CONTROL ROD DROP ACCIDENT (BWR)

Introduction

The ACRS list of generic items relating to light-water reactors includes Item IIA-2 "Control Rod Drop Accident (BWRs)". In status report number 4, dated April 16, 1976, the ACRS provided the following brief description of the scope and intent of this item.

> "Some uncertainties have arisen in previous calculations of this postulated accident, including the choice of negative reactivity insertion rate due to scram and the potential differences between a two dimensional and a three dimensional calculation. Particularly for the latter point, more precise theoretical comparisons may be required to resolve the matter although probabilistic considerations may be relevant."

The staff considers this generic item to be resolved, although work will continue on three-dimensional analyses as part of the on-going programs to improve methods of transient analysis. This report summarizes the history and status of the review of this topic. The discussion of the early phases related to reactivity insertion rates is brief since the documentation related to those reviews is extensive. The consideration of three-dimensional calculations is discussed in somewhat more detail since it has not been previously documented.

Reactivity Insertion

Initial concern with the suitability of the GE analysis of the rod drop accident (RDA) arose during 1971 as a result of generic studies on transient analyses by our consultant, Brookhaven National Laboratory (BNL). These studies indicated no general problems or disagreement with the GE analyses or results except in the area of

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control rod reactivity insertion rates. These reactivity insertion rates, and especially the scram reactivity, were substantially non-conservative because of the use of control rod scram movement rates faster than specified in the Technical Specifications and the use of inappropriate spatial reactivity functions (i.e., reactivity inserted as a function of the fractional insertion of control rods). The latter in particular was the subject of extensive BNL studies that examined the validity of reactivity representations in model geometries where spatial variables of importance were not explicitly described. Since, as will be discussed further in connection with the three-dimensional problem, the GE model is essentially a for of point kinetics (zero dimensional), the suitable representation of reactivity inputs is important.

As a result of the concerns arising from these studies, and in conjunction with some improvements in the GE model, GE reanalyzed the RDA. The reanalyses incorporated the suggested changes in the control rod reactivity functions, both rod movements in accord with Technical Specifications and spatial functions determined by methods comparable to BNL's. The calculations and results were reported in 1972-73 in NEDO-10527 for beginning of first cycle (BOC) in BWR 2s and 3s, in Supplement 1 for BOC in BWR 4s and 5s, and in Supplement 2 for both types of cores later in the first cycle and in the second cycle.

BNL also did a series of calculations of the rod drop accident using one- and two-dimensional (R, Z) kinetics codes and their own developed version of the GE model. The more significant of these calculations and comparisons were reported on in many BNL reports

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which received external distribution. These have all been distributed to the ACRS. Included were the following reports which discuss reactivity functions and methods, and transient analyses and check calculations.

"Rod and Scram Bank Simulation," BNL RP-1018, August 1971.

"Application of Reactivity Weight Factors to Reactor Transients," BNL RP-1020, March 1972.

"Rod Drop and Scram in Boiling Water Reactors, Part I," BNL RP-1021, April 1972.

"Rod Drop and Scram in Boiling Water Reactors, Part II," BNL RP-1027, July 1973.

"Gadolinia Shimmed BWR Rod Drop at Zero Power," and "Curtain Shimmed BWR Rod Drop at 10% Power," Progress Report for Reactor Safety Analytical Support, May 1973.

"Gadolinia Shimmed BWR Rod Drop at 10% Power," Progress Report, July 1973.

The review and check calculations and comparisons of the GE reanalysis were favorable. GE has switched to a suitable control rod reactivity representation and the comparisons with various BNL calculations were satisfactory. In our topical report review of NEDO-10527 several incomplete aspects of the review were noted, such as the 10% power and gadolinia core check calculations. These were subsequently completed (see the last three listed BNL reports) with

satisfactory comparisons. A final wrap up package involving a series of one-dimensional sensitivity calculations to relate the two-dimensional check calculations to a more appropriate range of dropped rod reactivity insertions was planned. This was not completed, however, and it has now been decided to await the completion of future three-dimensional studies (see below) before completing a formal wrap-up. However, we have concluded that the initial problem has been resolved and that the GE calculations are suitable.

Need for Systems to Minimize RDA

Meanwhile, the review of the reanalysis and the recognition that for a given control rod worth a higher fuel energy was predicted lead to an interest in an improved Rod Worth Minimizer (RWM) system to add assurance that a high reactivity worth, non-sequenced rod is not withdrawn to become a candidate for the accident. In response to this interest GE developed, in mid-1972, the Rod Sequence Control System (RSCS), essentially a hardwired version of the computer-controlled RWM. This, in particular in its most recent form, the Group Notch RSCS, has been approved and is (or is being) installed in BWR-4s.

This posed the question whether to backfit RSCS on BWR-2s and 3s. To answer this, the staff carried out (in early 1975) an independent analysis of the probabilities, individual and combined, of the multiple events that must occur in order that a rod drop accident exceed the staff acceptance criterion of 280 cal/gm in the hottest fuel pellet. This analysis was described in a June 1975 memo, "A Statistical Examination of the RDA in Some BWRs," and was presented to the ACRS at the March 23, 1976 Subcommittee meeting on reactor safety in Chicago. A copy of the memo is provided as Appendix A to this report.

The study determined that a reasonable (and quite possibly conservative) estimate of the probability of having an RDA exceeding 280 cal/gm is about 10^{-12} per reactor year, without including any factor for the use of a RWM or RSCS. This indicated a large margin to an acceptance criterion of 10^{-7} per reactor year, and allowed for considerable uncertainty in the input information or unforeseen interactions among elements of the analysis. In addition, all BWR 2 and later reactors do have RSCS and/or RWM systems and Technical Specification requirements for their use. This result was the technical basis for the decision not to require backfit of an RSCS on BWR 2s and 3s. It also, of course, indicates in general the low probability of encountering a limiting RDA.

A further indication of margins existing in the RDA is the relation of the maximum expected sequenced rod reactivity worth to the rod worth for which the criterion of 280 cal/gm would be exceeded. Typical maximum sequenced worths are less than 1% Δk. (See, for example, the Shoreham FSAR where the peak maximum sequenced worth, at cold clean conditions, is about 0.9% Δk and usually much less than that for other conditions). From the results of Supplements 1 and 2 of NEDO-10527 we see that rod worths to reach 280 cal/gm range from somewhat over 1.4 to over 2.1% Δk depending on time in cycle (BOC is lowest) and cycle and assuming scram at Tech Spec velocities (1.6 to 2.3 at measured scram velocities). A 0.9% Δk rod results in only about 150 cal/gm (assuming 1.4% Δk gives 280 cal/gm).

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<u>Importance of Dimensional Order</u>

Another possible RDA analysis problem was raised by an article by Birkhofer, Schmidt and Werner (to be referred to in this memo as BSW) in the October 1974 issue of Nuclear Technology comparing the results of three-dimensional with lower-order analyses. The main theme of this article may best be summarized by the final paragraph which states

> "The conclusion to be drawn is that space- and time-dependent effects in large LWR cores with space-dependent feedback can only be predicted <u>correctly</u> by performing genuine three-dimensional calculations. All lower dimensional simulations of the problem <u>may</u> produce non-conservative results." (underlines added)

The authors carried out some comparison calculations to come to this conclusion.

We reviewed this paper and sent requests to each vendor to also review and comment on it. We subsequently reviewed these comments, in particular those of GE, and evaluated the applicability of the BSW study to vendor analyses. Also, our consultant discussed the paper with the authors during a European trip. Since it has not been previously reported, a brief discussion follows of the applicability of the study and its conclusions to the LWR vendor's and especially GE's analysis models.

Using their own developed three-dimensional kinetics code (apparently fast running) BSW did parallel XYZ, RZ, XY, and R calculations on what was approximately a small (about 7 ft. diameter by 9 ft. high core) BWR at zero power. The calculations of primary interest were of the rapid removal

(less than 1 second) of an off-center control rod, which was made a center rod for RZ and R calculations. Results may be generally characterized by comparisons in two parts.

1. No Z component, i.e., XYZ vs XY and RZ vs R

2. Off-center to center, i.e., XYZ vs RZ and XY vs R

Both comparisons show a ratio of about 1.4 between the peak local energies of the higher and lower dimension results.

An important missing element in the study was the failure to consider sufficiently the role of auxiliary calculations when doing lower order calculations. (Perhaps if they had gone all the way and included a point-kinetics calculation in the study, this shortcoming might have become apparent since supplemental parameters would have been required.) A prime example is found in the no-Z component calculations. No axial peaking factor was supplied or considered (as any of the vendor calculations would have). For the geometry of the reactor analyzed, the appropriate peaking factor would have been approximately 1.4, the calculated discrepancy.

An illustration of vendor use of auxiliary results to augment a lower order calculation is the collection of Westinghouse design calculations described in the rod ejection report, WCAP-7588. The calculational model is effectively a zero-order, point-kinetics approach (actually a 1D-Z kinetics) with auxiliary calculations to supply conservative peaking factors and reactivity feedbacks. For example, transient peaking factors are calculated, statically, without reactivity feedback to maximize peaking, and feedback is

estimated with a full feedback-reduced power distribution to minimize feedback weighting. As expected, comparison with three-dimensional calculations (described in the WCAP) indicate a high degree of conservatism. In reference to the previously underlined "correctly" and "may" in the BSW conclusion this Westinghouse experience is an illustration that while lower order calculations will not directly give "correct" results and if not altered they "may" be non-conservative, suitable modifications can be introduced into the lower order analyses to provide suitably conservative results.

The GE analyses do consider Z component peaking. The GE design method (described in NEDO-10527 and Supplement 2) is a point-kinetics model with rod drop and scram reactivity functions from auxiliary three-dimensional, XYZ, calculations and power distribution functions and Doppler feedback weighting factors from concurrent static RZ multigroup calculations. (Moderator heating, particularly prompt heating from neutrons and prompt gammas, is not included in the model and this can be a significant conservatism.) The RZ power distribution calculation maintains the dropping control rod throughout the transient at one axial position, approximately the position at the time of the first large rise and turn around in power. Since, for the control rod reactivity worths of interest, there is a significant axial movement of the peak throughout the transient (see, for example, the "three"-dimensional power distribution figures in the previously referenced BNL reports, e.g., RP-1021), this model produces a conservatively large energy rise in the peak axial fuel pellet.

It might appear that of perhaps more interest to the GE BWR analysis is the off-center to center comparison. However, in this case it is evident that the BSW comparisons were complicated, and we believe rendered of doubtful direct value, by the changes in the reactor center local r[illegible] geometry which were apparently introduced into the proble[illegible]he B region in BSW) by the requirement to maintain a given value of control rod reactivity worth for the two configurations. These modifications cause changes in power distributions in the vicinity of the dropped rod thus changing peaking factors for obtaining peak enthalpies, Doppler feedback weighting factors, and evidently reactivity insertion rates (see reactivity insertion figures in BSW). Indeed the problem of simultaneously maintaining several important factors constant in one calculation while shifting basic geometry is a difficult one. However, in our view a suitable continuation of a BSW type study would have been to complete several more calculations examining consequences of geometry changes in detail to develop suitable auxiliary multipliers, weighting factors, etc., to apply to approximate or lower order geometry calculations.

The GE model, which as noted is essentially point-kinetics with several separate auxiliary calculations for reactivity and peaking input, does not encounter these difficulties. The rod and scram reactivity functions come from full XYZ calculations when required (see Supplement 2 to NEDO-10527), or from a suitable reduction to RZ when such a configuration is adequate (and in

nearly all of the cases of interest the maximum worth sequenced rods are central or central-like, i.e., not near boundaries and thus in significant regional flux-neutron importance gradients). The concurrent power distribution-Doppler weighting calculations are RZ. However, the local geometry and cross section distribution around the dropping rod are appropriately preserved. It is this modeling in the region of the rod which is of primary importance in determining feedback reactivity and peak energy deposition.

Conclusion

Our review has indicated that the GE model is not subject to the particular problems pointed out by and encountered in the BSW paper, and we continue to conclude that the GE analyses and results are suitable even though the model does not consist of a full three-dimensional space-time-feedback kinetics analysis.

However, we recognize that such an analytical tool would serve a useful role in confirming this evaluation. Analyses have been carried out with alternate methods only because the three-dimensional codes have been unavailable or too time consuming to operate. In particular they have not been available to the staff or its consultant. As part of the continuing generic review of transient analyses, the staff and BNL have recently acquired the new MIT-EPRI three-dimensional kinetics code, MEKIN, and are working to get it operational. Depending on its operating characteristics and practicality, it will be used to study problems of the type covered by the BSW paper.

Thus, with the perspective of the studies, reviews and evaluations regarding the original reactivity insertion rate problem and the question of dimensional order, and the margins existing both in the probability of an RDA exceeding the 280 cal/gm criterion and between generally expected maximum sequenced rod worth and that required to reach 280 cal/gm, it is our conclusion that the uncertainties identified by the ACRS generic item about the RDA analysis have been resolved.

APPEND.X I

A Statistical Examination Of The RDA In Some BWRs

Summary

This memo examines the probability arguments and related information analysis involved in decisions on requirements for additional elements (hardware or Tech Specs) to reduce the likelihood of having a BWR rod drop accident which might exceed the 280 cal/gm criteria. The memo presents the purpose for this examination, the statistical approach to be used, and develops a criterion for judgement (a rod drop exceeding 280 cal/gm should be less probable than 10^{-7} per reactor year). Previous studies, comments on the statistical methods used in these studies, and information sources explored to provide probabilities for these and the present study are outlined. The events which are required in order to have a rod drop which exceeds 280 cal/gm are examined and the conclusions reached from the information sources as to the probabilities for these events are given. The results of the study indicate that a conservative estimate, based on the information examined, of the probability of exceeding 280 cal/gm is on the order of 10^{-12} per reactor year. The uncertainty of this result is discussed and a possible maximum probability (10^{-7}) is developed. Based on these results it is concluded that additional hardware is not necessary. However, encouragement of RWM operability (through Tech Specs) is desirable.

Purpose

The purpose of this memo is to develop a technical background on the events involved in the BWR rod drop accident (RDA). This is to assist in decisions which need to be made on requirements for additional hardware or Tech Spec changes relating to the RDA for some of the operating BWRs. The reactors to be considered are the BWR 2 and 3 classes (Oyster Creek through Pilgrim) plus Vermont Yankee. These ten reactors do not have a Rod Sequence Control System (RSCS). A primary decision needed is whether these reactors should be required to install a RSCS or alternate equipment such as a template, which some applicants have claimed to be equivalent. (This memo will not address the merits of this equivalency argument.) A parallel question is the need for Tech Spec changes to require more certainly the operability of the Rod Worth Minimizer (RWM). The obvious intent of these requirements, if imposed, would be to reduce the probability of a RDA with excessive rod worth such that the 280 cal/gm criteria might be exceeded. Thus, the purpose of this memo is to examine the need for such added probability reduction for these 10 reactors.

This memo will not directly address this question for the 3 or 4 older BWRs (D-1, Humboldt, Big Rock, LaCrosse), where differing mechanisms, patterns and worth might produce different quantitative results. However, the analyses would be similar for those reactors, and quantitative results might well be expected to be of the same order.

It might be added that an additional purpose of this memo is to take another small step down the path of quantitative statistical analyses of reactor safety problems, and a reminder that in some cases at least the step is (apparently) not too difficult.

Approach

Since the basic nature of the decisions to be made is primarily probabilistic (at least so far as the decision is technical), i.e., are additional probability reducing elements needed, the approach to be used is an examination of the statistics of the involved events, singly and in combination. Thus, we will examine all the events required to produce a RDA above 280 cal/gm, estimate (hopefully conservatively) the probability of their occurrence, combine them to form an (conservative) estimate of the probability of the total event and compare the result with a criteria which we will develop to judge such an event for these 10 reactors. A detailed fault tree analysis of the mechanical failures of individual componets in the control rod - rod drive system will not be a part of this study. Instead, the analysis will be based on information on the failure (i.e., disconnect or sticking) of the rod system as a unit.

It is recognized that quantitative probabilistic analysis in this form is not currently a generally prevalent Regulatory approach, that previous attempts by GE in this area (Ref. 1) for the RDA have been disregarded, and that there have been numerous managerial statements about not rushing into WASH-1400 (Ref. 2) type reviews. Nevertheless,

this appears to be a problem area amenable to this type of analysis and one for which a decision is not otherwise obtainable (and has not been obtained) except by arbitrary fiat based on subjective probability feelings.

The memo will not discuss the accuracy of CE calculations of the RDA. It will be assumed that it is sufficiently accurate (conservative) for this type of study. The present review of the 3-D vs 2-D analysis variatión as applied to the GE analysis methods does not indicate the likelihood of an important inaccuracy, and the final results of this study would not appear to be sensitive to potential variations. The amorphous nature of some of the information presently readily available to us on some of the areas of the analysis and the effect of the results will be discussed later in the memo.

Criteria

A probabilistic study of this type requires a criterion in order to provide a basis for judgment. Just as probability studies are not prevalent analytical tools in NRC, criteria for use in such studies are also not prevalent. Thus, one will be developed for this study. The attempt will be to choose a criterion which, while not too unrealistically harsh, is appropriately conservative and easily defensible in the context of present NRC positions and precedents.

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The criterion will take the following form: For the ten reactors under consideration, the probability of a RDA exceeding 280 cal/gm shall be less than 10^{-7} per reactor (one) year. Otherwise, additional elements must be provided to attain such an objective.

There are three related Regulatory studies (known to me) offering pertinent information or viewpoints on this criterion.

The most directly useful of the Regulatory staff studies is reported in WASH-1270 (ATWS) (Ref. 3). This report establishes, in effect, a safety objective level such that ATWS becomes a problem if it can occur more probably than 10^{-7} per reactor year. The viewpoint of WASH-1270 can best be indicated by quoting from pages 16 and 17:

> "The staff believes this safety objective is met by requiring a design basis accident envelope that extends to very unlikely postulated accidents, and by establishing the further objective that accidents not included in the design basis envelope should have an average recurrence interval of at least a thousand years for all nuclear plants combined.
>
> For an anticipated population of about one thousand nuclear plants in the United States by the end of the century, the safety objective will require that there be no greater than one chance in one million per year for an individual plant of an accident with potential consequences greater than the Part 100 guidelines."

and from page 19:

> "The safety objective is that the likelihood of all accidents with significant consequences not included in the design basis envelope should not be greater than one chance in one million per year, i.e., should not occur with a failure rate

> greater than 10^{-6} per year. For the particular potential failure path of ATWS, the staff believes that a failure rate of the order of one tenth of the overall safety objective is an appropriate objective. Thus, (the probability per year of an ATWS event should be less than) 10^{-7}."

Note than that 10^{-7} was based on a thousand reactors in operation. If only ten had been involved, presumably the criterion would have been 10^{-5}.

WASH-1400 (Reactor Safety Study) does not directly develop such a criterion, but does produce probabilities for the occurrence of accidents in several consequence (release) categories. For BWR Category 1 accident sequences, the median probability of occurrence is 9×10^{-7} per reactor year (10^{-7} and 10^{-5} lower and upper (5%) bounds) and Category 2 is 20×10^{-7}. Since it is doubtful that the (non-sequenced rod) RDA above 280 cal/gm sequence would reach even Category 2 (less likely Category 1) ultimate release mechanisms, holding the probability of the accident to less than 10^{-7} per reactor year would seem not to affect the WASH-1400 results, and in particular, not even the release Category 1 results.

Note: For the BWR - "Category 1 involves a steam explosion in the reactor vessel in which about half the core is involved. The steam explosion ejects this half of the core from the containment. The resulting exposure of the finely dispersed molten fuel to an oxidizing atmosphere results in the largest release of radioactive material of all the accidents." The non-sequenced rod (maximum worth 2.5 - 3.0% Δk)

RDA would be expected to result in a maximum enthalpy of the order of 500 cal/gm in a small axial region of a few fuel pins, and a molten condition (the order of 300 cal/gm) in a limited axial region of several (the order of 16 out of a total of the order of 500) assemblies. While the staff has not been convinced by the mechanical disruption calculations performed for this enthalpy range (thus the 280 cal/gm limit), neither is it persuaded that this enthalpy distribution would necessarily lead to vessel rupture.

The third study, WASH-1318 (Pressure Vessel Reliability) (Ref. 4) is even less directly related to the desired criterion. However, it does develop the position that present knowledge leads to the conclusion that "the upper limit (99% confidence) probability of a "disruptive failure" event occurring in any one nuclear vessel during any service year falls within the range of 10^{-6} to 10^{-7}." By implication, this leads to the tacit assumption that since vessel rupture is not presently an analyzed accident that the 10^{-7} level might serve as a suitable criterion.

We may, thus, conclude from these three studies that there is a reasonable basis to conclude that the 10^{-7} value can be used as a suitably conservative criterion (and note again that it is a factor of 100 more conservative than the ATWS criteria).

Previous Studies

The probabilistic analysis of the RDA is not new. Just about every vendor - applicant reference to the RDA begins with a qualitative

discussion of the required events and a subjective statement about the total unlikelihood of the sequence. There have been (at least) three attempts at a quantitative analysis. Two were by staff members and the third by GE in the course of the RSCS development and review. The two staff reviews (Ref. 5 and 6) were cursory and intended only as initiating examinations. They contained little quantitative probability information (and most of that used was inappropriate) and contained fundamental errors in the statistical analysis. Except for passing mention they will not be considered further in this memo. The GE analysis, Ref. 1, (presented in Amendment 22 to the Peach Bottom FSAR and attached as Appendix A to this memo) is considerably more satisfactory. The probability analysis is fully quantitative and the statistical approach appears to be suitable. It is incomplete, however, primarily because of a lack of quantitative bases for the event probabilities. They are simply stated and no backup data is offered. The remainder of the report will present extensive comparisons with the GE analysis.

It might be noted that the two staff reports produced a "result" (based on my interpretation) of the order of 10^{-7} or less per reactor year of "too large" a RDA, while the GE analysis results were of the order of 10^{-19}. While this indicates a large variation in outlook, they all meet the criterion (none of them developed its own criteria).

Comments on Statistical Methods

The statistical models involved in this analysis are not complex, however, they must be considered with some care. As an example of potential pitfalls, consider the following problem:

Given: A years

B actions/year, thus, AB actions

C result 1s

D result 2s

Have: C/AB result 1s/action (probability of result /action)

D/AB result 2s/action

Probability is CD/A^2B^2 that there will be combined result 1 and 2 per action and:

CD/A^2B^2 combination/action x B action/year

Gives: CD/A^2B combinations/year

But also have: C/A result 1s/year

D/A result 2s/year

Giving: CD/A^2 combinations/year

Which is not the result above!

This may appear either paradoxical or obvious. The solution is that the second answer is for results occurring together (in the same

year), but not necessarily associated with the same action, as is the first answer. For the RDA, where the results are the (independent) disconnecting and sticking (and erroneous withdrawal) of a rod, the results must occur for the same rod being withdrawn (action), not just in the same reactor. The point is apparently sufficiently subtle that both staff reviews chose the wrong path and thus, had no chance of producing correct analyses. (Note that if there are n results, the answers will differ by B^{n-1}. For n equals 3 and 10^3 rod withdrawals per year, the answers would differ by 10^6.) The GE analysis is correct in this respect. Form one will be used in this memo. (It might also be noted that one of the staff reviews also introduced "duty cycle" time factors without recognizing correlations between failures within a "duty cycle".)

This memo will not delve into fault tree analyses for the mechanical failures leading to such phenomena as sticking and uncoupling. Attention will instead be confined to information on probabilities of the final event(s), e.g., uncoupling. The discussion will generally be in terms of events per rod withdrawals. The GE report made some limited display of fault tree development. It was not sufficient to catch the subsequently found strainer movement uncoupling mode of Dresden 2.

The basic assumption implicit in this (and the GE) analysis, involving the multiplication of the probabilities of the several individual events, each of which must occur, is that each failure (event)

rate is constant in "time" (and, of course, the several events are not correlated). In this case "time" is the action space (i.e., per rod withdrawal) previously discussed (e.g., uncoupling per rod withdrawal). Since we are dealing with the "rare" event, i.e., many observations (of rod withdrawal) and few failures, the appropriate distributions to be used appear to be Poisson (and exponential intervals). A brief but useful discussion in this area is given in Appendix A of WASH-1318 which is appended to this memo as Appendix B. Also given in this appendix is a discussion of the accuracy of estimating failure rate from limited observations based on these distributions (see also WASH-1270, pages 53-55), where it is shown that the chi-square distribution of the mean failure rate can be used to develop a confidence level for the failure rate estimate. Figure 2 of this appendix can be used to determine expected failure rates from observations and a desired confidence level. For example, zero observations of a failure lead to an estimate of five failures at a 99% confidence level and three failures at a 95% level (for the X number of withdrawals observed). The "desired" confidence level is, of course, debatable, WASH-1270 used 95, WASH-1318 used 99. For this memo, the available information is sufficiently amorphous that the difference does not materially affect results. Where a choice is made, it will be 95.

Information Sources

The information base drawn on for this report is far less complete and detailed than would be desired for a thorough, minimum uncertainty probability analysis. It consists primarily of (1) a computer printout

of "all" apparently related AO reports in its (OIE maintained) memory, (2) Semi-Annual Reports from utilities, (3) discussions with relevantly experienced project engineers in Operating Reactors, Operator Licensing, and Operation Evaluations (OIE) branches, and (4) previous years of experience dealing with the RDA. No attempt was made to extract detailed data from GE or utilities. As previously indicated the GE study contains little explicit failure data as a basis for the probabilities used.

An AO reporting system in which all failures relevant to the RDA were fully recorded into the computer system would, of course, constitute a suitable data source. There is ample indication that this is not the case, however. Discussions of insufficiencies in the data for individual failure areas will be presented later, but it might be noted as an example that known data on stuck rods and non-sequenced rod withdrawal appears to be missing from the data bank.

A more intensive and complete data retrieval was not attempted for this review because it does not appear necessary within the contexture of the results, criteria and conclusions, that a more precise result be produced at this time. This too will be discussed later.

Withdrawal Data

We will now examine the experience space we have to work in, i.e., the number of rod withdrawals which have occurred in relevant BWRs to provide failure rate information, and the expected frequency of withdrawals in the reactors which this memo addresses so that probabilities may be converted to yearly rates for comparison with the criterion.

We will consider only U.S. reactors since our information sources are limited to those. The reactors of interest are listed in Table 1, along with the number of rods, the year of startup and the approximate years of operation.

TABLE 1

Reactors

	# of Rods	Yr Start	Yr Exp.
OC	137	69	5
9M	129	69	5
Mont	121	70	4
Mill	145	70	4
D2	177	70	4
D3	177	71	3
QC1	177	71	3
QC2	177	72	2
VY	89	72	3
Pil	145	72	2
BF1	185	73	1
PB2	185	73	1
DA	89	74	1
BF2	185	74	1
PB2	185	73	1
Coop	137	74	1
Fitz	137	74	0
H1	137	74	0
	147 ave.		41 Total

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Examination of a number of semi-annual reports from reactors of various vintages indicates new reactors average about 30 startups a year and older ones ten or less.

Combining this information indicates that there is data from in excess of 10^5 rod withdrawals, and that for the reactors of this memo (the first ten in Table 1) the appropriate order of magnitude for the number of rods withdrawn per startup and per year is 10^2 and 10^3 respectiv

Events for RDA Above 280 cal/gm

We will begin the discussion of the events which must occur for the RDA above 280 cal/gm with a brief reminder of the relationship between the withdrawal aspects of the RDA and BWR operations.

Beginning from a fully shutdown reactor, startup proceeds by withdrawing rods one at a time but in proper patterns. The first half of the rods are pulled all the way in a checkerboard pattern (e.g., red squares) using 4 to 6 such groups each of which is withdrawn fully before the next subgroup is started. The second half of the rods (i.e., black squares) are withdrawn in smaller groups and are generally not fully withdrawn. See Figure 1 for a typical withdrawal pattern. It is this pattern which devices such as the RWM and RSCS are intended to assist in maintaining. Beginning from its most reactive state (i.e., cold, no xenon, maximum cycle time reactivity), the reactor would be critical with about half of the red rods withdrawn, about at hot standby with all the red rods out, and reach the order of 10% power with 10 or 20% of the black rod density withdrawn. At other times in the cycle or restarts from hot and/or xenon conditions these milestones would require

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MILLSTONE	02	06	10	14	18	22	26	30	34	38	42	46	50
51				1		2		1		2			
47			3		4		3		4		3		
43		1		2		1		2		1		2	
39	3		4		3		4		3		4		3
35		2		1		2		1		2		1	
31	4		3		4		3		4		3		4
27		1		2		1		2		1		2	
23	3		4		3		4		3		4		3
19		2		1		2		1		2		1	
15	4		3		4		3		4		3		4
11		1		2		1		2		1		2	
07			4		3		4		3		4		
03				1		2		1		2			

MILLSTONE	02	06	10	14	18	22	26	30	34	38	42	46	50
51					5		6		5				
47				13		14		14		13			
43			7		8		9		8		7		
39		13		15		16		16		15		13	
35	5		8		10		11		10		8		5
31		14		16		17		17		16		14	
27	6		9		11		12		11		9		6
23		14		16		17		17		16		14	
19	5		8		10		11		10		8		5
15		13		15		16		16		15		13	
11			7		8		9		8		7		
07				13		14		14		13			
03					5		6		5				

1. COMPLETE WITHDRAWALS INDICATED IN EACH COLUMN BEFORE GOING TO NEXT COLUMN

2. WITHDRAW GROUPS 1, 2, 3 AND 4 RODS INDIVIDUALLY FROM 0 TO 48 IN THE FOLLOWING ORDER

1 – (22–27), (30–35), (38–27), (30–19),
(22–11), (14–19), (06–27), (14–35),
(22–43), (30–51), (38–43), (46–35),
(46–19), (38–11), (30–03), (14–03),
(06–11), (06–43), (14–51)

2 – (30–27), (22–19), (14–27), (22–35),
(30–43), (38–35), (46–27), (38–19),
(30–11), (22–03), (14–11), (06–19),
(06–35), (14–43), (22–51), (38–51),
(46–43), (46–11), (38–03)

3 – (26–31), (34–23), (26–15), (18–23),
(10–31), (18–39), (26–47), (34–39),
(42–31), (50–23), (42–15), (34–07),
(18–07), (10–15), (02–23), (02–39),
(10–47), (42–47), (50–39)

4 – (26–23), (18–31), (26–39), (34–31),
(42–23), (34–15), (26–07), (18–15),
(10–23), (02–31), (10–39), (18–47),
(34–47), (42–39), (50–31), (50–15),
(42–07), (10–07), (02–15)

3. WITHDRAW RODS IN OTHER GROUPS BY BANK, KEEPING RODS OF A GROUP WITHIN A FEW NOTCHES OF EACH OTHER.

4. FOR ROD INSERTIONS, REVERSE THE ORDER.

RED
↑
↓
BLACK

GROUP	"WITHDRAW TO POSITION" STEPS									
	1	2	3	4	5	6	7	8	9	10
1	48									
2	48									
3	48									
4	48									
5	24	48								
6	12	24	36	48						
7	12	24	30	34	38	40	48			
8	6	10	14	18	24	30	38	48		
9	6	10	14	18	24	30	38	48		
10	4	8	12	16	22	28	38	48		
11	4	8	12	16	22	28	38	48		
12	4	8	12	16	22	28	38	48		
13		12	24	36	48					
14			6	10	14	18	24	30	34	38
15				6	10	14	18	22	26	30
16						4	6	10	14	18
17						4	6	10	14	18

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~~Figure A.3-1~~ Control Rod Withdrawal Sequence A

FIGURE 1

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greater rod withdrawal. Above approximately 10% power, the RDA can not exceed 280 cal/gm because of both the more prompt Doppler feedback in this power range and the impossibility of achieving high rod reactivity worth with the relatively low rod density, even with erroneous rod patterns. Based on the observations from the semi-annual reports, as a general rule, having gone above the 10% power level, the reactor will be back in the RDA range again only following shutdown (usually by scram), i.e., it will operate in the power range for a period of time then shutdown and start over again. (This is not an inevitable course, but appears to be predominate. Variations, where the full complement of rods is not fully inserted in order to stand by in the "zero power" range, would not appear to result in conditions sufficiently different to invalidate the analyses.)

The events which must all occur (emphasize <u>all</u>) to exceed 280 cal/gm are outlined in Table 2 and described briefly here. These required events are (essentially) described in many GE presentations, including the attached Appendix A. They are arranged (or rearranged) here for convenience of presentation. The assumption is that a rod is (being) withdrawn and the events in the table must (subsequently) occur on that rod.

I. Disconnect

A. The rod must be or become disconnected from the drive either by never being coupled, by becoming unlatched or by a break in the system such as a broken index tube.

B. This disconnect must occur with the rod in the upper quarter or less of the core (or it must subsequently be moved there) (see II B) since most of the reactivity worth of the rod occurs during the initial part of the withdrawal.

C. The disconnect must not be discovered (e.g., by over-travel coupling tests) and remedied (see also III. D).

II. <u>Stuck</u>

A. The rod must stick as the drive is moved away.

B. The rod must stick in the upper quarter or less if the core, as discussed IB.

C. The drive must be lowered at least a third of the core length away from the stuck rod.

III. <u>Errors</u>

A. The operator must select and withdraw a wrong (non-sequence) rod. Sequenced rods do not have sufficient worth to exceed 280. It might be noted that in some cases rods are withdrawn in several steps (i.e., not fully withdrawn bank positions for some rod groups). For some of these rods, the error, sufficient to exceed 1.5% Δk, might be withdrawing the rod too far rather than an incorrect selection.

B. The RWM or,

C. The second operator (which has been allowed as an alternate for the RWM) must allow the wrong selection to go uncorrected.

D. The rod must be withdrawn in spite of potential warmings of being uncoupled, which are sometimes available, such as no response on the nuclear instrumentation to rod motion.

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IV. High Worth Potential

A. The erroneously pulled rod must have a high potential worth. While it varies somewhat with the reactor and time in cycle, as well as the drop velocity and scram time, a RDA must have a reactivity worth greater than about 1.5% Δk to exceed 280. Many of the possible erroneously pulled rods would not have that worth, e.g., many core edge rods (see also V.C).

V. Drop-Timing

A. The stuck rod must drop.

B. The rod must drop when the reactor is nearly critical (if it is far subcritical, the reactivity worth potential of the rod would not be sufficient) and less than the order of 10% power. Above this power, 280 cal/gm can not be attained.

C. The rod must drop (within that critical to 10% power time frame) when the withdrawn rod pattern enhances the rod worth so that it approaches its maximum worth. Generally, only error rods near (next to) the first rods pulled in a group, or some of the first of the black (beyond 50% rod density) rods have sufficiently high worth, and they lose that high worth as the pattern withdrawal progresses.

Probabilities for Events

We will now proceed through the events of the RDA and assign probabilities to them, and compare these probabilities with those assigned by GE. These probabilities are given in Table 2. The probabilities

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will be discussed (and listed in Table 2) individually and in groups under the group event headings previously given. The group probabilities in particular are an attempt at a conservative interpretation of the apparent information and do not necessarily represent a direct use of the individual components. Note that the units for the first 3 groups are per withdrawal, and for the fourth group per erroneous withdrawal (this is a conditional probability, given a non-sequence withdrawal has occurred). The fifth group has a time fractional probability based on operation from the time the rod is withdrawn until it is eventually inserted.

Table 2

Probabililties For Events

	GE Group	GE Individual	This Report Group	This Report Individual	Maximum
I. Disconnect (/w)	10^{-6}		$2x10^{-5}$		$2x10^{-4}$
A. disconnect		10^{-6}		10^{-4}	
B. upper 1/4		-		$3x10^{-1}$	
C. not discover		-		10^{-2}	
II. Stuck (/w)	10^{-4}		10^{-3}		10^{-2}
A. stuck		10^{-4}		10^{-3}	
B. upper 1/4		-		$3x10^{-1}$	
C. lower drive 1/3		-		-	
III. Errors (/w)	$8x10^{-9}$		10^{-4}		10^{-2}
A. operator select		$2x10^{-3}$		$2x10^{-3}$	
B. 2nd operator		$2x10^{-3}$		10^{-1}	
or C. RWM		or $2x10^{-3}$		10^{-3}	
D. withdraw		$2x10^{-3}$		10^{-1}	
IV. High Worth Potential (/ew)	$7x10^{-4}$		10^{-1}		$4x10^{-1}$
A. high worth		7.10^{-4}		10^{-1}	
V. Drop-Timing (/dsew)	$7x10^{-3}$		$2x10^{-3}$		$6x10^{-3}$
A. drop		1		10^{-1}	
B crit to <20%P		10^{-1}		10^{-2}	
C. pattern enhance		$7x10^{-2}$		10^{-2}	
VI. Rods/year (w/yr)	10^{3}-10^{4}?		$2x10^{3}$		$2x10^{3}$
Total (above 280/yr)	$\sim 10^{-19}$		$\sim 10^{-12}$		$\sim 10^{-7}$

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I. Disconnect. The information sources indicate that there have been no disconnects which have not been detected immediately (before operations could lead to a RDA). The 95% confidence level, zero occurrence rate estimate (see Figure 2 of Appendix B) is thus 3 occurrences out of 2×10^5 withdrawals or a rate of 1.5×10^{-5} per withdrawal.

There have been 7 uncouplings in Dresden-2. These all occurred at the fully withdrawn position (unlatching from a moved strainer) and were immediately detected by the over-travel coupling check. One uncoupled rod was found in Millstone following a rod change. It was discovered in coupling checks before startup. Using these 8 uncouplings (14 at a 95% confidence level) would result in an uncoupling probability of about 10^{-4}. However, this should be combined with a probability of not detecting the uncoupling. Since procedures call for coupling checks under conditions when such events might occur, it seems reasonable to assign a probability of the order of 10^{-2} for such a failure to detect, giving a total of 10^{-6} for undetected uncouplings.

No attempt has been made to analyze breaks in the system leading to disconnects. GE has assigned a 10^{-6} probability to this event based on its not having occurred to date (incidentally, this indicates that GE considers the number of withdrawals to total over 10^6).

Thus, based on the available record, it would appear that a probability of 2×10^{-5} per withdrawal, based primarily on the observation of no non-detected disconnects, is a suitably conservative value for this review. This gives no credit for having to occur in the upper quarter of the core.

Note that the assumption is made that the events of sticking and disconnect are independent. The available information does not contradict

this assumption since no occurrence in which one has caused the other has been observed. This subject will be mentioned later under uncertainties.

II. Stuck. The computer AO report systems give almost no information on stuck rods. They are apparently not generally considered as AOs since when they occur, they can be treated "normally" as inoperable rods, and thus do not appear on AO lists. The semi-annual reports examined also indicate little evidence for stuck rods.

The other information sources (verbal), however, indicate that stuck rods appear not to be uncommon (although the differentiation between stuck rod and inoperative drive is unclear). GE uses 10^{-4} per withdrawal for this probability, stating it would result, if at all at this level, from warped channel boxes.

Because of the lack of precise data, a satisfactory statistical analysis can not be carried out. However, what would appear to be a suitably conservative estimate will be made of 10^{-3} per withdrawal. Note that this implies more than one stuck rod per reactor year. Once again the factors for occurrence in the upper quarter of the core will not be used.

III. Errors. The computer AO reports also provide no information on operator error histories. Although at least one case (in VY) is known in which a non-sequenced rod was pulled without being halted by the second operator (the RWM was not in operation), this case is not, nor is any other, listed by the computer. Thus we have no reliable record of actually withdrawn non-sequenced rods. As an obvious corollary to this

we also have no record of single operator errors which were caught by the second operator or RWM. Thus, we will have to consider more generalized reviews of operators actions.

Appendix III of WASH-1400 delves into the question of human error. While a directly comparable case is not discussed, my review of that material indicates that it is not incompatible with a value of 10^{-3} (per withdrawal) for the operator error rate for the type of complexity and stress level of the withdrawal operation. The material would also appear to assign a lesser degree of reliability to the checking operation (second operator).

The information from experienced staff personnel (OLB, OEB) indicates that numerous startups have been observed without an operator error. (For example, 50 startups without error observed, which by the statistical analysis at a 95% confidence level would estimate an error rate of 3 in over 5000 withdrawals or less than 10^{-3} per withdrawal.) The information appears generally compatible with a value of 10^{-3}. These sources do not indicate a very high degree of confidence in the second operator (as it has generally been implemented). The values assigned by GE in Appendix A are 2×10^{-3} per withdrawal for the operator error and the same value for the second operator.

Based on this type of judgment information, this memo has assigned an error rate of 10^{-3} per withdrawal operation to the operator and 10^{-1} to the second operator. Since rod withdrawal for some rods requires more than one operation (rods moved to not fully out bank positions), this estimate is increased by a factor of 2, to 2×10^{-3}, to account for

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this increase in operations. A value of 10^{-1} has also been assigned to other potential withdrawal errors such as not observing the non-reaction of the nuclear instrumentation to the non-movement of the control rod as the drive is actuated. GE apparently assigns a value of $2x10^{-3}$ to such an error, although the assignment is very unclearly stated.

This memo will not attempt to develope a well justified value to be assigned to the RWM. (That is a task for the EI&CSB.) Based on personnel judgment, a value of the order of 10^{-3} per withdrawal appears reasonable.

Because of this amorphous nature of the information in this area, an overall estimate for the withdrawal error has been taken at what appears to be a conservative level of 10^{-4} per withdrawal. This basically ignores any contribution from the RWM. Note that 10^{-4} implies the order of 1 withdrawal error per year for ten operating reactors. This appears compatible (but somewhat conservative) with known errors. However, the GE error rate, which is of the order of 10^{-8} is possibly less so.

It should be noted that any potential difference between a wrong rod selection error and an error of withdrawing a rod too far when in group bank operation has been ignored. This is believed to be justified both because the basic probabilities are not fundamentally different and because the significant excessive withdrawal error operations are only a small fraction of the total withdrawal operation. Similarly, for the same reasons, the complications introduced by the occasional entering into the RDA range from the higher power range by inserting rods has been ignored. This will be mentioned later in the discussions of uncertainty and RWM.

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IV. High Worth Potential. It is evident that a number of possible non-sequenced rods will not have sufficient reactivity worth to exceed the order of 1.5% Δk required to exceed 280 cal/gm. For example, while pulling red rods, a wrong red rod might be withdrawn. This might have a slightly larger worth than normal, but would not (to the best of our present understanding) exceed 1.5% Δk. Other potential candidates for non-excessive worth might be core edge rods or rods not near first rods pulled in a group. We do not have precise quantitative information in this area, however. Our normal review of rod worths for the RDA focuses on the maximum possible rod worths and not on those of somewhat lesser magnitude. Furthermore, we no longer even examine non-sequenced rods since the assumption is that only sequenced rods are involved in the RDA. Thus, the estimate is largely subjective, based on past experiences and discussions and augmented by several BNL calculations. The estimate is that 10^{-1} of the erroneously withdrawn rod have sufficient reactivity worth. (Note that this is a conditional probability, given an erroneously withdrawn rod, and the units are per erroneous withdrawal.) It might be noted that the GE value of 7×10^{-4} (which is deduced from Figure J.4.13.7 of Appendix A) is obviously for a BOL curtain core. The RDA is somewhat more severe for a given rod worth in a reload core (which is of concern in this memo) and approaches that for a BOL shaped Gd core. Thus if GE had considered a reload core for the analyses the rod worth to exceed 280 would be lowered and the potential for exceeding that worth increased.

The 1.5% Δk value for 280 cal/gm is only slightly sensitive to variations in the drop velocity and scram velocity parameters for reload cores. It is somewhat more sensitive in first cycle curtain cores and the velocities included as parameters in the GE analysis (Figure J4.13.7 of Appendix A). For reload cores, the possible range of velocity for either drop or scram can result in only about a 10% change in enthalpy, which is equivalent to about 0.1% Δk change in rod worth. The available information on rod worth distribution is not sufficiently precise to consider this as a parameter of this analysis.

V. <u>Drop-Timing</u>. It is unlikely that every stuck rod would always drop. However, there appears to be no clear basis for developing a probability for the occurrence. Thus, we list a probability of 10^{-1} on Table 2, but basically assume a probability of 1 in developing a total probability for this group. GE also assumed 1. (Note that we again have a conditional probability with units of per disconnected, stuck, erroneous withdrawal.)

In considering the timing of the occurrence of the rod drop, it will be assumed, for the moment, that the probability is constant over the time period from withdrawal to eventual insertion (after power operation).

This is conservative if the stuck rod is more liekly to drop during the increased vibration time of power operations. It is not conservative if the drop is more correlated to occurring soon after the drive is withdrawn from the stuck rod.

To exceed 280 cal/gm the rod must drop after the reactor is nearly critical (usually after more than 1/4 of the rods have been withdrawn) and before reaching the order of 10% power. Typically the reactor is in

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this regime less than 8 hours per startup. With 10 startups per year this gives a 10^{-2}, (8 x 10/8000), probability of dropping during this critical time frame.

The high reactivity worth rod will have that high worth only during a part(s) of the withdrawal sequence. There is a sawtooth maximum worth pattern as a function of time (number of rods withdrawn). As was discussed under High Worth Potential, we have less than fully detailed knowledge of the magnitudes of this function. However, we believe it can be estimated as the order of 10^{-2} for the fraction of the critical time that high worth exists for a given rod.

Combining these factors (conservatively), we will estimate unit probability for the drop, 10^{-1} for the drop occurring in the critical time frame, and 2×10^{-2} for the drop occurring in that time frame when the pattern enhances the rod worth. This gives a total of 2×10^{-3}. The probability for the critical time frame has been increased from 10^{-2} to 10^{-1} in an attempt to account for a possibility of early drop correlation. The pattern enhancement factor has been increased by a factor of 2 to account for uncertainties in our knowledge of the pattern worths.

The GE value of 7×10^{-3} is not exactly directly comparable. It is for the hot startup condition only. (The breakup in Table 2 is arbitrary.) The final result is also for that condition only.

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VI. Rods/Year. As previously discussed, the estimated startups per year is 10, giving the order of 10^3 withdrawals per year. This has been increased to $2x10^3$ to include some consideration for the multiple steps per withdrawal for some rods. The value stated by GE in Appendix A is 10^3. However, 10^4 appears to be used in their analysis.

Results

The final probability resulting from the product of these assigned probabilities is about 10^{-12} RDA events per reactor year exceeding the 280 cal/gm criteria. This can be compared with the GE value of about 10^{-19}, which, as previously mentioned, was for hot startup conditions in a BOL curtain core. Changing the GE results to bring these differences in assumptions into agreement, would still leave several orders of magnitude difference in results. Principal differences are an order of magnitude each for disconnect and stuck probabilities and 4 orders of magnitude for errors. These differences are not surprising since we attempted to be conservative when the information available to us was less than complete. Our information and analyses does not directly refute the GE results, however.

This result of 10^{-12} is, of course, a factor of 10^5 less than the criteria of 10^{-7}. The conclusion from this is that an RSCS (or its equivalent) system is not required for the 10 reactors under consideration.

Uncertainty of Results and Possible Maximum Probability

As has been noted throughout this memo, the analyses is less precise than desired, or could be achieved with further detailed exploration of other possible information sources. Even though the attempt has been

made to be conservative where information is incomplete, the question might be asked as to whether the information should be made more precise by pursuing a more complete data base with GE and the utilities. The answer appears to be that it is unnecessary for the purposes of this memo. The question might also be asked as to whether there might be important hidden (or forgotten) characteristics of one of the events which would completely invalidate the assumed modeling of the probabilities of the event, and thus, should details of the mechanisms involved be more closely explored. This answer appears to be the same as for the first question. These answers are the result of the magnitude of difference between the calculated probability and the criterion, and the lack of dominance of the resultant probability by any one of the required events. Thus the sign of the comparison remains unchanged for large changes in the components.

In developing the event group probabilities, each one was considered uncertain because of potentially incomplete information, but values expected to be conservative were "chosen". In answering the question of need for further information of improvement, it is useful to push the extent of conservatism even further. If resulting probabilities are greater than could reasonably be expected from augmented data, such a search is unnecessary. An example of such a push is given in Table 2 under Maximum, and discussed as follows:

I. The "Disconnect" probability which had been based on no known occurrences of operation with a disconnected rod is increased by an order of magnitude to $2x10^{-4}$. For a 95% confidence level estimate,

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this would indicate that the order of 30 uncorrected disconnections have occurred (in the upper 1/4 of the core).

II. The "Stuck" probability which had been based on only verbal information sources is increased by an order of magnitude. This rate of 10^{-2} per withdrawal is equivalent to having one stuck rod (not drive) per startup (in the upper 1/4 of the core), and data would have to indicate a history of about 400 stuck rods.

III. The "Error" probability which had been based on subjective evaluations is increased by 2 orders of magnitude. This rate of 10^{-2} per withdrawal is also equivalent to the order of one erroneously withdrawn rod (selection, backup, non-responsive withdrawal) per startup and a history of 400 occurrences.

IV. The "High Worth Potential" which had been estimated based on less than full information on worth frequency distributions less than maximum is increased by a factor of 4. This rate of 4×10^{-1} high worth rod per erroneous withdrawal would be equivalent to nearly all "black" rods having high worth.

V. The "Drop-Timing" probability which had also been estimated with less than full information on pattern worth distributions is increased by a factor of 3. The rate of 6×10^{-3} may be viewed as being composed of the same conservative assumption of a drop probability of 1, in a critical to 20% power range of 10^{-1}, along with a probability for dropping when the rod is enhanced at the appropriate "sawtooth" (time-withdrawal) interval which is 6×10^{-2}, an increase of 6 over the expected value.

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The total in this example, of course, is less than 10^{-7}. The event probabilities are beyond those to be expected from further exploration of data. Thus, further exploration is unnecessary.

The example is also a view of possible accommodation of model deficiencies. That is, if as yet hidden or possibly insufficiently explored aspects of the events (e.g., a new uncoupling mechanism or the previously mentioned entering of the RDA region from the power region by rod insertion) were to lead to (future) increased probabilities, these increases could apparently be significant without exceeding the criteria. A slightly alternate view of this is that any of the events could be at a probability of one and the total probability would still be less than 10^{-7}. Thus, if the constant occurrence rate assumption of the model were sufficiently invalid that one of the events were to increase to unit probability, the criteria would not be exceeded. Note that this also includes the case for one event causing another event, i.e., a stuck rod causing a decoupled rod, and thus the lack of independence resulting in unit probability for the second event. Thus it appears to be unnecessary to further explore details of mechansims involved.

Conclusions

The approach developed in this memo has shown that even when conservatively accounting for imprecision in information and modeling, the probability of an RDA above 280 cal/gm, for the 10 reactors without an RSCS, is less than a suitably conservative criteria. Thus no further information is needed to conclude that an RSCS is not required for these reactors, so far as this decision is based on a technical argument.

The RWM

A parallel question exists as to requirements (Tech Spec) for RWM operability. The analysis in this memo has essentially ignored any contribution of the RWM to the total probabilities. The 10^{-4} per withdrawal error rate assumed a second operator error rate of 10^{-1} rather than a RWM error rate of 10^{-3} (which, it may be recalled, was rather arbitrarily assigned). The history of withdrawal errors, if it were known, is somewhat influenced by the RWM, however. It is (also) imprecisely known, but estimated that the order of half of the startups which have occurred have been with the RWM in operation. Early operability of the RWM was very poor and a large percentage (approaching 100%) of the startups involved the second operator. Subsequent changes in Tech Specs for some reactors, requiring greater operability have evidently resulted in increased efforts to achieve operability, which have apparently met with success. For example, Pilgrim, for which RWM operability is required (without exception), has apparently achieved nearly 100% operability. Other non RSCS reactors, for which a less restrictive Tech Spec (requiring operability for the withdrawal of the first twelve rods) has been issued, have apparently achieved similar operability results. The latter Tech Spec requirement is not in itself a logical requirement, since it is after the twelth rod that the RDA is of importance. However, it too seems to have resulted in an upgrading of the RWM availability sufficiently to make operability a "likely" condition. To approach the RWM assigned probability of 10^{-3} rather than the 10^{-1} rate of the second operator would require a reasonably high level of operability. To decrease the

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10^{-1} rate to about 10^{-2}, 2×10^{-3} or 10^{-3} would require operability of 90, 99 or 99.9% respectively. Something in this range appears to be achievable.

The results of the analyses in this memo do not seem to demand the extra factor of 10^{-1} to 10^{-2} which could be achieved from the RWM. Nevertheless, it is a system which exists and which can be useful with relatively little effort. The extra factor which it introduces serves as an additional buffer for the RDA and should not be ignored. It would seem to be particularly useful in assuring the correct rod patterns when entering the RDA region from the power region. Thus, it is recommended that Tech Spec requirements be maintained to assure a reasonable degree of operability.

References

1. Amendment 22 to the Peach Bottom 2,3 FSAR, October 1972.

2. WASH-1400, Reactor Safety Study, Draft, August 1974.

3. WASH-1270, Anticipated Transients Without Scram for Water-Cooled Power Reactors, September 1973.

4. WASH-1318, Analysis of Pressure Vessel Statistics from Fossil-Fueled Power Plant Service and Assessment of Reactor Vessel Reliability in Nuclear Power Plant Service, May 1974.

5. Memo: Rod Drop Accident for BWRs, G.F. Owslay, May 1972.

6. Memo: Assessment of a BWR Rod Drop Occurrence, A. Serkig, August 1972.

Appendix A

Taken from Amendment 22 - Peach Bottom FSAR

A probabilistic approach to Design Basis Accidents enables designers not only to see the consequences of an accident, but it also affords the opportunity of easily determining which components aid most in reducing the probabilities of such an accident. For this reason, the probabilistic approach has been used as an aid in investigating the design basis CONTROL ROD DROP ACCIDENT (CRDA) and the resultant choice of a system to further reduce the pro-

bability of significant consequences due to the event. The analysis presented here concentrates primarily on the most probable modes of failures leading to a CRDA. Where probabilities cannot be calculated from historical data, deliberately conservative values have been used to avoid contentions over particular numbers. The overall probability for the CRDA will be seen to be very small despite the compounding of conservative assumptions.

Figure J.4.13.5 is a fault tree model of a control rod stuck in the core ready for a drop. Three primary branches combine to produce the stuck rod "ready for a drop" state. These are: (1) a CONTROL ROD IS STUCK IN THE "NEAR FULL" OR "FULL IN" POSITION; (2) a CONTROL ROD SEPARATED FROM THE CONTROL ROD DRIVE (CRD); AND (3) a CONTROL ROD DRIVE MOVED ABNORMALLY. The heavy lines in Figure J.4.13.5 indicate the most probable path of failure which could lead to a CRDA.

The first of the three conditions which must necessarily exist for there to be a potential for a CRDA is that a control rod would be stuck in the "near full"or "full in" position. This could happen in several ways, the most probable one would be where the fuel channel is warped, which could be conservatively estimated to have a probability of 10^{-4} failures/insertion. Another way would be that a loose object could enter the channel and jam the rod in a fixed position. However, the fact that the control rod weighs nearly 200 pounds and the low probability of loose objects lodging in channels in an operating reactor seem to restrict this mode of failure; in fact, from the historical evidence that no such failures have occurred, a conservative probability of 10^{-6} failures/insertion for this mode of failure can be assumed. Assuming the other modes of sticking have a much lower probability than the previously discussed ones, the total probability for a control rod to stick would be essentially 1.0×10^{-4} failures/insertion.

The second condition contributing to the rod drop accident requires that the CRD be disconnected from the control rod. The most likely mode of failure in the separation of the control rod drive from the rod itself would be through the breaking of the index tube. Since this failure has never occurred with the present design, it is justifiable to say that the probability of its happening would be no larger than 10^{-6} failures/withdrawal. Assuming that the probability of failure of the control rod would be much smaller than that of the index tube, it has not been incorporated into this analysis. Since there must be multiple failures present for the rod actually to be uncoupled without the operator's knowledge, the probability for this mode of failure would be small in comparison to that of the index tube breaking.

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Consequently, the probability of a control rod becoming separated from its drive is approximately 10^{-6} failure/ withdrawal.

The third and final condition necessary in order for a control rod to drop, is that the control rod drive must have been moved abnormally. A purely mechanical failure could not cause this condition without several other accompanying failures. This product of failures leads to a probability which is much smaller than that of the other mode of failure, namely, the operator error. In order for an operator to withdraw a control rod drive which is disconnected from the control rod, he would have to ignore or not receive the various signals indicating a malfunction in the drive. Even if the operator ignores an alarm and withdraws the uncoupled drive, the accident is of no consequence unless the rod is an out-of-sequence rod. There is a Rod Worth Minimizer (RWM) which acts as an automatic check on the operator's drive selection. Since this report is to look into the improvement in reliability of the addition of a Rod Sequence Control System (RSCS), both cases of operator error (with and without the RSCS) will be investigated. In order to be conservative in the calculation, a probability of 2×10^{-3} errors/withdrawal has been used for both the operator's selection of an out-of-sequence rod and for the checking performed by the RWM (or second reactor operator), the joint probability being 4×10^{-6} errors/withdrawal. To arrive at the probability of a drive withdrawal for the power level where only the RWM is in effect, the probability of 4×10^{-6} errors/withdrawal is multiplied by 2×10^{-3} errors/operation (which is a conservative estimate of the probability of an operator to ignore a signal or alarm which indicates a system malfunction). This produces a probability of 8×10^{-9} errors/withdrawal that the operator withdraws an out-of-sequence rod and ignores any indications of system problems. At the power level when the RSCS is also in operation the probability of withdrawing an out of sequence rod is a factor of 10^{-3} smaller (assuming the hard wired RSCS is twice as reliable as the RWM).

The probability that all three required conditions are present during a particular period of time is the product of the probabilities of each condition. Thus, the probability of an out-of-sequence control rod becoming stuck, disconnected, and its drive withdrawn would be 8×10^{-19} failures/withdrawal, with only the RWM being in effect. With both the RWM and the RSCS, the probability of a control rod being ready to drop is 8×10^{-22} failures/ withdrawal. Assuming 1000 withdrawals per year per reactor, the probabilities become 8×10^{-15} failures/year and 8×10^{-18} failures/year respectively.

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For mechanical reasons, the probability of a rod becoming unstuck is not independent of the probability of its being stuck. In order to eliminate the complexities of this interdependency calculation of probability, the probability of a stuck rod unsticking will be conservatively assumed to be unity. This means that if a rod sticks it will drop.

From the above argument, the probability of a control rod sticking and then falling is 8×10^{-18} failures/year for low power levels (when both the RSCS And the RWM are operating) and 8×10^{-15} failures/year when the RWM only is in effect. However, in the Design Basis Accident, it is necessary to assume that there is a rod drop. It is easily seen that this overwhelming assumption has exaggerated the importance of this accident.

Figure J.4.13.6 shows that there are three reactor states used in the antlysis of the CRDA. These are: (1) cold-startup (reactor at 20 degrees C, atmospheric pressure, and 10^{-8} of rated power); (2) hot-startup (reactor at saturated temperature, operating pressure and 10^{-6} of rated power); and (3) 10% of rated power. No higher power levels are investigated since the effects are negligible in that no calculated fuel rod perforation occurs (peak fuel enthalpy is less than 170 cal/gm). The probability of a reactor being at cold startup can be calculated using the nominal startup time (reference 7). The time span from the initiation of rod withdrawal to the hot-standby (power) condition is 7.5 hours per startup. In order to be conservative and to take into account that there may be conditions requiring the operator to slow the startup procedure, a factor of 2 will be used. Thus, a reactor is in the cold-startup condition 15 hours/startup. Assuming four cold-startups a year, the probability that a reactor is in the cold-startup state, is 6.9×10^{-3}. The hot-startup condition requires 2.8 hours to pass through for each startup. On the average, there are eleven hot startups/year which gives a total of 61.6 hours/year in the hot-startup condition, or a probability of 7×10^{-3}. The third condition of any consequence is 10% power. The time that a reactor is in this state will be very conservatively assumed to be 1000 hours/year. Thus, the probability of a reactor being at 10% power will be 0.114.

Figure J.4.13.5 shows that the decision tree spreads into three groups. Only the effects of the CRDA at hot-startup will be discussed because it is the worst case. Figure J.4.13.7 shows the decision tree, culminating in the number of failed fuel pins. The analyses of failed fuel and rod worths are for a curtain core; however, the results will not be significantly different for a gadolina core. The rod worths are broken down into four groups. Assuming

that there has been an operator error, and that an out-of-sequence rod has been withdrawn, the probability of an out-of-sequence rod having a rod worth of 0 to 1% $\Delta K/K$ is 5.9×10^{-2}. The probability that the out-of-sequence rod would have a worth of 1% to 2% $\Delta K/K$ is 0.893. Only a specific configuration of withdrawn rods will produce a rod worth in the 2% to 3% range. The probability associated with this configuration is 4.7×10^{-2}. Considering the possibility of the operator and the instrumentation making multiple errors, it is necessary to multiply the probability of attaining a multiple error configuration by the probability of the operator making a second error and by the probability of the instrumentation allowing a second error to be made. The probability of a configuration to produce rod worths in the range of 3% to 4.5% $\Delta K/K$ is 1.7×10^{-3}, whereas, the probability of an operator making a second error is 10^{-2} (the higher probability accounts for the possibility of interrelations between errors). The probability associated with the RWM allowing the error is taken to be 10^{-2} (same reasoning as for the operator) and the RSCS is also taken to have a second-failure probability of 10^{-2}. Thus, the probability for rod worths in the 3% to 4.5% $\Delta K/K$ range is 1.7×10^{-7} without the RSCS and 1.7×10^{-9} with it.

The next parameter of concern is the velocity at which the control rod drops. Experiments have been performed (the average rod drop velocity was 2.73 feet/second) and the probabilities are as shown in Figure J.4.13.7.

The scram time (90% insertion) is the next parameter of interest. The data for actual control rod scrams in operating reactors was used to obtain the probabilities given in Figure J.4.13.7. The scram time technical specification is 4 seconds for 90% insertion.

By using the different combinations of parameters shown in Fig.J.4.13.7, the values of peak enthalpy (reference 5) can be obtained. Also, these same parameters provide the needed inputs to computer codes which project the number of fuel pins perforated. Both the enthalpy and fuel damage numbers appear in Figure J.4.13.7.

The effect of the modification on dose at the site boundary can now be assessed. The actual magnitude of the release is affected by many factors besides the mechanics of failure and nuclear excursion discussed here: decontamination factor, isolation valve closure time, main steam and recirculation flow at the time of the accident, condenser outleakage, and meteorology are a few of these parameters which add uncertainty to the final result. However, all of these factors which are not actually related to the magnitude of the excursion have been taken at the highly conservative

values used in the FSAR analysis. This approach will demonstrate the specific effect of the modification quite clearly.

The offsite dose is calculated from the highest 24-hour whole-body gamma dose (essentially the same as the course-of-the-accident dose) tabulated in Table 14.4.3 of the Peach Bottom FSAR. This value of 2.3×10^{-5} Rem is equivalent to 6.9×10^{-8} Rem per perforated rod.

In Figure J.4.13.8 the ordinate of a point on the curve is the probability per reactor-year that the dose to a hypothetical individual at the site boundary, assuming the least favorable meteorological conditions, will exceed the value of the abscissa due to the postulated control rod drop accidents. The low probability that there will be any release at all is equal to the probability that there will be a rod drop times the probability that there will be any fuel damaged, given the rod drop occurs. Figure J.4.13.8 does not include any of the cases resulting in peak fuel enthalpies in excess of 425 cal/gm: under such conditions existing physics models cannot be used to predict the exact dynamics of fission product release. From Figure J.6.13.7 the probability of such circumstances is no greater than 4×10^{-25} per reactor-year with only the RWM in operation and no greater than 4×10^{-29} with both the RWM and the RSCS.

The expected value of fencepost dose due to postulated control-rod drop accidents can be calculated from the distributions shown in Figure J.4.13.8 by integrating the respective curves. The result is 3×10^{-22} millirem per year before the modification and 3×10^{-25} millirem per year after the modification. The same hypothetical individual at the site boundary would expect to receive approximately 140 millirem per year from natural "background" and other manmade radiation sources, as reflected in the "natural radiation" curve of Figure J.4.13.8.

It is concluded that the addition of the Rod Sequence Control System acts further to reduce the impact of the CRDA, but from the effects of an accident whose impact on public health and safety was already trivially small.

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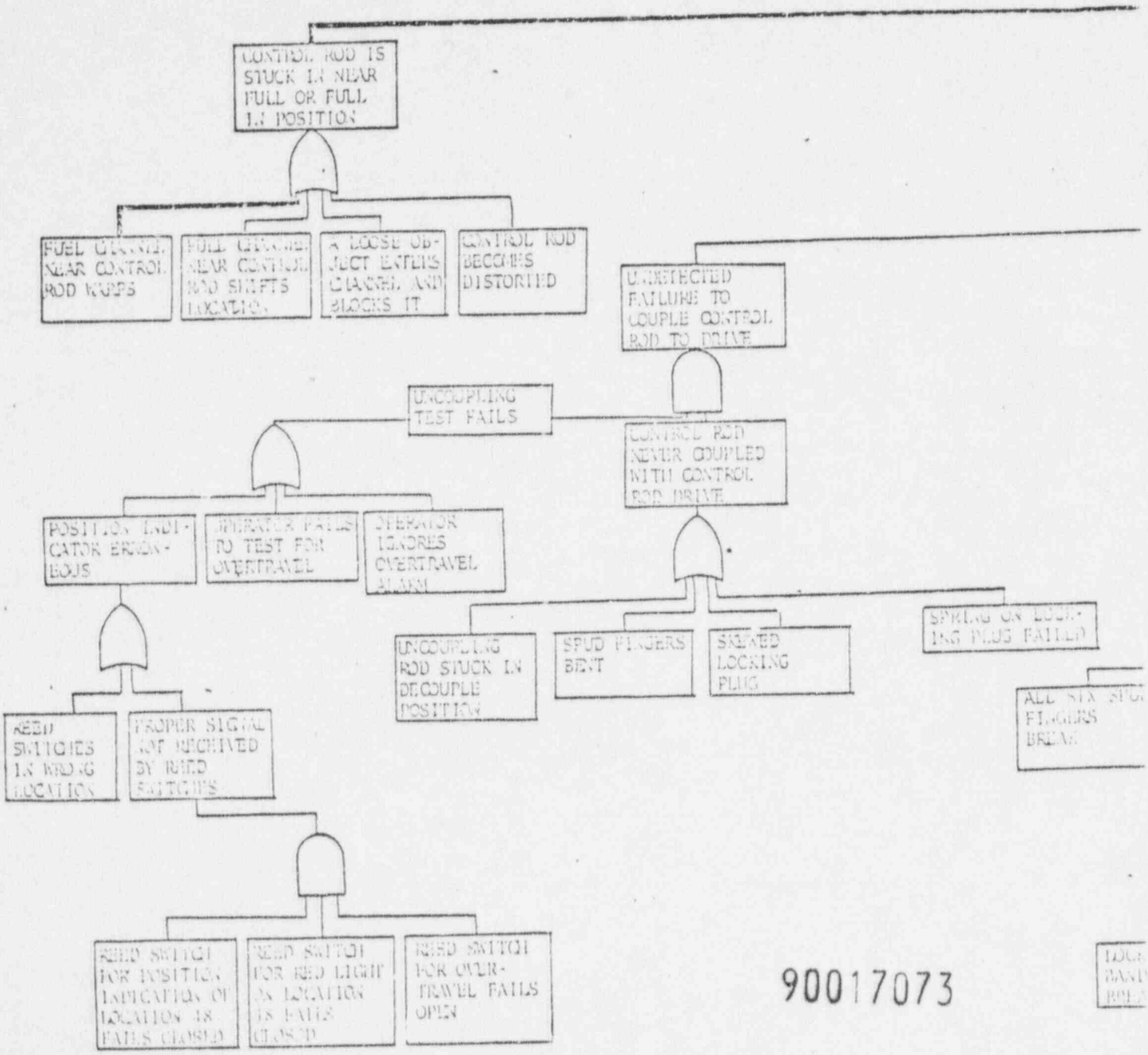
CONTROL ROD IS STUCK IN NEAR FULL OR FULL IN POSITION
FUEL CHANNEL NEAR CONTROL ROD WARPS
FUEL CHANNEL NEAR CONTROL ROD SHIFTS LOCATION
A LOOSE OBJECT ENTERS CHANNEL AND BLOCKS IT
CONTROL ROD BECOMES DISTORTED
UNDETECTED FAILURE TO COUPLE CONTROL ROD TO DRIVE
UNCOUPLING TEST FAILS
CONTROL ROD NEVER COUPLED WITH CONTROL ROD DRIVE
POSITION INDICATOR ERRONEOUS
OPERATOR FAILS TO TEST FOR OVERTRAVEL
OPERATOR IGNORES OVERTRAVEL ALARM
UNCOUPLING ROD STUCK IN DECOUPLE POSITION
SPUD FINGERS BENT
SKEWED LOCKING PLUG
SPRING ON LOCKING PLUG FAILED
ALL SIX SPUD FINGERS BREAK
REED SWITCHES IN WRONG LOCATION
PROPER SIGNAL NOT RECEIVED BY REED SWITCHES
REED SWITCH FOR POSITION INDICATION OF LOCATION 48 FAILS CLOSED
REED SWITCH FOR RED LIGHT ON LOCATION 48 FAILS CLOSED
REED SWITCH FOR OVERTRAVEL FAILS OPEN

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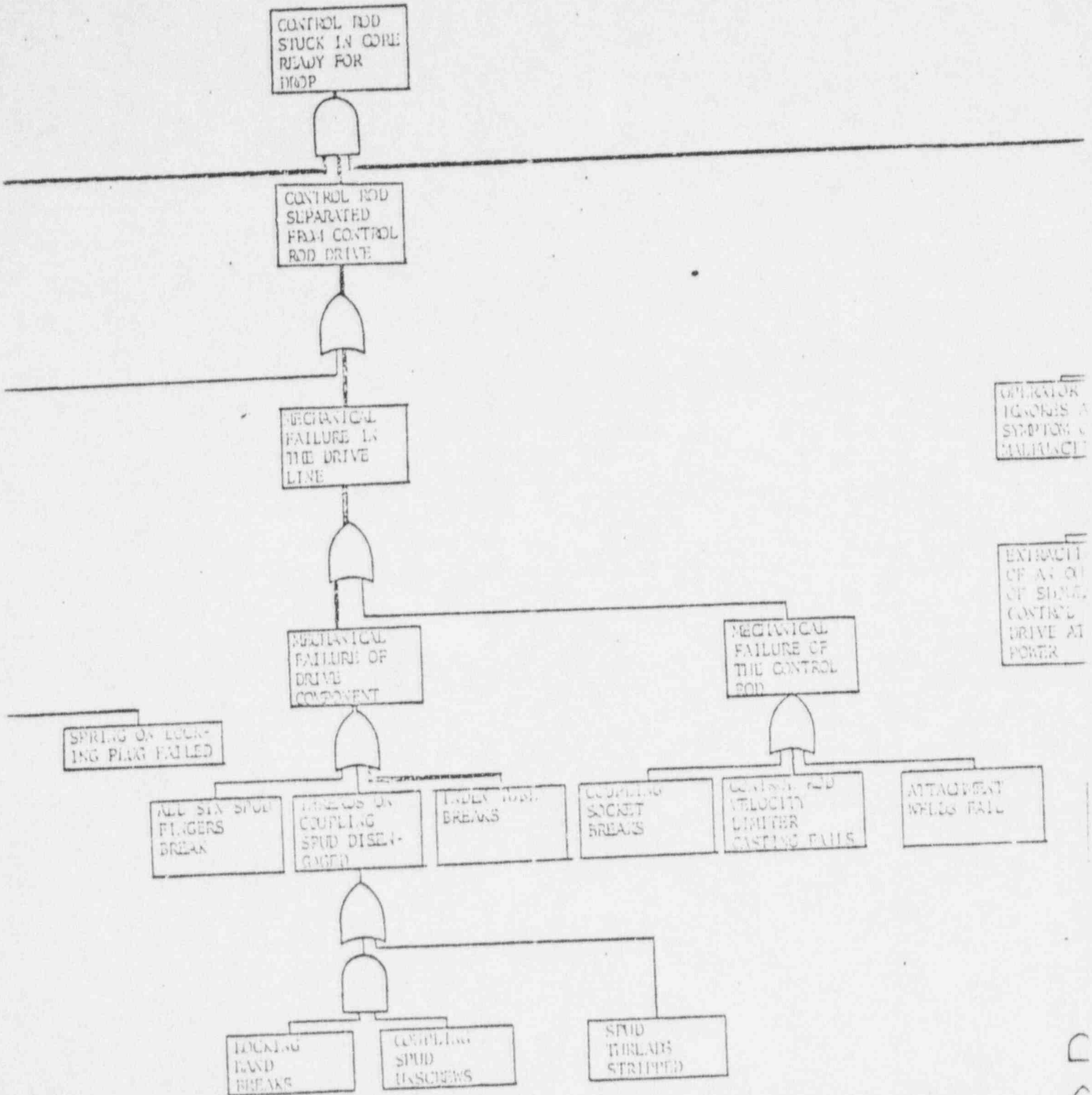
CONTROL ROD STUCK IN CORE READY FOR DROP
CONTROL ROD SEPARATED FROM CONTROL ROD DRIVE
MECHANICAL FAILURE IN THE DRIVE LINE
MECHANICAL FAILURE OF DRIVE COMPONENT
MECHANICAL FAILURE OF THE CONTROL ROD
SPRING ON LOCKING PLUG FAILED
ALL SIX SPUD FINGERS BREAK
THREADS ON COUPLING SPUD DISENGAGED
BREAKS
COUPLING SOCKET BREAKS
VELOCITY LIMITER CASTING FAILS
ATTACHMENT WELDS FAIL
LOCKING BAND BREAKS
COUPLING SPUD UNSCREWS
SPUD THREADS STRIPPED

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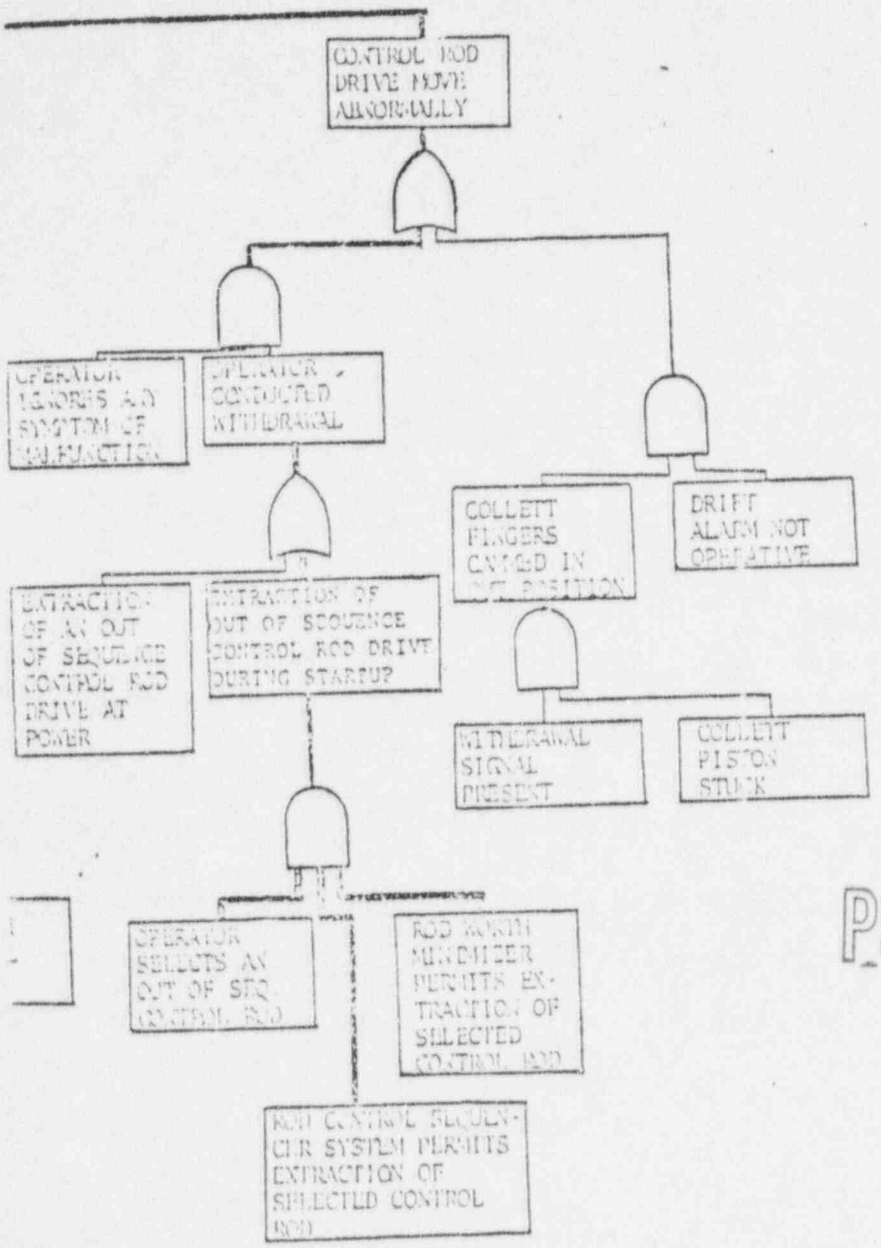

Means all inputs must be present for the result to take place.

Means only one of the inputs must be present for the result to take place.

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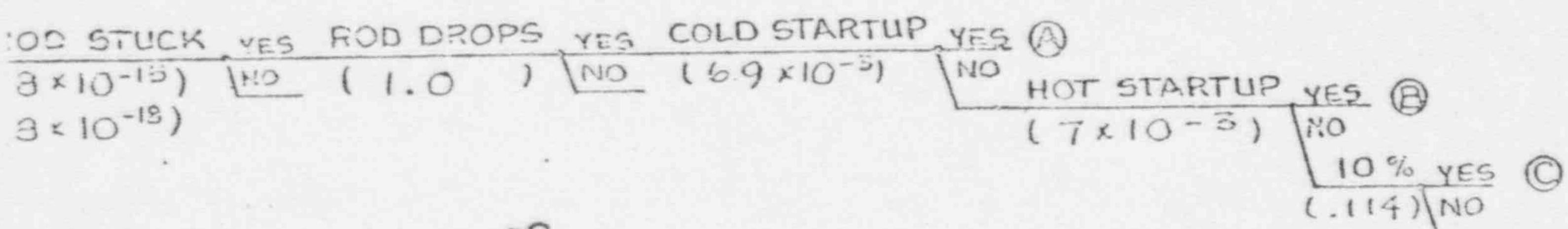
PHILADELPHIA ELECTRIC
PEACH BOTTOM ATOMIC PO
UNITS 2 AND 3
FINAL SAFETY ANALYSI

Fault Tree Model
a Control Rod Stuck
Core Ready for a

FIGURE J.4.13.5



ROD STUCK
3×10⁻¹⁵)
3×10⁻¹⁸)
YES
NO
ROD DROPS
(1.0)
YES
NO
COLD STARTUP
(6.9×10⁻⁵)
YES Ⓐ
NO
HOT STARTUP
(7×10⁻³)
YES Ⓑ
NO
10 %
(.114)
YES Ⓒ
NO

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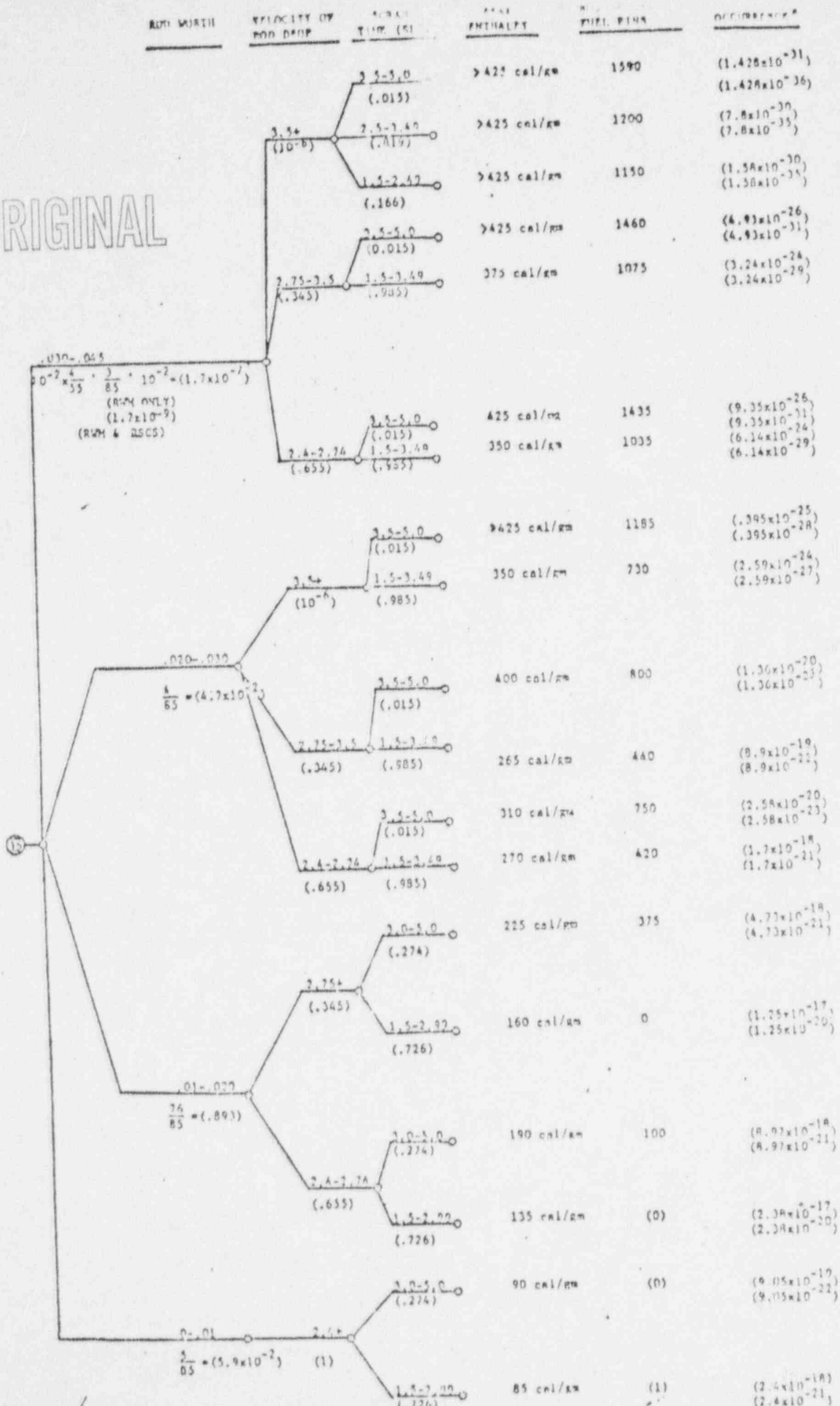

Fig J...

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Appendix B

Taken from Appendix A of WASH-1318

STATISTICAL ANALYSIS OF PRESSURE VESSEL "FAILURE" EVENTS

Introduction

The objective of this discussion is to provide the basis for defining and inferring the values of probabilities of failure events from service experience data available on non-nuclear vessels in fossil-fueled power plant service. Statements of probability derived from observation of the frequency of events must necessarily have associated with them a statement of the confidence level. Also, the calculated probabilities should be considered to be the upper limits at the stated confidence level that can be supported by the mathematical procedures.

Discussion

When rare events are distributed in time, the assumption is usually made that (a) the number of events in a fixed time interval has a Poisson distribution and (b) the intervals between events have an exponential distribution. Two types of situations exist in which the exponential distribution holds on both a theoretical and experimental basis. If components are "run in" or subjected to preservice examinations to eliminate those with manufacturing defects, and degraded components are replaced or repaired, the failure rate may be assumed as constant and the intervals between failures as exponentially distributed. The same distribution law holds for complex systems where individual

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components are replaced (or restored to the original condition) as soon as a failure occurs.[13]

A complex system may fail from any one of a number of mechanisms or modes. The details of one or more mechanisms or modes may be known with some degree of certainty, but no extent of analysis will assure that all failure mechanisms and modes are included in a supposedly complete description of the system. The probability of failure of the system during the time interval [0,T] is given by

$$F(T) = 1 - \exp\{-\sum_i \int_o^T h_i\,(t - \tau_i)\,dt\} \tag{1}$$

where:

$h_i\,(t)$ = hazard function for the ith failure mode

τ_i = time at which ith mode was known to be "good."

If "i" is sufficiently large (i.e., >5), then the term $\sum_i h_i\,(t-\tau_i)$ is approximately constant, say λ. Consequently,

$\sum_i \int_o^T h_i\,(t-\tau_i)\,dt = \lambda T$, and equation (1) becomes

$$F(T) = 1 - \exp\{-\lambda T\} \tag{2}$$

The density function of time to failure, f(T) is then

$$dF(T)/dT = \lambda \exp\{-\lambda T\} \tag{3}$$

and the density function is exponential.

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This argument is the justification for the general assumption that times-to-failure for complex systems may be assumed to be exponentially distributed without knowledge of the details of the several failure mechanisms or modes.

To demonstrate that the above argument holds in reality, a Monte Carlo calculation was performed. A complex system was assumed to fail by each of 10 different mechanisms all of which were assumed to have Weibull distributions (instead of exponential distributions) of times-to-failure. The constants for several Weibull distributions were selected as shown in Table A. The cumulative distribution for the Weibull function is

$$F(t) = 1 - \exp\{-(\lambda t)^n\} \tag{4}$$

TABLE A

Failure Mechanism	λ	n	Mean Time-to-Failure
1	3.14×10^{-3}	1.2	300
2	1.47×10^{-3}	1.4	620
3	1.95×10^{-3}	1.6	460
4	1.53×10^{-3}	1.8	580
5	1.64×10^{-3}	2.0	540
6	2.11×10^{-3}	2.2	400
7	2.61×10^{-3}	2.4	340
8	1.39×10^{-3}	2.6	640
9	1.78×10^{-3}	2.8	500
10	2.35×10^{-3}	3.0	380

A sample of 200 intervals between failure was selected by the random or Monte Carlo process and the results were plotted as shown in Figure 1. The selection of the abcissa, $-\ln(1-F(t))$, and the ordinate, t, was made because if the overall distribution was exponentially distributed then the result on such a plot should be a straight line with a slope of γ, since

$$F(t) = 1 - \exp\{-\gamma t\} \tag{5}$$

$$-\ln(1-F(t)) = \gamma t. \tag{6}$$

The relationship between γ and the mean times-to-failure for the Weibull distribution is

$$\frac{1}{\gamma} = \sum_i \frac{1}{t_i} = \frac{1}{45.02}$$

where t_i = mean time to failure for "ith" Weibull failure mechanism.

The straight line in Figure 1 for the exponential distribution with $\gamma = 45.02$ shows a good fit to the data generated by the Monte Carlo calculation. Thus, although no individual failure mechanism was assumed to be exponentially distributed, the overall failure rate for the complex system is exponentially distributed.

Reference 14/, which provides the basis for the discussion that follows, furnishes additional confirmation of the use of the exponential distribution as a time-to-failure model.

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FIGURE 1

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The exponential probability density function

$$f(t;\lambda) = \begin{cases} \lambda e^{-\lambda t}, & t \geq o \text{ and } \lambda > o \\ o, & \text{elsewhere} \end{cases} \tag{7}$$

that is the most commonly used time-to-failure distribution, plays a central role in reliability, comparable to that of the normal distribution.

The hazard function for an exponentially distributed variate is

$$h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \tag{8}$$

Thus the probability of failure during a specified time interval is a constant depending only on the length of the interval and is the same irrespective of whether the component is in its early period of operation or has previously survived 10 years, 20 years, or 30 years. The parameter λ is referred to as the failure rate.

The time-to-failure for a component is exponentially distributed if the componeevent happens independently at a constant rate. Frequently, however the time-to-failure distribution for a component is not exponentially distributed over its <u>entire</u> life, but the in-usage portion is exponentially distributed. The time-to-failure for the <u>component in-usage</u> is then exponentially distributed, because the applicable hazard function is

constant. This is so, even if the time-to-failure distribution over the total life of the component is far from exponential. The exponential distribution is considered more appropriate as a time-to-failure model for a complex system than for its component parts. Reference[15/], which deals specifically with nuclear reactors, provides additional material on estimation of failure rates. This reference indicates that a mathematically acceptable method of estimating failure rates is to use data from the operating history of the reactor or similar type vessels. The method can be used if the failures are assumed to be distributed as a Poisson process with gamma-distributed waiting times to failure and exponentially-distributed interarrival times. These asumptions are valid if it can be assumed that maintenance and repair result in no wear-out effect.[16/]

The degree of accuracy obtained in estimating a failure rate is directly related to the amount of operating data available. As an example, if a component does not fail in 15 years, estimating its failure rate presents some difficulty. However, its failure rate is less than for a component that failed once in 15 years. Using this concept and the above-stated distribution theory, an upper confidence limit may be placed on a component that has no observed failures in an observation time T. The lower 1-α confidence interval on the population failure rate ν, given as a probability statement, is

$$P\,[0 \leq \nu \leq \chi^2_{1-\alpha}(2n+2)/2T] = 1-\alpha \qquad (9)$$

where:

α = probability that an observation from the hypothesized population will randomly occur outside the confidence interval,

n = number of failures observed in time interval T,

$\chi^{2}_{1-\alpha}(2n+2)$ = $1-\alpha$ percentile of a chi-square distribution with $2n+2$ degrees of freedom,

ν = population value of failure rate per year.

References[17], [18], and [19] provide more details on failure rate confidence intervals.

The above formula from Otway, et al.[15], is a special case of a much more general formula developed in this discussion to compute the confidence intervals for the parameters in Poisson and binomial processes. The basic assumption is that the quantities of interest are the limits that define an interval of the frequency corresponding to the stated confidence level, conditional on the observations. A distribution function,[20] $G(r|c, \alpha)$ is defined as follows:

$$G(r|c, \alpha) = P(\lambda \leq r|c,\alpha) = \int_0^r g(\lambda|c,\alpha)\, d\lambda, \quad \lambda \geq 0 \qquad (10)$$

where

$P(\lambda \leq r|c, \alpha)$ = the probability that the frequency, λ, is less than or equal to r, conditional upon c the observed count, and α, the hypothesis of the prior density of the frequency.

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The density function, $g(\lambda|c, \alpha)$ is assumed to be given by Bayes' theorem[21/] in the following form:

$$g(\lambda|c, \alpha) = \frac{f(\lambda) \quad h(c|\lambda)}{\int_0^\infty f(\lambda) \quad h(c|\lambda)\, d\lambda} \tag{11}$$

For a Poisson process

$$h(c|\lambda) = \frac{\lambda^c e^{-\lambda}}{c!} \text{ for } \begin{cases} c = 0, 1, 2 \ldots \\ \lambda > 0 \end{cases} \tag{12}$$

where:

λ = the true frequency or the expected number of events during the period of observation.

c = the observed number of events.

The density function $f(\lambda)$ is conveniently chosen to be a gamma density, i.e.,

$$f(\lambda) = \frac{\beta(\beta\lambda)^K e^{-\beta K}}{\Gamma(K+1)} \quad \text{for } \lambda \geq 0,\ \beta \geq 0,\ K \geq 0 \tag{13}$$

This density has two parameters β and K so that appropriate choices for their numerical values can represent a variety of assumptions concerning the "a priori" distribution of λ. In particular, the choice $\beta = 0$ and $K = 0$ corresponds to the assumption that λ is uniformly distributed between zero and infinity. This means that no particular value of λ is favored over another prior to obtaining the statistical data.

By combining these considerations, the density function, $g(\lambda|c, \beta, K)$ is a gamma density.

$$g(\lambda|c, \beta, K) = \frac{(1 + \beta)^{c+K+1}}{(c+K+1)} \lambda^{c+K} e^{-\lambda(1+\beta)} \qquad (14)$$

The value of the integral of this density function is equal to the chi-square integral whenever K = 0 or a positive integer. The relationship[22/] is

$$P(\lambda \leq r|c, \beta, K) = P\ [\chi^2 \leq 2r(1+\beta)|\nu = 2\ (c+K+1)] \qquad (15)$$

This formula becomes identical to Otway, et al.[15/] when $\beta = 0$ and K = 0. Therefore, Otway, et al. are presumed to have made the "a priori" assumption that λ is distributed uniformly between zero and infinity.

This "a priori" assumption is seemingly a safe and most reasonable one since it really amounts to an unprejudiced attitude with respect to what λ might be prior to collecting the data.

A short listing of values derived from the above formula for the case of the "a priori" assumption that λ is uniformly distributed between zero and infinity, as utilized in the determination of failure probabilities stated in this report, is shown in Table B. (For values other than those listed refer to Figure 2.)

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TABLE B

STATISTICALLY-INFERRED NUMBER

OF OCCURRENCES

Observed Number of Occurrences - n_o	Confidence Level 90%	95%	99%
0	2.3	3.0	4.6
1	3.9	4.7	6.6
2	5.3	6.3	8.4
6	10.5	11.8	14.5
13	18.9	20.6	24.1
100	114.2	118.2	125.4

A Monte Carlo calculation was performed to verify the values from the above table for zero observed events. Monte Carlo "experiments" on digital computers utilize their capability to make random selection of numbers in order to produce results completely independent of analytical methods. However, the sample size selected must necessarily be finite, and consequently the resulting values will not be equal exactly to those achieved by analytical methods using the infinite limit.

Poisson processes were selected at random with λ's uniformly distributed between zero and infinity and a count was obtained on each process. If the count was zero, then the corresponding λ was added to the sample;

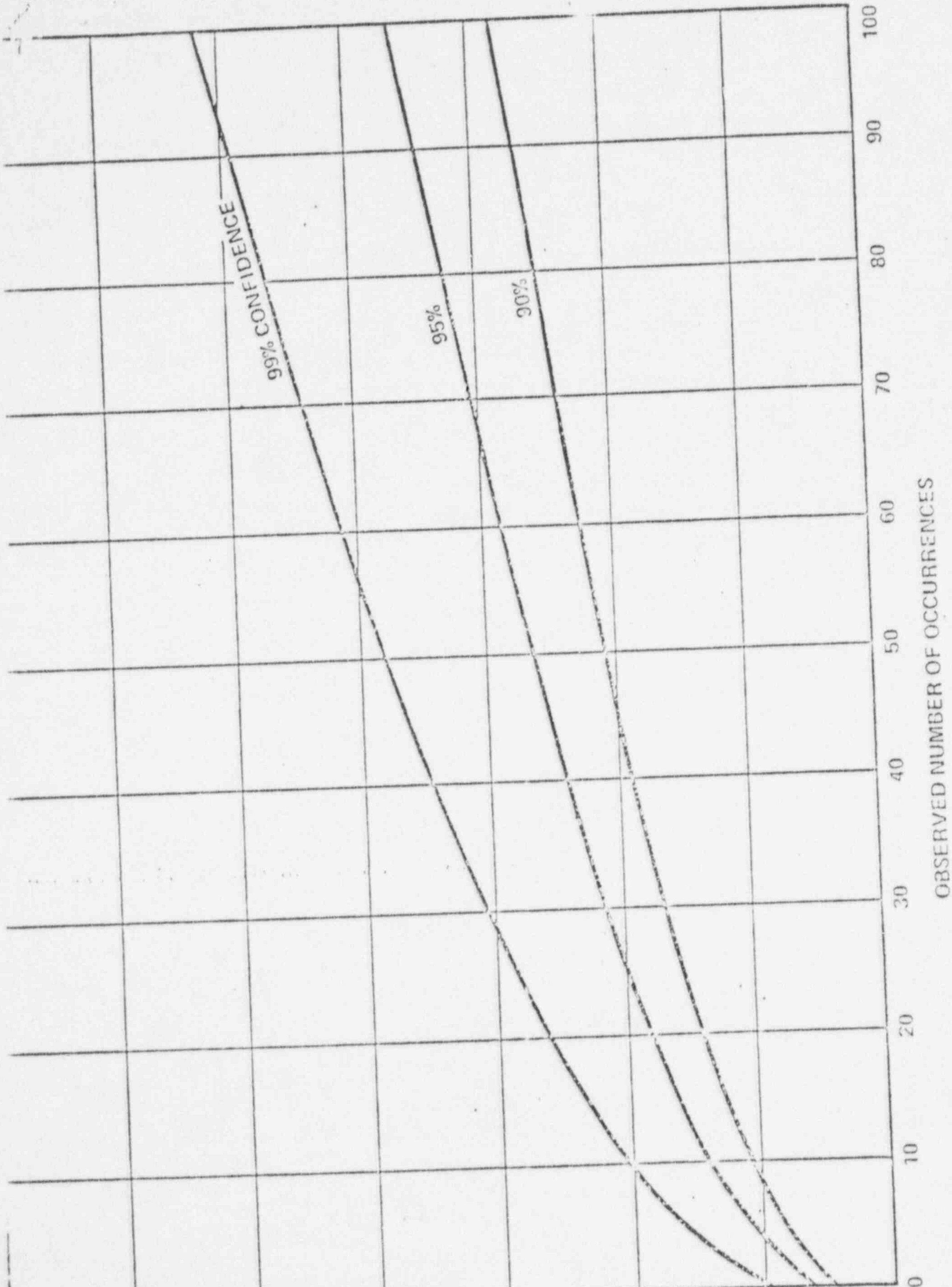

FIGURE 2 STATISTICALLY INFERRED EXPECTED NUMBER OF OCCURRENCES
FROM OBSERVED NUMBER OF OCCURRENCES

otherwise the process was discarded. This procedure was carried out until 20,000 frequencies had accumulated. The results are tabulated in Table C.

TABLE C

Upper Limit on Frequency, λ	Number of Frequencies Less than Upper Limit	Percentage of Total-Confidence Value
2.3026	18023	90.115
2.9957	19027	95.135
4.6052	19804	99.020

The upper limits listed are the values, to five significant figures, from Table B for 90%, 95%, and 99% confidence and for zero observed events. Obviously, the results demonstrate that the Monte Carlo and analytical values are in excellent agreement.

These results are judged to be an adequate demonstration that the methods discussed can be applied to estimate the range of failure probabilities from statistical service histories of pressure vessels, for the case of either a finite number of observed failure events or the absence of any failure event.