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CRITERIA FOR COMBINATIONS OF EARTHQUAKE AND/OR OTHER TRANSIENT RESPONSES

N. M. Newmark

R. P. Kennedy

PREAMBLE

The intent of the methods proposed for combinations of transient, dynamic responses is to achieve a nonexceedance probability of approximately 84% for the peak combined response of the system, component, or element considered. This goal is achieved by compliance with any one of the following criteria, or any alternative method that meets the intent stated above, provided that the intensity of loads or accelerations for each input are conservatively represented (approximately at the level of the 84th percentile, or the mean plus one standard deviation, of the expected input intensity).

CRITERION 1

Dynamic or transient responses of structures, components, and equipment arising from combinations of dynamic loading or motions may be combined by SRSS provided that each of the dynamic inputs or responses has characteristics similar to those of earthquake ground motions, and that the individual component inputs can be considered to be relatively uncorrelated; i.e., the individual dynamic inputs or responses considered are either from independent events or have random peak phasing. This similarity involves a limited number of peaks of force or acceleration (not more than 5 exceeding 75% of the maximum, or not more than 10 exceeding 60% of the maximum), with approximately zero mean and a total duration of strong motion (i.e., exceeding 50% of the maximum) of 10 seconds or less.

Explanation. Since earthquake motions in various directions produce responses which are combined conservatively by the use of SRSS, the descriptions of dynamic or transient inputs are based on those applicable to earthquake motions. The coefficient of correlation for these is less than 0.4, and the pattern of peaks is based on Table 2 of Circular 672 of the U.S. Geological Survey describing earthquake ground motions for use in the design of the Alaska oil pipeline.

The probability distribution for the responses to earthquake motions is based on the concepts underlying U.S. NRC Regulatory Guide 1.60, where the standard deviation is 30 to 40% of the median value.

It was proved some decades ago that modal responses to earthquake motions may be conservatively combined by SRSS methods with the same degree of conservatism as that of the motions. If each of such responses is considered to be at the level of mean plus one standard deviation, the SRSS value is also at this level. For the same reasons, responses from the three component directions of earthquake motions may also be conservatively combined by SRSS methods.

CRITERION 2

When response time histories are available for all multiple dynamic loadings being combined, SRSS methods may be used for peak combined response when CDF calculations, using appropriate assumptions on the range of possible time lags between the response time histories, show the following criteria are met:

1. There is estimated to be less than approximately a 50% conditional probability that the actual peak combined response from these conservatively defined loadings exceeds approximately the SRSS calculated peak response, and

2. There is estimated to be less than approximately a 15% conditional probability that the actual peak combined response exceeds approximately 1.2 times the SRSS calculated peak response.

APPENDIX C

ADDITIONAL SAMPLE RESULTS

COMBINATION OF ARTIFICIAL RESPONSES IN FREQUENCY DOMAIN

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PURPOSE

- To generate artificial responses with specific fourier spectra:

  Amplitude of predominent frequencies

  Frequency bandwidth

  Relative position of predominent frequencies

- To combine artificial responses with random time lag

  (Unif. distribution)

- To compute the CDF of the combined peaks

- To compare with the SRSS value

- To obtain non-exceedance probability of SRSS

DERIVATION OF FORMULAS

$$R(T) = \sum_{K=1}^{N} \left(A_K \cos(KW_0T) + B \sin(KW_0T)\right)$$

$$= \sum_{K=L}^{N} C_K \cos(KW_0T - \phi_K)$$

To generate artificial R(T):

- $C_K = N_K \cdot E^{-(KW_0 - \mu_W)} / 2\sigma_W^2$

  Where N_K is a random number between 0 and $C_{K,MAX}$

- μ_W -- The predominant frequency

- σ_W^2 -- Frequency band variance

- ϕ_K -- Random phase angle

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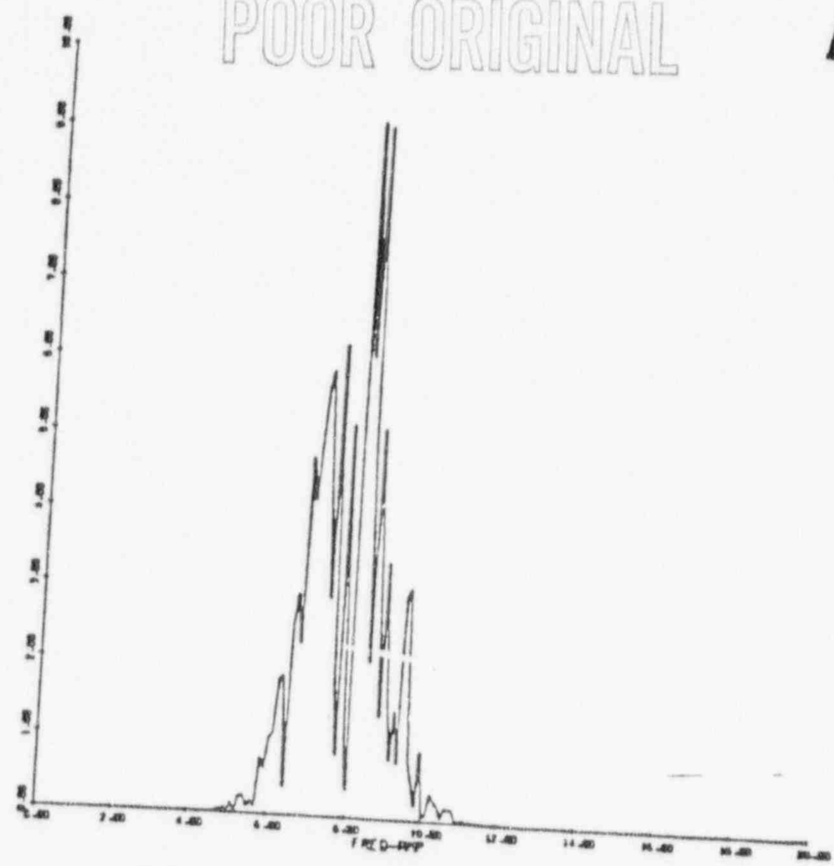
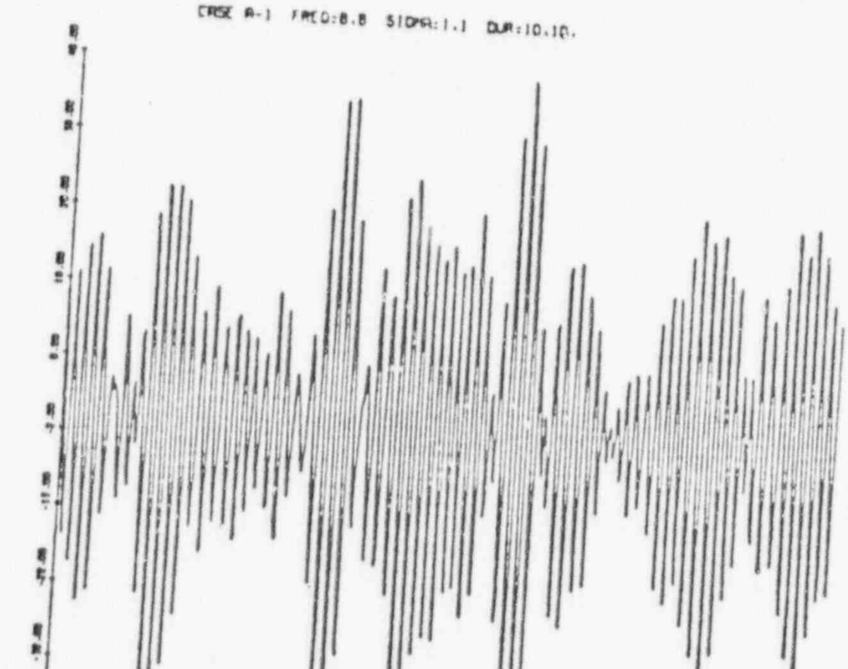


CASE A-1 FREQ:8.8 SIGMA:1.1 DUR:10.10.



CASE A-1 FREQ:8.8 SIGMA:1.1 DUR:10.10.

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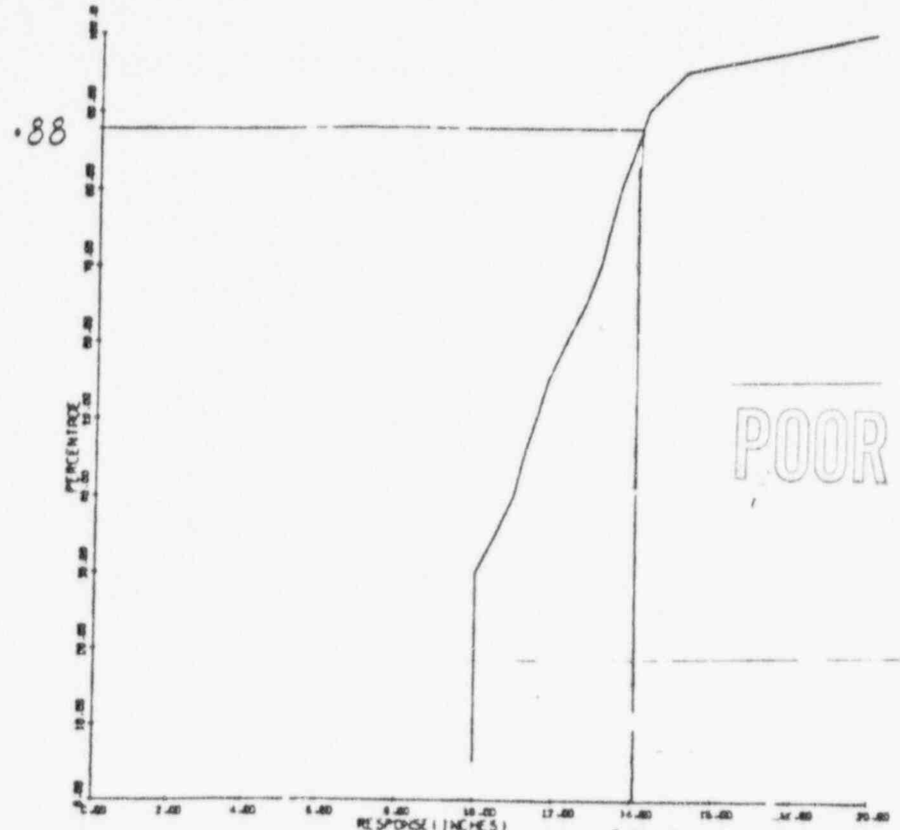
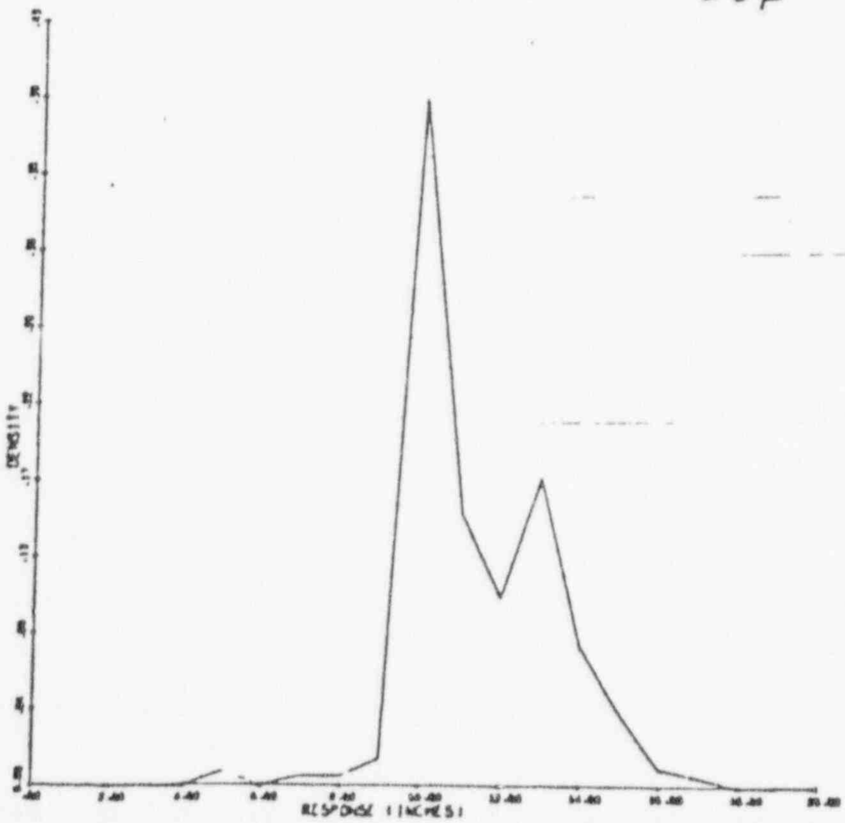


CASE A-1 FREQ:8.8 SIGMA:1.1 DUR:10.10.

CDF



CASE A-1 FREQ:8.8 SIGMA:1.1 DUR:10.10.

DENSITY

POOR ORIGINAL

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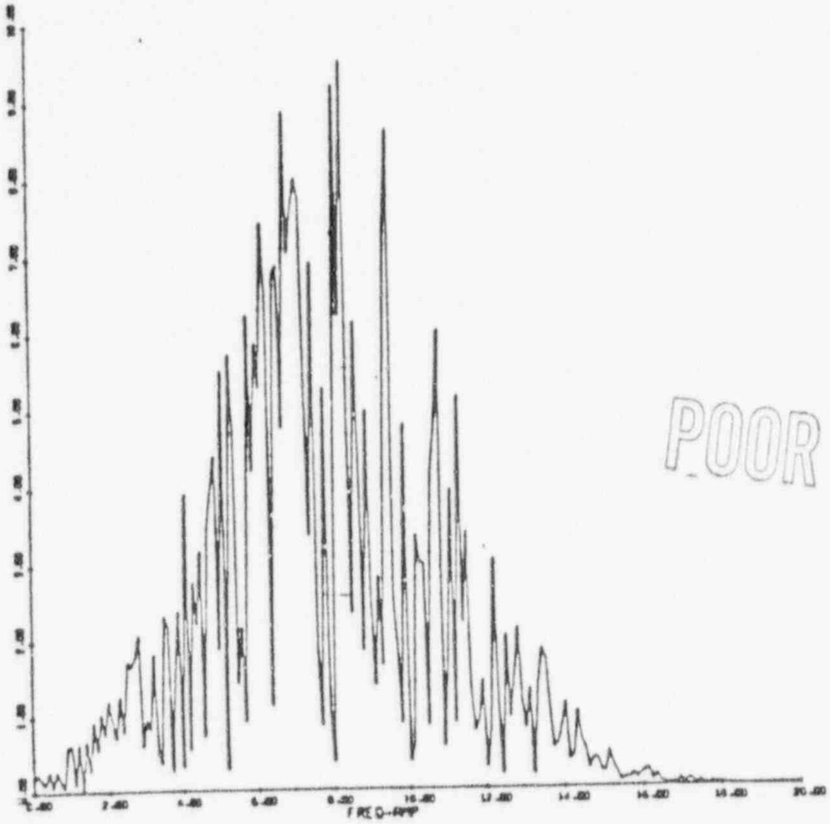
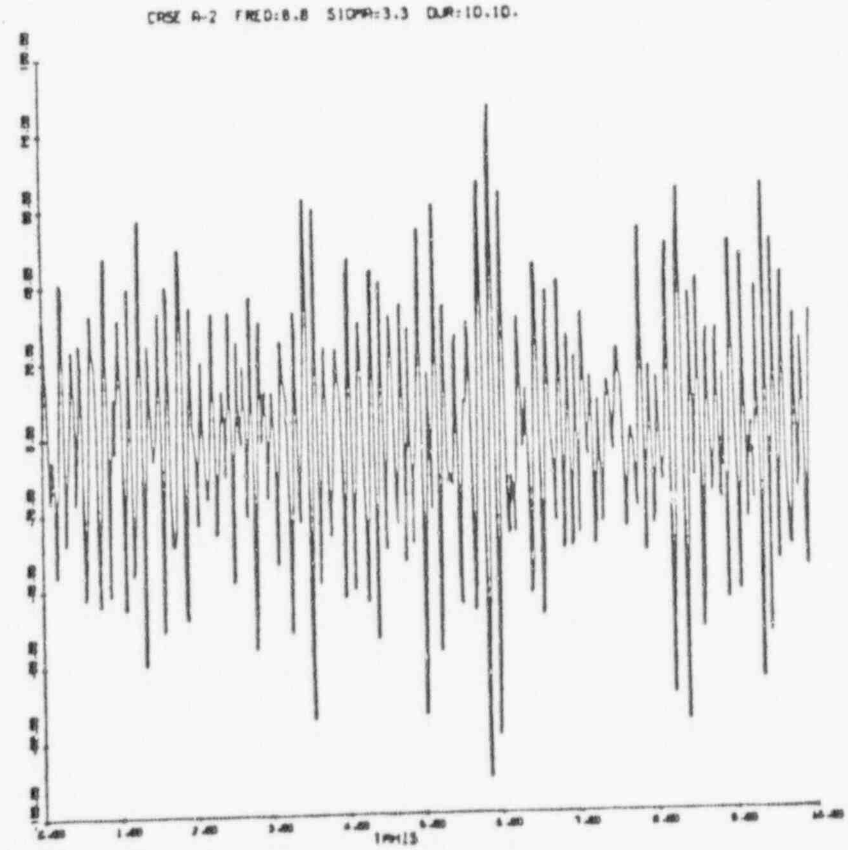


CASE A-2 FREQ:8.8 SIGMA:3.3 DUR:10.10.

POOR ORIGINAL



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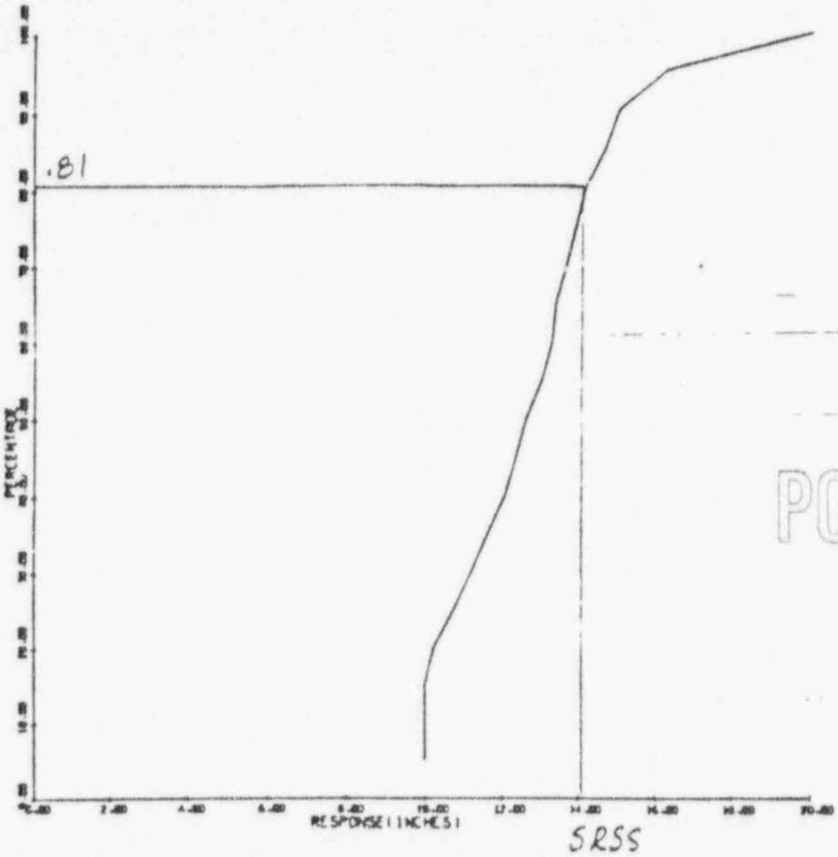
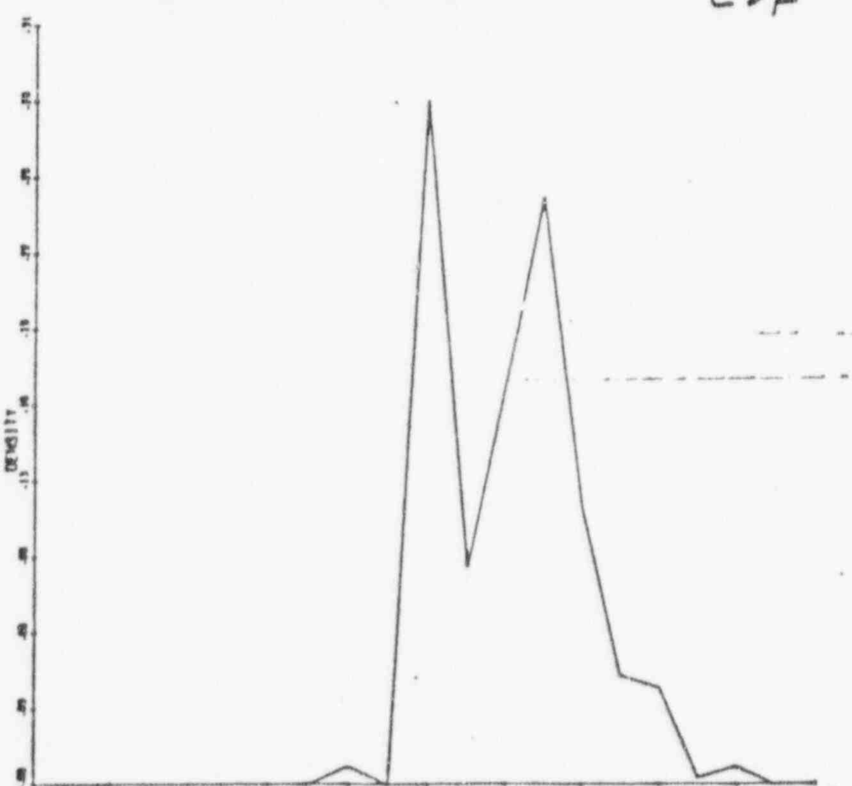


CASE A-2 FREQ:8.8 SIGMA:3.3 DUR:10.10.

CDF



CASE A-2 FREQ:8.8 SIGMA:3.3 DUR:10.10.

Density

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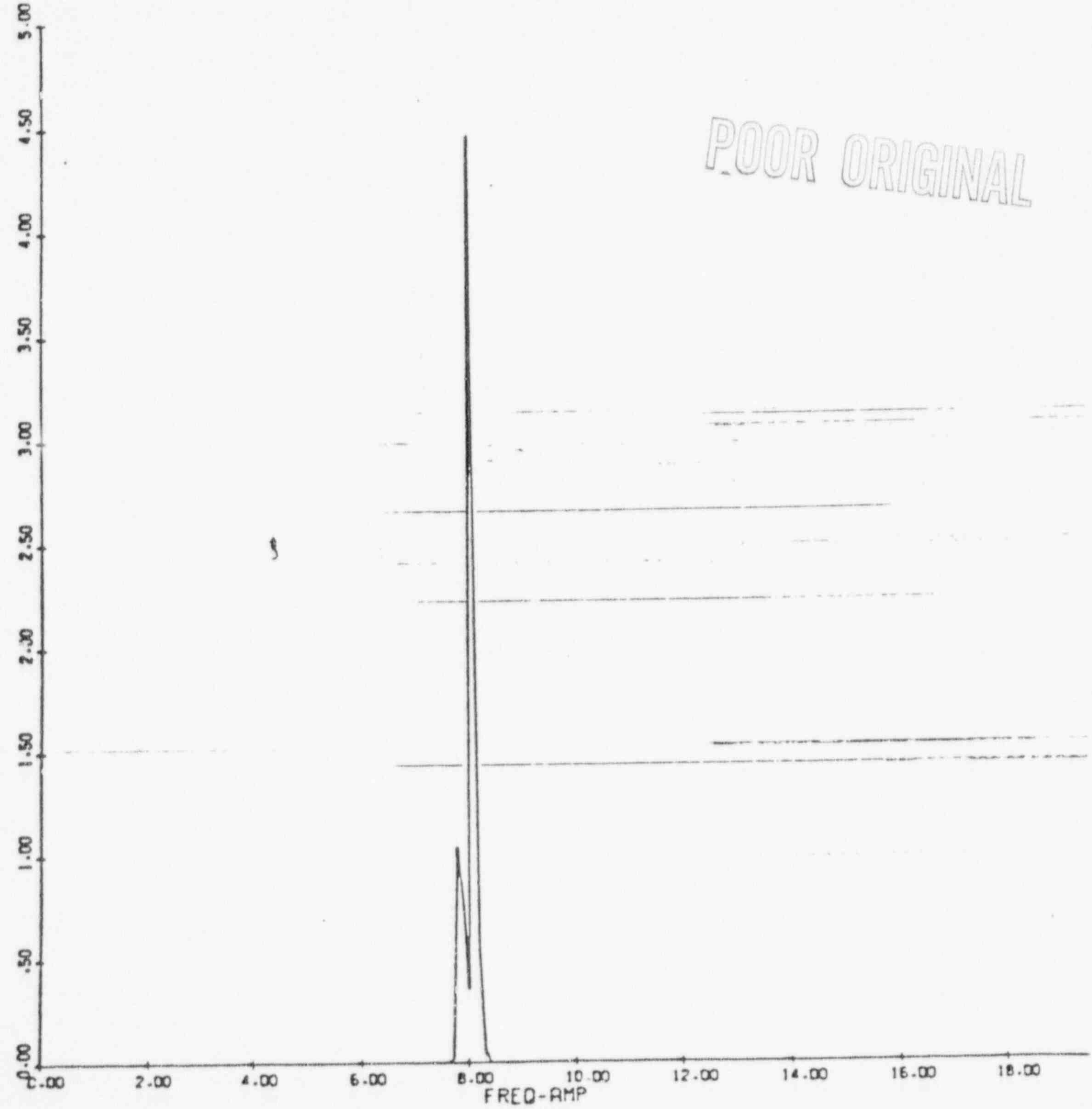

POOR ORIGINAL

CASE A-1 FREQ=8.8 SIGMA= .1 DUR=10.10.

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CASE A-1 FREQ=8.8 SIGMA=.001 DUR=10.10.

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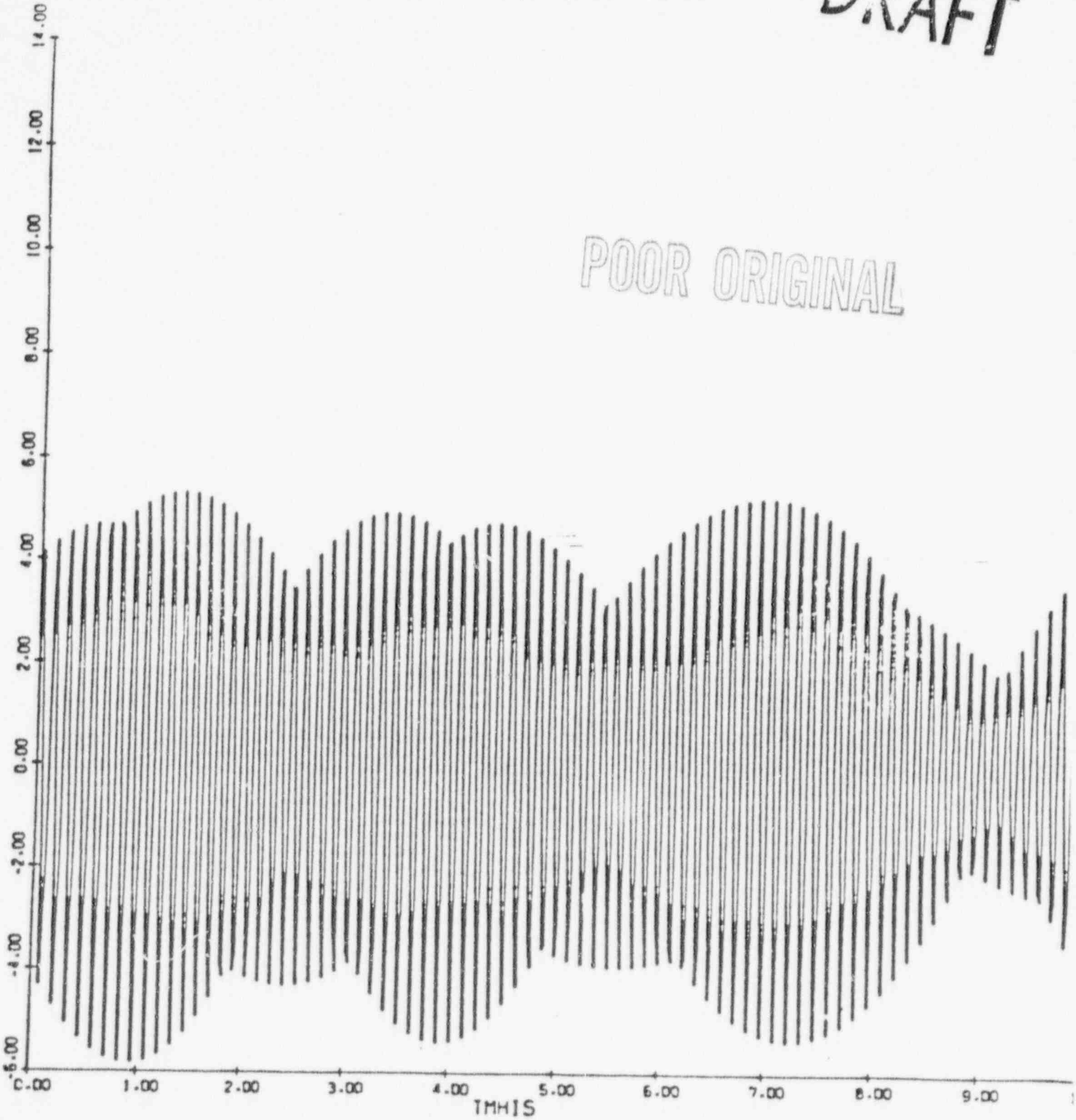

POOR ORIGINAL

CASE A-1 FREQ=8.8 SIGMA= .1 DUR=10.10.

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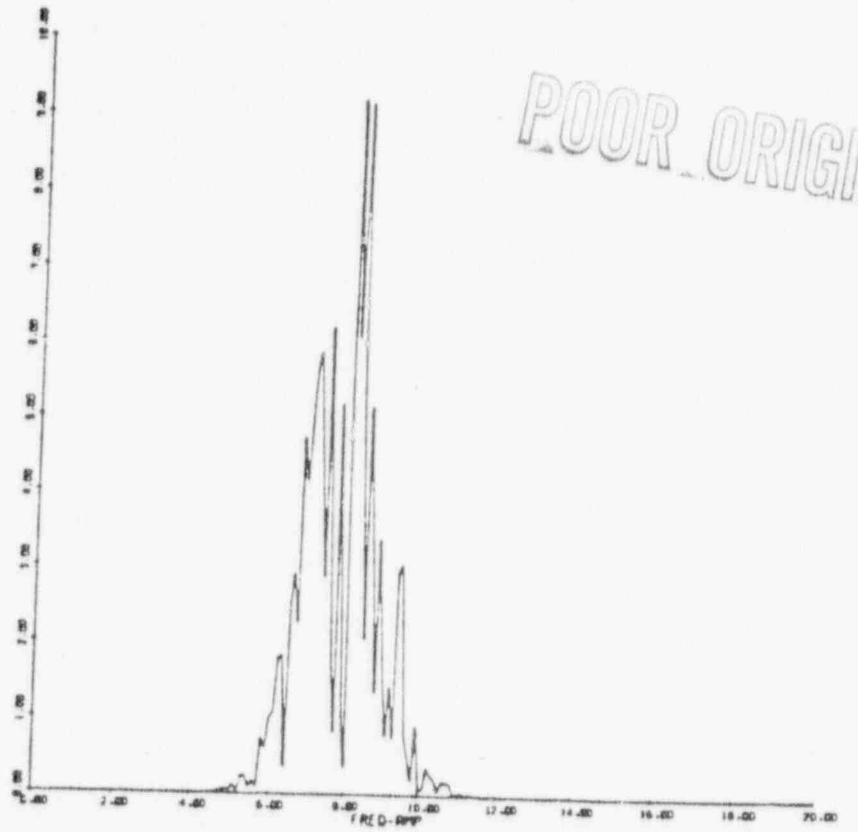
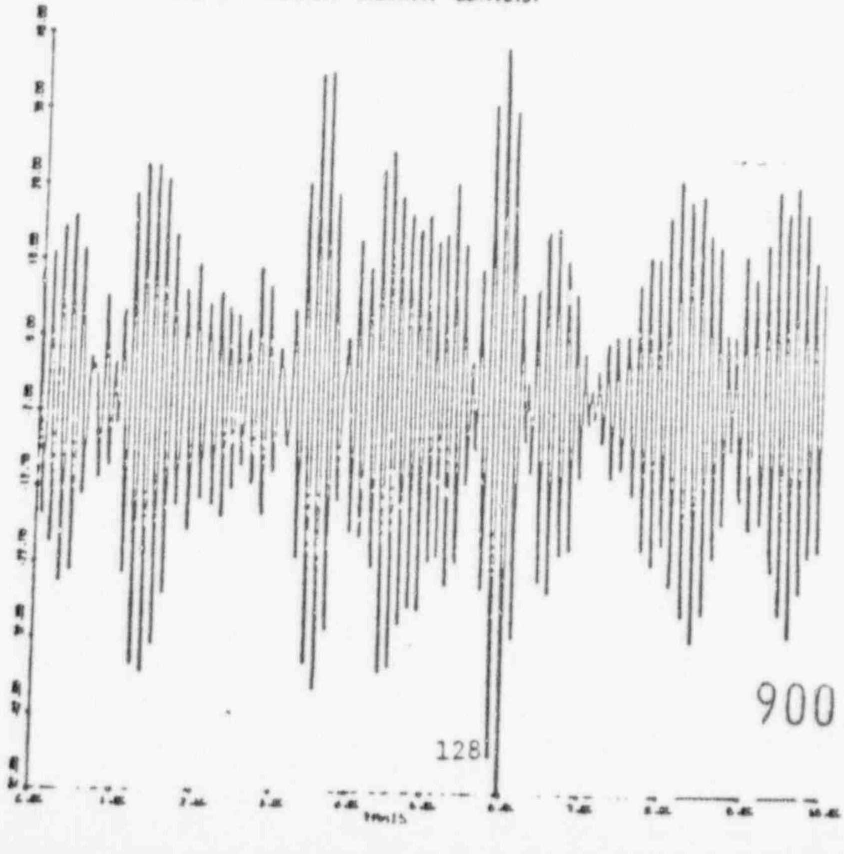
POOR ORIGINAL



CASE B-1 FREQ=8.8 SIGMA=1.1 DUR=10.5.



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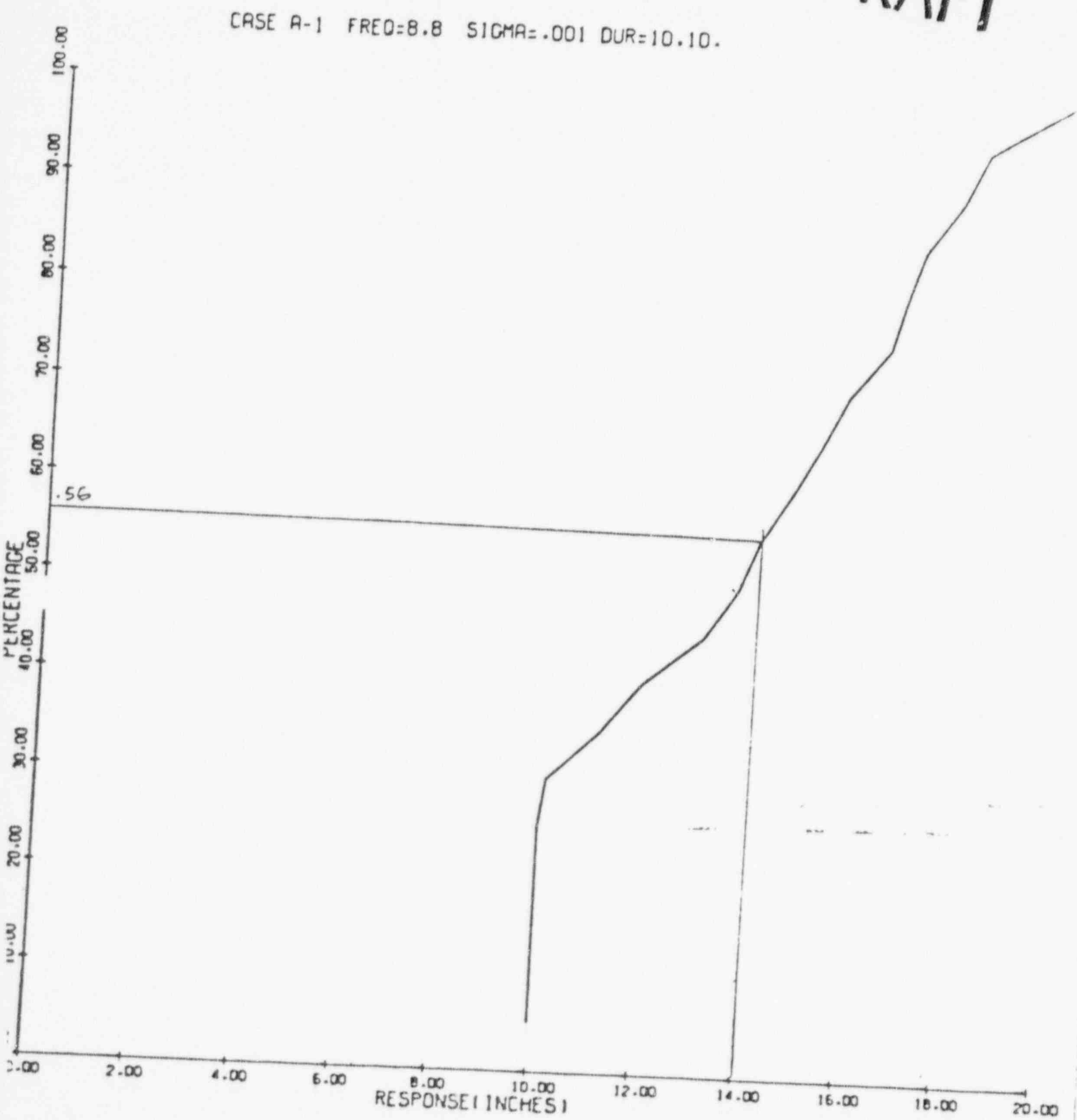

CASE A-1 FREQ=8.8 SIGMA=.001 DUR=10.10. CDF

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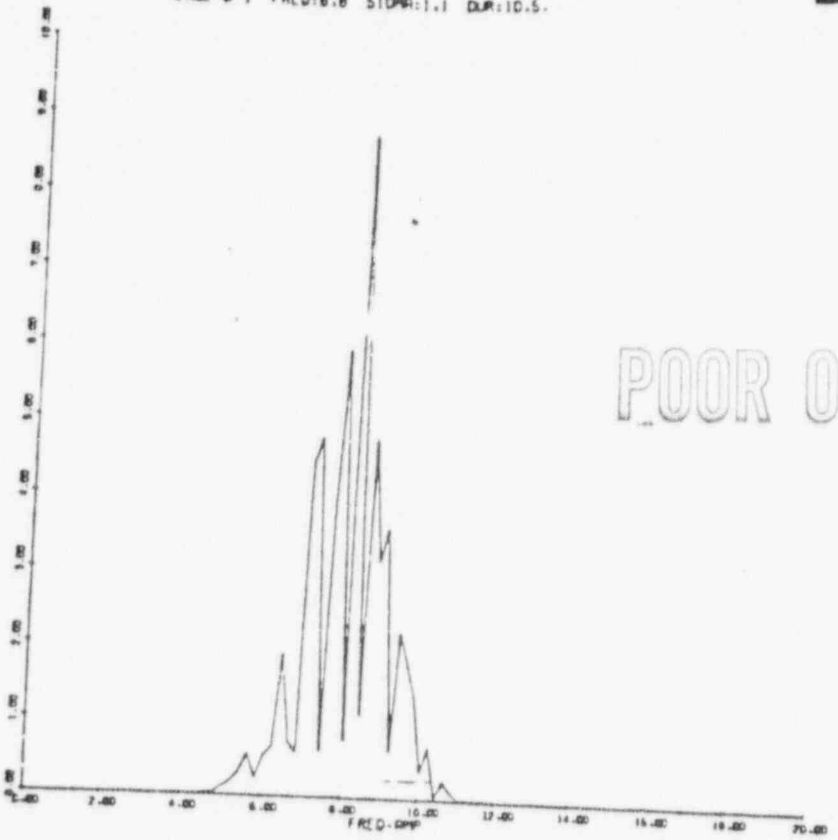
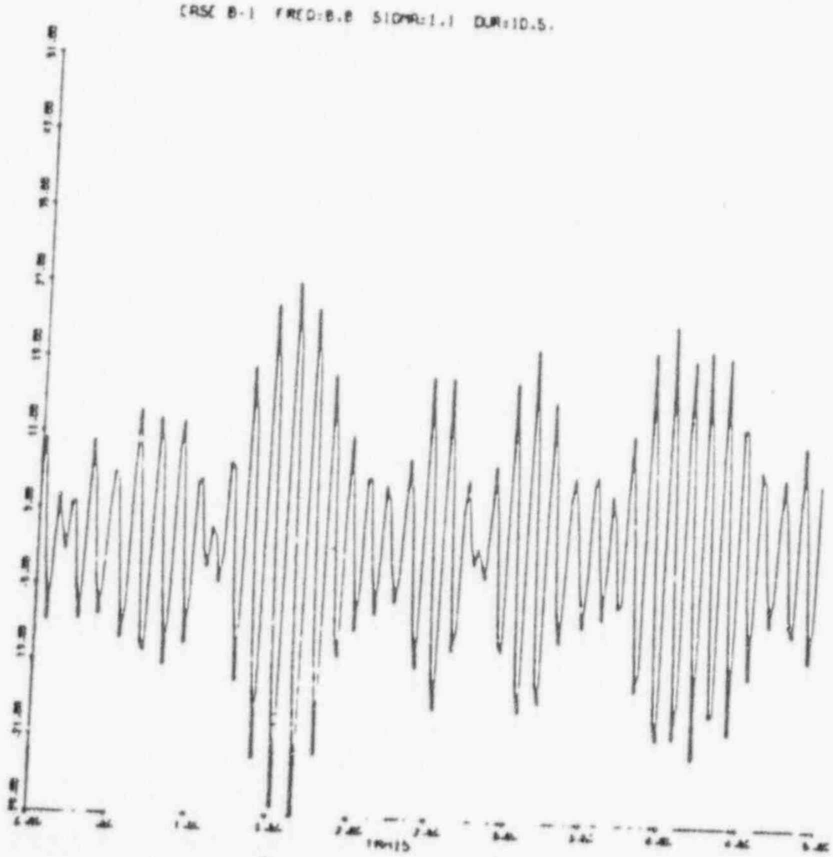
CASE B-1 FREQ:8.8 SIGMA:1.1 DUR:10.5.



POOR ORIGINAL

CASE B-1 FREQ:8.8 SIGMA:1.1 DUR:10.5.



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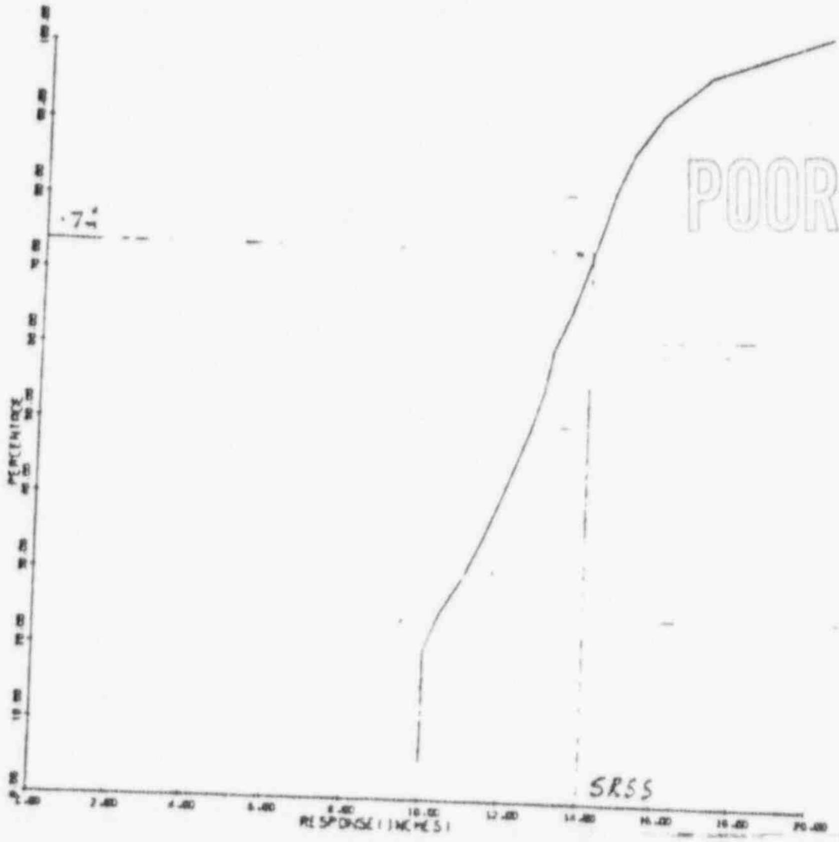

CASE B-1 FREQ:8.8 SIGMA:1.1 DUR:10.5.

CDF

POOR ORIGINAL

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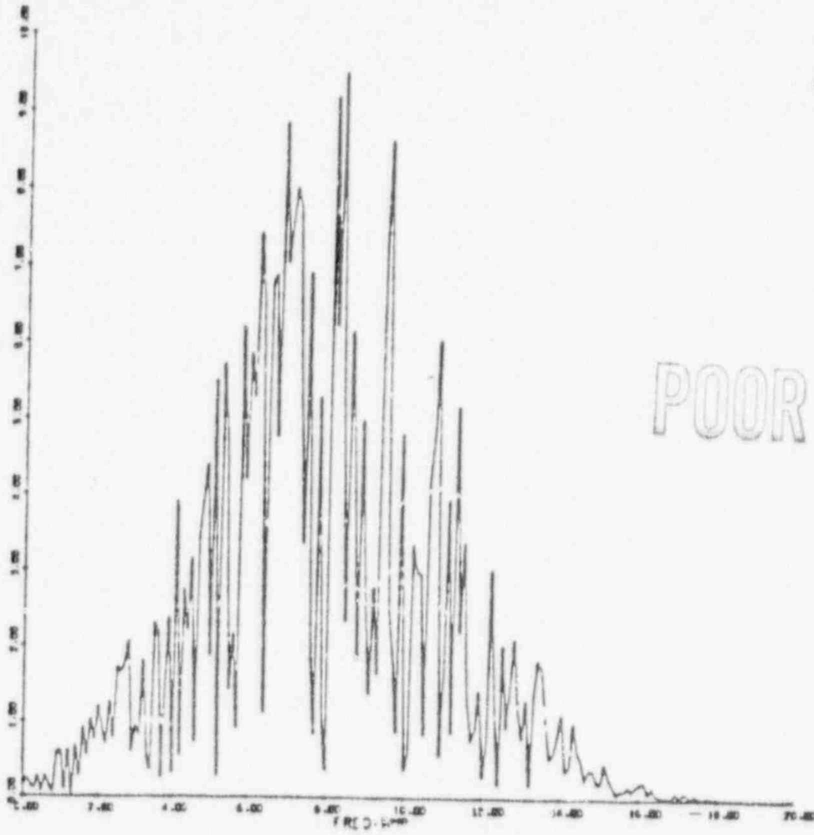
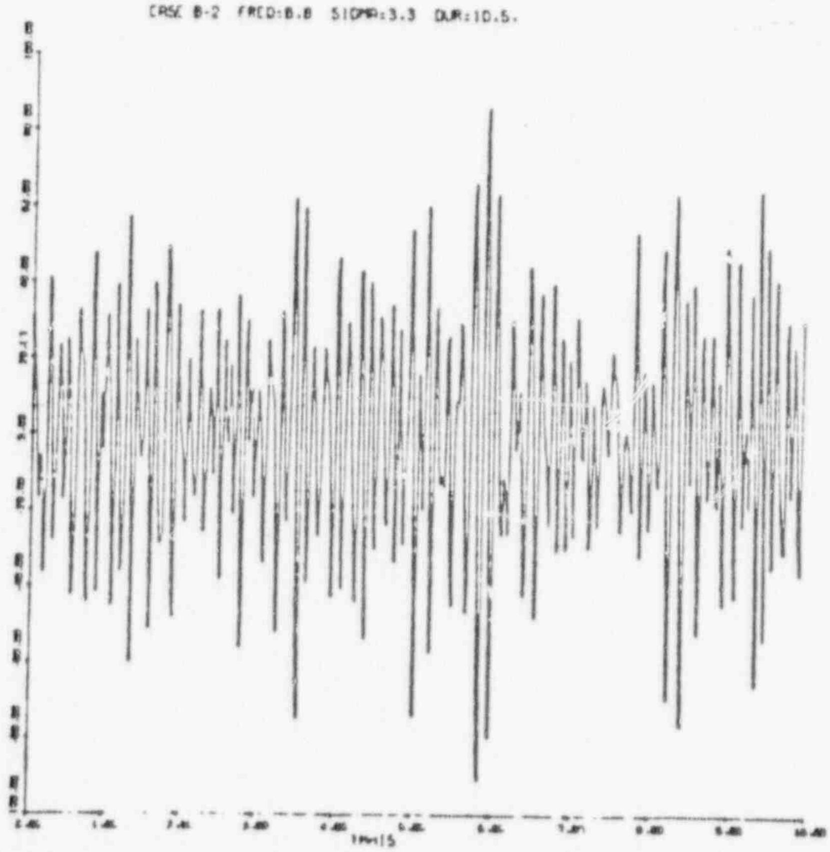


CASE B-2 FREQ:8.8 SIGMA:3.3 DUR:10.5.



CASE B-2 FREQ 8.8 SIGMA:3.3 DUR:10.5.

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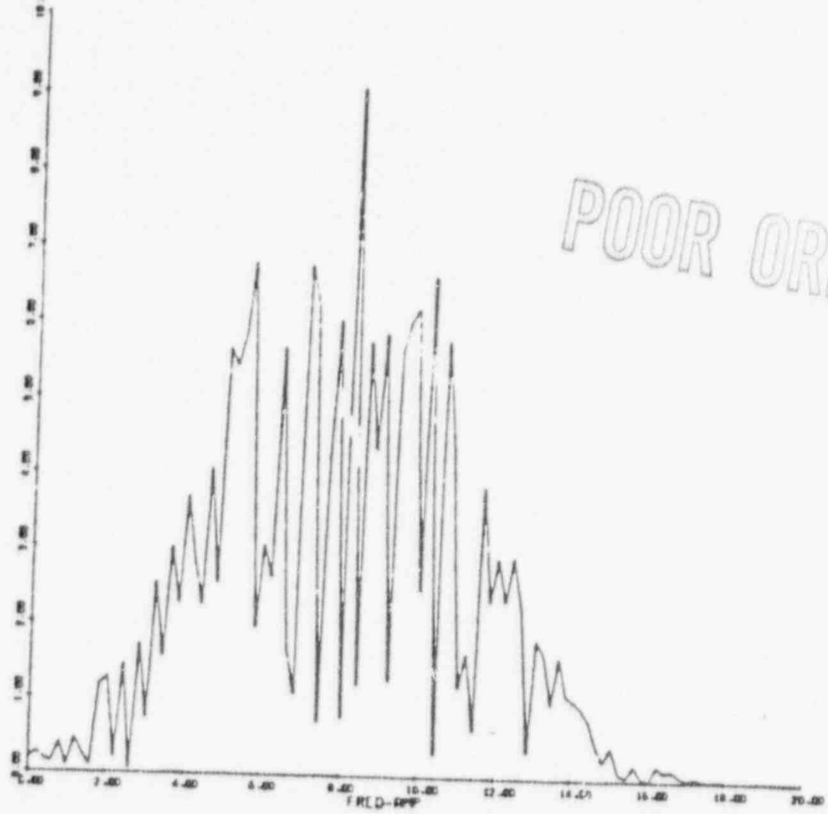
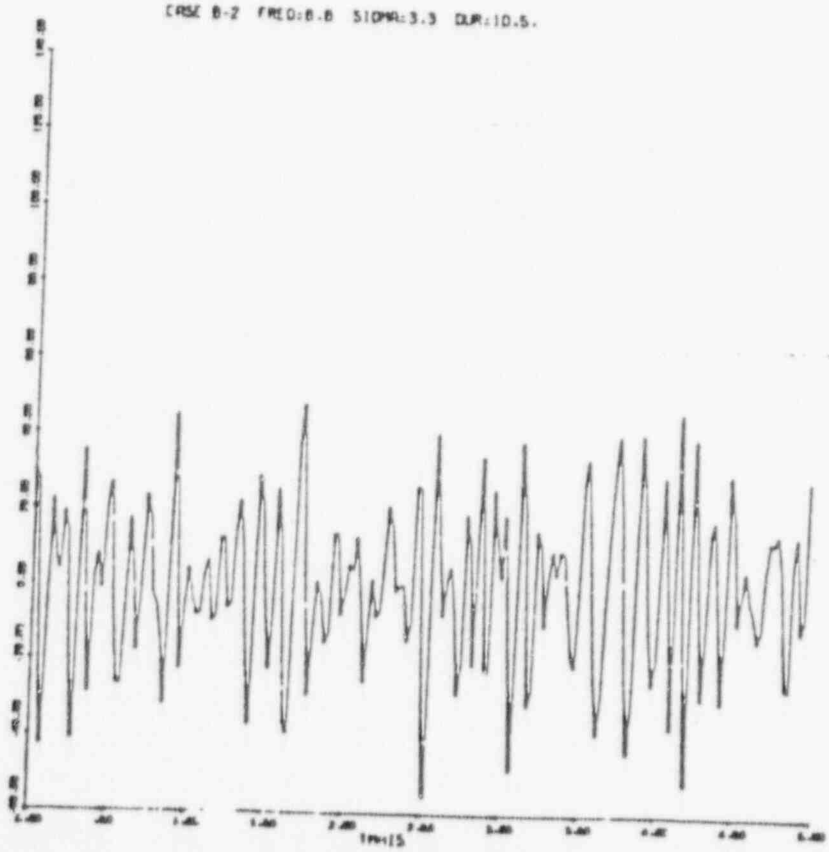
POOR ORIGINAL



CASE B-2 FREQ:8.8 SIGMA:3.3 DUR:10.5.



CASE B 2 FREQ:8.8 SIGMA:3.3 DUR:10.5.

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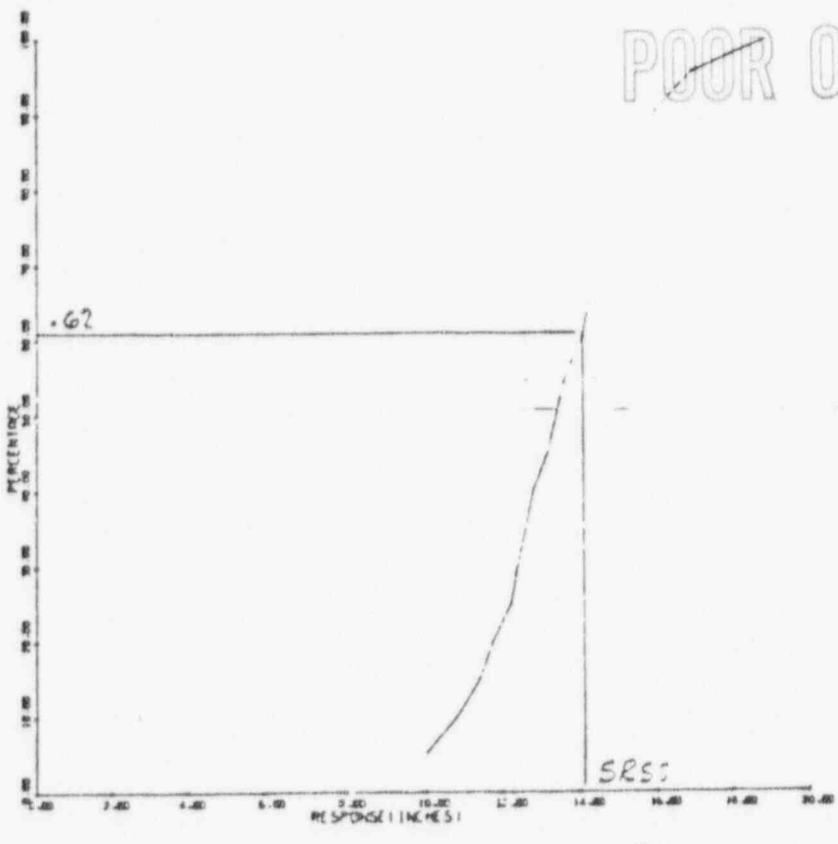
POOR ORIGINAL



CASE B-2 FREQ:8.8 SIGMA:3.3 DUR:10.5. CDF

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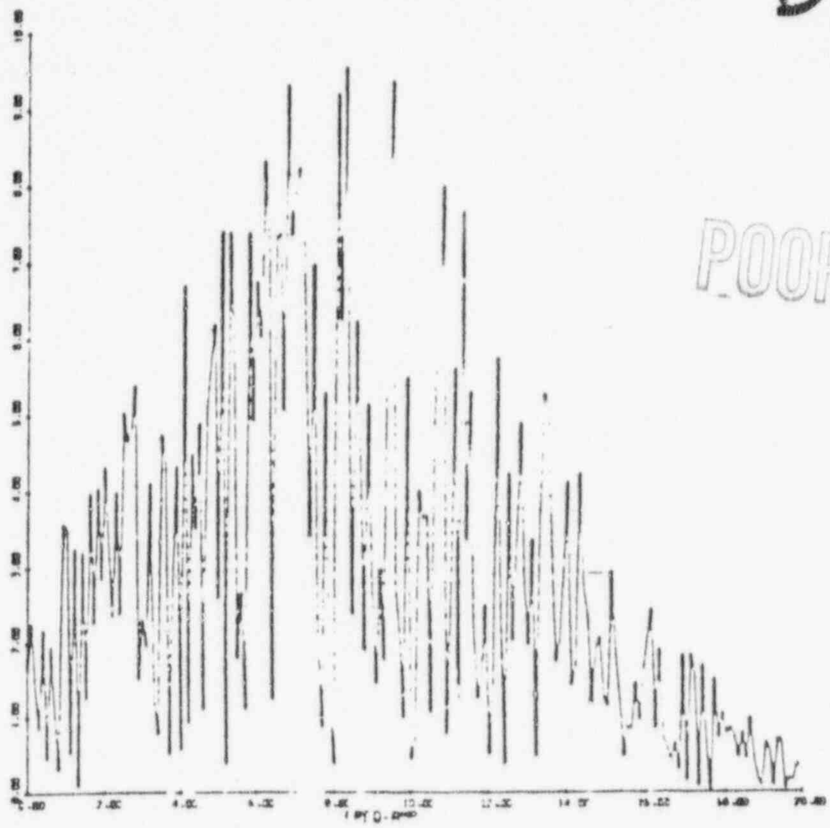
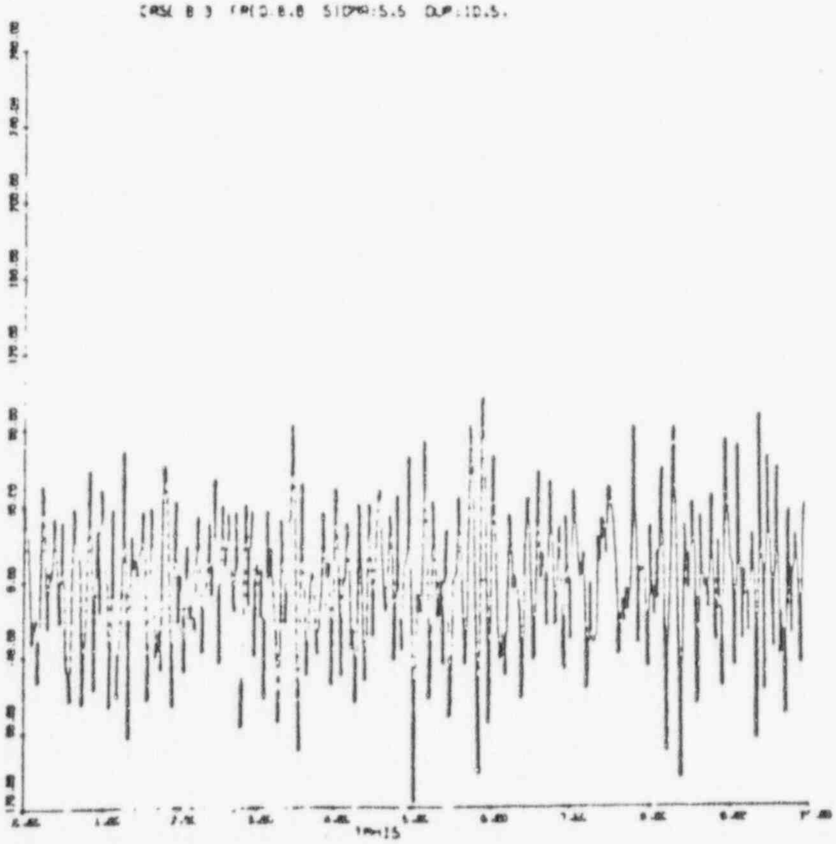


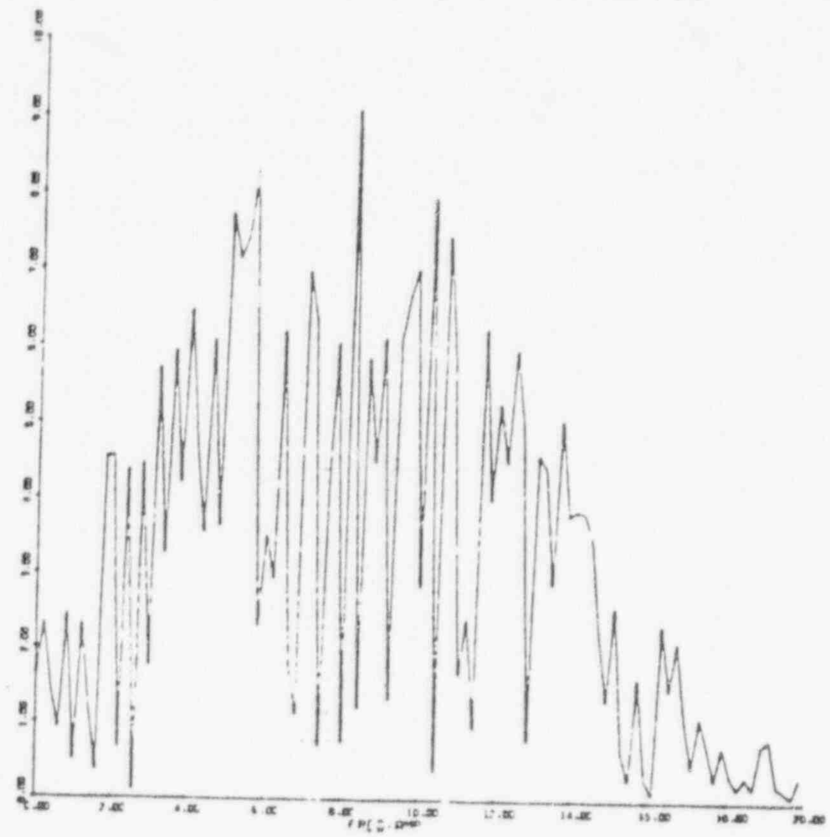
CASE B 3 FREQ:8.8 SIGMA:5.5 DUR:10.5.



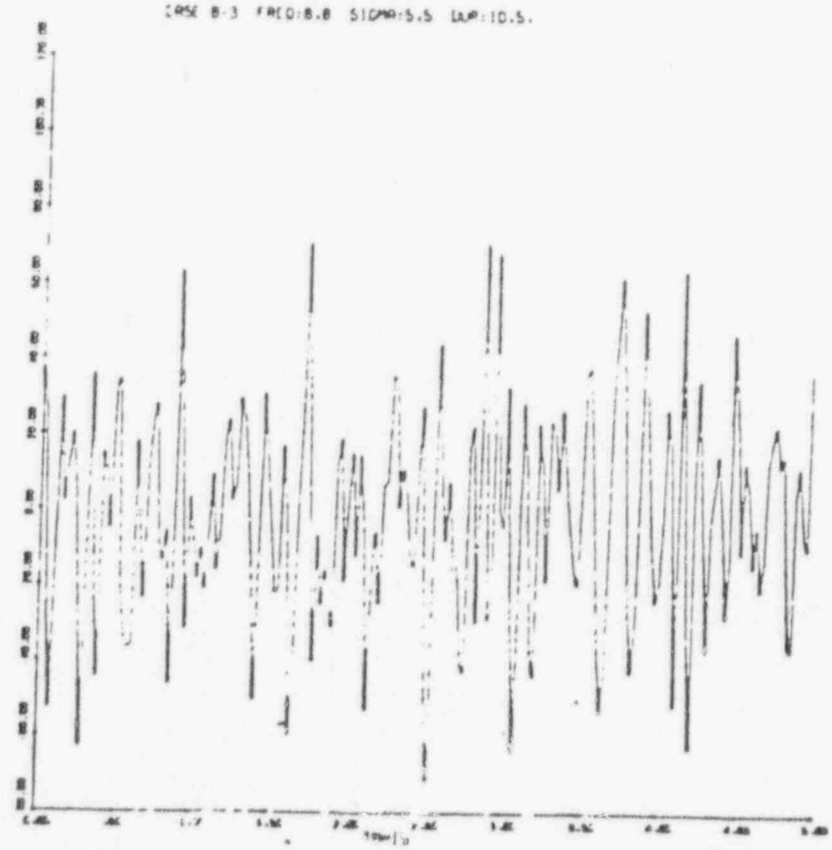
CASE B 3 FREQ 8.8 SIGMA 5.5 DUR 10.5.

3

POOR ORIGINAL DRAFT



CASE B-3 FREQ:8.8 SIGMA:5.5 DUR:10.5.



CASE B-3 FREQ:8.8 SIGMA:5.5 DUR:10.5.

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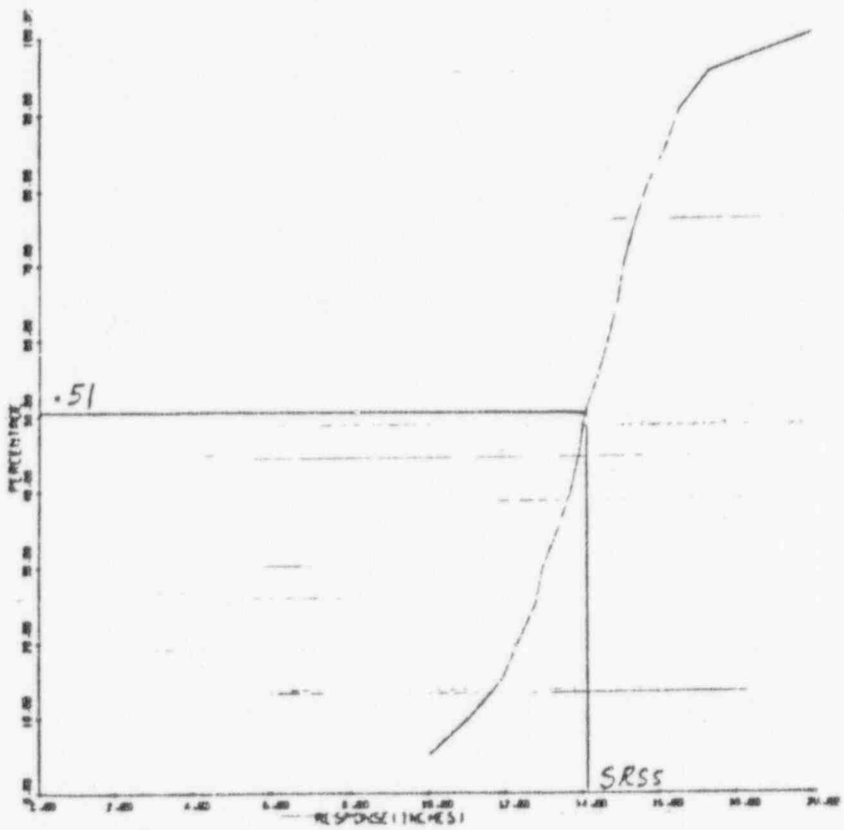
POOR ORIGINAL



CASE B-3 FREQ:8.8 SIGMA:5.5 DUR:10.5. CDF

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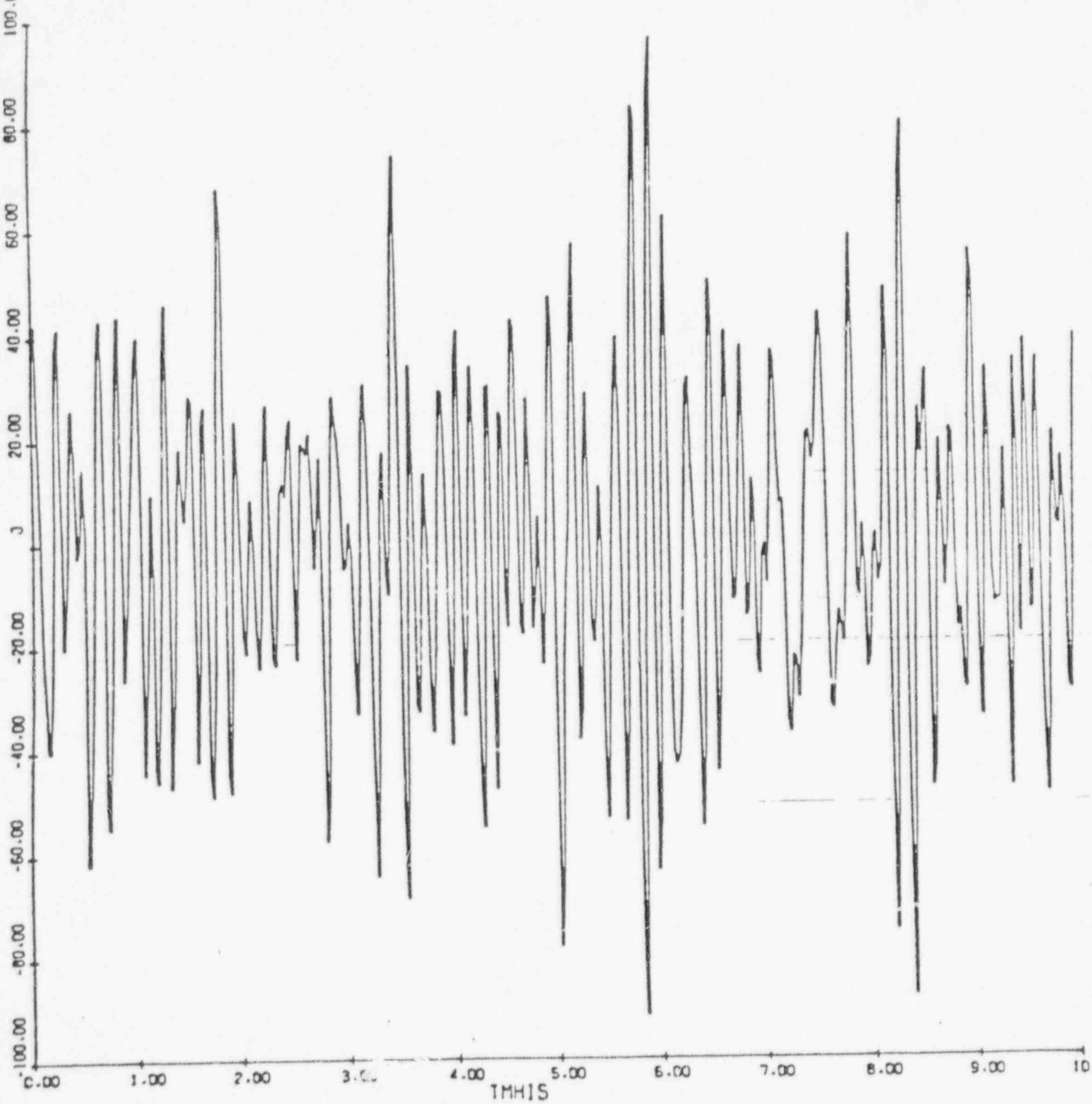

CASE L'-2 FREQ=6.12 SIGMA=3.3 DUR=10.5.

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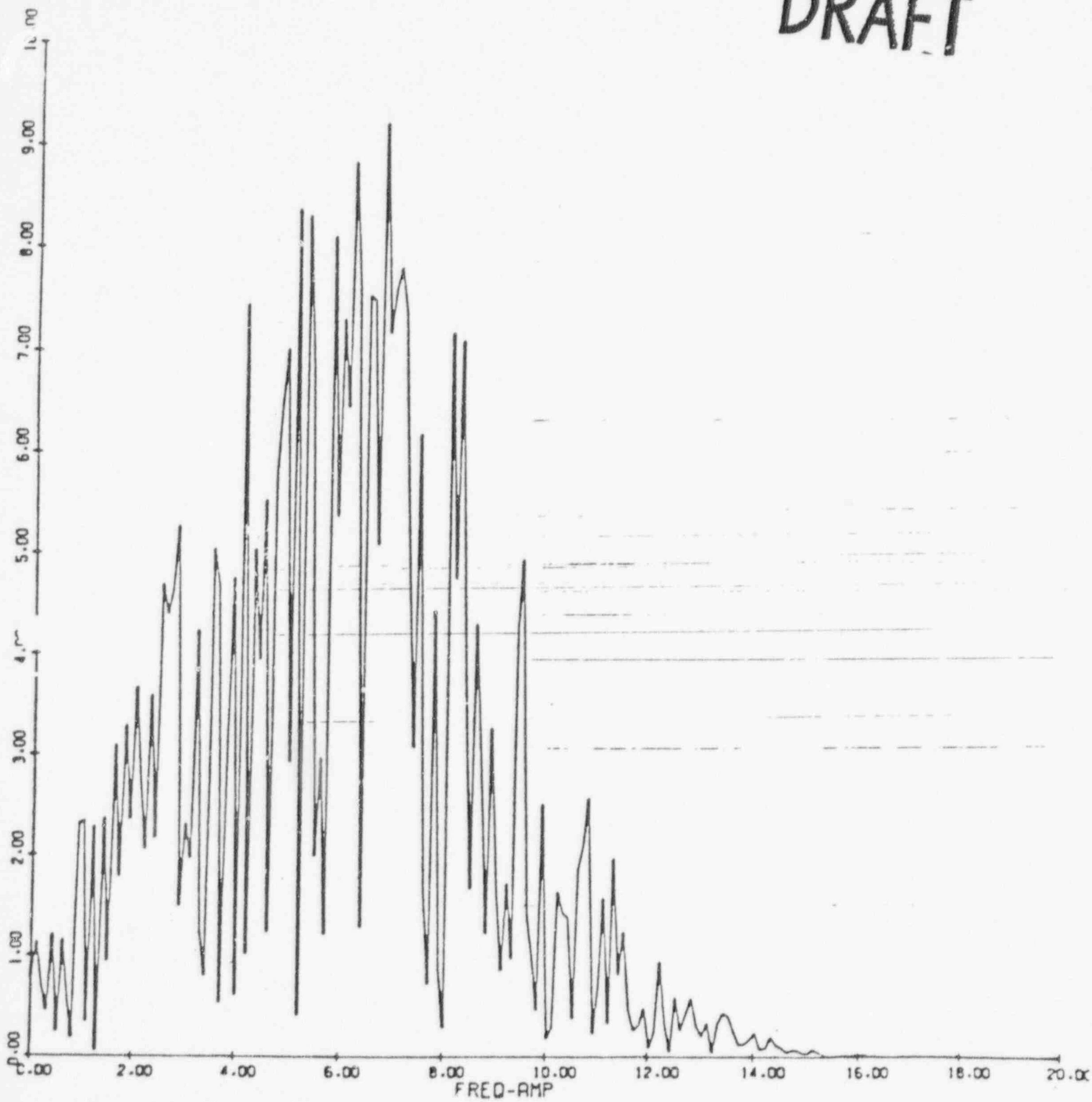

CASE C-2 FREQ=6.12 SIGMA=3.3 DUR=10.5

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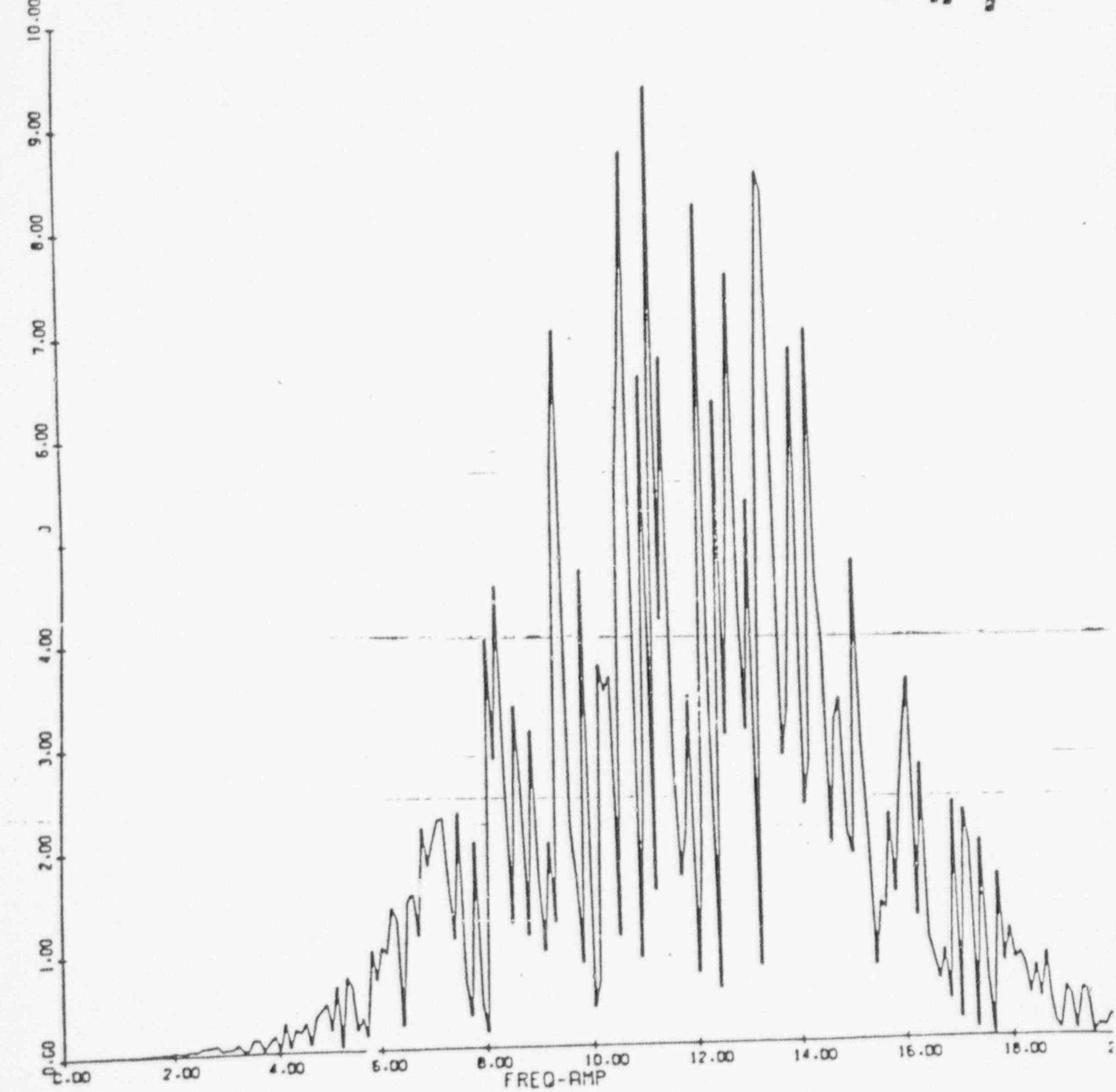

CASE C-2 FREQ=6.12 SIGMA=3.3 DUR=10.5.

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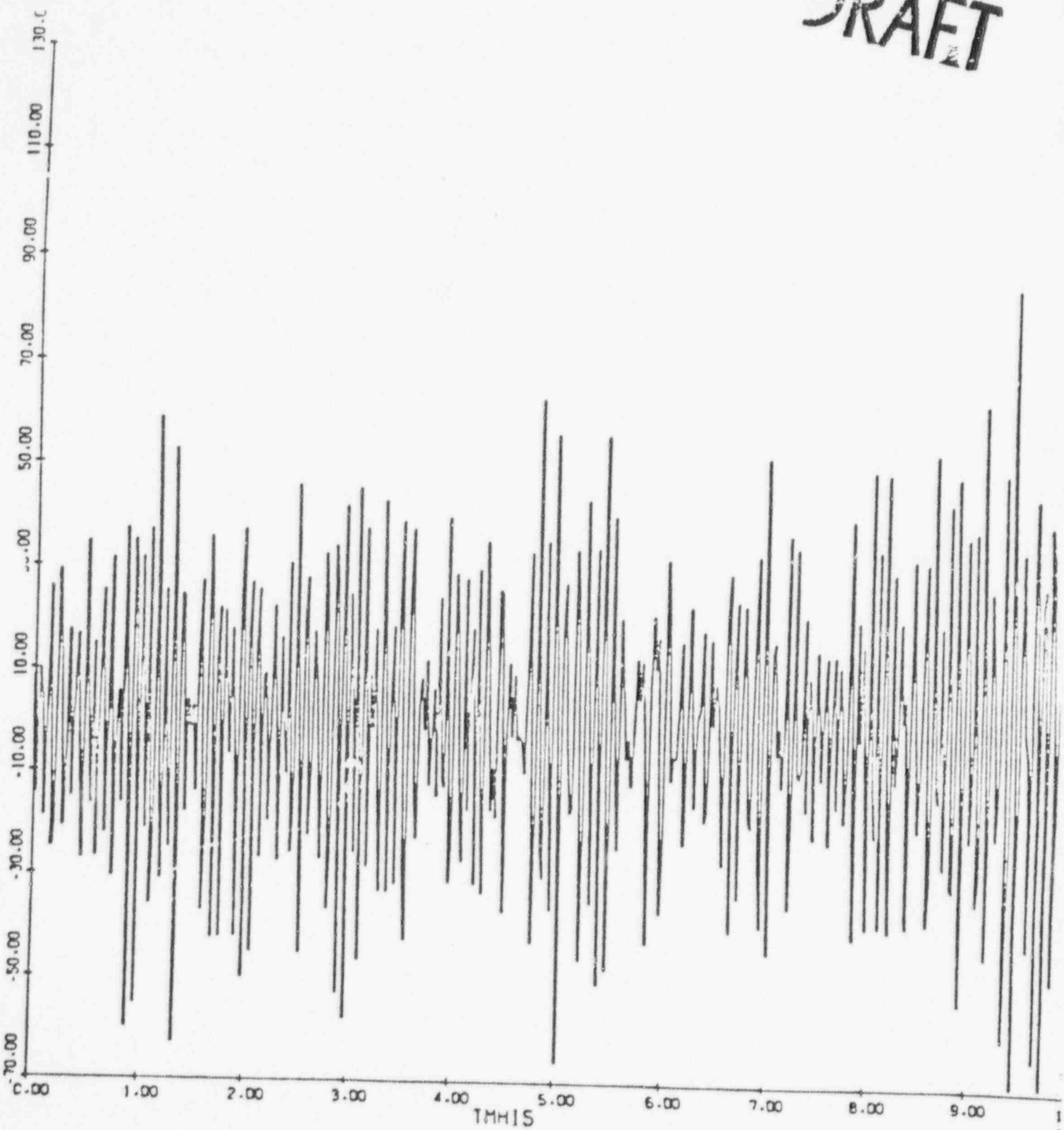

CASE D-2 FREQ=6.12 SIGMA=3.3 DUR=10.5

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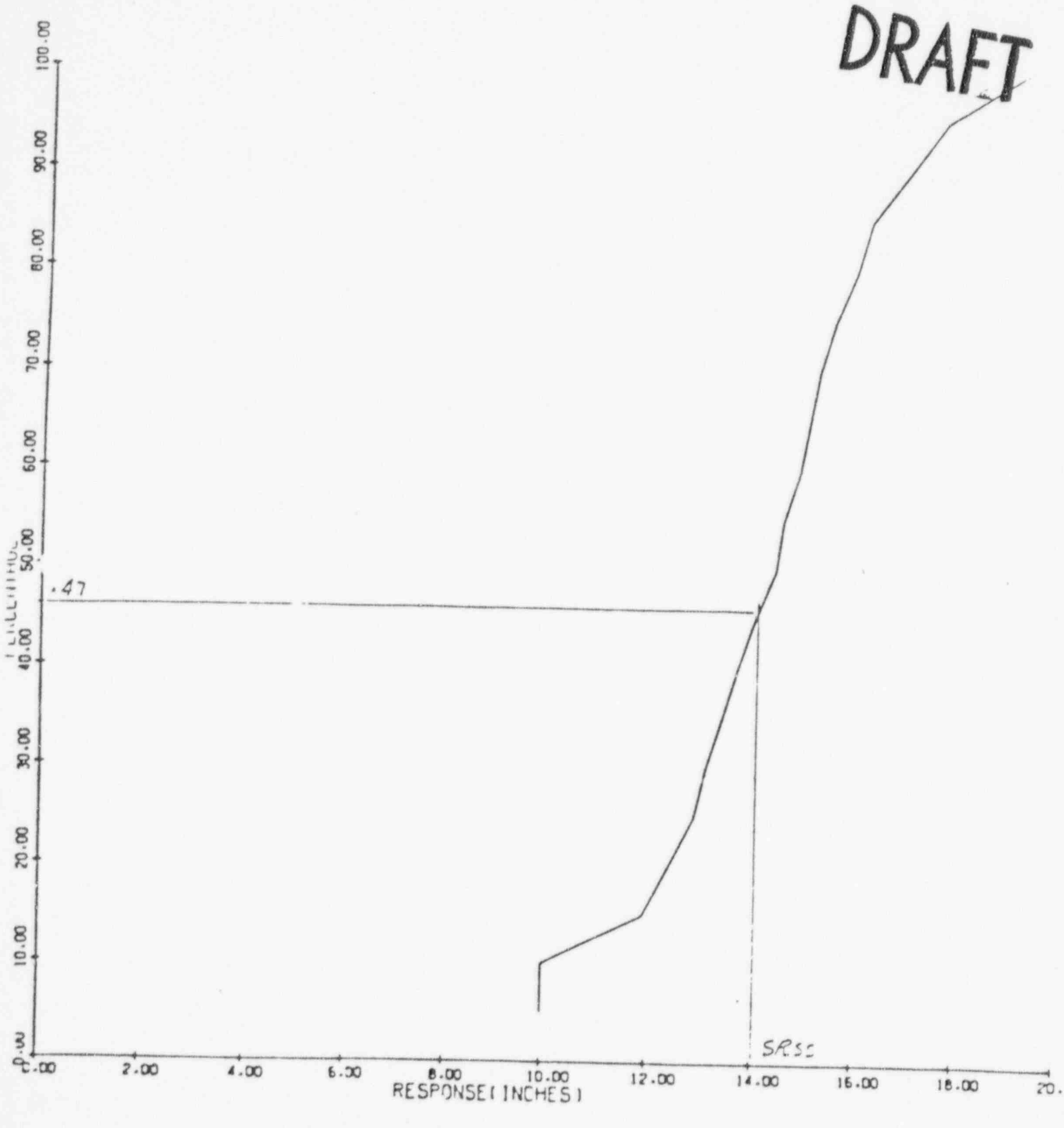

CASE D-2 FREQ=6.'2 SIGMA=3.3 DUR=10.5

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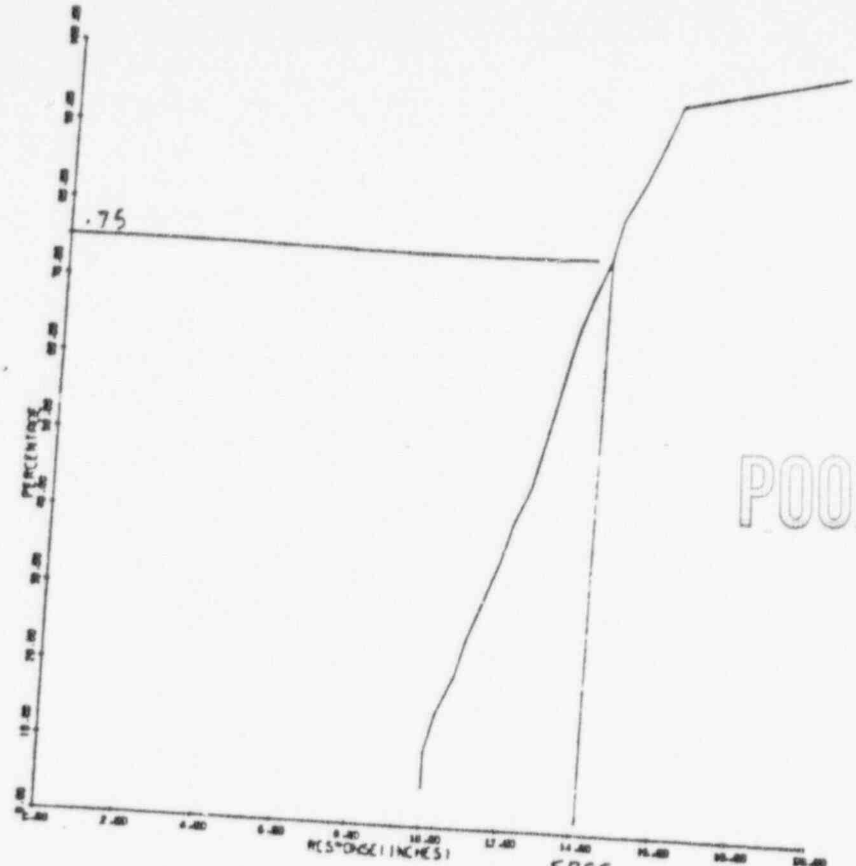
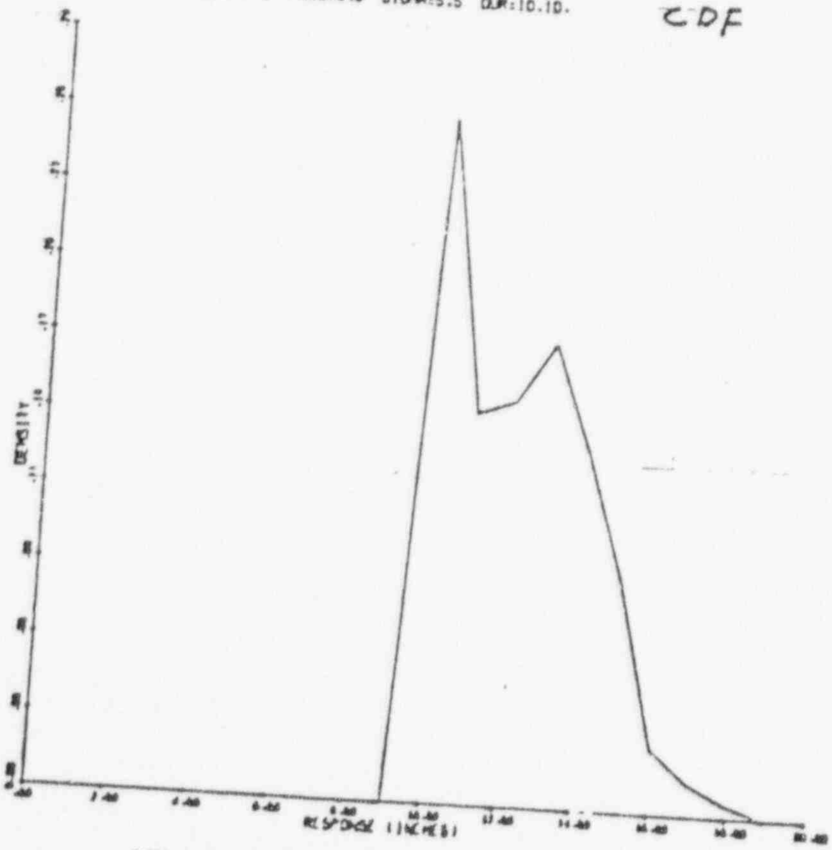
POOR ORIGINAL



SRSS

CASE A-3 FREQ:8.8 SIGMA:5.5 DUR:10.10.

CDF



CASE A-3 FREQ:8.8 SIGMA:5.5 DUR:10.10.

Density

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COMBINATION OF ARTIFICIAL RESPONSES IN TIME DOMAIN

PURPOSE

- To generate artificial responses with time dependent parameters: Occurrence time of peaks; probability of peak occurrences; envelope shape of response curve; sparsity of peaks; filtering frequencies.
- Fourier transform.
- Combine responses with time lag.
- Plot CDF of combined peaks.
- Locate non-exceedance probability of SRSS.
- Formulation

$$R(T) = \int_0^T H(T-t)\ \psi\ (t)\ V\ (t)\ D\ t$$

$H(T)$ - Frequency control filter (impulse response)

$\psi(T)$ - Shape function

$V(T)$ - Random number

Experimental functions

$$H(T) = \frac{1}{\omega_\phi} e^{-5\omega_\alpha t} \operatorname{Sin}\alpha_d t$$

$$\psi(T) = E^{-\alpha T} - E^{-\beta T}$$

$V(T)$ = Gaussian random number or unif. dist. random number.

- RESULTS:

1. Two identical responses, Dur = 10 sec

   Unif. dist. of peak

   No filter

   MAX peak at ± 5 sec

   P (R < SRSS) = 0.29

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2. Two identical responses, Dur = 10 sec

   Gaussian dist. of peak

   No filter

   Max. peak at ± 5 sec

   P (R < SRSS) = 0.60

3. Two identical responses, Dur = 10 sec

   Gaussian dist. of peak

   Filter with impulse responses

   Max peak at 3 sec - 6 sec

   P (R < SRSS) = 0.72

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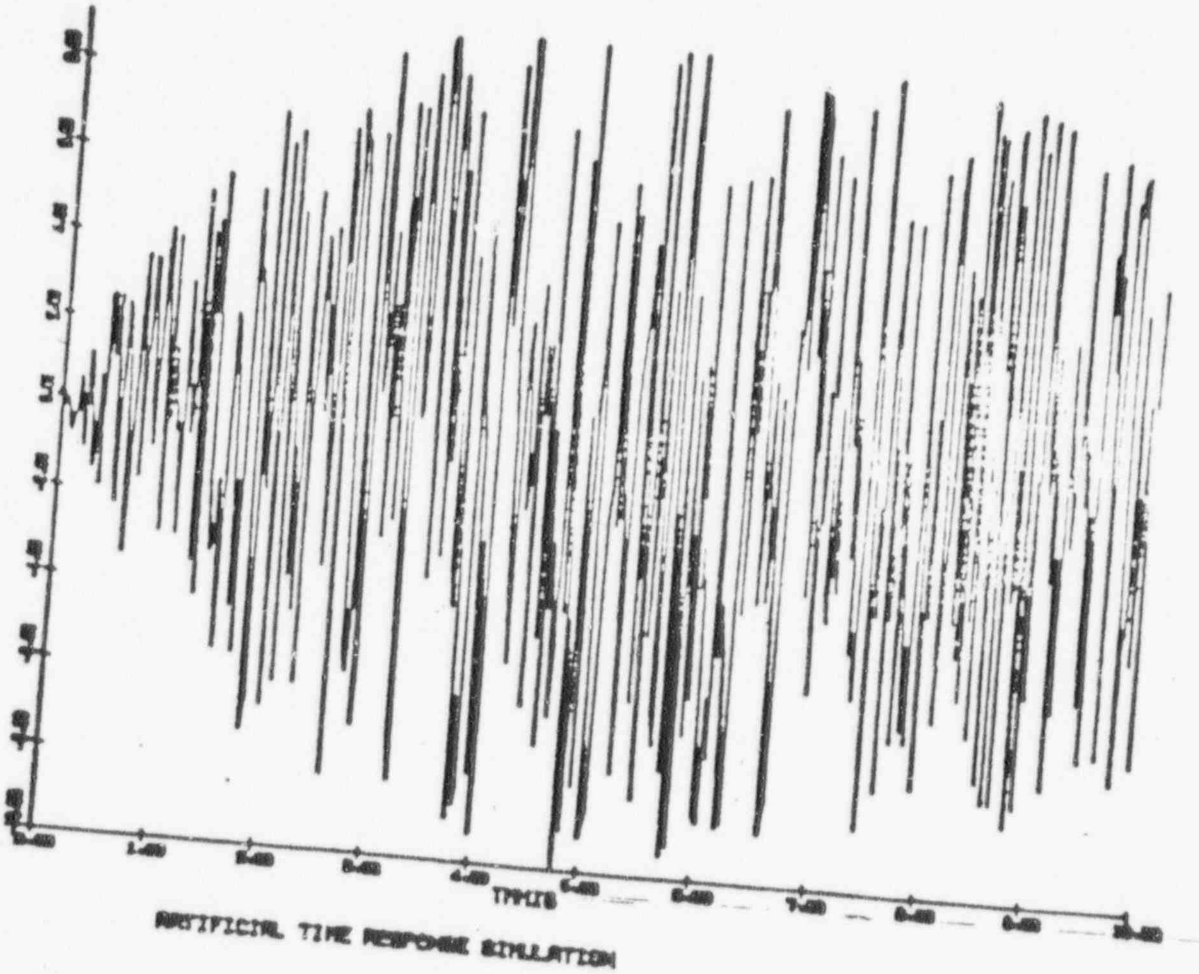

ARTIFICIAL TIME RESPONSE SIMULATION

1. UNIF. DIST OF PEAK
NO FILTER

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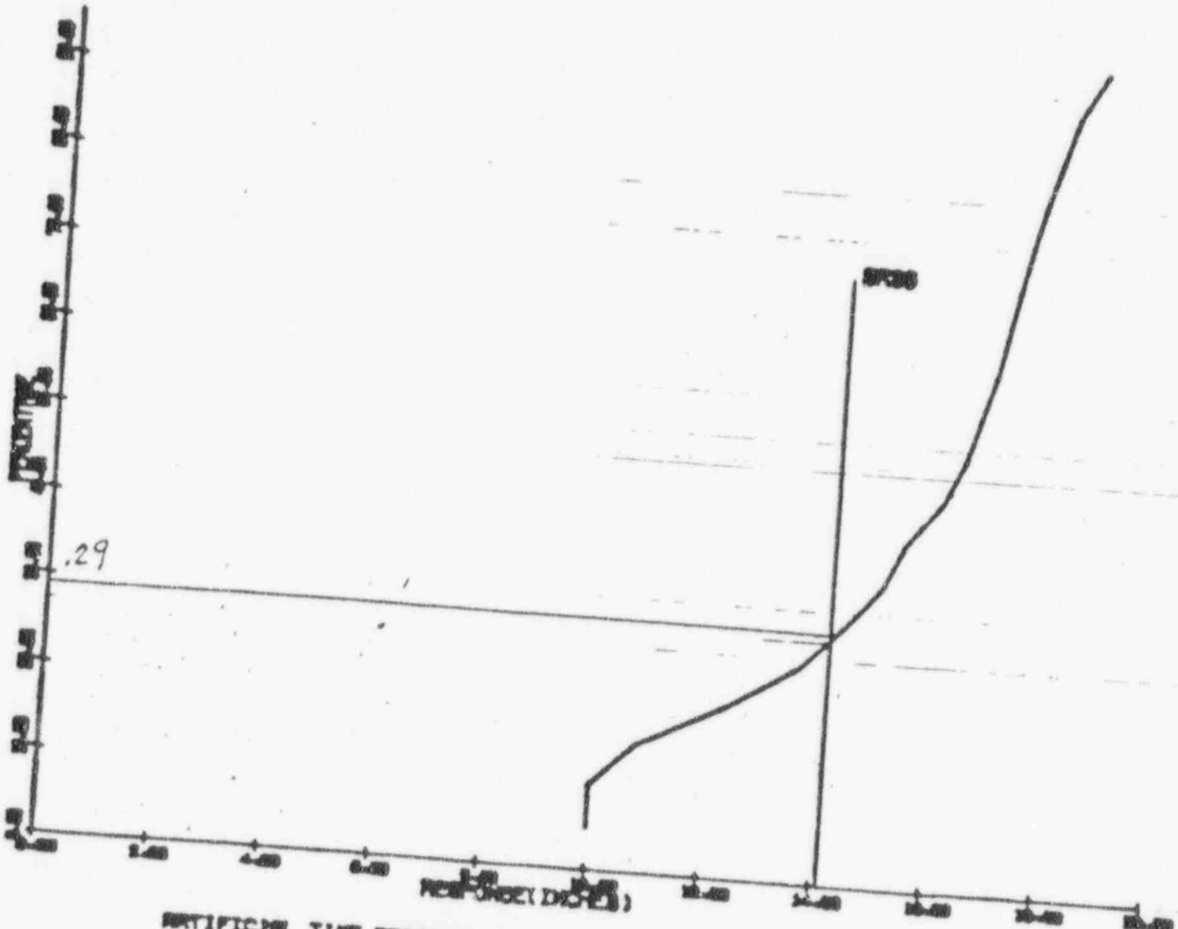
POOR ORIGINAL



ARTIFICIAL TIME RESPONSE SIMULATION

1. UNIF. DIST. OF PEAK CDF
NO FILTER

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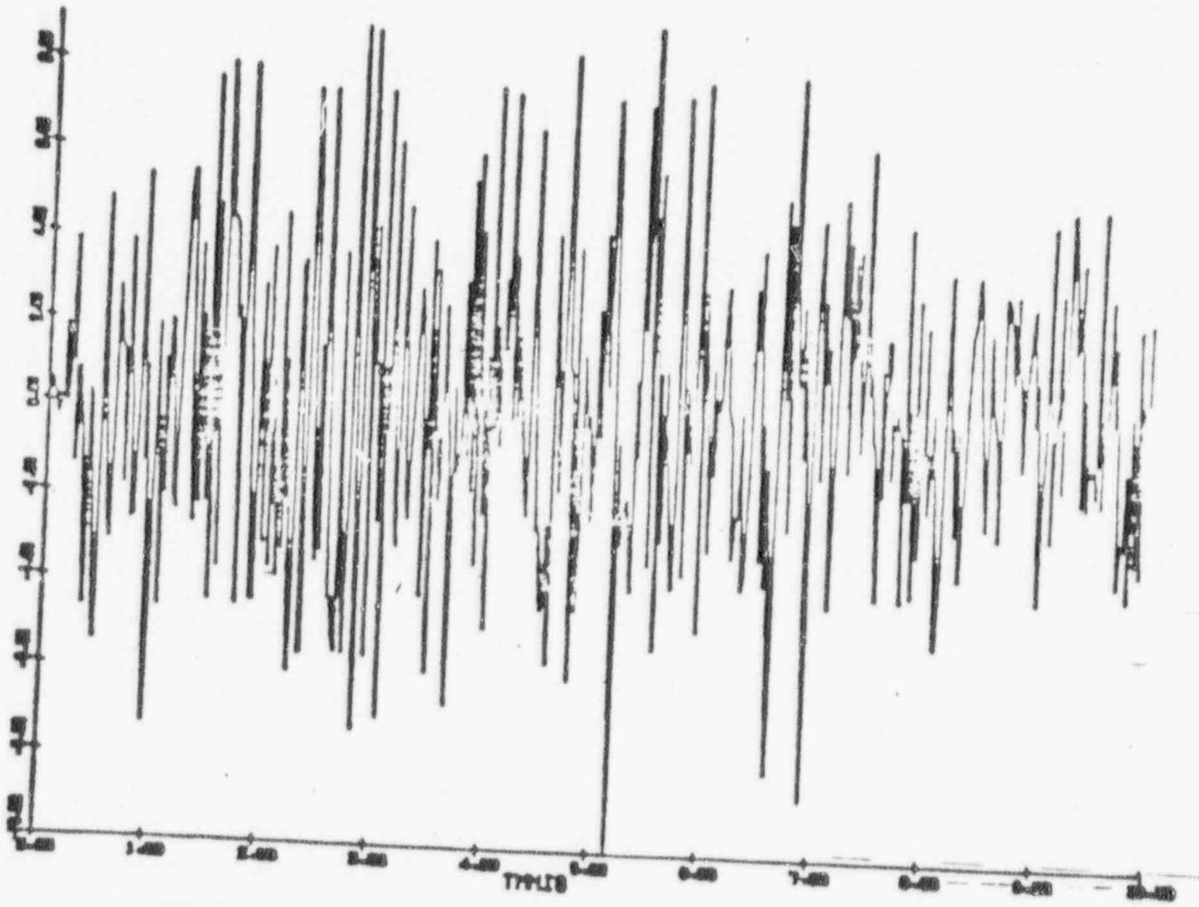

ARTIFICIAL TIME RESPONSE SIMULATION

2. GAUSSIAN DIST OF PEAK
NO FILTER

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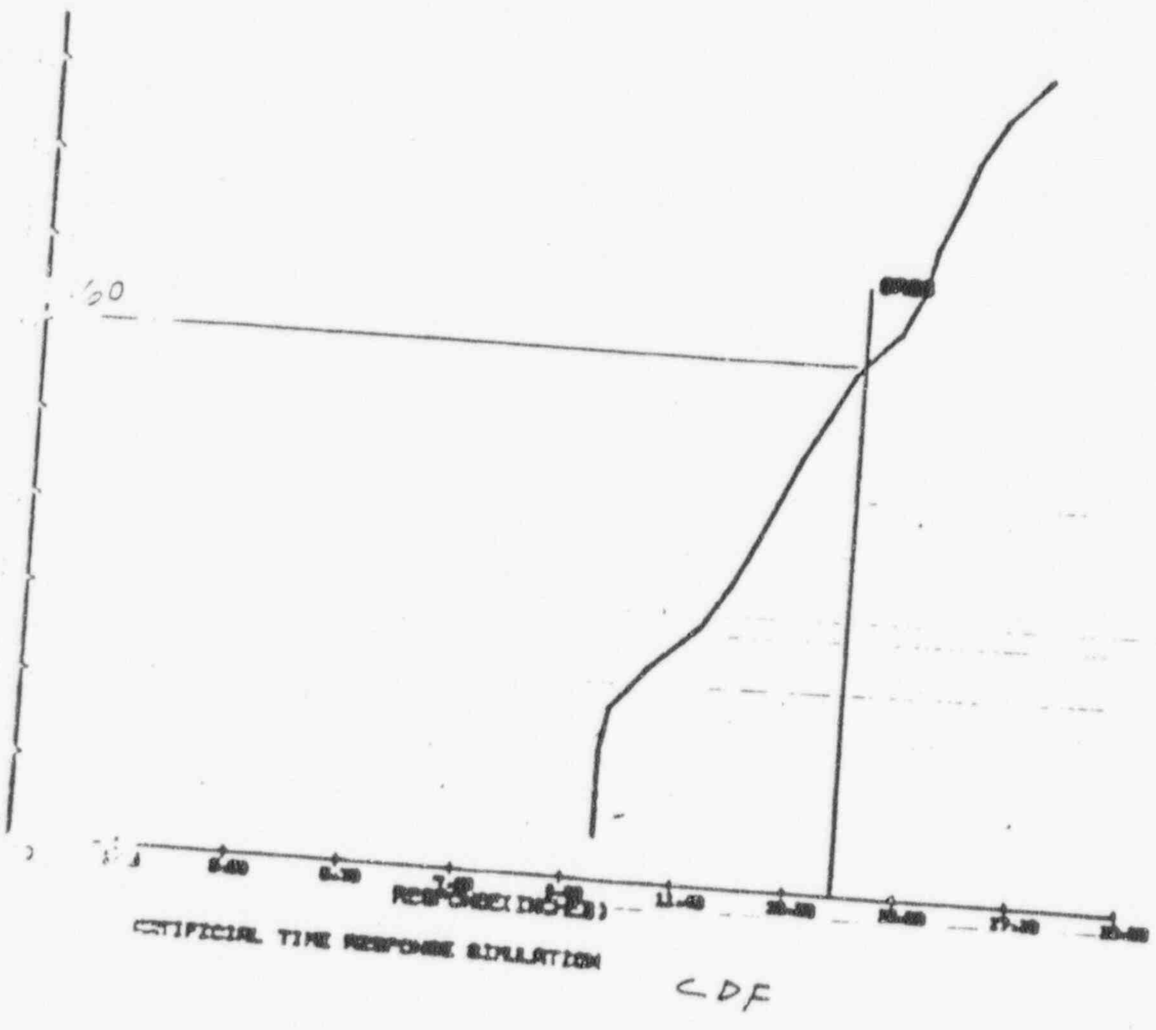

2. GAUSSIAN DIST. OF PEAK
NO FILTER

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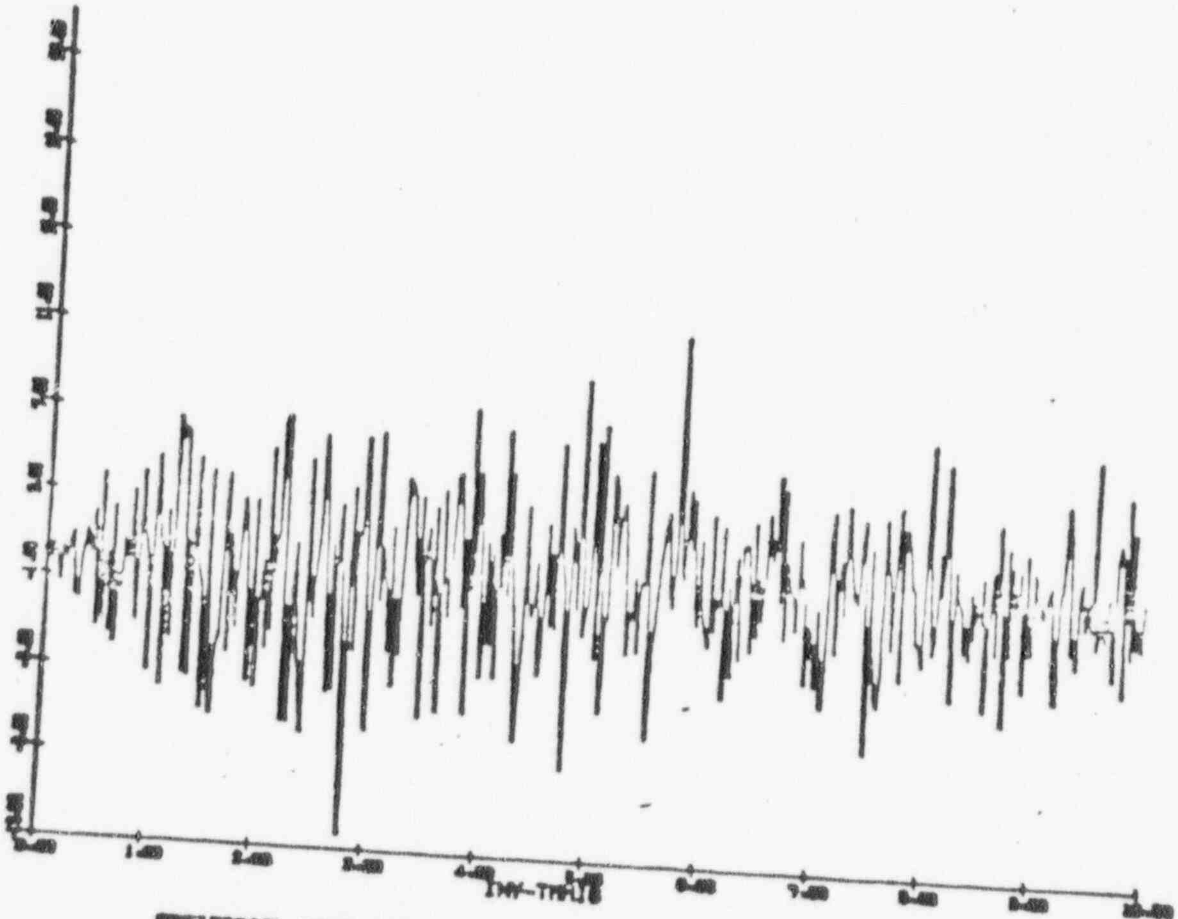
POOR ORIGINAL



ARTIFICIAL TIME RESPONSE SIMULATION

3. GAUSSIAN PEAK DIST.
FILTERED

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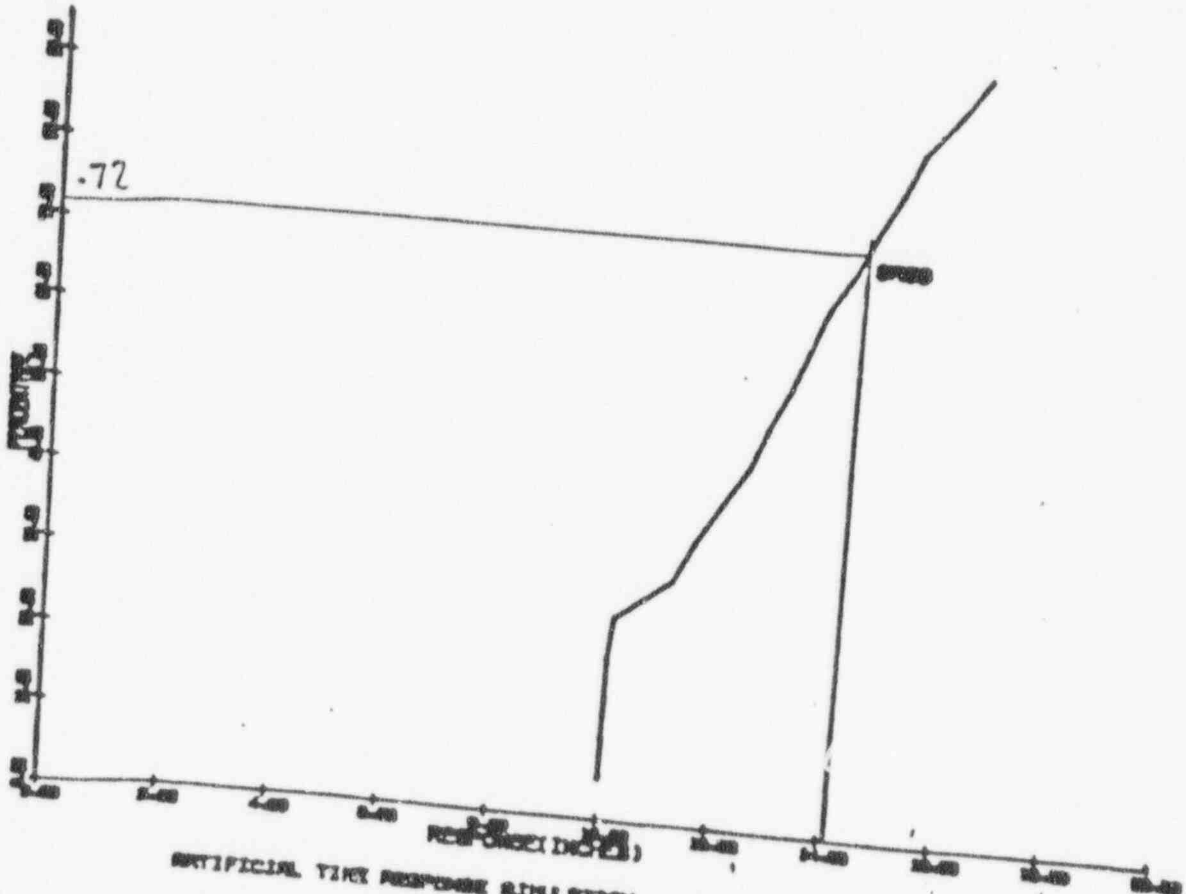
POOR ORIGINAL



ARTIFICIAL TIME RESPONSE SIMULATION CDF

3. GAUSSIAN PEAK DIST.
FILTERED

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APPENDIX D

REVIEW OF SRSS METHOD FOR COMBINING LOADS FOR MARK II PLANTS

INTRODUCTION

General Electric's technical basis for applying the square root of the sum of the squares method when combining dynamic loads (or rather peak responses) for MARK II plants were presented in a General Electric Report NEDE-24101-P. The Structural Analysis Group of Brookhaven National Laboratory, on behalf of the Mechanical Engineering Branch/DSS of the Nuclear Regulatory Commission, has made a review of the two criteria for load combinations presented in the report. In addition, a limited number of dynamic response data provided by General Electric (GE) was utilized to obtain load combination values for MARK II plants. The conclusions drawn from this review are thus constrained by the available data which consisted of relatively long duration earthquakes ($\approx$ 10 - 15 sec) with one or more short duration CHUG or SRV loads. Generic conclusions regarding criteria I and II of NEDE-24010-P will be given in a final report to be issued shortly. This report is limited to the load combination results obtained from the use of the MARK II data provided by GE.

BACKGROUND

MARK II plants may be subjected to simultaneous dynamic load occurrences such as earthquake (OBE or SSE), loss of coolant events (LOCA), and suppression pool dynamic loadings (SRV or CHUG). The response of each load is computed

*Work performed under the auspices of the U. S. Nuclear Regulatory Commission.

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separately. The peaks of these responses must subsequently be combined in a rational manner so that a safe design is achieved. The GE report justifies the use of the SRSS method for combining dynamic responses by the following factors:

(1) The calculation of the conditional probability level, based upon the condition that the events and maximum calculated response occur, if not exceeding the SRSS load combination value, has a mean non-exceedance value of 86 percent. In addition, 98.4 percent of all load combinations had a non-exceedance probability of greater than 50 percent.

(2) The very low probability of the simultaneous occurrence of the individual events which produce the dynamic loads being combined.

(3) In comparing the SRSS with the absolute sum rule for combining dynamic loads, the difference in calculated reliability is very small. For the representative cases included in this report, the reliability value range for SRSS is 0.99999425 to 0.99999750 and for absolute sum is 0.99999680 to 0.99999750.

(4) Demonstration that even in those few cases where non-exceedance probability of the SRSS fell below 50 percent, the effect on structural reliability is minimal.

The limited review reported here by Brookhaven National Laboratory (BNL) is restricted to statement (1). The response functions used for combinations were provided by GE in digitized form. These are identified in Table I.

Only the first eleven sets were actually selected for the BNL combination studies because of the relative longer duration time of these particular SRV and CHUG loads. The reason for this is based upon our previous generic studies which showed that for cases involving response combinations with signals having large differences in time duration, SRSS values would have high non-exceedance probabilities. Thus, the very short duration cases were not considered.

TABLE I

TIME HISTORIES FOR RPV & INTERNALS

SET	RESPONSE	DELTA T	DURATION	ELEMENT	FORCE	UNIT	RECORDS
1	VERTICLE OBE	0.004	14.998	GUIDE TUBE	AXIAL	KIPS	375
2	VERTICLE OBE	0.004	14.996	SHROUD SUPPORT	AXIAL	KIPS	375
3	VERTICLE SSE	0.004	14.996	GUIDE TUBE	AXIAL	KIPS	375
4	VERTICLE SSE	0.004	14.996	RPV SKIRT	AXIAL	KIPS	375
5	VERTICLE SSE	0.004	14.996	SHROUD SUPPORT	AXIAL	KIPS	375
6	AXISYMETRIC CHUG	0.001	0.999	GUIDE TUBE	AXIAL	KIPS	100
7	AXISYMETRIC CHUG	0.001	0.999	RPV SKIRT	AXIAL	KIPS	100
8	AXISYMETRIC CHUG	0.001	0.999	SHROUD SUPPORT	AXIAL	KIPS	100
9	AXISYMETRIC SRV (7.5 HZ)	0.001	0.999	GUIDE TUBE	AXIAL	KIPS	100
10	AXISYMETRIC SRV (7.5 HZ)	0.001	0.999	RPV SKIRT	AXIAL	KIPS	100
11	AXISYMETRIC SRV (7.5 HZ)	0.001	0.999	SHROUD SUPPORT	AXIAL	KIPS	100
12	HORIZONTAL OBE	0.010	14.980	GUIDE TUBE	SHEAR	KIPS	150
13	HORIZONTAL OBE	0.010	14.980	GUIDE TUBE	MOMENT	K-IN	150
14	HORIZONTAL OBE	0.010	14.980	SHROUD SUPPORT	SHEAR	KIPS	150
15	HORIZONTAL OBE	0.010	14.980	SHROUD SUPPORT	MOMENT	K-IN	150
16	HORIZONTAL OBE	0.010	14.980	RPV SKIRT	SHEAR	KIPS	150
17	HORIZONTAL OBE	0.010	14.980	RPV SKIRT	MOMENT	K-IN	150
18	HORIZONTAL SSE	0.010	14.980	GUIDE TUBE	SHEAR	KIPS	150
19	HORIZONTAL SSE	0.010	14.980	GUIDE TUBE	MOMENT	K-IN	150
20	HORIZONTAL SSE	0.010	14.980	SHROUD SUPPORT	SHEAR	KIPS	150
21	HORIZONTAL SSE	0.010	14.980	SHROUD SUPPORT	MOMENT	K-IN	150
22	HORIZONTAL SSE	0.010	14.980	RPV SKIRT	SHEAR	KIPS	150
23	HORIZONTAL SSE	0.010	14.980	RPV SKIRT	MOMENT	K-IN	150
24	AXISYMETRIC CHUG (20 HZ)	0.001	0.497	GUIDE TUBE	SHEAR	KIPS	50
25	AXISYMETRIC CHUG (20 HZ)	0.001	0.497	GUIDE TUBE	MOMENT	K-IN	50
26	AXISYMETRIC CHUG (20 HZ)	0.001	0.497	SHROUD SUPPORT	SHEAR	KIPS	50
27	AXISYMETRIC CHUG (20 HZ)	0.001	0.497	SHROUD SUPPORT	MOMENT	K-IN	50
28	AXISYMETRIC CHUG (20 HZ)	0.001	0.497	RPV SKIRT	SHEAR	KIPS	50
29	AXISYMETRIC CHUG (20 HZ)	0.001	0.497	RPV SKIRT	MOMENT	K-IN	50
30	AXISYMETRIC SRV (7.5 HZ)	0.001	0.597	GUIDE TUBE	SHEAR	KIPS	60
31	AXISYMETRIC SRV (7.5 HZ)	0.001	0.597	GUIDE TUBE	MOMENT	K-IN	60
32	AXISYMETRIC SRV (7.5 HZ)	0.001	0.597	SHROUD SUPPORT	SHEAR	KIPS	60
33	AXISYMETRIC SRV (7.5 HZ)	0.001	0.597	SHROUD SUPPORT	MOMENT	K-IN	60
34	AXISYMETRIC SRV (7.5 HZ)	0.001	0.597	RPV SKIRT	SHEAR	KIPS	60
35	AXISYMETRIC SRV (7.5 HZ)	0.001	0.597	RPV SKIRT	MOMENT	K-IN	60

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METHOD OF CONSTRUCTING NON-EXCEEDANCE CUMULATIVE DENSITY FUNCTION (CDF)

The SRSS value of combining several dynamic responses is given by the following expression:

$$R_{SRSS} = R_{1m}^{2} + R_{2m}^{2} \ldots R_{nm}^{2}$$

where R_{1m}, R_{2m} ... R_{nm} are the peak responses of dynamic responses R_1, R_2 ... R_n, respectively. Furthermore, the absolute values of these response peaks are generally used to compute the value of R_{SRSS}. The responses are conditionally assumed to occur concurrently but with random starting times (τ) as shown in Figure 1.

For each set of random values of τ, the responses are combined algebraically at corresponding time steps. The absolute maximum value of the combination is recorded as one sample combination result. Thus, with sufficient sample points, a CDF curve can be constructed and the non-exceedance probability (NEP) of R_{SRSS} can be located on the curve.

In contrast to BNL's approach of construction of the CDF curve, GE separates the responses into two parts, i.e., plus and minum. Thus, in GE's calculation, there are two SRSS values and two CDF curves. Consequently, the NEP of SRSS also has two values which will be higher than the corresponding BNL values for the NEP of SRSS.

In constructing the response CDF curve, two salient factors must be considered: (1) the density function of the random starting time, and (2) the interval that is to be spanned after the starting time of the responses. It appears from the results that the variation of the density function of the random starting time plays an insignificant role in the NEP of R_{SRSS}. Thus, in

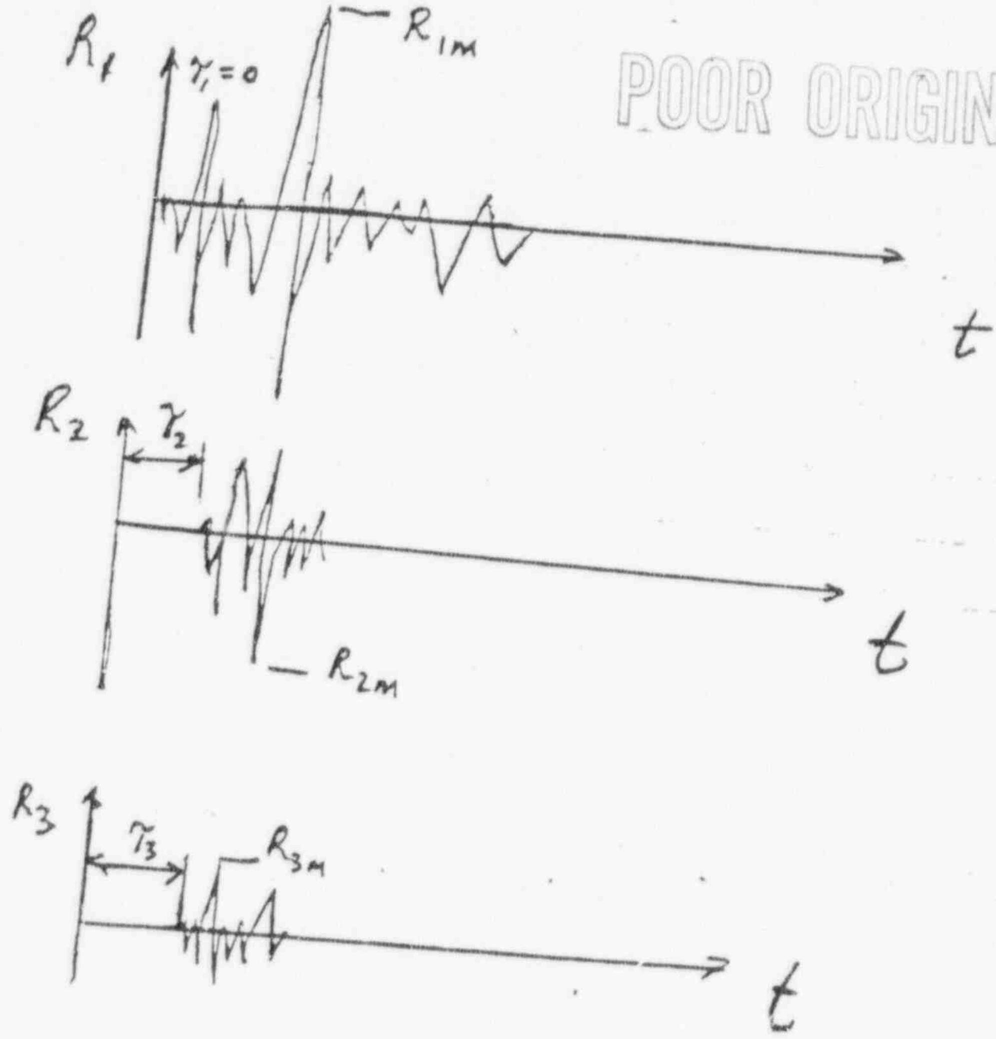

FIGURE 1

the final verification computer runs, normal and uniform distributions were used. With regard to the selection of the starting time τ_i, it appears that for combining responses of approximately equal durations, the random starting time of the first response can be assumed as zero, while the remaining responses can start uniformly between zero and the end of the first response. On the other hand, if the first response has a much longer duration than that of the second response, then the assumption of uniform distribution of τ between zero and the end of the first (long) response will result in higher (and perhaps less conservative) NEP values of SRSS. Thus in the verification computer runs carried out at BNL, the starting time of the second and subsequent responses were assumed to be normally distributed about the occurrence time of the peak of the first response as the mean and with the duration of the remaining reponses as the standard deviation (see Figure 2).

RESULTS

Five cases of response combinations were examined. Each of these cases involved digitized data obtained from General Electric and can be identified from Table I. Specifically, these involve the following combinations of three sets of signals:

(1) guide tube axial sets, i.e., sets 1 + 6 + 9, OBE + CHUG + SRV

(2) guide tube axial, i.e., sets 3 + 6 + 9, SSE + CHUG + SRV

(3) shroud support axial, i.e., sets 2 + 8 + 11, OBE + CHUG + SRV

(4) shroud support axial, i.e., sets 5 + 8 + 11, SSE + CHUG + SRV

(5) RPV skirt axial, i.e., sets 4 + 7 + 10, SSE + CHUG + SRV

Three additional cases involving subsets of two responses were also investigated. These involve:

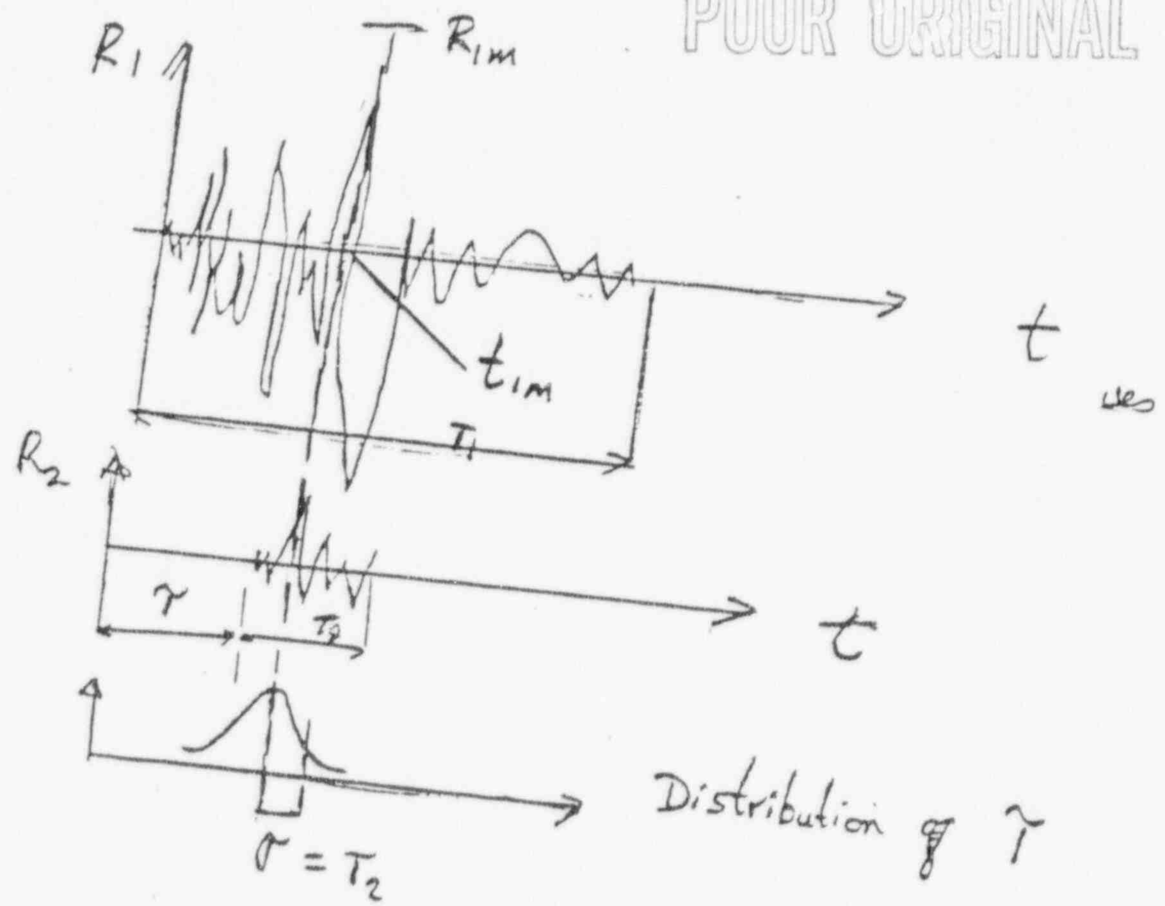

FIGURE 2

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(6) guide tube axial, i.e., sets 6 + 9, CHUG + SRV

(7) shroud support axial, i.e., sets 8 + 11, CHUG + SRV

(8) RPV skirt axial, i.e., sets 7 + 10, CHUG + SRV

Results for these evaluations are given in Table II. For the first five cases, the average values of NEP of SRSS are in the range of 86 percent, while for the last three cases, the average is approximately 76 percent. The last three cases involve only the combinations of SRV and CHUG which have similar maximum peak values and time durations. As can be seen in Table II, the supplied GE SSE and OBE data had low σ/R_{max} values (where σ is the square root of the sum of the squares of the digitized values and R is the maximum value), while the CHUG and SRV data had higher σ/R_{max} values. This is the reason for selecting these cases and accounts for the lowering of the NEP of the SRSS. The general significance of the σ/R_{max} parameter will be discussed in greater detail in our final reports.

In addition to the information supplied to us, as shown in Table I, we were also recently given information by Sargent and Lunday regarding the responses for the main steam line, the feedwater line, and the residual heat removal line. We did not receive this information in digitized form and hence no SRSS check with our methods could be made at this time. It is to be noted, however, that the information supplied by Sargent and Lunday indicated similar characteristics to those of Table I.

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TABLE II

Comb. No.	Comp. No.	Peak at (Time)	SRSS	Dur. (sec)	No. of Peaks > 75% max	No. of Peaks > 60% max	$\frac{\sigma}{R_{max}}$	Prob(R_{comb} < SRSS)
1 guide tube axial	OBE	30.7 (4.14)	315.2	15	13	58	.296	.87
	CHUG	206.9 (.06)		1	7	9	.332	
	SRV	235.8 (.07)		1	6	10	.352	
2 guided tube axial	SSE	61.0 (4.14)	319.57	15	13	59	.294	.875
	CHUG	206.9 (.06)		1	7	9	.332	
	SRV	235.8 (.07)		1	6	10	.352	
3 shroud support axial	OBE	31.6 (4.14)	280.3	15	13	56	.294	.86
	CHUG	194.6 (.16)		1	7	9	.340	
	SRV	199.3 (.12)		1	9	17	.396	
4 shroud support axial	SSE	62.9 (4.14)	285.56	15	13	57	.294	.86
	CHUG	194.6 (.16)		1	7	9	.340	
	SRV	199.3 (.12)		1	9	17	.396	
5 RPV skirt axial	SSE	326.6 (4.14)	1406.74	15	13	50	.295	.87
	CHUG	990.8 (.16)		1	7	10	.328	
	SRV	943.7 (.12)		1	12	17	.348	

TABLE II, continued
Sub-Case of Response

Comb. No.	Comp. No.	Peak at (Time)	SRSS	Dur. (sec)	No. of Peaks > 75% max	No. of Peaks > 60% max	$\frac{\sigma}{R_{max}}$	Prob(R_{comb} < SRSS)
6 guided tube axial	CHUG	206.9 (.06)	.313.7	1	7	9	.332	.795
	SRV	235.8 (.07)		1	6	10	.352	
7 shroud support axial	CHUG	194.6 (.16)	278.5	1	7	9	.340	.75
	SRV	199.3 (.12)		1	9	17	.396	
8 RPV skirt axial	CHUG	990.8 (.16)	1368.3	1	7	10	.328	.77
	SRV	943.7 (.12)		1	12	17	.348	

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APPENDIX E

COMBINATION OR REAL RESPONSES

- Digitize response curves from:
  - Westinghouse Report
  - G. E. Report
- Fourier transformation to investigate frequency content
- Combine responses with time lag (unif. dist.)
- Plot CDF of the combined peaks
- Locate non-exceedance probability of SRSS

TABLE 6.2

Responses With Non-Zero Mean

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CASE NO.	RESPONSES	SOURCE	P(R ≤ SRSS)	REMARKS
1	LOCA NO. 22 SSE NO. 22	W	.84	SKEWED + " 0
2	LOCA NO. 23 SSE NO. 23	W	.72	" + " +
3	LOCA NO. 24 SSE NO. 24	W	.98	" + " -
4	LOCA NO. 25 SSE NO. 25	W	.48	" - " -
5.	LOCA NO. 26 SSE NO. 26	W	.89	" + " -
6.	LOCA NO. 28 SSE NO. 28	W	.68	" + " 0
7.	LOCA NO. 29 SSE No. 29	W	.81	" + SPARSE " 0
8.	LOCA NO. 30 SSE NO. 30	W	.98	" - " + SPARSE
9.	LOCA NO. 31 SSE NO. 31	W	.98	" - " + SPARSE
10.	FIG. B-36 OBE FIG. B-37 SRV	G.E.	.71	SKEWED 0 0

W - WESTINGHOUSE
G.E. - GENERAL ELECTRIC

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CONCLUSION:

1. Sparsity of peaks:

   $P(R < SRSS)$ increases with sparsity of peaks.

2. Skewness of mean response

   $P(R < SRSS)$ increases with opposite skewness but decreases with same skewness.

3. $0.48 \leq P(R \leq STRSS) \leq 0.98$

   From ten cases

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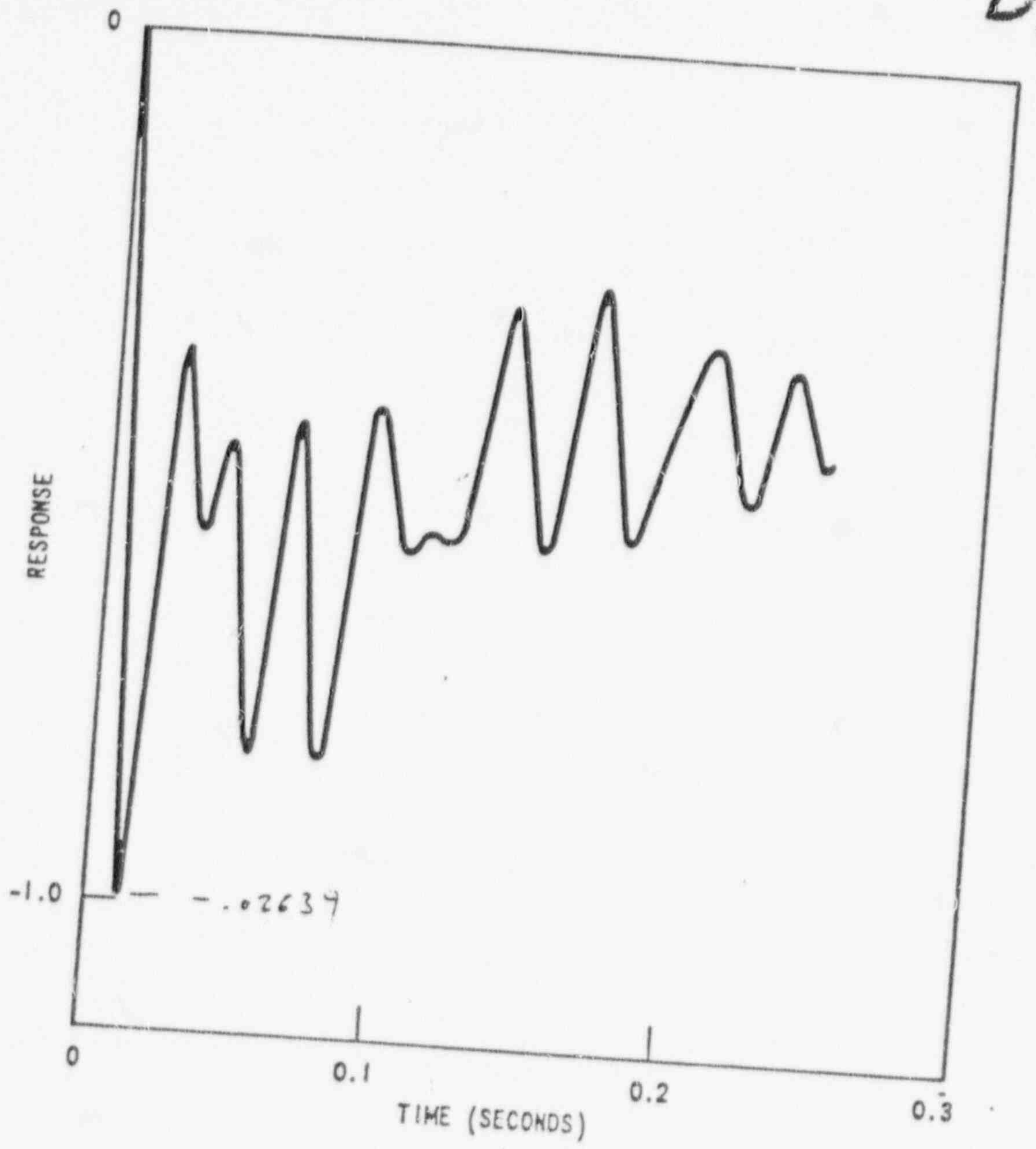

Figure B-29. LOCA Response Number 25
(Original)

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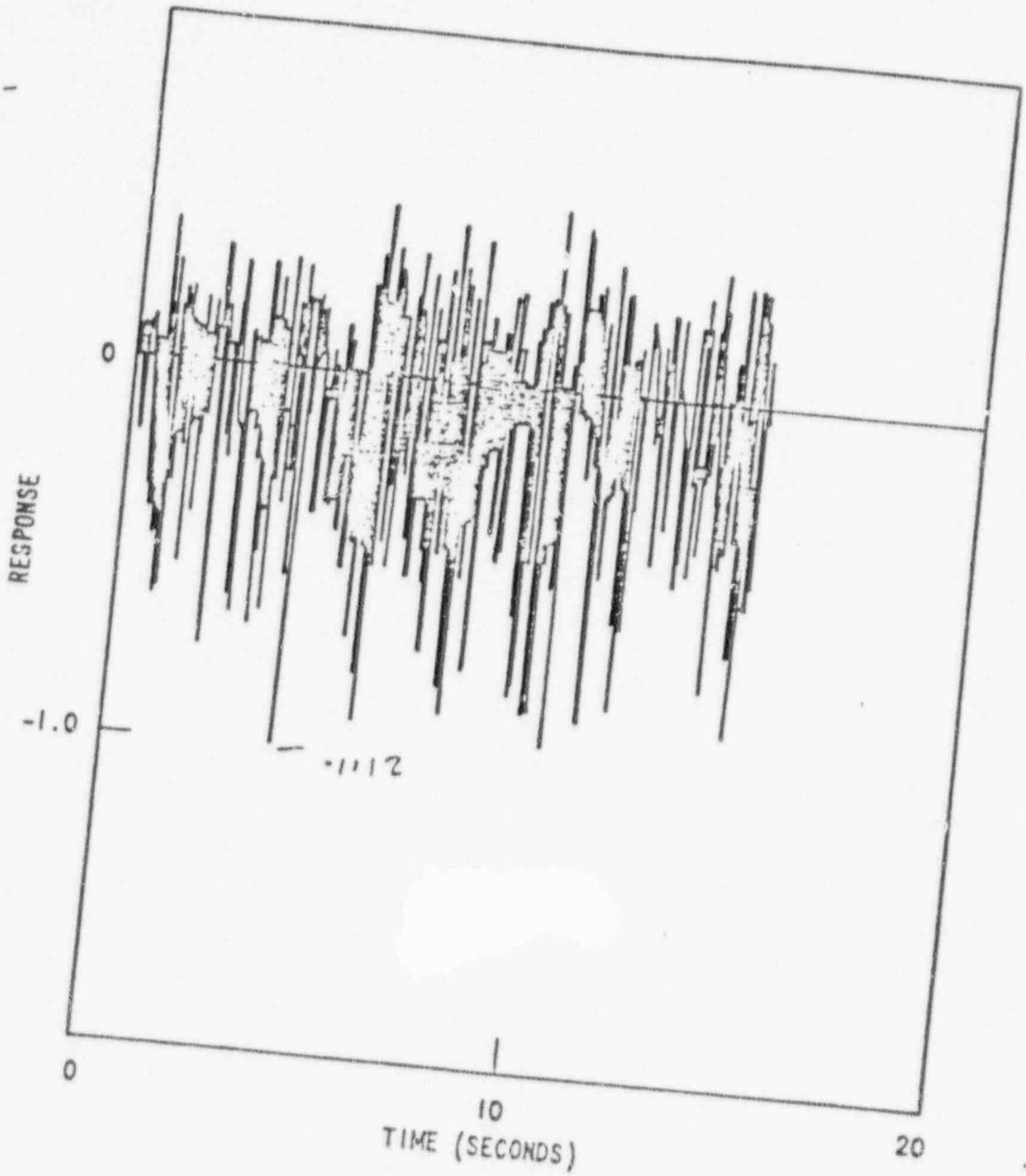

Figure B-30. SSE Response Number 25 (original)

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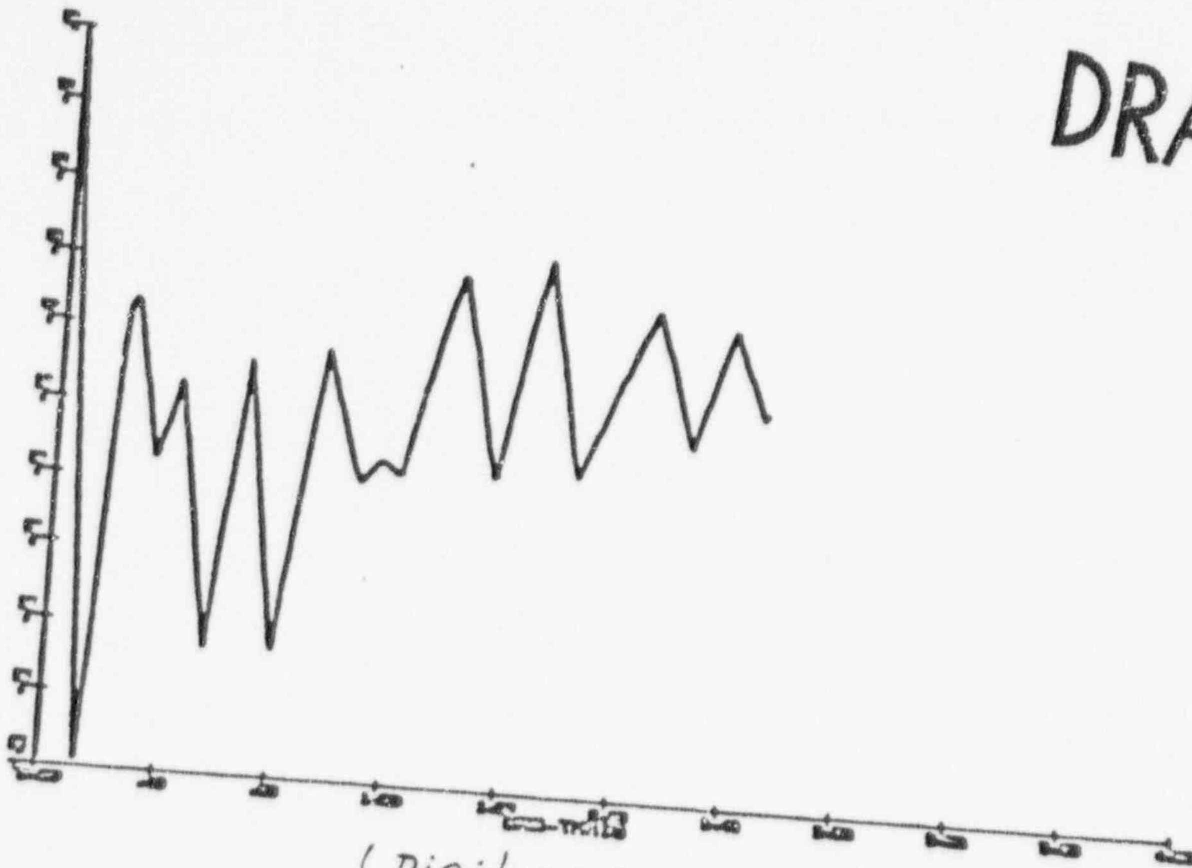
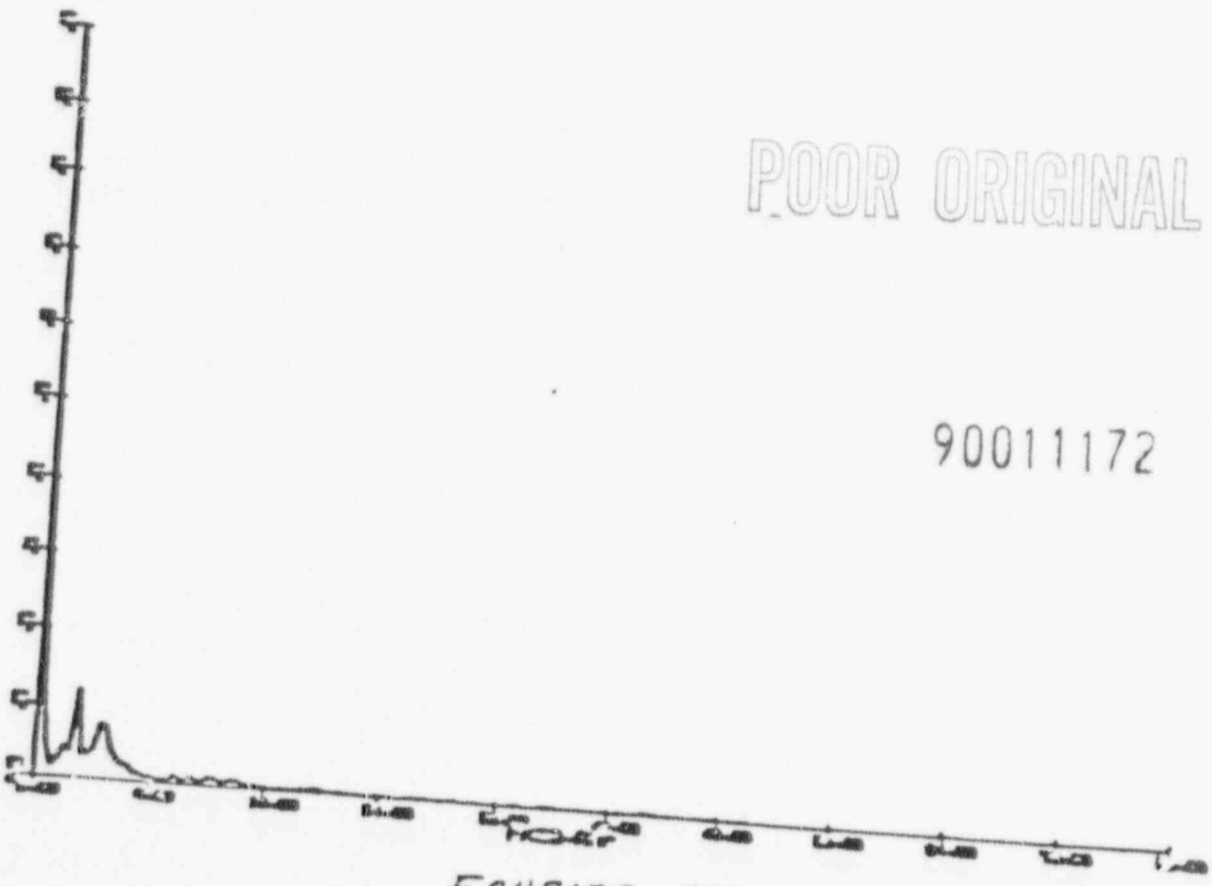


(Digitized)



POOR ORIGINAL

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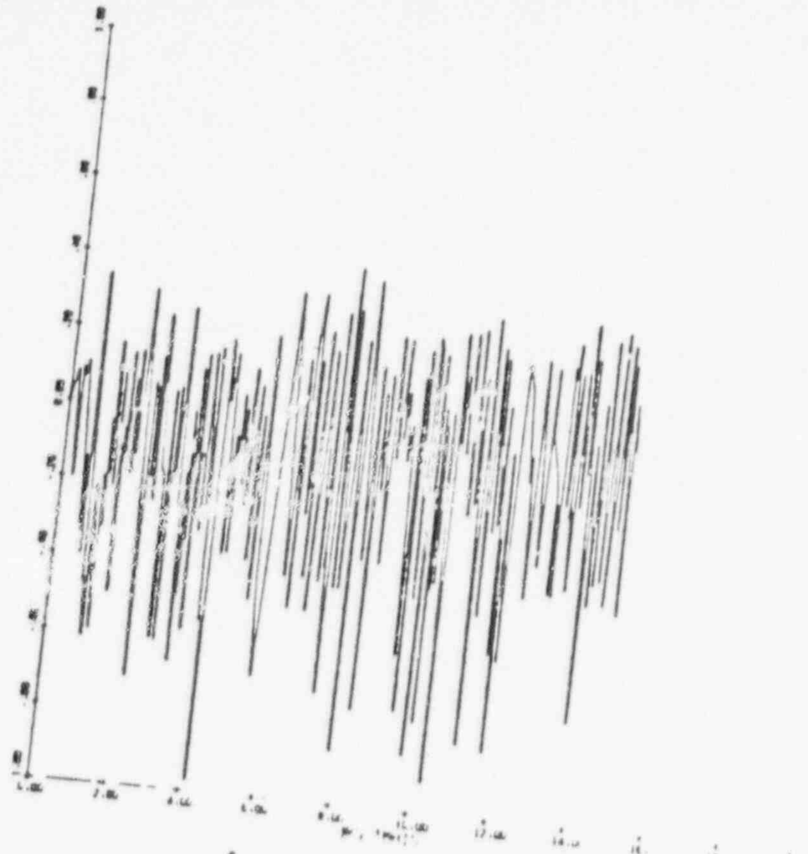
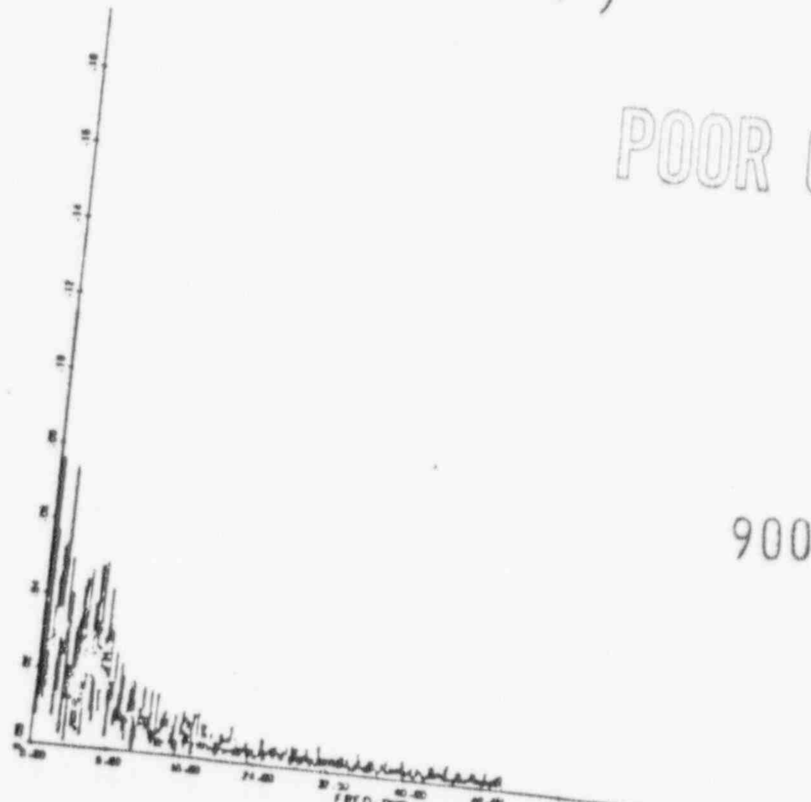
FOURIER SPECTRUM



(Digitized)



FOURIER SPECTRUM

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POOR ORIGINAL

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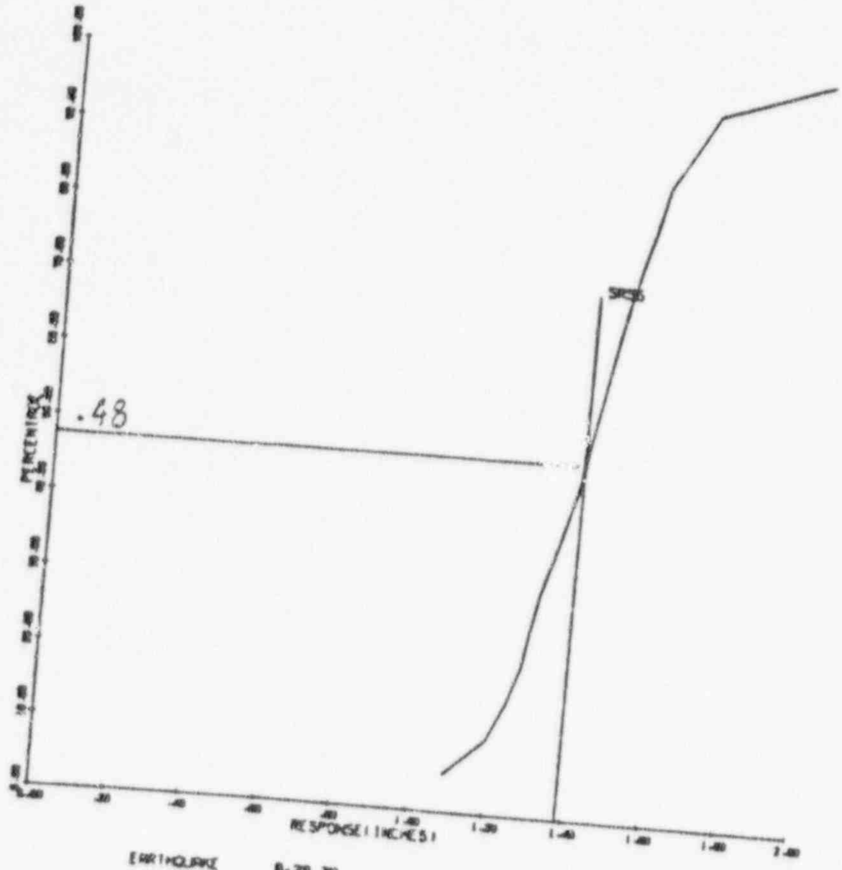
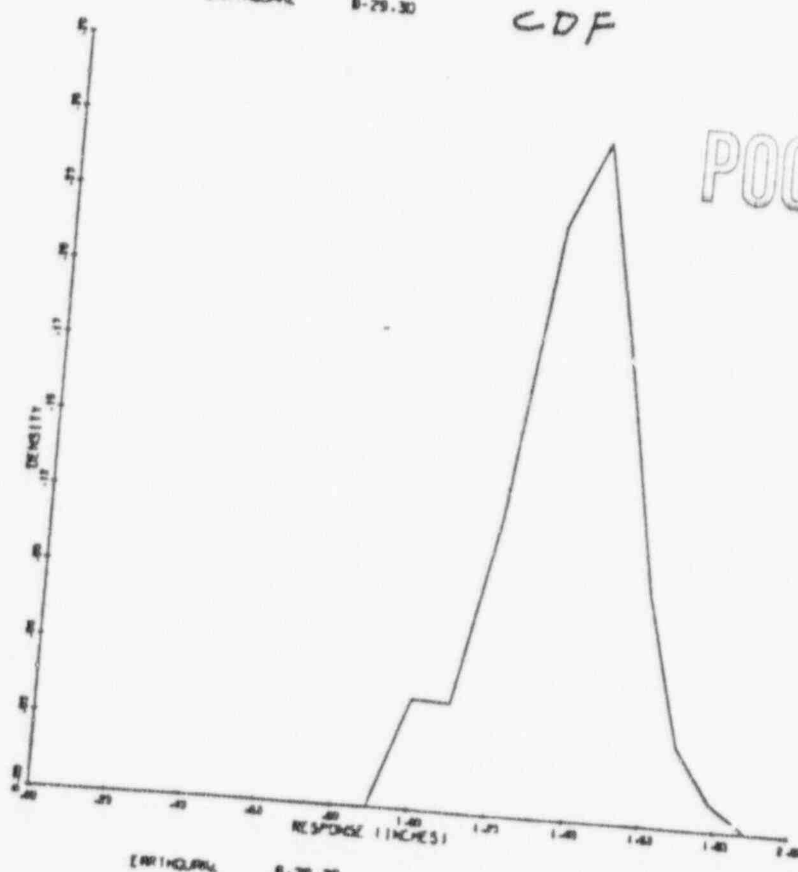

PERCENTAGE
.48
SRSS
RESPONSE (INCHES)
EARTHQUAKE B-29.30

CDF


DENSITY
RESPONSE (INCHES)
EARTHQUAKE B-29.30

DENSITY

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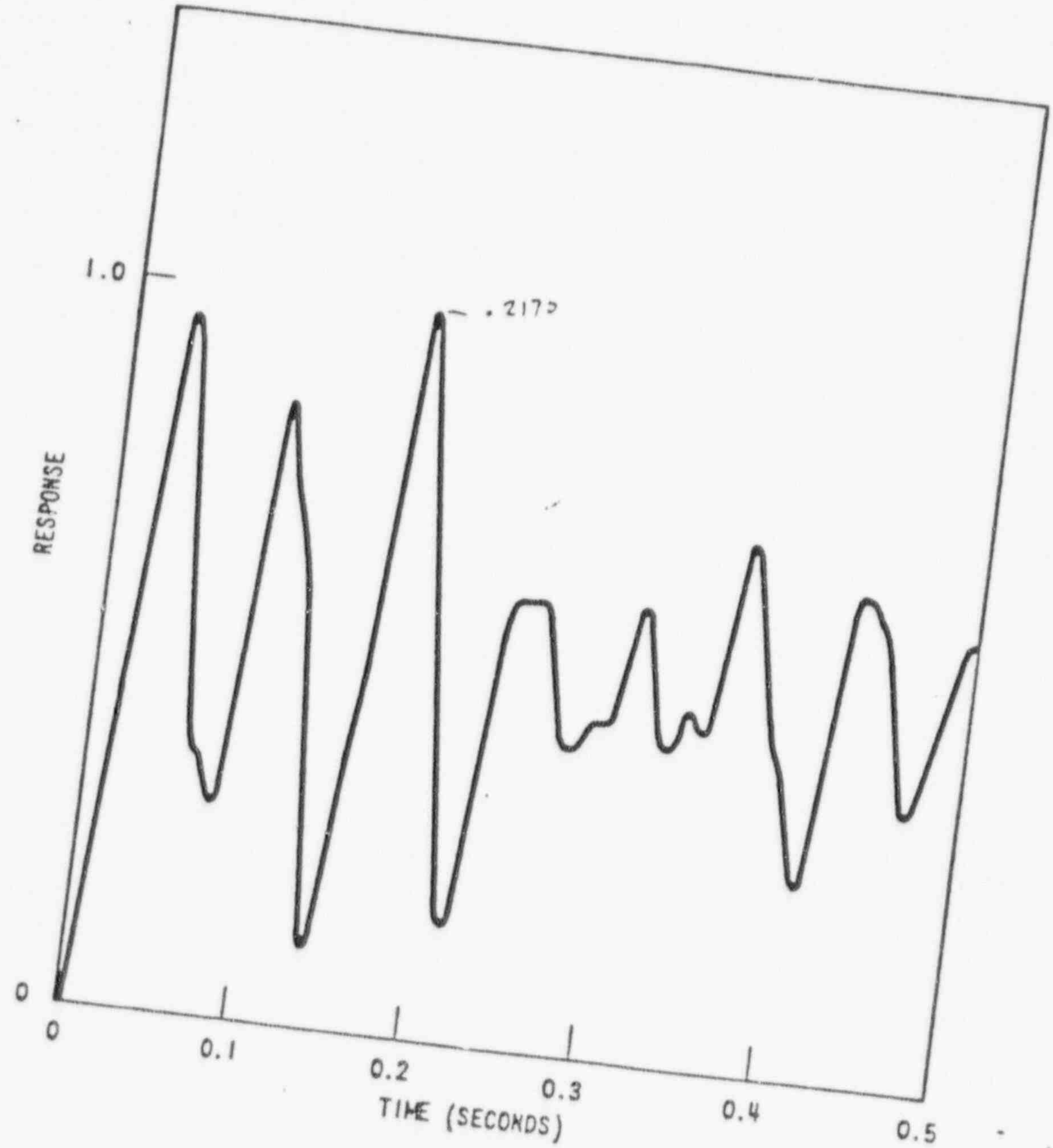

Figure B-23. LOCA Response Number 22
(original)

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POOR ORIGINAL

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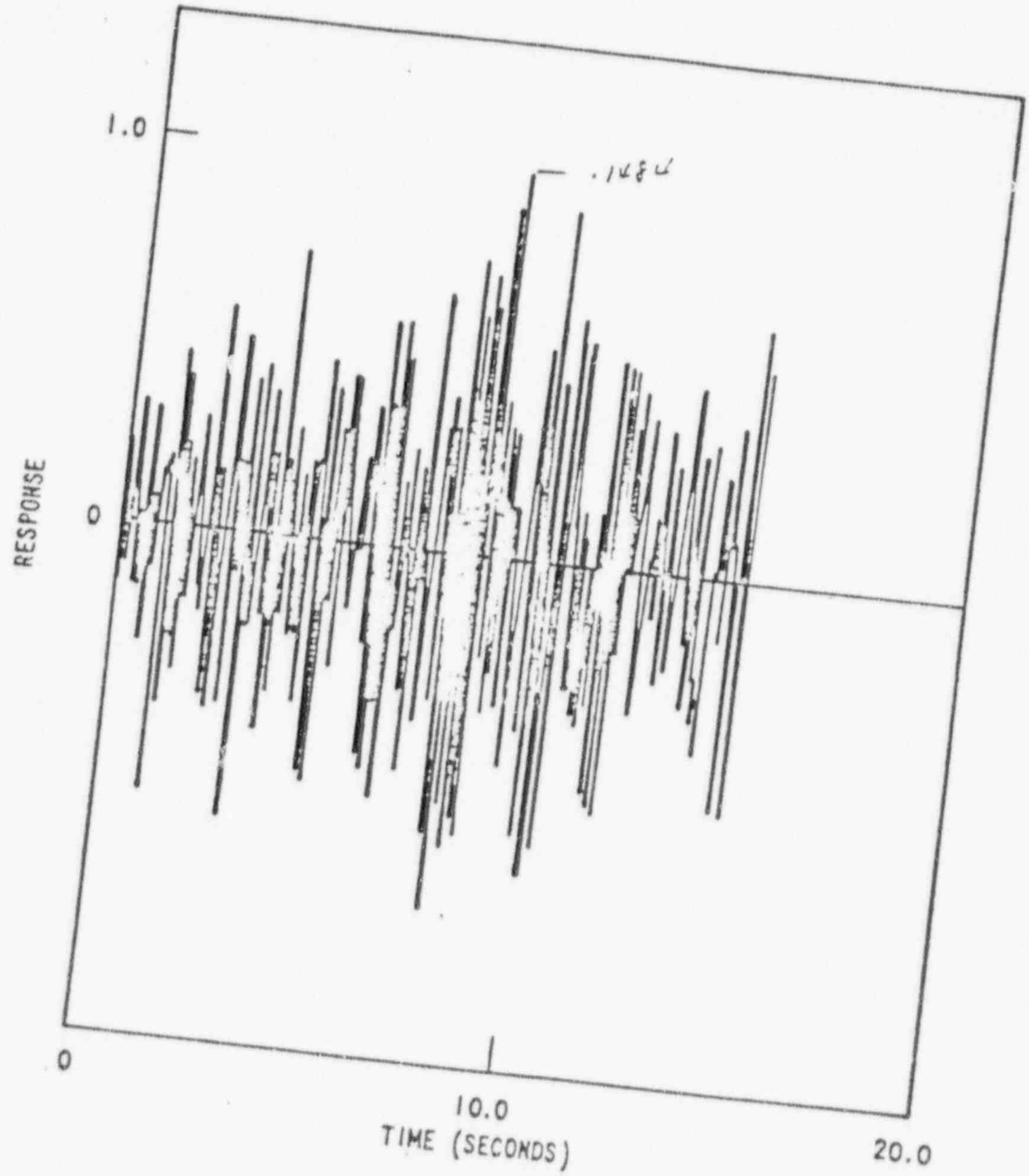

Figure B-24. SSE Response Number 22

(original)

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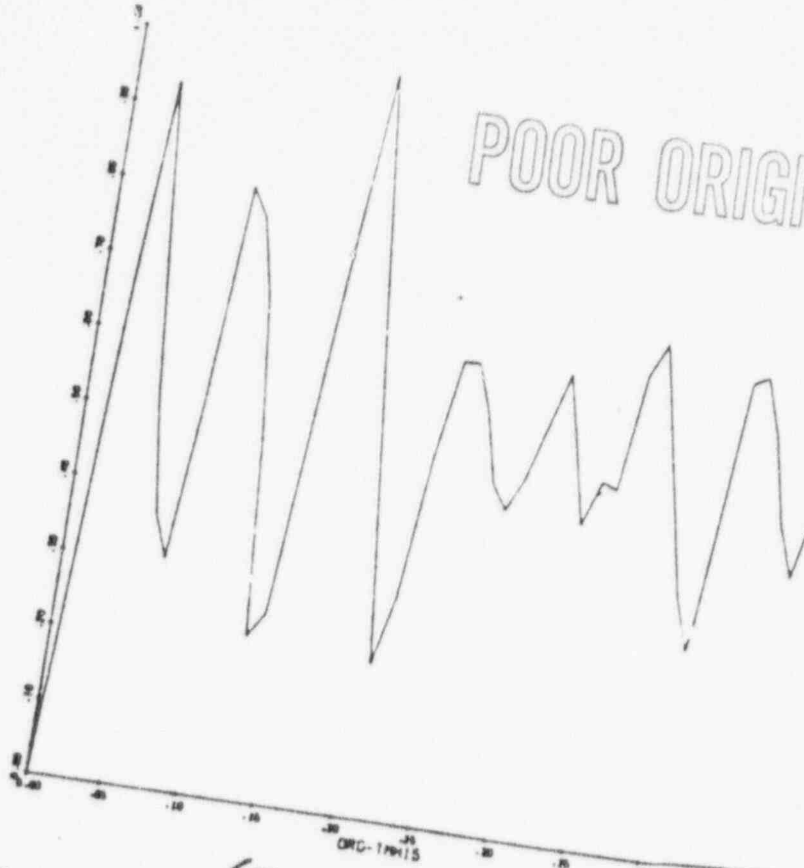
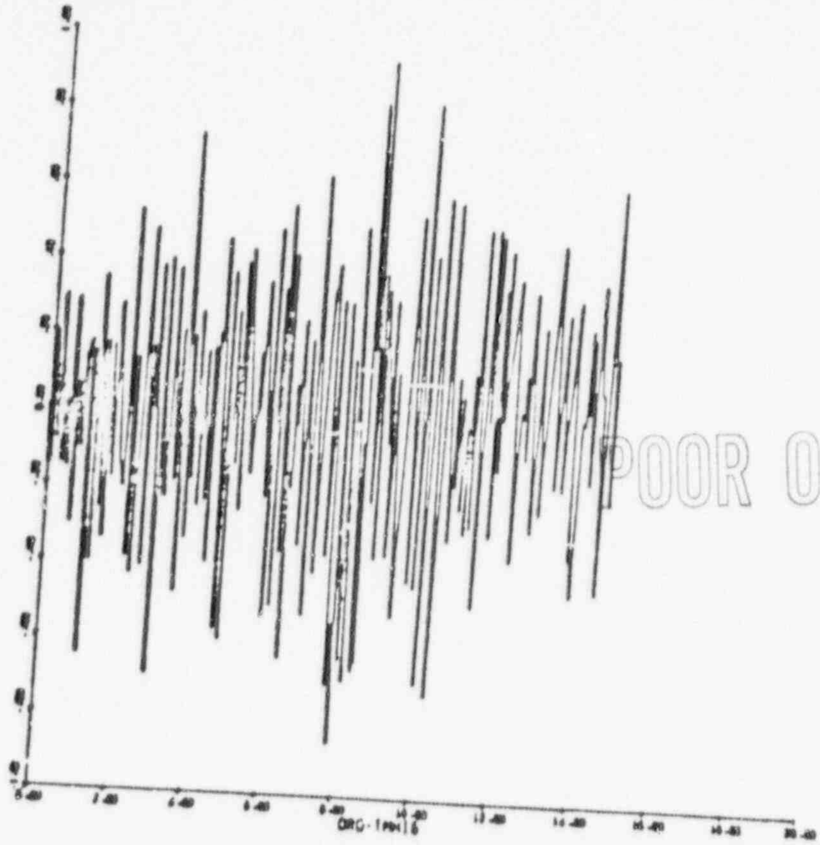


(Digitized) B23

FOURIER SPECTRUM

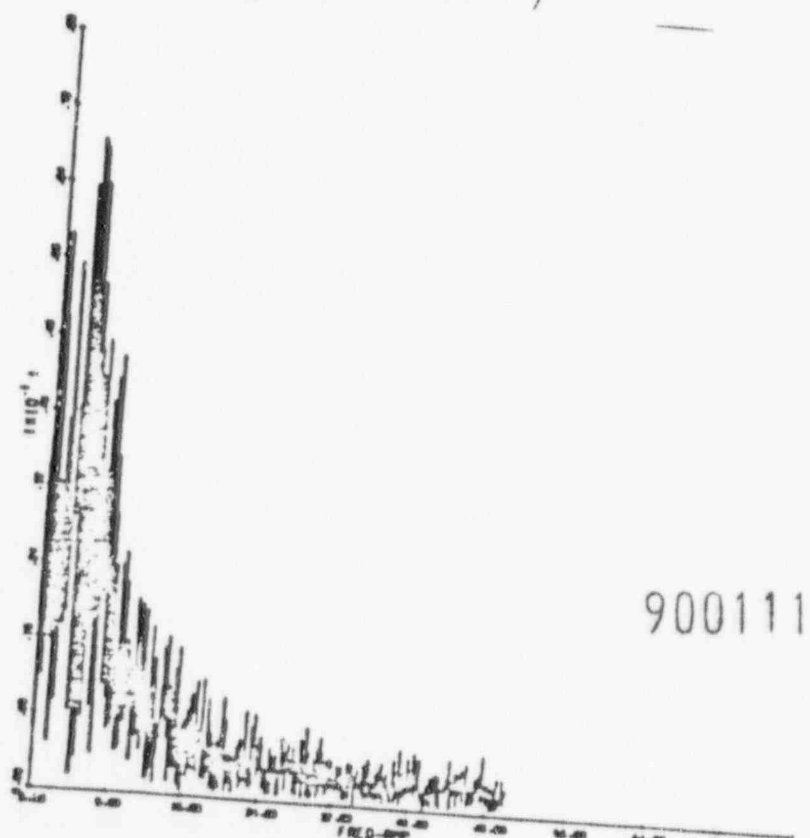
Dr



POOR ORIGINAL

(Digitized)

324



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FOURIER SPECTRUM

APPENDIX F

THE DISTRIBUTION OF THE LARGEST AMPLITUDE OF A LINEAR COMBINATION OF TWO SINUSOIDS

Samuel C. Kao*

Consider the linear combination of two sinusoids:

$X_{(t)} = \alpha_1 \sin\omega_1 t + \alpha_2 \sin(\omega_2 t + \theta)$ where $\alpha_1, \alpha_2 > 0$ are fixed, $\omega_i \overset{L}{\sim} N(\mu_i, \sigma_i^2)$ and $\theta \overset{L}{\sim}$ Unif $(0, 2\pi)$; $i = 1,2$. Let $R(\ell) = \max\limits_{0 \leqq t \leqq \ell} |X_{(t)}|$, then the question is what is the distribution of $R(\ell)$. From the preceding argument, we see that:

$$(1) \quad \begin{cases} R_{(\ell)} \approx \max\limits_{0 \leqq t \leqq \ell} C_{(t)} \\ C_{(t)} = \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \cdot \cos[(\omega_2 - \omega_1)t + \theta]} \end{cases}$$

Therefore, the cumulative distribution function, $F_{(y)} = \Pr(R_{(\ell)} < y)$ for $(\alpha_1 + \alpha_2) \geqslant y \geqslant |\alpha_1 - \alpha_2|$, can be derived as in what follows. From (1) we have that:

$$\left\{R_{(\ell)} \geqslant y\right\} \Longleftrightarrow \left\{\sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \cdot \cos[(\omega_2 - \omega_1)t + \theta]} \geqslant y \text{ for some } t \text{ in } [0,\ell]\right.$$

or:

$$\left\{\cos[(\omega_2 - \omega_1)t + \theta] \geqslant \frac{y^2 - (\alpha_1^2 + \alpha_2^2)}{2\alpha_1\alpha_2} \text{ for some } t \text{ in } [0,\ell]\right\}$$

Since it is clear that, for $y \leq \sqrt{\alpha_1^2 + \alpha_2^2}$,

$$(2) \Pr\left(\cos[(\omega_2 - \omega_1)t + \theta] \geqslant \frac{y^2 - (\alpha_1^2 + \alpha_2^2)}{2\alpha_1\alpha_2} \text{ for some } t \text{ in } [0,\ell] \text{ given } \omega_1, \omega_2\right)$$

$$= \frac{\pi - n}{\pi} + \frac{1}{2\pi} \text{ minimum } (2n, |\omega_2 - \omega_1|\ell),$$

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*Department of Applied Mathematics, Brookhaven National Laboratory.

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where $0 \leq n = \cos^{-1} \dfrac{|y^2 - (\alpha_1^2 + \alpha_2^2)|}{2\alpha_1\alpha_2} \leq \dfrac{\pi}{2}$,

and (2) equals to:

$$1 - \frac{n}{\pi} + \frac{1}{2\pi} \cdot \left\{ 2n \cdot I_{(|\omega_2 - \omega_1|\ell \geq 2n)} + |\omega_2 - \omega_1|\ell \cdot I_{(|\omega_2 - \omega_1|\ell < 2n)} \right\}$$

we have:

(3) $$\Pr\left(\cos[(\omega_2 - \omega_1)t + \theta] \geq \frac{y^2 - (\alpha_1^2 + \alpha_2^2)}{2\alpha_1\alpha_2} \text{ for some } t \text{ in } [0,\ell] \right)$$

$$= E_{\omega_1,\omega_2} \left\{ 1 - \frac{n}{\pi} + \frac{1}{2\pi} \cdot \left[2nI_{(|\omega_2 - \omega_1|\ell \geq 2n)} + |\omega_2 - \omega_1|\ell\, I_{(|\omega_2 - \omega_1|\ell < 2n)} \right] \right\}$$

$$= 1 - \frac{n}{\pi} + \frac{1}{\pi} \cdot \Pr\left(|\text{Standard Normal}| \leq \frac{2n}{\ell \cdot \sqrt{\sigma_1^2 + \sigma_2^2}} \right) + \frac{\ell \cdot \sqrt{\sigma_1^2 + \sigma_2^2}}{2\pi} E|Z| I\left(|Z| \leq \frac{2n}{\ell \cdot \sqrt{\sigma_1^2 + \sigma_2^2}} \right)$$

where $Z \overset{L}{\sim} N_{(0,1)}$

$$= 1 + \frac{n}{\pi} - \frac{2n}{\pi} \cdot \phi\left(\frac{2n}{\ell\sqrt{\sigma_1^2 + \sigma_2^2}} \right) + \frac{\ell\sqrt{\sigma_1^2 + \sigma_2^2}}{\sqrt{2\pi^3}} \cdot \left[1 - e^{-\frac{2n^2}{\ell^2(\sigma_1^2 + \ _2^2)}} \right]$$

Therefore for (i) $y \leq \sqrt{\alpha_1^2 + \alpha_2^2}$ (i.e. SRSS),

(4) $$F_{(y)} = \frac{2n}{\pi} \phi\left(\frac{2n}{\ell\sqrt{\sigma_1^2 + \sigma_2^2}} \right) - \frac{n}{\pi} - \frac{\ell\sqrt{\sigma_1^2 + \sigma_2^2}}{\sqrt{2\pi^3}} \cdot 1 - e^{-\frac{2n^2}{\ell^2(\sigma_1^2 + \sigma_2^2)}}$$

where:

$$0 \leq n = \cos^{-1} \frac{|y^2 - (\alpha_1^2 + \alpha_2^2)|}{2\alpha_1\alpha_2} \leq \frac{\pi}{2}.$$

For case that $(\alpha_1 + \alpha_2) \geq y > \sqrt{\alpha_1^2 + {}_2^2}$, we have instead of (2) that:

(5) $\Pr\left(\cos[(\omega_2 - \omega_1)t + \theta] \geq \dfrac{y^2 - (\alpha_1^2 + \alpha_2^2)}{2\alpha_1\alpha_2}\right.$ for some t in $[0,\ell]$ given $\left.\omega_1, \omega_2\right)$

$$= \frac{n}{\pi} + \frac{1}{2\pi} \cdot \text{minimum} \left(2(\pi-n),\ |\omega_2 - \omega_1|\ell\right)$$

$$= \left[\frac{n}{\pi} + \frac{|\omega_2 - \omega_1|\ell}{2\pi}\right] I\left(|\omega_2 - \omega_1|\ell < 2(\pi - n)\right) + I\left(|\omega_2 - \omega_1|\ell > 2(\pi - n)\right).$$

Therfore we have:

(6) $\Pr\left(\cos[(\omega_2 - \omega_1)t + \theta] \geq \dfrac{y^2 - (\alpha_1^2 + \alpha_2^2)}{2\alpha_1\alpha_2}\right.$ for some t in $\left.[0,\ell]\right)$

$$= \frac{n}{\pi} \cdot \Pr\left(|Z| \leq \frac{2(\pi - n)}{\ell\sqrt{6_1^2 + 6_2^2}}\right) + \frac{\ell\cdot\sqrt{6_1^2 + 6_2^2}}{2\pi} \cdot E|Z|I\left(|Z| \leq \frac{2(\pi - n)}{\ell\sqrt{6_1^2 + 6_2^2}}\right)$$

$$+ 1 - \Pr\left(|Z| \leq \frac{2(\pi - n)}{\ell\sqrt{6_1^2 + 6_2^2}}\right)$$

$$= 2 - \frac{n}{\pi} - 2\left(1 - \frac{n}{\pi}\right)\Phi\left(\frac{2(\pi - n)}{\ell\sqrt{6_1^2 + 6_2^2}}\right) + \frac{\ell\sqrt{6_1^2 + 6_2^2}}{\sqrt{2\pi^3}} \cdot \left[1 - e^{-\frac{2(\pi - n)^2}{\ell^2(6_1^2 + 6_2^2}}\right.$$

This then gives that, for $\sqrt{\alpha_1^2 + \alpha_2^2} < y \leq \alpha_1 + \alpha_2$,

(7) $$F_{(y)} \simeq \frac{n}{\pi} + 2\left(1 - \frac{n}{\pi}\right) \cdot \Phi\left(\frac{2(\pi - n)}{\ell\sqrt{6_1^2 + 6_2^2}}\right) - \frac{\ell\sqrt{6_1^2 + 6_2^2}}{\sqrt{2\pi^3}} \cdot \left[1 - e^{-\frac{2(\pi - n)^2}{\ell^2(6_1^2 + 6_2^2)}}\right] - 1,$$

where n is as defined in (4).

Note that Φ in (4) and (7) denotes the cumulative distribution function of the standard normal distribution.

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Remark 1. The density function for $R_{(\ell)}$ can be obtained immediately from differentiating $F_{(y)}$ with respect to y.

Remark 2. The density function for $R_{(\ell)}$ will be seen to be asymmetric unless $\mu_1 = \mu_2$ and $6_1 = 6_2 = 0$.

Remark 3. The values from using (4) and (7) match very close to the values from simulation using 5,000 repetitions. We are to consider the case that $|\mu_1 - \mu_2|$, 6_1 and 6_2 are small relative to μ, which indicates the minimum of μ_1 and μ_2.

Letting $\omega = \dfrac{\omega_1 + \omega_2}{2}$ it may be shown that:

$$X_{(t)} = \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \cos [(\omega_1 - \omega_2) t - \theta]} \cdot \sin \frac{\omega_1 + \omega_2}{2} t + \Phi_{(t)}$$

where:

$$\Phi_{(t)} = \tan^{-1} \left\{ \frac{\alpha_1 \sin [(\omega_1 - \omega) t] + \alpha_2 \sin [(\omega_2 - \omega) t + \theta]}{\alpha_1 \cos [(\omega_1 - \omega) t] + \alpha_2 \cos [(\omega_2 - \omega) t + \theta]} \right\}$$

Then if it holds also that $|\mu| \cdot \ell > 2\pi$, we have the approxiamtion that:

$$\max_{0 < t < \ell} |X_{(t)}| \approx \max_{0 < t < \ell} C_{(t)}$$

Furthermore, this approximation becomes closer as $|\mu|\ell$ becomes larger.

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