF_6, (2) hydrolysis and storage of UO_2F_2, and (3) transfer of UO_2F_2 to a feed makeup column. A flow diagram of these steps is shown in Fig. 1.

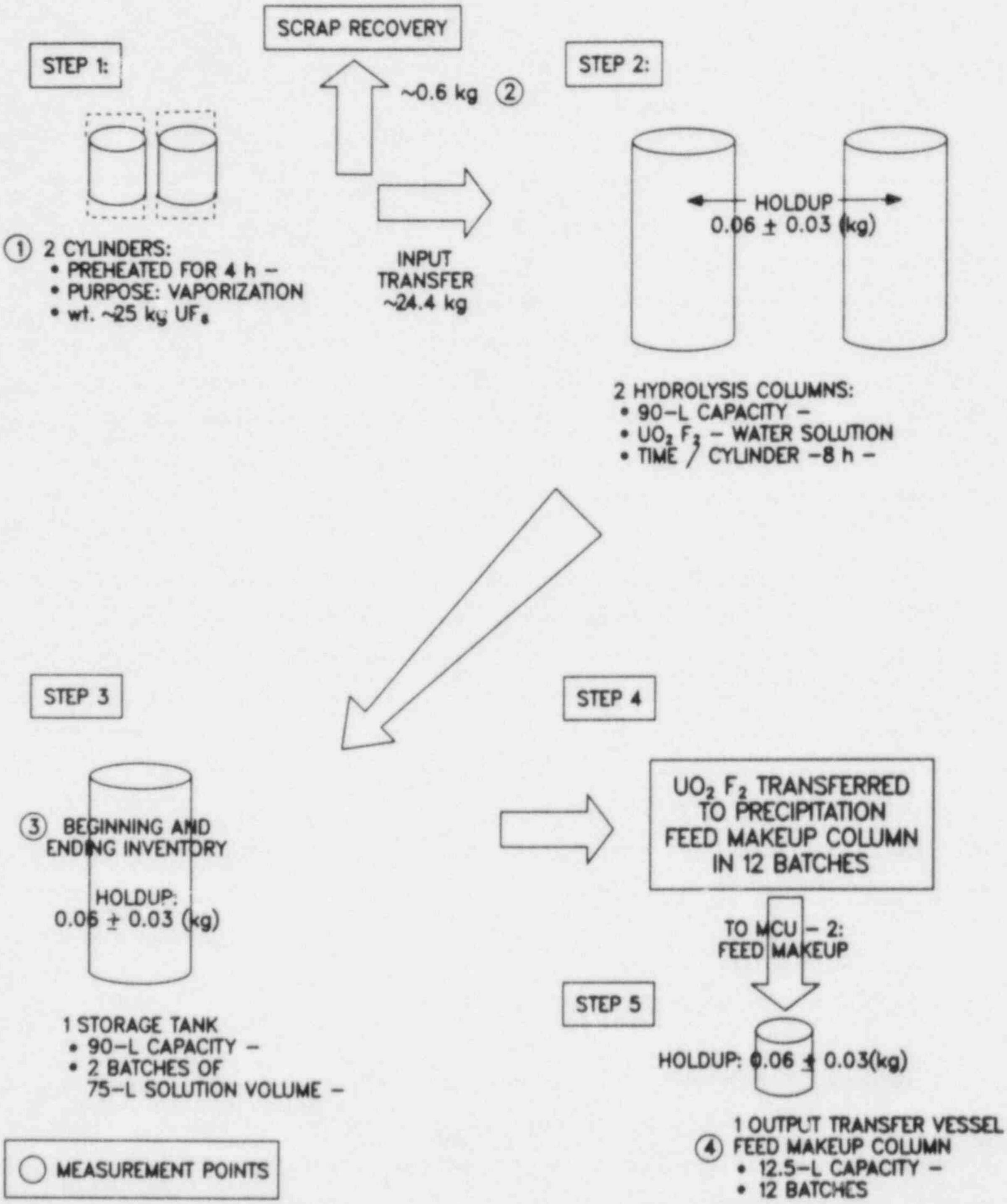


Fig. 1.
Preheat to Storage Material Control Unit
in a UF_6-to-U_3O_8 conversion process.

These three steps complete the cycle in the MCU. The volume of material transferred to the feed makeup column (step 4) is measured in the adjoining MCU (step 5). Holdup of uranium is assumed to take place in the transfer vessel at the same rate as steps 2 and 3.

In the first step two cylinders of UF_6 are preheated for 4 h to vaporize the contents. Operational constraints limit the net weight of material in each cylinder, including the heel, to $<\sim 25.0$ kg. The heel of ~ 0.6 kg is sent to scrap recovery. Because of process variations, the true volume of liquid in the cylinder will not be exactly the same at the start of each run. This process variability is translated into our computer model as a uniform distribution of weights between 24.0 and 25.0 kg. The two measurements in step 1 are the gross weight (cylinder weight plus material transferred plus heel) and the weight of the cylinder plus heel. The amount of material transferred is estimated by the difference between the two measurements.

When the heating is complete, the UF_6 is transferred to two hydrolysis columns where a UO_2F_2 water solution is produced (step 2). Each column has a 90-L capacity and the hydrolysis requires ~ 8 h. We assume that the average amount of uranium held up in each column is ~ 0.06 kg with a standard deviation of 0.03 kg.

When hydrolysis is complete, the UO_2F_2 is transferred to a 90-L-capacity storage tank for ~ 12 h (step 3). Measurements of beginning and ending inventories are taken. Holdup quantities are assumed to have the same properties as in step 2.

III. INVENTORY DIFFERENCE EQUATION

The ID equation is defined as the sum of the difference between beginning and ending inventories of SSNM plus the difference between incoming and outgoing transfers. The following nomenclature is used to write the ID equation for the MCU discussed in Sec. II.

C: Measured concentration of uranium in UO_2F_2 storage (kg of uranium/L).

V: Measured solution volume in UO_2F_2 storage (L).

M: Measured weight of UF_6 in cylinder (kg).

m: Weight of UF_6 in cylinder heel (kg).

G: Uranium fraction in cylinder (kg of uranium).

v: Measured volume in feed makeup column (L).

n: The number of days in an inventory period ($n = 60$).

I_{1j}: Inventory, defined for beginning and ending inventories as $I_{1j} = C_{1j}V_{1j}$ and $I_{2j} = C_{2j}V_{2j}$, respectively.

T_{1j}: Input transfer given by

$$T_{1j} = \sum_{k=1}^{2} G_{kj}(M_{kj} - m_{kj}) \quad .$$

T_{2j}: Output transfer given by

$$T_{2j} = \sum_{k=1}^{2} (C_{kj} \sum_{l=1}^{6} v_{klj}) \quad .$$

ε_{ij}: A random error having a normal distribution with mean 0 and variance $\sigma^2_{\varepsilon_{ij}}$, where the "i" subscript identifies a measurement within a specific "day," such as beginning or ending inventory, and the "j" subscript indicates a specific day ($j = 1, 2, 3, \ldots, n$). For example, $\varepsilon_{C_{23}}$ would identify the random error associated with a concentration measurement (C) as part of the ending inventory ($i = 2$) on the third day ($j = 3$).

η_i: A random error having a normal distribution with mean 0 and variance $\sigma^2_{\eta_i}$, where the error does not change over the inventory period. This error is referred to as a "calibration" error because the variance of η contributes to day-to-day ID correlations and originates from instrument calibrations. Jaech[5] calls η a "systematic error."

All of the measured quantities contain ε and η error terms. The error models for M and m are given by

$$M = 25.0 + \varepsilon_M + \eta_M \quad \text{and}$$

$$m = 0.5917 + \varepsilon_m + \eta_M \quad .$$

The error models for G, C, v, and V are given by

$$G = 0.676(1 + \varepsilon_G + \eta_G) \quad ,$$

$$C = 0.22(1 + \varepsilon_C + \eta_C) \quad ,$$

$$v = 12.5(1 + \varepsilon_v + \eta_v) \quad , \quad \text{and}$$

$$V = 75.0(1 + \varepsilon_V + \eta_V) \quad .$$

The ID equation can be written

$$ID_j = T_{1j} + I_{1j} - I_{2j} - T_{2j} \quad .$$

The development of the error structure of ID_j values is given in Appendix A. A breakdown of the errors is given in Table I.

From Table I, we can write the variance-covariance matrix for ID_j (j = i, 2, ..., 60) as

$$\Sigma_{ID} = \begin{bmatrix} 0.1076 & -0.0003 & 0.0294 & \cdots & 0.0294 & 0.0294 & 0.0294 \\ -0.0003 & 0.1076 & -0.0003 & \cdots & 0.0294 & 0.0294 & 0.0294 \\ \vdots & & & & & & \\ 0.0294 & 0.0294 & 0.0294 & \cdots & 0.1076 & -0.0003 & 0.0294 \\ 0.0294 & 0.0294 & 0.0294 & \cdots & -0.0003 & 0.1076 & -0.0003 \\ 0.0294 & 0.0294 & 0.0294 & \cdots & 0.0294 & -0.0003 & 0.1076 \end{bmatrix}$$

TABLE I

MATERIALS BALANCE ERROR STRUCTURE

Term	Variance	Covariance for Successive Balances	Covariance for Nonsuccessive Balances
T_{1j} (transfer in)	0.0060	0.0011	0.0011
$I_{1j} - I_{2j}$ (difference between inventories)	0.0593	-0.0297	0.0000
T_{2j} (transfer out)	0.0423	0.0283	0.0283
ID_j (inventory difference)	0.1076	-0.0003	0.0294

The expected values of ID and σ_{ID} may deviate from their true values because of holdup, process variation, and loss. Holdup, identified in the tanks depicted in Fig. 1, is assumed to be normally distributed with mean 0.06 kg of uranium and standard deviation 0.03 kg of uranium. Changes in process holdup contributions are accounted for by a term added to the ID equation.

We assume the net weight of material in the cylindrical tank has a uniform distribution between 24 and 25 kg. This process variation is included in the ID.

We assume that any loss of material takes place in the hydrolysis tank. The amount is added to the term $(I_{1j} - I_{2j})$ in the ID equation.

IV. TEST PROCEDURES

For this report, we considered two approaches to developing sequential tests for detecting materials loss. These approaches are based on the analysis of dependent ID sequences. One approach is to deal with the dependent data directly and use either the CUSUM test statistic or a truncated version of the Sequential Probability Ratio Test (SPRT).[6] The other approach is to transform the dependent sequence into an independent sequence

and use either Page's[7] or a power-one test statistic.[8] Cobb[9] has shown that, in general, the latter two statistics are more powerful than the SPRT. We did not consider other tests described in an earlier report[10] after determining them to be less effective in detecting materials loss.

A comparison between CUSUM and SPRT revealed small differences in effectiveness for both dependent and independent data. For this reason and for ease of computation, we have chosen CUSUM rather than SPRT as a reference test.

Because both Page's and power-one tests assume independent data, we transformed the dependent ID data into an independent sequence before applying these tests. Procedures available for transforming a dependent sequence of data to an independent sequence include the difference statistic given by Jaech,[5] the innovation sequence of Stewart,[11] and the ITMUF sequence of Woods et al.[12] Recently Sellinschegg[13] used recursive residuals[14] as yet another procedure to obtain independent data. We have found Sellinschegg's approach to be most expedient and used the recursive residual technique to carry out transformations. A description and example are given in Appendix B.

A. CUSUM Test

A CUSUM test is often a generic title given to any sequential testing procedure that involves cumulative sums. Lucas,[15] for example, defines what we call Page's test as a CUSUM test. Because our test statistic is based only on cumulative sums and their standard deviations, we call it a CUSUM test statistic.

The CUSUM statistic is formed from accounting data by taking cumulative ID sums. The CUSUM test is based on limits determined by multiplying the ID standard deviations by an appropriate constant chosen by simulation for an FAP = 0.05. If the test statistic exceeds its limit, we accept the loss hypothesis.

Consider an ID sequence $\{ID_i\}$ and a corresponding CUSUM sequence

$$CUSUM_i = \sum_{k=1}^{i} ID_k \quad (i = 1, 60) \ .$$

Recall that for the example system the variance-covariance matrix Σ for the ID sequence is symmetric with entries $\sigma_{ii} = 0.1076$, $\sigma_{i,i+1} = \sigma_{i-1,i} = -0.0003$, and $\sigma_{ij} = 0.0294$ elsewhere.

The variance term corresponding to the nth term in the CUSUM sequence is given by the matrix product $\underset{\sim}{1}'\Sigma_n\underset{\sim}{1}$, where $\underset{\sim}{1}$ is a column vector consisting of ones. For example, the first three variances in the CUSUM sequence are

$$\sigma_1^2 = 0.1076 \quad ,$$

$$\begin{aligned}\sigma_2^2 &= [1 \quad 1]\begin{bmatrix} 0.1076 & -0.0003 \\ -0.0003 & 0.1076 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= [\,0.1073 \quad 0.1073\,]\begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= 0.2146 \quad , \quad \text{and}\end{aligned}$$

$$\begin{aligned}\sigma_3^2 &= [1 \quad 1 \quad 1]\begin{bmatrix} 0.1076 & -0.0003 & 0.0294 \\ -0.0003 & 0.1076 & -0.0003 \\ 0.0294 & -0.0003 & 0.1076 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= [\,0.1367 \quad 0.1070 \quad 0.1367\,]\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= 0.3804 \quad .\end{aligned}$$

The CUSUM test consists of comparing each CUSUM statistic, $\sum ID_i$, with the threshold $c\sigma_k$. If $\sum ID_i > c\sigma_k$, the hypothesis of loss is accepted: otherwise, the hypothesis of no loss continues not to be rejected.

Thresholds are obtained by finding a constant c such that c times the sequence of standard deviations gives a FAP $\alpha = 0.05$. A value of $c \sim 6$ was determined from a set of 1000 runs with zero losses. Results gave a 5% FAP; that is, 50 runs out of 1000 rejected the zero-loss hypothesis.

B. Page's Test

Page's test is applied to accounting data by (1) transforming the observed ID sequence $\{ID_i\}$ to an independent sequence $\{ID_i^*\}$ by the method of recursive residuals; (2) calculating Page's statistic

$$T_i = \text{Max}\,\{T_{i-1} + ID_i^* - k,\ 0\} \quad ,$$

where $T_0 = 0$; and (3) comparing each T_i with a threshold h and accepting the hypothesis of materials loss when $T_i \geq h$. In effect the statistic T_i represents the largest cumulative sum of IDs over all previous accounting periods.

The parameters k and h denote the reference value and decision interval, respectively. There are many different sets of k and h values that will give the same FAP but will also give different values of another parameter, $CARL_0$. $CARL_0$ is defined as the average number of days that elapse before action is unnecessarily taken. $CARL_0$ is the average number of days the hypothesis of no loss is rejected using only the 50 rejected runs out of 1000 tested. The target value of $CARL_0$ was arbitrarily set at $\sim$30. We determined values of h and k by trial and error under a no-loss situation to obtain a FAP = 0.05 and a $CARL_0$ $\sim$30. In the example system, for FAP = 0.05, we found that k = 0.50 and h $\sim$ 5.07 came close to meeting the FAP, $CARL_0$ criteria. Further details of these criteria are given in Sec. V and an example of this procedure is given in Appendix B.

C. Power-One Test

The power-one test was initially proposed by Robbins[16] as a procedure that accepts the alternative hypothesis H_1 with probability one when H_1 is true and testing can continue indefinitely. Because of the requirement to make a decision on materials loss at least every 60 days, the truncated form of the test used in this report cannot attain a probability of one for diversion detection. Nevertheless, we consider this test procedure to be one of the most powerful sequential procedures for loss detection.

For materials accounting applications, the appropriate test statistic is the cumulative sum of the transformed ID sequence. The test statistic is defined as

$$S_n^* = \sum_{i=1}^{n} ID_i^*/\sigma_i^* \quad .$$

In this expression, ID^* and σ^* are transformed values. Thresholds are obtained from a formula given by Robbins and Siegmund[8] as

$$b_n = \{(n + P)[A^2 + \ln (\frac{n}{P} + 1)]^{1/2}; \; A^2 = -2 \ln \gamma; \; P > 0\} \quad ,$$

where P and γ are parameters determined to obtain a FAP = 0.05 and $CARL_0 \sim 30$.

As in Page's two-parameter case, we could have selected many different values of γ and P; however, for FAP = 0.05, $\gamma \sim 0.33$ and B $\sim$ 25.0 came closest to meeting the FAP, $CARL_0$ criteria.

An example of the power-one sequence is given in Appendix B.

V. TEST COMPARISON

A. Background

Our objective is to determine the "best" statistical procedures for detecting protracted losses, considering that these procedures must be compatible with the conditions specified by the MC&A Reform Amendment and implemented in operating nuclear facilities. Among all test statistics, the criteria we used for determining a most preferred test are

(1) the test statistic should have the highest detection probability for any protracted loss and

(2) the test statistic should be able to detect protracted losses in the shortest time from the onset of loss.

These criteria are not satisfied by one test. Instead, the test that is optimal for each criterion depends on the loss scenario. Page's test is best when protracted losses occur near the end of the testing period. Power-one performs better if the losses occur early.

Conclusions concerning test procedures are drawn from results simulated from the example system given in Sec. II. One thousand sets of IDs with a correlation structure representative of the example system were created and analyzed for each of the 60-day balance periods. We imposed different loss strategies and compared sequential tests. Before discussing these results, we give some specialized definitions.

Detection probability (DP) is the fraction of 1000 repeated trials in which a loss is detected. In a zero-loss situation, this fraction is defined as the FAP and is arbitrarily set at either 1% or 5%.

Average run length (ARL) is, for the general case, the expected (average) number of balance periods that elapse before the test procedure accepts the loss hypothesis.

In this study we are interested in the average number of balance periods that occur from the onset of loss before detection. In addition, we wish to condition the run length calculation in the event that detection occurs. Thus, we call our parameter the conditional average run length (CARL) and define it for a specified loss situation as the average number of days from the onset of loss until action is taken. The computation is based on the subset of 1000 runs that require ≤ 60 days before detection.

Both Page's and power-one tests require two parameters that can be uniquely determined for specified FAP and $CARL_0$. By trial and error, we obtained test parameters under the zero-loss hypothesis for FAP ~ 0.05 and $CARL_0 \sim 30$. The computation of $CARL_0$ involves only those 50 cases out of the 1000 runs involving a decision error.

B. Loss Strategies

Loss strategies include protracted losses over a long period of time (days 11-60), protracted losses near the beginning of the period (days 6-30), protracted losses at the end of the period (days 31-60), and a sequence of six abrupt losses every tenth day (days 10, 20, 30, 40, 50, 60).

C. DP and CARL Curves

DP and CARL curves are given for each loss strategy in Figs. 2-10. Power-one has the highest DP values for uniform losses that take place in the 1-30, 11-60, and 6-30 time periods, and Page's test performs best for uniform loss in the 31-60 period and for the set of six equally spaced abrupt losses (see Figs. 2-6). Page's test has the lowest CARL for almost all loss strategies (see Figs. 7-10).

The power-one test uses all previous data in each step and obtains much larger values of DP for the early loss strategy. The power-one test performs more poorly when losses occur near the end of the inventory period.

The above discussion is summarized in Figs. 2 and 3, where uniform losses in periods 11-60 and 6-30 reveal power-one outperforming Page, and in Figs. 4 and 5, where uniform losses in periods 31-60 and repeated abrupt losses in periods 10, 20, 30, 40, 50, and 60 show Page's test performing better than the power-one test.

Lucas and Crosier[17] recognized the weakness of Page's test in detecting early losses. They propose a "fast initial response" (FIR) procedure that allows earlier detection capabilities. Using this methodology led to higher FAPs. We suggest further study for improving Page's test to detect early losses.

A protracted loss scenario in the time period 1-30 will give correspondingly smaller DP values than similar loss scenarios where losses begin after the first time period. In addition, the use of FAP = 0.01 instead of 0.05 will result in smaller DP values. The DP curves for a 1-30 loss scenario with FAP = 0.01 is shown in Fig. 6. This curve depicts DP values noticeably smaller than those given in Figs. 2-4 for comparable amounts of total loss.

Page's test gives better results for CARL in those cases shown in Figs. 7-10. These results indicate that the behavior of CARL is nearly antagonistic with DP. Thus, there is no unique test that satisfies the criteria of highest DP and lowest CARL over all loss strategies.

Neither Page's nor the power-one test meets the criteria for all loss scenarios. Page's test is better if CARL is important and/or the protracted loss is in the latter part of the inventory period. Page's test also excels in the case of abrupt losses. The power-one test is better if protracted

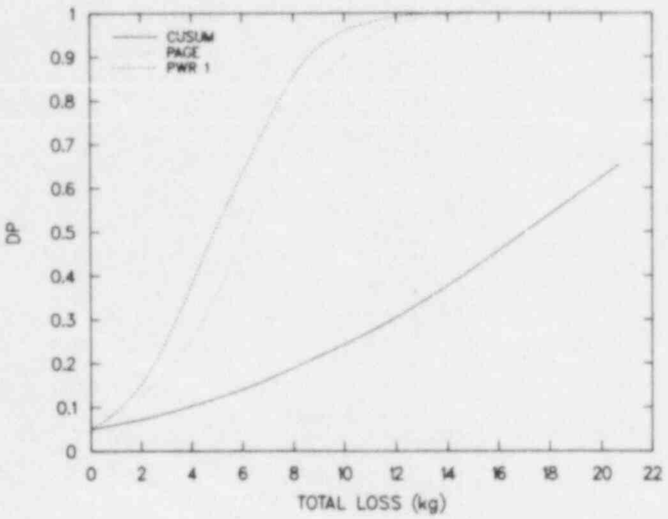


Fig. 2.
DP curves for protracted losses over 50 time periods from periods 11-60.

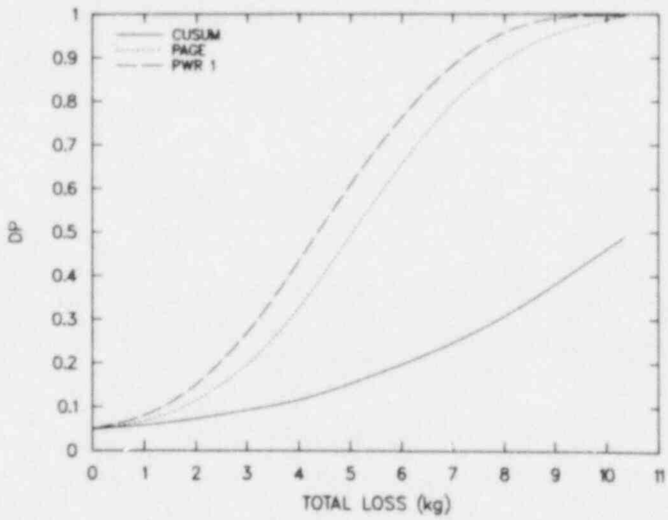


Fig. 3.
DP curves for protracted losses over 25 time periods from periods 6-30.

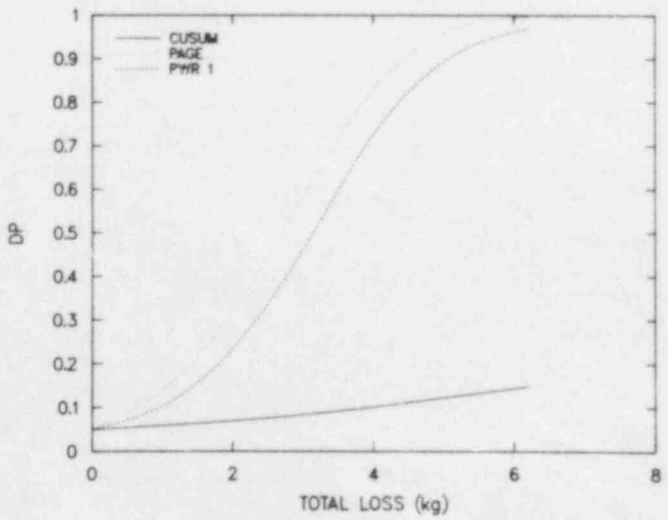


Fig. 4.
DP curves for protracted losses over 30 time periods from periods 31-60.

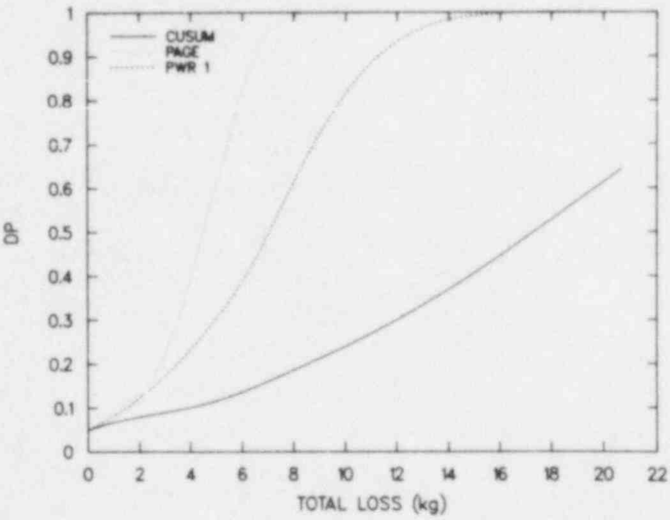
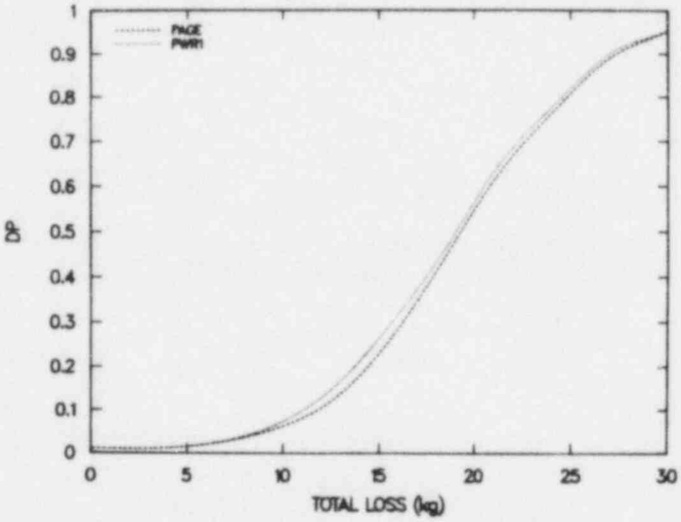


Fig. 5.
DP curves for six equally spaced abrupt losses at time periods 10, 20, 30, 40, 50, and 60.

Fig. 6.
DP curves for protracted losses after 30 time periods from periods 1-30 (FAP = 0.01).



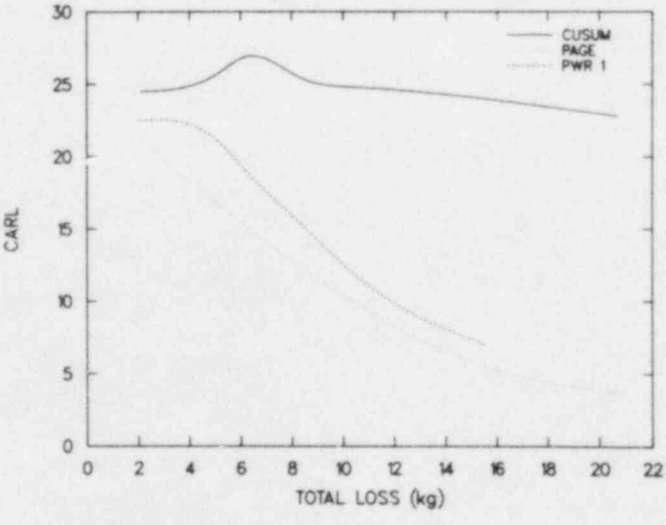


Fig. 7.
CARLs for protracted losses over 50 time periods from periods 11-60.

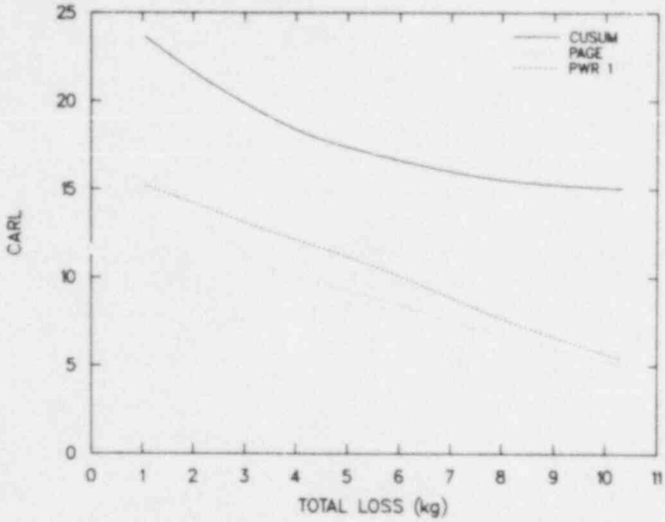


Fig. 8.
CARLs for protracted losses over 25 time periods from periods 6-30.

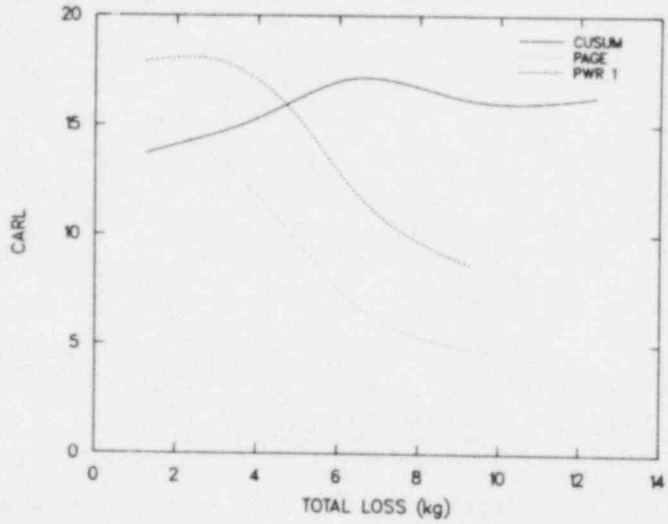


Fig. 9.
CARLs for protracted losses over 30 time periods from periods 31-60.

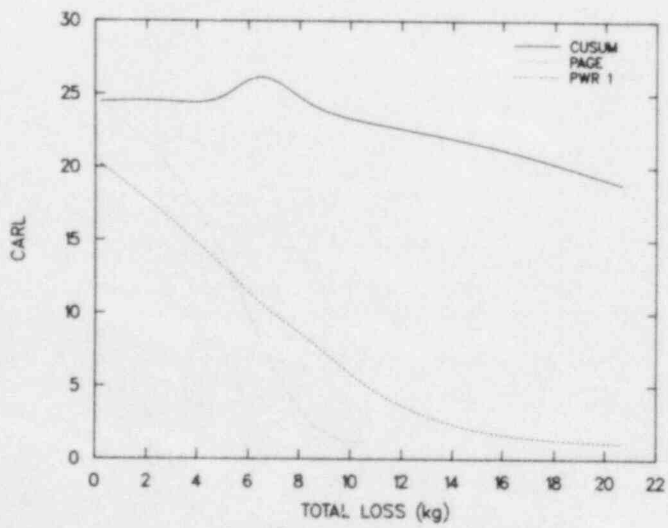


Fig. 10.
CARLs for six equally spaced abrupt losses at time periods 10, 20, 30, 40, 50, and 60.

losses occur at the beginning of the inventory period. These conclusions are in agreement with results given by Avenhaus, Beedgen, and Seilinschegg.[18]

Values of CARL were omitted from the curves under the zero-loss condition because of the change in definition. Values of $CARL_0$ for zero loss are 27.52, 32.22, and 30.14 for the CUSUM, Page, and power-one tests, respectively.

D. Rule Compliance

The DP curve provides a mechanism for determining what constitutes an amount of material that is as low as reasonably achievable (ALARA) for compliance with the MC&A Reform Amendment. To detect a recurring loss with 90% power of detection, one finds the loss corresponding to 0.90 in the DP curve. The ALARA quantity is obtained from the sequential test that gives the highest DP values. There is no single "best" test; therefore, the most preferred sequential test depends upon the particular loss scenario.

Recommended ALARA values for different types of loss strategies are given in Figs. 11-15. Values of ALARA range from 4.6 to 26.8 kg and go higher as the initial loss approaches the first time period (day 1) and go lower as the initial loss approaches the last time period (day 60). As shown in Fig. 15, the special case of FAP = 0.01 and losses during periods 1-30 give the largest ALARA quantity.

As depicted in Fig. 16, ALARA values increase as values of FAP decrease. Figure 16 also shows that ALARA quantities offering 90% detection probability range from $\sim$9.3 kg with α = 0.05 to 15 kg with α = 0.004 for protracted losses during period 11-60.

VI. EFFECTS OF HOLDUP AND PROCESS VARIATION

Two important effects of holdup and process variation (hpv) are as follows:

(1) If hpv is present and is modeled and compared with a model in which hpv is not present, few differences exist between DP curves except in the case of spaced abrupt losses.

(2) If hpv is present but not modeled, the FAP can increase sharply.

These effects emphasize the importance of including hpv in the model.

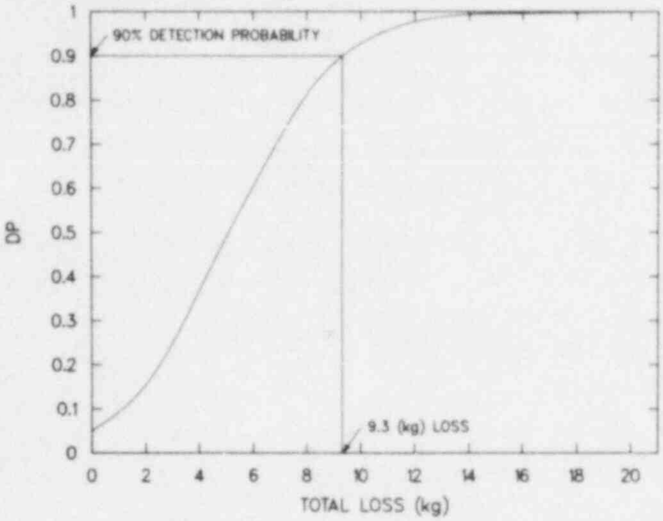


Fig. 11.
Compliance with Rule 70.83(b)(2): ALARA for protracted losses over time periods 11-60 is a 9.3-kg SSNM total loss determined by the power-one test.

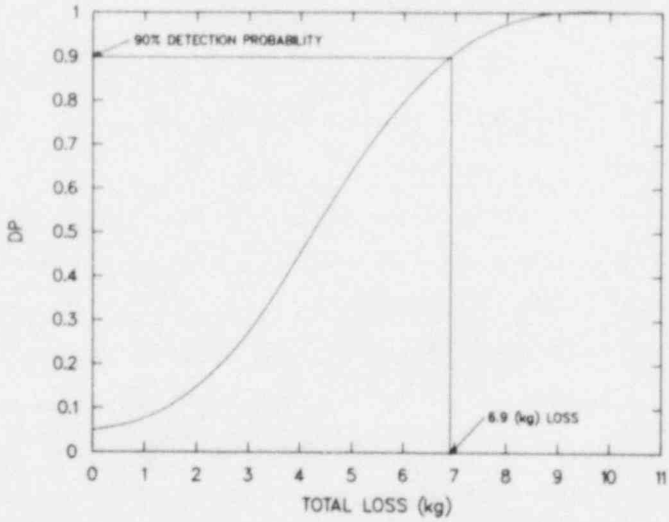


Fig. 12.
Compliance with Rule 70.83(b)(2): ALARA for protracted losses over days 6-30 is a 6.9-kg SSNM total loss determined by the power-one test.

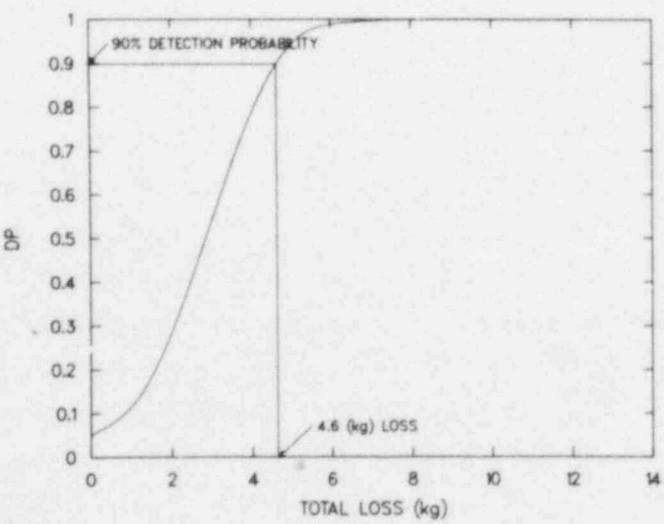


Fig. 13.
Compliance with Rule 70.83(b)(2): ALARA for protracted losses over days 31-60 is a 4.6-kg SSNM total loss determined by Page's test.

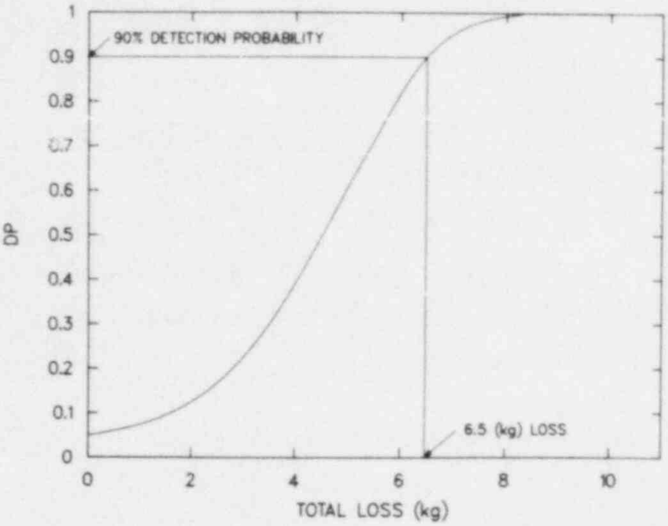
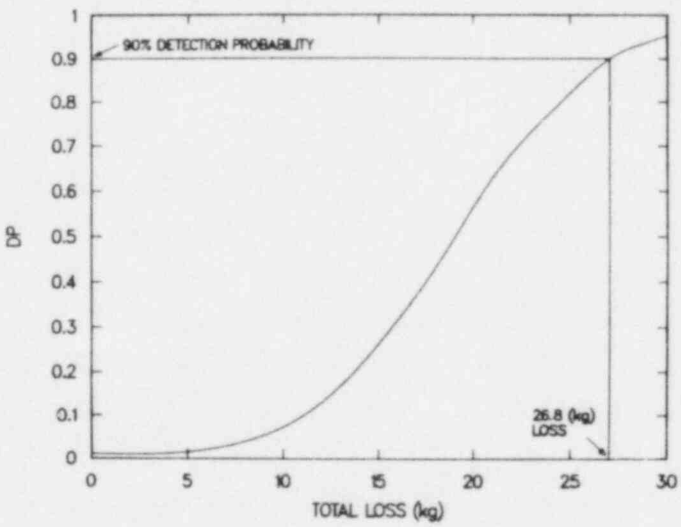


Fig. 14.
Compliance with Rule 70.83(b)(2): ALARA for six equally spaced abrupt losses at time periods 10, 20, 30, 40, 50, and 60 is a 6.5-kg SSNM total loss determined by Page's test.

Fig. 15.
Compliance with Rule 70.83(b)(2): ALARA for protracted losses over days 1-30 is a 26.8-kg SSNM total loss determined by Page's test.



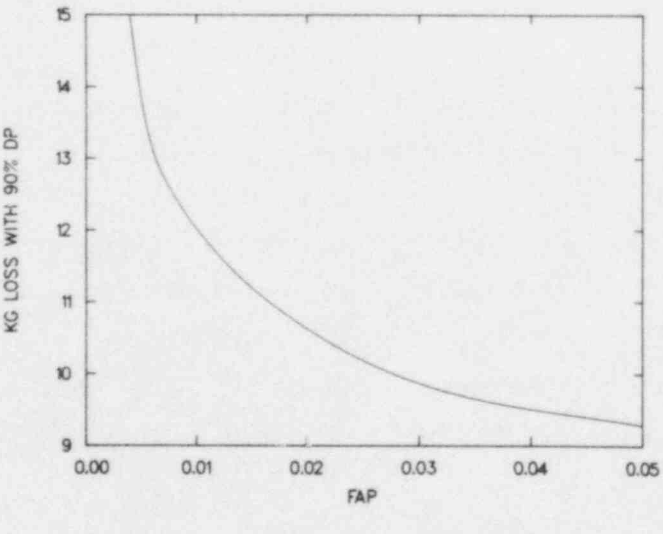


Fig. 16.
The effect of decreasing FAPs on the ALARA for protracted losses during periods 11-60.

The effects of hpv were studied by comparing values of DP and CARL from simulations using models in which hpv was and was not present. We determined that only in the spaced abrupt loss scenario for Page's test was DP substantially lower for the model that included hpv.

The effects of hpv were also studied by comparing FAPs using thresholds obtained from a simulation without hpv on simulated data that included hpv. We determined for this study that if hpv is not included in the model but is present in the MCU, the FAP increases from 0.05 to 0.11.

VII. SUMMARY AND CONCLUSIONS

This report is intended to provide licensees with the analytical tools necessary to comply with Section 70.83(b)(2) of the MC&A Reform Amendment as propo. . by the NRC. This regulation is concerned with the detection of recurring materials losses in a single-space, multiple-time framework. Sequential tests are preferred in this setting because they detect possible materials losses faster than other tests (for example, tests made for a fixed time period). Also sequential tests often will pinpoint the time of change more accurately than other tests.

The sequential tests presented in this report are truncated versions of Page's and power-one tests and an ad hoc sequential analog of fixed-length tests, which we call CUSUM. We believe that these tests are user-oriented because computer programs can eventually be made available to facilitate their use. This report contains examples to clarify all of the necessary computations.

The test procedures may be useful in satisfying the rule requirements. Their effectiveness is based on a simulation of a MCU in a UF_6-to-U_3O_8 conversion process that has both realistic holdup and process variation contributions.

Table II summarizes recommended test procedures. These recommendations are based on the ability of the tests to meet the criteria of (1) shortest average detection time from the onset of loss and (2) highest DP. There is no one unique test that satisfied both criteria simultaneously. Instead the most preferred test varies depending on the particular loss scenario.

TABLE II

SUMMARY OF TEST RECOMMENDATIONS
BASED ON LOSS SCENARIOS AND CRITERIA

Loss Scenario (Constant Loss) (Days)	Shortest Average Detection Time from Onset of Loss	Highest Detection Probabiity
1-30	Page	Power-one
6-30	Page	Power-one
11-60	Page	Power-one
31-60	Page	Page
10, 20, 30, 40, 50, 60	Page	Page

In addition, as seen in Table II, the two criteria may not be met within the same loss scenario by a single test. For example, power-one satisfies criterion (1) better than Page's test for constant losses in days 6-30, but Page's test excels for criterion (2).

The licensee has no idea of the loss scenario; consequently, we recommend further investigations to determine a test that would be best suited to satisfy the criteria. The extent to which the FAP and DP curves will be affected would be part of that further study.

In accordance with the rules, the amount of recurring loss to be detected at the 90% probability level must be "as low as reasonably achievable" (ALARA). Values of ALARA depend upon the loss scenario. Estimates of ALARA are obtained from DP curves using the appropriate sequential test as given in Table II. Results for this study are given in Table III.

Another important result of this report concerns the effects of hpv on the FAP and DP. When hpv is present and appropriately modeled in the ID and compared with hpv not present, then FAP and DP values are nearly equal for all loss scenarios except equally spaced abrupt losses. In this scenario, Page's test gives substantially lower DP values for the modeled hpv. If hpv was present and not modeled, the FAP rose from 0.05 to 0.11 indicating the importance of including hpv in the model.

TABLE III

ALARA VALUES FOR CERTAIN LOSS SCENARIOS

Loss Scenarios Days (Time Period)	ALARA Amounts (kg)
1-30	26.8
6-30	6.9
11-60	9.3
31-60	4.6
10, 20, 30, 40, 50, 60	6.5

APPENDIX A

MATERIALS BALANCE VARIANCE CONTRIBUTIONS

Measurements used in the example MCU and their associated variances are given in Table A-I. Variances for the inventory difference (ID) terms given in Table I of Sec. III are obtained as follows:

<u>Input Transfer</u>

Given

$$
\begin{aligned}
T_{1j} &= \sum_{k=1}^{2} G_{kj}(M_{kj} - m_{kj}) \\
&= (0.676)(1 + \varepsilon_{G_{1j}} + \eta_{G_{1j}})(25 + \varepsilon_{M_{1j}} + \eta_{M_{1j}} - 0.5917 - \varepsilon_{m_{1j}} - \eta_{M_{1j}}) \\
&\quad + (0.676)(1 + \varepsilon_{G_{2j}} + \eta_{G_{2j}})(25 + \varepsilon_{M_{2j}} + \eta_{M_{2j}} - 0.5917 - \varepsilon_{m_{2j}} - \eta_{M_{2j}}) \\
&= (0.676)[(1 + \varepsilon_{G_{1j}} + \eta_{G_{1j}})(24.4083 + \varepsilon_{M_{1j}} - \varepsilon_{m_{1j}}) \\
&\quad + (1 + \varepsilon_{G_{2j}} + \eta_{G_{2j}})(24.4083 + \varepsilon_{M_{2j}} - \varepsilon_{m_{2j}})] \ .
\end{aligned}
$$

The estimate of the variance contribution for the jth day is obtained by propagation of error as

$$
\begin{aligned}
\hat{\sigma}^2_{T_{1j}} &= 4(0.676)^2\sigma^2_{\varepsilon_M} + 2(0.676)^2(24.4083)^2\sigma^2_{\varepsilon_G} \\
&\quad + 4(0.676)^2(24.4083)^2\sigma^2_{\eta_G} \\
&= 0.0060 \ .
\end{aligned}
$$

TABLE A-I

MEASUREMENTS AND THEIR VARIANCES

Measurement	$\sigma_\varepsilon^2(\text{Kg}^2\ \text{U})$	$\sigma_\eta^2(\text{Kg}^2\ \text{U})$
M	9.0×10^{-6}	1.0×10^{-6}
m	9.0×10^{-6}	1.0×10^{-6}
G	9.0×10^{-6}	1.0×10^{-6}
C	9.0×10^{-6}	1.0×10^{-6}
v	1.0×10^{-4}	2.5×10^{-5}
V	1.0×10^{-4}	2.5×10^{-5}

The covariance between measured transfers on days j and j' ($j \neq j'$) is estimated by

$$\hat{\sigma}_{T_{1j},T_{1j'}} = 4(0.676)^2(24.4083)^2\sigma^2_{\eta_G}$$

$$= 0.0011 \quad .$$

Inventory Difference

Given

$$I_{1j} - I_{2j} = C_{1j}V_{1j} - C_{2j}V_{2j}$$

$$= (0.22)(1 + \varepsilon_{C1} + \eta_{C1})(75.0)(1 + \varepsilon_{V1} + \eta_{V1})$$

$$-(0.22)(1 + \varepsilon_{C2} + \eta_{C2})(75.0)(1 + \varepsilon_{V2} + \eta_{V2}) \quad .$$

The estimate of the variance contribution for the jth day is given by

$$\hat{\sigma}^2_{I_j} = 2(0.22)^2(75.0)^2(\sigma^2_{\epsilon_C} + \sigma^2_{\epsilon_V})$$

$$= 0.0593 \quad .$$

Successive IDs are correlated because the ending inventory of day j is equal to the beginning inventory of day j + 1.

The estimate of covariance between successive IDs j and j + 1 is given by

$$\hat{\sigma}_{j(j+1)} = -(0.22)^2(75.0)^2(\sigma^2_{\epsilon_C} + \sigma^2_{\epsilon_V})$$

$$= -0.0297 \quad .$$

The estimate of covariance between any two nonsuccessive IDs, j and j' (j' $\neq$ j, j' $\neq$ j + 1), is zero. The η error drops out when taking differences; consequently, only inventory measurements on successive days are correlated.

Output Transfer

Given

$$T_{2j} = \sum_{i=1}^{2} [C_i \sum_{k=1}^{6} v_{ik}] \quad .$$

The estimate of the variance contribution for the output transfer on the jth day is given by

$$\hat{\sigma}^2_{T_{2j}} = (0.22)^2(12.5)^2[12(\sigma^2_{\varepsilon_V} + \sigma^2_{\eta_V} + \sigma^2_{\varepsilon_C} + \sigma^2_{\eta_C})$$

$$+ 60(\sigma^2_{\eta_V} + \sigma^2_{\varepsilon_C} + \sigma^2_{\eta_C}) + 72(\sigma^2_{\eta_V} + \sigma^2_{\eta_C})]$$

$$= (0.22)^2(12.5)^2[12(1.0 \times 10^{-4}) + 72(9.0 \times 10^{-6})$$

$$+ 144(2.6 \times 10^{-5})] = 0.0423 \quad .$$

The covariance between days j and j' ($j \neq j'$) is estimated as follows: The estimate of variance for the jth day can be written

$$\hat{\sigma}^2_{T_{2j}} = [(0.22)(12.5)]^2[72\sigma^2_{\varepsilon_C} + 12^2(\sigma^2_{\eta_V} + \sigma^2_{\eta_C}) + 12\sigma^2_{\varepsilon_V}] \quad .$$

Covariances between output transfers on days j and j' will involve precisely the same η terms as any other two days; consequently, covariance terms are equal. Thus, $\sigma_{ij} = \sigma_{i'j'}$ ($i' \neq i$, $j' \neq j$, $i \neq j$, and $i' \neq j'$).

Using the identity, for a general variable X,

$$\mathrm{Var} \sum_{i=1}^{n} (X_i) = \sum_{i=1}^{n} \mathrm{Var}\,(X_i) + n(n-1)\,\mathrm{Cov}\,(X_i, X_j) \quad ,$$

we get

$$7.5625[72n(9.0 \times 10^{-6}) + (12n)^2(2.6 \times 10^{-5}) + 12n(1.0 \times 10^{-4})]$$

$$= 0.0423n + n(n-1)\sigma_{T_{2i}T_{2j}} \quad ,$$

which reduces to

$$(\sigma_{T_{2i}T_{2j}} - 0.0283)n - (\sigma_{T_{2i}T_{2j}} - 0.0283) = 0 \quad \text{or}$$

$$\sigma_{T_{2i}T_{2j}} = 0.0283 \quad .$$

Finally, the ID terms in Table I in Sec. III are obtained by adding the corresponding variance and covariance contributions.

APPENDIX B

EXAMPLES OF SEQUENTIAL TEST PROCEDURES

Formulas are given for the recursive residual procedure for transforming an ID sequence into an independent sequence, Page's test statistic and limits, and the power-one test and limits. An example of a sequence with just four terms is given to clarify the presentation.

I. RECURSIVE RESIDUALS

For convenience, let x_i denote the ith term in the ID sequence. Recursive residuals are defined as the sequence $\{y_1, y_2, \ldots, y_n\}$ and determined as follows. Let

$$\underset{\sim}{x}_n = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix},$$

and assume that x_n follows a multivariate normal distribution[19] with mean vector $\underset{\sim}{0}$ and variance-covariance $\sum_n$, where

$$\underset{\sim}{\mu}_n = 0 \quad \text{and} \quad \sum_n = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix}.$$

The recursive residuals are given by

$$y_i = x_i - E(x_i | x_1, x_2, \ldots, x_{i-1})$$

$$= x_i - \sum_{j=1}^{i-1} \beta_{ij} x_j$$

$$= x_i - \sum_{j=1}^{i-1} \beta_{ij} x_j \quad (i = 1, 2, \ldots, n) \quad ,$$

where

$$\beta_{in} = \begin{bmatrix} \beta_{i1} \\ \beta_{i2} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{in} \end{bmatrix} = \Sigma_{21,n} \Sigma_{11,n}^{-1} \quad ,$$

and the decomposition of Σ is given by

$$\Sigma_n = \begin{bmatrix} \Sigma_{11,n} & \Sigma_{12,n} \\ \Sigma_{21,n} & \Sigma_{22,n} \end{bmatrix} .$$

The variance-covariance matrix of $\underset{\sim}{y}$ is

$$\Sigma_{y,n} = B_n \Sigma_n B_n' = \begin{bmatrix} \sigma_{y_1}^2 & 0 & 0 & \ldots & 0 \\ 0 & \sigma_{y_2}^2 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & \sigma_{y_n}^2 \end{bmatrix} \quad ,$$

where B is the lower triangular matrix defined by

$$B = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -\beta_{21} & 1 & 0 & \cdots & 0 \\ -\beta_{31} & -\beta_{32} & 1 & \cdots & 0 \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ -\beta_{n1} & -\beta_{n2} & -\beta_{n3} & \cdots & 1 \end{bmatrix} .$$

The y_i are independent because $B_n \Sigma_n B_n'$ is diagonal.

II. EXAMPLE

The first 4 values of the first 1000 runs simulated for the MCU given in Sec. II are defined as

$$\underset{\sim}{x}_4 = \begin{bmatrix} 0.37 \\ -0.91 \\ -0.14 \\ -0.06 \end{bmatrix} .$$

The variance-covariance of $\underset{\sim}{x}_4$ is

$$\Sigma_4 = \begin{bmatrix} 0.1076 & -0.0003 & 0.0294 & 0.0294 \\ -0.0003 & 0.1076 & -0.0003 & 0.0294 \\ 0.0294 & -0.0003 & 0.1076 & -0.0003 \\ 0.0294 & 0.0294 & -0.0003 & 0.1076 \end{bmatrix} .$$

n = 1

$$\underset{\sim}{x}_1 = 0.37; \quad \underset{\sim}{y}_1 = 0.37; \quad \sigma_{y1} = \sigma_{x1} = \sqrt{0.1076} = 0.33;$$

$$S_1 = \frac{0.37}{\sqrt{0.1076}} = 1.13 \quad .$$

n = 2

$$\underset{\sim}{x}_2 = \begin{bmatrix} 0.37 \\ -0.91 \end{bmatrix}; \quad \beta_{21} = \frac{-0.0003}{0.1076} = -0.0028$$

$$\underset{\sim}{y}_2 = x_2 - \beta_{21} x_1 = -0.91 - (-0.0028)(0.37) = -0.91$$

$$\Sigma_{y2} = B_2 \Sigma_2 B_2' = \begin{bmatrix} 1 & 0 \\ 0.0028 & 1 \end{bmatrix} \begin{bmatrix} 0.1076 & -0.0003 \\ -0.0003 & 0.1076 \end{bmatrix} \begin{bmatrix} 1 & 0.0028 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1076 & -0.0003 \\ 0 & 0.1076 \end{bmatrix} \begin{bmatrix} 1 & 0.0028 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1076 & 0 \\ 0 & 0.1076 \end{bmatrix}$$

$$S_2 = \frac{y_2}{\sigma_{y2}} = \frac{-0.91}{\sqrt{0.1076}} = -2.774 \quad .$$

n = 3

$$\underset{\sim}{x}_3 = \begin{bmatrix} 0.37 \\ -0.91 \\ -0.14 \end{bmatrix} \quad ;$$

$$\beta_3' = \begin{bmatrix}\beta_{31} & \beta_{32}\end{bmatrix} = \begin{bmatrix}0.0294 - 0.0003\end{bmatrix}\begin{bmatrix}0.1076 & -0.0003 \\ -0.0003 & 0.1076\end{bmatrix}^{-1}$$

$$= \begin{bmatrix}0.2732 & -0.0020\end{bmatrix} \quad ;$$

$$y_3 = x_3 - \beta_{32}x_2 - \beta_{31}x_1 = -0.14 - (-0.0020)(-0.91) - (0.2732)(0.37)$$

$$= -0.24 \quad ;$$

$$\Sigma_{y3} = B_3\Sigma_3B_3' = \begin{bmatrix}1 & 0 \\ 0.0028 & 1 \\ -0.2732 & 0.0020\end{bmatrix}\begin{bmatrix}0 & 0.1076 & -0.0003 & 0.0294 \\ 0 & -0.0003 & 0.1076 & -0.0003 \\ 1 & 0.0294 & -0.0003 & 0.1076\end{bmatrix}$$

$$\begin{bmatrix}1 & 0.0028 & -0.2732 \\ 0 & 1 & 0.0020 \\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0.1076 & 0 & 0 \\ 0 & 0.1076 & 0 \\ 0 & 0 & 0.0996\end{bmatrix} \quad ;$$

$$S_3 = \frac{-0.24}{\sqrt{0.0996}} = 0.76 \quad .$$

<u>n = 4</u>

$$\underset{\sim}{x}_4 = \begin{bmatrix}0.37 \\ -0.91 \\ -0.14 \\ -0.06\end{bmatrix} \quad ;$$

$$\beta_4' = [\beta_{41} \quad \beta_{42} \quad \beta_{43}]$$

$$= \begin{bmatrix} 0.0294 & 0.0294 & -0.0003 \end{bmatrix} \begin{bmatrix} 0.1076 & -0.0003 & 0.0294 \\ -0.0003 & 0.1076 & -0.0003 \\ 0.0294 & -0.0003 & 0.1076 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0.2967 & 0.2740 & -0.0831 \end{bmatrix} \quad ;$$

$$y_4 = x_4 - \beta_{43}x_3 - \beta_{42}x_2 - \beta_{41}x_1$$

$$= -0.06 - (00.08)(-0.14) - (0.27)(-0.91) - (0.30)(0.37)$$

$$= 0.09 \quad ;$$

$$\Sigma_{y4} = B_4 \Sigma_4 B_4' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0028 & 1 & 0 & 0 \\ -0.2732 & 0.0020 & 1 & 0 \\ -0.2967 & -0.2740 & 0.0831 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.1076 & -0.0003 & 0.0294 & 0.0294 \\ -0.0003 & 0.1076 & -0.0003 & 0.0294 \\ 0.0294 & -0.0003 & 0.1076 & -0.0003 \\ 0.0294 & 0.0294 & -0.0003 & 0.1076 \end{bmatrix} \begin{bmatrix} 1 & 0.0028 & -0.2732 & -0.2967 \\ 0 & 1 & 0.0020 & -0.2740 \\ 0 & 0 & 1 & 0.0831 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1076 & 0 & 0 & 0 \\ 0 & 0.1076 & 0 & 0 \\ 0 & 0 & 0.0996 & 0 \\ 0 & 0 & 0 & 0.0907 \end{bmatrix} \quad ;$$

$$S_4 = \frac{0.09}{\sqrt{0.0907}} = 0.30 \quad .$$

The independent transformed sequence is

$$\underset{\sim}{y}_4' = [0.37, -0.91, -0.24, 0.09] \quad .$$

The standard deviations of $\underset{\sim}{y}_i$ are {0.3280, 0.3280, 0.3156, 0.3012}. The standardized sequence is given by y_i/σ_{y_i} or {1.13, -2.77, -0.76, 0.30}.

III. PAGE'S TEST

Parameters for Page's test in the MCU example are $h \sim 5.1$ and $k \sim 0.5$. Page's transformed sequence (called SITMUF in Ref. 12) is given by

$$T_n = \text{Max}\,\{T_{n-1} + \frac{y_n}{\sigma_n} - k;\ 0\}$$

$$T_0 = 0 \quad .$$

The sequence of test statistics for the first four time periods is given by

$$T_1 = (1.13 - 0.50) = 0.63$$

$$T_2 = 0$$

$$T_3 = 0$$

$$T_4 = 0 \quad .$$

All values of T_i are <5.1, the control limit. To detect a loss on the first period, the observed ID would have to be over five times its standard deviation. For this example, the first ID would have to be at least (5.1)(0.33) = 1.68 kg--a large amount of material--to accept the loss hypothesis. This example points out the weakness of sequential testing for early losses in the inventory period.

IV. POWER-ONE TEST

The test statistic is given by

$$S_n = \sum_{i=1}^{n} \frac{y_i}{\sigma_i} .$$

The sequence in this example is

$$S_4 = \{1.13, -1.64, -2.40, -2.10\} .$$

Power-one control limits are given, according to Robbins and Siegmund,[8] as

$$b_n = \{(n + P)[A^2 + \ln (\frac{n}{P} + 1)]\}^{1/2}; \; A^2 = -2\ln\gamma; \; P > 0 \quad (n = 1, 2, ...)$$

where P = 25 and $\gamma = 0.33$ in the MCU example. The sequence of control limits for the first four time periods is

$$\underset{\sim}{b}' = \{b_1, b_2, b_3, b_4\} = \{11.35, 11.61, 11.86, 12.12\} .$$

As in Page's test, an exorbitant loss of material at an early stage would go undetected using the power-one test.

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Sequential tests are required for detecting protracted (trickle) losses of strategic special nuclear materials from a single materials control unit (MCU). We compared applicable tests including modified versions of Page's test and power-one procedures. We used simulated data from a MCU in a conversion/fabrication process that took into account process variations, materials holdup, and measurement uncertainties. Comparisons were made over a 60-day accounting period under different loss scenarios. Some important findings include

(1) No single test procedure is best for all diversion scenarios.
(2) Power-one procedures are best for protracted losses that occur early in the accounting period and Page's test is best for late loss occurrence.
(3) If holdup and process variations are not included in the Inventory Difference model but are present in the process, then assuming steady-state conditions, false-alarm probabilities can double.

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