PDR 40-8728

University of Idaho
College of Mines & Earth Resources
Department of Geology
Moscow, Idaho/83843

Circa 10/30/79

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Mr. Ron Kauffman
Nuclear Material Safety and Safeguards
U.S. Nuclear Regulatory Commission
7915 Eastern Avenue
Silver Springs, Maryland 20555

Dear Ron:

This letter presents my assessment of the Application and Environmental Report for Source Material License; Docket No. 40-8728 for uranium in-situ leaching at the Leuenberger Site, Converse Co., Wyoming, as a joint venture by Teton-Nedco. The Company proposes to leach in-situ two aquifer zones containing uranium ore bodies. The intent of the above Report is to show that these two aquifer zones are hydraulically separate with no interconnections which would permit transfer of low-quality leaching waters to adjoining aquifers.

The report presents two techniques to substantiate the hypothesis that the uranium production zones are not hydraulically connected; 1) geological cross-sections interpreted from geophysical well logs and 2) the results of two pump tests on the proposed production zones. Each of these techniques and subsequent data will be addressed separately in conjunction with questions or discrepancies which I have concerning them.

GEOLOGY, STRUCTURE AND LOCAL STRATIGRAPHY INTERPRETED FROM GEOPHYSICAL LOGS.

The Report has very sparse specific geological information for the area in question. Some discussion presents a description of soil profiles but very little information is given concerning geologic units within the proposed production areas. A general stratigraphic column with varying geologic descriptions for each formation is presented in Table II.5.01, but this column is not corrected to be site specific for the Leuenberger Site. At least 45 test holes and piezometers were drilled in the study area and these geologic logs should have been recorded and presented in this report. I disagree with the statement on Page II-23 in that "Figures II.5.03A through II.5.03C are geologic cross-sections showing the ore zones of interest". The cross-sections which are presented in this Report are geophysical cross-sections which would be strengthened by geological interpretations. Geophysical logs indicate changes in lithology, well construction and interstitial water quality; they are not indicative of stratigraphic changes exclusively.

Geophysical logs presented on the "Geologic cross-sections" include the gamma ray, spontaneous potential and resistance and resistivity logs. The logs are not described specifically so one does not know if the RES log is indicative of a single point or differential resistance log while the resistivity logs pre-

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sented for some of the holes are not identified as to whether they are normal, lateral, or micrologs, etc. The fact that the resistivity logs parallel or mirror-image the resistance logs (example: well PN5 L514 on C-C') would indicate that the resistivity logs are of relatively short spacings, possibly the short normal. Specific sonde types should be given for any logs presented.

Even though geologic information is sparse, the geophysical logs can be interpreted to provide considerable information on hydrostratigraphic units in this area. A brief description follows as to interpretational techniques for the logs.

The gamma ray log measures gross gamma activity and is generally a lithology and stratigraphic indicator. It is usually indicative of clay content so indirectly it is a permeability indicator; however, for this area the gamma ray log is most useful for delineating the natural radioactivity of the ore body. The spontaneous potential log measures natural potentials which occur between the borehole fluid and the surrounding rock material. There are two types of spontaneous potentials; 1) the streaming potential which is the electromotive force caused by an electrolyte moving through a permeable medium, particularly indicative of aquifer zones which may be gaining or losing water, which is an indicator for head differences within the borehole, and 2) an electrochemical potential caused by current flow through dissimilar materials.

The Teton-Nedco Report does not describe the drilling techniques for the test holes but the SP logs indicate that the fluid in the borehole at the time of logging was less saline or fresher than the formation waters. The reason for this assessment is that we have positive deflections associated with shale zones and negative deflections across sand zones. This relationship is not always consistent throughout all of the geophysical cross-sections which may indicate that water quality relationships do change. The SP logs will be discussed as each cross-section is analyzed.

The resistance log, which I assume to be a point resistance log, has a small radius of investigation and is consequently a good lithology indicator and may also be used for delineating fractures within the borehole materials. The resistance log ordinarily can be used as a caliper log to show small changes in borehole rugosity which is an indicator of stratum competency. All of the geophysical logs presented in this report were presumably taken through PVC casing so that casing effects should be a constant throughout the log. The PVC casing is slotted in the production zones and this change in the casing condition is evident on the SP and resistivity logs (example: PN5 L306 A-A'). No descriptions are given for the resistivity logs presented on the cross-section so one can only guess as to what type of resistivity tool was used to take the logs. As stated before, the resistivity logs parallel the resistance logs which would indicate possibly a short normal resistivity log. In general, this uncertainty should not be crucial because at least one resistivity log is presented with every drill hole and provides information concerning hydrostratigraphy.

<u>Geologic cross-section A-A'</u>: This cross-section shows the correlation of the aquifer zones across the study area and locates and delineates the integrity

of the confining beds proposed to occur between the aquifer zones. The resistance logs are very useful in delineating the aquifer zones because there is an increase in resistance across the aquifer zone which is a function of the insolating properties of the sand grains. Relatively poor water quality in these aquifers would tend to decrease the resistivity responses but not to the level of the clay or siltstone complex above the aquifer zone. The natural gamma log is not very definitive in delineating lithology in cross-section A-A' except in those areas of the ore body. As examples, note the responses or lack of them on the proposed contacts of the Idaho aquifer, N and M Aquifers and Basal aquifers of PN5 L306 and PN5 L303. In general, I feel that the geophysical logs do delineate the proposed hydrostratigraphy of the study area but lack some credence in drillholes where responses are less definitive and where no geological information is presented to substantiate the picks. An example of this lack of definition would be the choice of the Idaho aquifer on PN5 L308 on cross-section A-A' which is really based only on the resistance log as correlated with PN5 L180. Both have similar configurations but are based on very slight excursions on the resistance logs whicn is a function of the variable scaling on the log presentation. When one examines these scales for the geophysical logs, it should be noted that the scales do change from drillhole to drillhole. An example of this occurs on PN5 L473 for the resistance log which has a scale of 5 ohms per division, compared to PN5 L303 which has a resistance scale of 10 ohns per division compared to PN5 L180 of 20 ohms per inch. Possibly the scaling rations relating inches and divisions may be similar but this is not evident because of the differences in the chart paper on these logs. When hydrostratigraphic boundaries as important as confining beds versus aquifer zones must be delineated by one geophysical log as shown in PN5 L180, the scaling relationship of the log becomes quite important. In drillholes such as these, supporting geological evidence (absent here) is very important.

Geologic cross-section A-A' shows continuity of aquifer zones and confining beds with the exception of the Idaho aquifer which appears to thicken and thin by as much as 40 feet. The correlations seem reasonable and would indicate that there are no major off-sets or unconformities within the cross-section, assuming that the assigned elevations are accurate. According to Page II-46 of the Report, this point may be in question.

The correlations presented on the geophysical cross-section indicate that the clay-siltstone regions of the drillholes are defined as confining beds. In order to be a confining bed, the clay or siltstone material must be relatively impervious to ground-water flow and not allow interconnection between the aquifer zones. The geophysical logs indicate that the postulated confining bed appears to have variations within itself. The confining bed between the Idaho aquifer and the N aquifer appears to have more variations than the confining bed between the N aquifer and the M aquifer. An example of this is shown in PN5 L152. PN5 L473 and PN5 L158 also show changes in characteristic resistivity responses of the confining bed between the N and M aquifers in addition to variations within the silt and claystone materials between the N and Idaho aquifers. Based on the geophysical logs alone, one would be safe to say that there are hydrostratigraphic differences between the confining beds and the aquifer zones, but there is some risk in saying that the confining beds have consistent responses indicative of homogeneous impervious material. Again, the scale variations of the logs becomes

important in the assessment of relative changes of characteristics.

Geologic cross-section B-B': This cross-section is an east-west presentation of stratigraphy which, in my opinion, does not show significant thinning of the production zone (N aquifer) which could give a negative boundary on the aquifer test. I am in agreement with their correlations between the confining and aquifer beds but I am again concerned with the variations of the resistivity and resistance logs of the upper-confining bed between the Idaho and N aquifers. This is particularly evident in PN5 L159, L516 and L33. In these three holes, the geophysical logs are presented with three different scales which are 25 ohms per inch, I believe, and 10 ohms per division and 15 ohms per inch, all of which may be equal or not. Geophysical logs generally indicate continuity of the hydrostratigraphy though the characteristic SP and resistivity responses in the aquifer zones appear to change in the easterly direction exemplified by PN5 L391. There is a discrepancy on this drillhole because it is labeled as L301 on Figure II 5.02, "Location of Drillholes Used to Construct Geological Cross-Sections". Table II.6.02 lists an elevation of approximately 5200' for PN5 L301 which does not agree with the elevation given on cross-section B-B' of Figure II 5.03B. Consequently, I assume that PN5 L391 is correct on the geophysical cross-section and incorrect on Figure II 5.02

Again, it would be very useful and important to have geologic information presented in addition to the geophysical logs. Responses of the geophysical logs do vary laterally which indicate variations in hydrostratigraphic characteristics important to the question of aquifer interconnection.

Geologic cross-section C-C': Geologic cross-section C-C' is the only north-south cross-section shown of the study area. In general, the log responses on these drillholes are typical of the other two cross-sections presented with the exception of the SP log which starts to show a reversed trend on drillholes PN5 L30, L33, L38, and L45. Changes of water quality within the aquifer zones relative to the borehole fluid would cause such a reversal in the SP response. Unfortunately, no water quality data are presented for these test holes.

As in the other cross-sections, the integrity of the confining bed between the Idaho and N aquifers appears to be variable as indicated by the resistivity logs of PN5 L30 and L33. The letters N, M and L are also presented on PN5 L30 with no indication as to what they refer to. If N and M indicate the N and M aquifer zones, then the M is not located at the proper depth as shown by the geophysical logs. There is no suggestion as to what the L could represent on the cross-section.

There appears to be some discrepancy with depths to casing on PN5L 314WW on this geologic cross-section C-C'. Written on the resistance log is "Bottom of Casing at A Depth of 470'", yet the log responses would indicate that the bottom of casing would be at 460'. In consulting Table II.6.02 for well completion data, assuming that PN5 L314 is the same well as PN5 L314WW, then the casing bottom would be at 480'. Well-completion data on Table II.6.02 also indicate that the screened interval is between 500 and 540' which would not agree with the geophysical logs of PN5 L314WW which has the basal aquifer delineated

between 480 and 500'. I do not feel that one can interpret the location of the Basal aquifer based only on the geophysical log responses in this hole. Delineation of this Basal aquifer and the construction of PN5 L314 is important because this information is used in the pump test as evidence for aquifer isolation characteristics.

AQUIFER CHARACTERISTICS VIA PUMP TESTS

The Report describes separate aquifer tests conducted on both the M and N aquifers using multiple observation wells within the tested aquifer zone and one observation well in each of the adjoining aquifers. In the case of the M aquifer pumping test, the Report indicates that it continued for 48 hours at a "constant discharge rate of 44 gallons per minute" (Page II-37). When one checks Appendix D for the actual pumping rate, it can be seen that the average pumping rate might have been 44 gallons per minute whereas the actual pumping rate flucturated quite wildly from 66.5 gallons per minute down to 50 back to 60 down to 49, down to 45, back up to 46, down to 42, etc. throughout the test.

Figure II 6.03 shows the location of wells used to determine aquifer characteristics and ground-water quality but this figure has some discrepancies included on it. There are two Well No. 306's and there is Well No. 300 which is not referred to at all in the Table of Well-Completion Data (Table II.6.02) and is not referred to in either of the pump tests or included in water-quality data.

Aquifer characteristics for aquifer M are determined by the various Theis and Jacob techniques for the observation wells within the zone of pumping. Important aspects of this test would be to prove or disprove the interconnection of the M aquifer zone with either the N aquifer or the Basal aquifer zones which are separated from the production zone with the siltstone-claystone confining beds. In attempting to prove this important relationship of hydraulic independence, only one well within each adjacent zone was monitored during the pump test. An unnumbered figure is shown in Appendix D which gives the relationships between the N aquifer water levels, Basal aquifer, and barometric pressure changes with respect to time during the M pump test. Fluctuations do occur in the N and Basal aquifers during the test on aquifer M which the authors describe as barometric and Noordbergum effects. Because of this interpretation being so important for the assessment of the well responses, one must be satisfied that the test was conducted long enough, at a high enough pumping rate, with enough monitoring wells used to delineate these effects.

The effect of barometric changes on artesian aquifers is an inverse relationship. The data indicate that the barometric pressure is decreasing during the test, so one would expect an increase in the water levels of artesian aquifers. A slight increase is seen on the piezometric levels of the N and Basal aquifers of the M pump test. The barometric effects may tend to mask any drawdown relationships of the adjacent zones.

The Noordbergum effects should initially cause an increase of water level in the Basal zone because of the loading effect of the upper aquifer pumping.

"Due to the decreasing pore water pressures in the upper layer, this layer will attract water from the upper part of the lower layer. The loss of water involves a decrease in volume of the upper part of the lower layer. All elements of a circular ring around the origin will decrease in volume, thus producing a radial displacement towards the center to keep the ring closed. For the lower parts of the layer, this radial displacement presents a loading due to which a tendency for compression will be present. Since the pore water opposes volume compression of the soil, this results in an increase of the pore water pressure"; Page 368 of Elastic Storage of Aquifers by Arnold Verruijt in Flow Through Porous Media by Roger J. M. De Wiest, Academic Press, 1969.

The essence of the Noordbergum effect requires leakance, yet the report indicates that the best fit curve for the data is the non-equilibrium type curve for non-leaky isotropic aquifers (Page I-42). It is my opinion that the test on the M aquifer was not conducted at a high enough pumping rate for a sufficiently long enough period of time to substantiate these hypotheses. Because the isolation of these aquifer zones is very critical, and injection withdrawal rates and times will be greater than those tested here, I would advise an extended pump test at carefully measured constant rates to prove aquifer interconnection or the lack of it under operational conditions.

The Report indicates that the measurements of the elevations of the wells were not sufficiently accurate to produce piezometric maps of the aquifer zones. Consequently, one may have valid concerns with respect to other elevations proposed within this Report. An example of this concern may be expressed with respect to the geologic cross-sections presented with the geophysical logs. The continuity of the hydrostratigraphic correlations is based upon the presentation of the relative elevations of the geophysical logs with respect to each other. A 20' off-set in any of those elevations would indicate an off-set on the correlation which may indicate some sort of unconformity or fault in the area which would be a concern relating to interconnection to aquifer zones. With respect to the fuzziness of the elevation data presented in this Report, it would be justifiable to ask for accurate well-head elevations so that accurate correlations can be made of piezometric configurations and hydrostratigraphic contacts. Preliminary assessment of the piezometric levels indicates variations in transmissivity of significant magnitude to be a concern. Accurate potentiometric maps should be produced for each of the zones in question.

The pump test on the N aquifer gives a constant Q value in the text but does not indicate any discharge values within the Appendix D. The question of constant versus average discharge arises. Constant discharge rates are critical to valid interpretations of T values. This pump test indicates the presence of a negative boundary to the east which is explained as a thinning of the sand zone. Geophysical logs presented in the cross-sections do not suggest a thinning of the sand zone so I would be concerned as to the character of this negative boundary. Again, the interconnection of the N aquifer, the Idaho aquifer and the M aquifer is addressed with a figure in the Appendix showing changes in water level with time with respect to barometric pressure. This figure, which again has no number, is mislabeled by calling the upper aquifer the N aquifer when actually it is the Idaho aquifer. These data between the M aquifer and the barometric pressure indicate

a reverse relationship between these two parameters which the text describes as being more of an effect of the Noordbergum effect again. Importance of the conclusion of aquifer zone isolation must be stressed again and the question asked, "Do these two pump tests indicate that condition?"

SUMMARY

In summary, my analysis of this Report reveals the following:

1) The lack of drilling and geologic information for correlating the geophysical logs produces interpretive risks.

2) The lack of information concerning the types of geophysical tools run to obtain the resistance and resistivity logs produces some interpretive risks.

3) The inaccuracies of well-head elevations making hydrostratigraphic correlations "off" by a potential 10 to 20 feet in either direction.

4) Likewise, piezometric levels should be presented accurately and inaccuracies in well-head elevations introduce risks of error.

5) Geophysical log responses can be correlated with more assurance when all scales are the same - this apparently is not the case here.

6) Very little data are presented on the claystone/siltstone confining layers. The characteristics of these layers are essential for the assessment of ore body separation - yet the Report does not describe these zones at all in the study area. The geophysical logs suggest variations within the strata which be assessed in more detail, possibly by coring in the vicinity of PN5 L473.

7) Piezometric levels presented for the M and N aquifers are the same, yet they are separated by approximately 100' of siltstone and claystone. This would strongly suggest some interconnection. Water chemistry data are also very similar for the two aquifers which suggests interconnection.

My recommendations concerning this Report are:

1) Produce accurate piezometric maps for all aquifers.

2) Provide additional geologic information for the wells.

3) Conduct another pump test on the M and N zones at a <u>constant</u> rate typical of their predicted withdrawal rates and for a minimum of one week duration. Constant pumping rate is a requirement for theoretical validity.

In general, the greatest risk with making decisions on the basis of the

present data is the contamination of surrounding ground waters because of the lack of integrity of the confining beds, or the presence of off-sets with associated fractures which are not defined due to poor elevation control. Water quality, piezometric levels and geophysical logs should also provide supporting information on the isolation of the ore zones, but instead, the present level of effort indicates inconsistancies of the data.

If you wish to discuss my comments with me, please do not hesitate to call.

Sincerely,

Muriel S Robinette

Muriel S. Robinette
Assistant Professor
Geology

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Similar expressions may be obtained for other quantities, such as the displacement components.

The integration constants A_1, A_2, and B_1, which in general depend upon the parameters ξ and s, have to be determined from the boundary conditions along the surface $z = 0$. When these constants have been found, the solution of the problem is complete. However, mathematical difficulties may arise in the evaluation of the integrals in the s and ξ planes, depending upon the nature of the boundary conditions. Only in cases with very simple boundary conditions can the integrals be evaluated in closed form. In Section 6 an application of the method described above will be discussed.

6. The Noordbergum Effect

As an example, the approximate solution of the problem of a pumping test in the upper layer of a two-storied leaky artesian aquifer of infinite radial extent will be determined (Fig. 4). It is assumed that the lower layer of the aquifer tends towards infinity in negative z direction. Storage in the semi-impermeable layer separating the two stories of the aquifer will be disregarded, which means that this layer is assumed to be incompressible. Although a correct formulation of the problem has to account for the coupling of the flow phenomena in the two layers, it will be assumed that the pore water pressures in the upper layer satisfy Hantush's equation. This assumption seems reasonable if the replenishment from the lower layer is small. The above assumptions make it possible to use the method of Section 5 for the solution of the problem in the lower layer, by using the solution for the upper layer as a boundary condition. Furthermore, it will be assumed that the total vertical load acting upon the lower layer does not change, so that the incremental stress σ_{zz} vanishes along the surface $z = 0$. It is also assumed that along the surface $z = 0$, the shear stress σ_{rz} vanishes. The coefficient of permeability of the upper layer of the aquifer will be denoted by K_1 and its thickness by b_1. The coefficient of permeability of the lower layer of the aquifer will be denoted by K_2. The coefficients of permeability of the two semi-impermeable layers will be denoted by K' for the upper one and K'' for the lower one. Their respective thicknesses will be denoted by b' and b''. For the pore water pressure in the upper layer, the symbol σ_1 will be used. The symbols σ and σ_{ij} refer to the pore water pressure and the components of total stress in the lower aquifer.

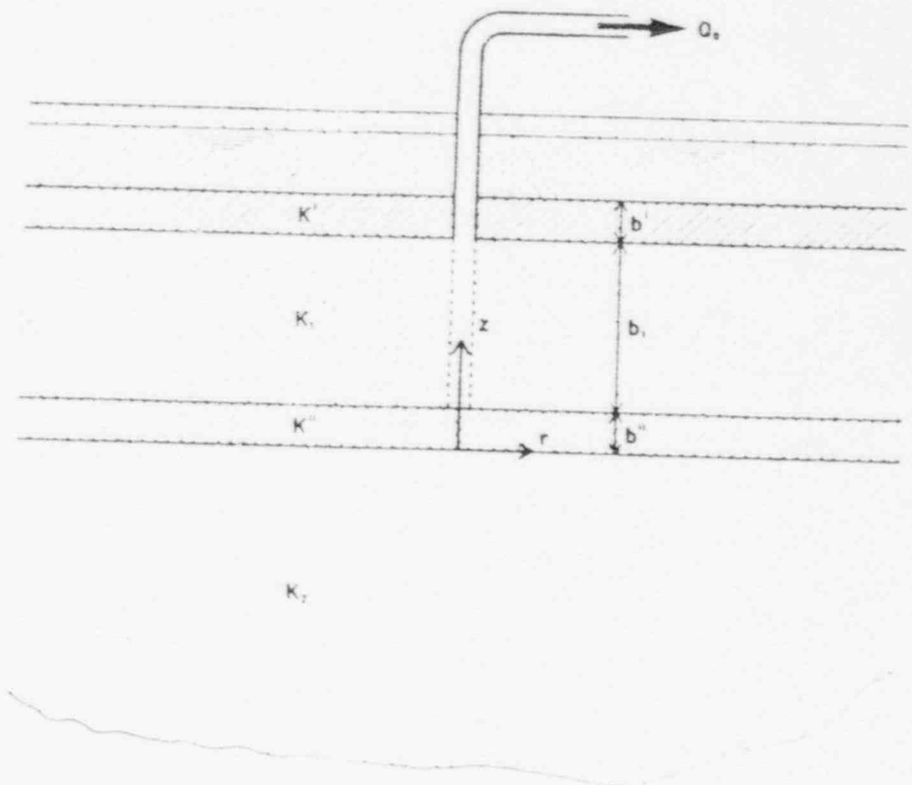

FIG. 4. Well in the upper layer of a two-storied leaky artesian aquifer.

The boundary conditions for the lower layer are:

$$z = 0: \quad \sigma_{zz} = 0 \tag{120}$$

$$z = 0: \quad \sigma_{rz} = 0 \tag{121}$$

$$z = 0: \quad -(K_2/\rho g)(\partial\sigma/\partial z) = (K''/\rho g)(\sigma - \sigma_1)/b'' \tag{122}$$

The condition (122) follows from the continuity of the vertical component of the specific discharge along the surface $z = 0$. Using the notation $L = b''K_2/K''$, this condition can be written as

$$z = 0: \quad \partial\sigma/\partial z = -(\sigma - \sigma_1)/L \tag{123}$$

where σ_1 is assumed to be given by Hantush's formula, Eq. (94).

In view of the general expressions (118) and (119) for the st sses

σ_{zz} and σ_{rz}, it immediately follows from the conditions (120) and (121) that

$$\xi A_1 - \xi A_2 - B_1 = 0 \tag{124}$$

$$\xi A_1 + (\xi^2 + s/c)^{1/2} A_2 = 0 \tag{125}$$

In order to find the condition for the constants A_1, A_2, and B_1, resulting from the boundary condition (123), it is necessary to have an integral representation of g_1, because σ and $\partial\sigma/\partial z$ are given as integral representations. The Laplace transform of g_1 is given by (92),

$$\bar{g}_1 = -(P/s)\,K_0(\omega r) \tag{126}$$

where for simplicity the notation $P = Q_0\rho g/(2\pi K_1 b_1)$ is introduced and where

$$\omega^2 = B^{-2} + \rho g(\alpha' + n\beta')s/K_1 = (s + c'/B^2)/c' \tag{127}$$

The constant $c' = K_1/[\rho g(\alpha' + n\beta')]$ is called the *consolidation coefficient* of the upper layer of the aquifer. The Hankel transform of (126) is (Bateman, 1954),

$$\bar{g}_{1-h} = -(P/s)\int_0^\infty r J_0(r\xi)\,K_0(\omega r)\,dr = -P/[s(\omega^2 + \xi^2)]$$

$$= -Pc'/[s(s + c'\xi^2 + c'B^{-2})]$$

Hence, one may write

$$g_1 = (2\pi i)^{-1}\int_0^\infty \xi J_0(r\xi)\,d\xi \int_{\gamma-i\infty}^{\gamma+i\infty} \{-Pc'/[s(s + c'\xi^2 + c'B^{-2})]\}\exp(st)\,ds \tag{128}$$

The boundary condition (123) now gives, with (116), (117), and (128),

$$(\lambda + 2\mu)(s/c)(\xi^2 + s/c)^{1/2} A_2 + 2\mu\xi^2 B_1$$
$$= -L^{-1}\{(\lambda + 2\mu)(s/c)A_2 + 2\mu\xi B_1 + Pc'/[s(s + c'\xi^2 + c'B^{-2})]\} \tag{129}$$

The equations (124), (125), and (129) together form a system of three equations with three unknowns. Solving for A_1, A_2, and B_1 gives

$$A_1 = \frac{Pc'}{2\mu\xi sL}\,\frac{(\xi^2 + s/c)^{1/2}}{N(s,\xi)}, \qquad A_2 = -\frac{Pc'}{2\mu sL}\,\frac{1}{N(s,\xi)}$$

$$B_1 = \frac{Pc'}{2\mu sL}\,\frac{(\xi^2 + s/c)^{1/2} - \xi}{N(s,\xi)} \tag{130}$$

where

$$N(s,\xi) = (s + c'\xi^2 + c'B^{-2})\{(2ms/c)[(\xi^2 + s/c)^{1/2} + L^{-1}]$$
$$- \xi(\xi + L^{-1})[(\xi^2 + s/c)^{1/2} - \xi]\}$$

and where $m = (\lambda + 2\mu)/4\mu$. Substitution of the values for the constants A_1, A_2, and B_1 given in (130) into Eq. (116) yields the following expression for the pore water pressure

$$\sigma = -\frac{Pc'}{2\pi i L}\int_0^\infty \xi J_0(r\xi)\,d\xi$$

$$\times \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\left[\begin{array}{l}\{(2ms/c)\exp[z(\xi^2 + s/c)^{1/2}]\\ \quad - \xi[(\xi^2 + s/c)^{1/2} - \xi]\exp(z\xi)\}\exp(st)\end{array}\right]}{\left[\begin{array}{l}s(s + c'\xi^2 + c'B^{-2})\{(2ms/c)[(\xi^2 + s/c)^{1/2} + L^{-1}]\\ \quad - \xi(\xi + L^{-1})[(\xi^2 + s/c)^{1/2} - \xi]\}\end{array}\right]}\,ds \tag{131}$$

The problem is now reduced to the evaluation of the integrals appearing in Eq. (131). Since it seems impossible to obtain a general expression in terms of elementary functions, some special cases will be considered in the sequel.

Steady State Solution

When $t \to \infty$, the system will tend towards a steady state. The pore water pressures in this steady state are most easily obtained by using a theorem from the theory of the Laplace transformation (Carslaw and Jaeger, 1947, p. 256) which states that

$$\lim_{t\to\infty}\sigma = \lim_{s\to 0} s\bar{\sigma} \tag{132}$$

where $\bar{\sigma}$ is the Laplace transform of σ. The theorem holds under the condition that the limit in the right-hand member exists. Evaluation of this limit with (131) gives

$$\lim_{t\to\infty}\sigma = -\frac{Pc'}{L}\int_0^\infty \xi J_0(r\xi)\,d\xi$$

$$\times \lim_{s\to 0}\frac{\left[\begin{array}{l}\{(2ms/c)\exp[z(\xi^2 + s/c)^{1/2}]\\ \quad - \xi[(\xi^2 + s/c)^{1/2} - \xi]\exp(z\xi)\}\end{array}\right]}{\left[\begin{array}{l}(s + c'\xi^2 + c'B^{-2})\{(2ms/c)[(\xi^2 + s/c)^{1/2} + L^{-1}]\\ \quad - \xi(\xi + L^{-1})[(\xi^2 + s/c)^{1/2} - \xi]\}\end{array}\right]}$$

or

$$\sigma_\infty = \lim_{t\to\infty}\sigma = -\frac{P}{L}\int_0^\infty \frac{\xi J_0(r\xi)\exp(z\xi)}{(\xi^2 + B^{-2})(\xi + L^{-1})}\,d\xi \tag{133}$$

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No explicit expression for this integral is known to the author. Numerical evaluation is possible by writing $\xi = \rho B^{-1}$. Then the formula (133) can be written in the following dimensionless form:

$$\frac{\sigma_\infty}{P} = -\frac{B}{L}\int_0^\infty \frac{\rho J_0(\rho r/B)\exp(\rho z/B)}{(\rho^2+1)(\rho+B/L)}\,d\rho \qquad (134)$$

In Fig. 5, numerical values are represented graphically for the case $B/L = 0.35$.

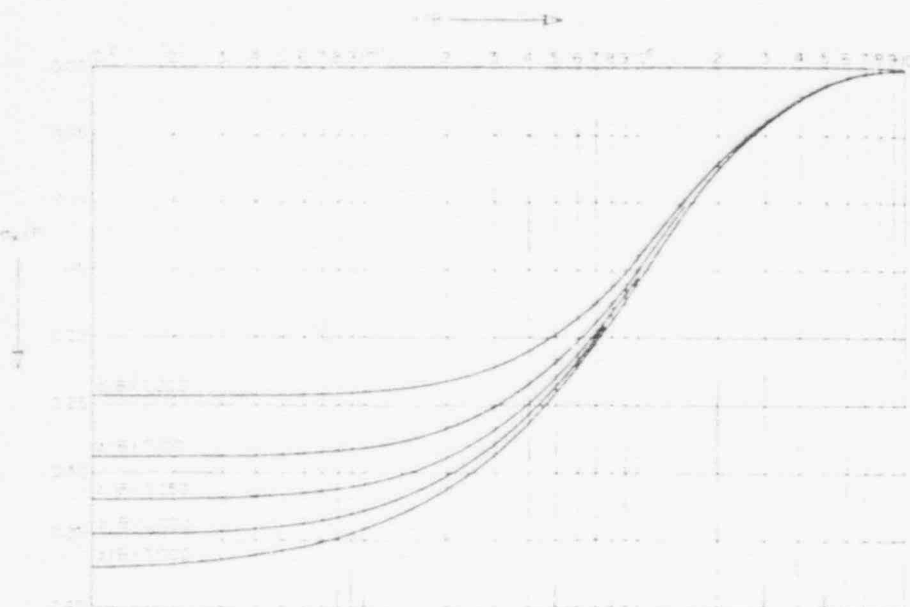

FIG. 5. Final values of the drawdown in the lower layer of a two-storied leaky artesian aquifer under the influence of a well in the upper layer.

Solution for Small Values of the Time

From the definition of the Laplace transformation,

$$\bar{\sigma} = \int_0^\infty \exp(-st)\,\sigma\,dt$$

it follows that when s is very large, the integral will mainly consist of contributions for small values of the time because the factor $\exp(-st)$ will tend to zero very rapidly. Therefore, an approximation suitable for small values of the time is obtained by taking s large in the transformed function.

When s is very large, the exponential form $\exp[z(\xi^2 + s/c)^{1/2}]$ in Eq. (131) will vanish because $z < 0$. Hence (131) reduces to

$$\sigma = [Pc'/(2\pi iL)]\int_0^\infty \xi^2 J_0(r\xi)\,d\xi \int_{\gamma-i\infty}^{\gamma+i\infty} (s/c)^{1/2}[(1+\xi^2 c/s)^{1/2} - \xi(c/s)^{1/2}]$$
$$\times \exp(z\xi)\exp(st)\,s^{-1}(s + c'\xi^2 + c'B^{-2})^{-1}\{(2ms/c)(s/c)^{1/2}[(1+\xi^2 c/s)^{1/2}$$
$$+ L^{-1}(c/s)^{1/2}] - \xi(\xi + L^{-1})(s/c)^{1/2}[(1+\xi^2 c/s)^{1/2} - \xi(c/s)^{1/2}]\}^{-1}\,ds$$

Expansion in decreasing powers of s gives

$$\sigma = [Pc'c/(4\pi imL)]\int_0^\infty \xi^2 J_0(r\xi)\exp(z\xi)\,d\xi$$
$$\times \int_{\gamma-i\infty}^{\gamma+i\infty} s^{-3}\{1 - (\xi + L^{-1})(c/s)^{1/2} + \cdots\}\exp(st)\,ds$$

The integrals in the complex s plane are well-known inverse Laplace transforms (Bateman, 1954). Hence,

$$\sigma = [Pc'c/(2mL)]\int_0^\infty \xi^2 J_0(r\xi)\exp(z\xi)$$
$$\times \{\tfrac{1}{2}t^2 - (8/15)\,\pi^{-1/2}(\xi + L^{-1})\,c^{1/2}t^{5/2} + \cdots\}\,d\xi$$

and since $z < 0$, the above integrals are well-known convergent integrals which happen to be of the Laplace transform type. Evaluation of these integrals gives

$$\frac{\sigma}{P} = \frac{c'c}{2mL}\left\{\frac{t^2}{2}\,\frac{2z^2 - r^2}{(z^2+r^2)^{5/2}} - \frac{8t^2}{15}\left(\frac{ct}{\pi}\right)^{1/2}\frac{6z^3 - 9r^2z}{(z^2+r^2)^{7/2}}\right.$$
$$\left. - \frac{8t^2}{15}\left(\frac{ct}{\pi}\right)^{1/2}\frac{2z^2 - r^2}{L(z^2+r^2)^{5/2}} + \cdots\right\}$$

Using the notation $R = (z^2 + r^2)^{1/2}$, this result can be written in the following form,

$$\frac{\sigma}{P} = \frac{1}{4m}\,\frac{c'ct^2}{R^4}\left\{\frac{2z^2 - r^2}{RL}\right.$$
$$\left. - \frac{16}{15\pi^{1/2}}\left(\frac{ct}{R^2}\right)^{1/2}\frac{-3zL(2z^2 - 3r^2) + R^2(2z^2 - r^2)}{R^2L^2} + \cdots\right\} \qquad (135)$$

It appears that a series of increasing powers of t is obtained. This series is useful for small values of the time. Since the first term is positive when $2z^2 - r^2 > 0$, it follows that the pore water pressure initially increases in the region for which $2z^2 - r^2 > 0$. This region is the interior of the cone $r = -z\sqrt{2}$, $(z < 0)$.

The result that in a certain region the pore water pressures initially

increase during a pumping test may be surprising at a first glance. However, this phenomenon may be explained from the elastic behavior of the aquifer as follows. Due to the decreasing pore water pressures in the upper layer, this layer will attract water from the upper part of the lower layer. The loss of water involves a decrease in volume of the upper part of the lower layer. All elements of a circular ring around the origin will decrease in volume, thus producing a radial displacement towards the center to keep the ring closed. For the lower parts of the layer, this radial displacement presents a loading due to which a tendency for compression will be present. Since the pore water opposes volume compression of the soil, this results in an increase of the pore water pressure.

Similar effects have been obtained in several types of problems in three-dimensional consolidation (Mandel, 1950, 1953; Cryer, 1963; De Leeuw, 1964; Schiffman, 1965). Moreover, experimental evidence in support of the theoretical results obtained by Cryer for a solid sphere of saturated soil has been reported by Gibson *et al.* (1963) and Verruijt (1965).

The present computations show increasing pore pressures in a pumping test in a two-storied aquifer. In several pumping tests this initial increase was, in fact, observed first at the village of Noordbergum in the Netherlands. Theoretical and experimental results will be compared below.

Numerical Solution for $m = \frac{1}{2}$ *and* $c' = c$

Although an analytical evaluation of the integral expression (131) seems impossible, numerical results may be obtained by means of a computer. However, numerical evalutation of the integral in the complex s plane presents the difficulties that along the path of integration, the integrand is an oscillating function because of the factor $\exp(st)$. Furthermore the convergence is rather slow. Therefore, it is reasonable to try to convert the path of integration using the theorems of the theory of functions. For simplicity it will be assumed that $c' = c$ and that $m = \frac{1}{2}$ (which means that Poisson's ratio is assumed to be zero).

With $c' = c$ and $m = \frac{1}{2}$, Eq. (131) reduces to

$$\frac{\sigma}{P} = -\frac{1}{2\pi i}\frac{c}{L}\int_0^\infty \xi J_0(r\xi)\,d\xi \times \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\{(s/c)\exp[z(\xi^2+s/c)^{1/2}] - \xi[(\xi^2+s/c)^{1/2}-\xi]\exp(z\xi)\}\exp(st)}{s(s+c\xi^2+cB^{-2})\{(s/c)[(\xi^2+s/c)^{1/2}+L^{-1}] - \xi(\xi+L^{-1})[(\xi^2+s/c)^{1/2}-\xi]\}}\,ds \quad (136)$$

Introducing the dimensionless variables

$$\rho = \xi B,\quad y = sB^2/c,\quad \eta = r/B,\quad \zeta = -z/B,\quad \psi = B/L,\quad \tau = ct/B^2$$

Eq. (136) can be written in the following form

$$\frac{\sigma}{P} = -\frac{\psi}{2\pi i}\int_0^\infty \rho J_0(\eta\rho)\,d\rho \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\{y\exp[-\zeta(\rho^2+y)^{1/2}] - \rho[(\rho^2+y)^{1/2}-\rho]\exp(-\zeta\rho)\}\exp(\tau y)}{y(y+\rho^2+1)\{y[(\rho^2+y)^{1/2}+\psi] - \rho(\rho+\psi)[(\rho^2+y)^{1/2}-\rho]\}}\,dy$$

By putting $x = y + \rho^2$, it appears that the numerator and the denominater of the integrand have a common factor $x^{1/2} - \rho$, and the following simplified result is obtained

$$\frac{\sigma}{P} = -\frac{\psi}{2\pi i}\int_0^\infty \rho J_0(\eta\rho) \times \exp(-\tau\rho^2)\,d\rho \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\{(x^{1/2}+\rho)\exp(-\zeta x^{1/2}) - \rho\exp(-\zeta\rho)\}\exp(\tau x)}{(x-\rho^2)(x+1)[x+(\psi+\rho)x^{1/2}-\rho^2]}\,dx \quad (137)$$

First the integral in the complex x plane will be considered. The singularities of the integrand are a branch point in the origin ($x = 0$) and poles of order one in the points $x = \rho^2$, $x = -1$, $x = x_1$, and $x = x_2$, where x_1 and x_2 are the zeros of $x + (\psi + \rho)x^{1/2} - \rho^2$. The branch point $x = 0$ makes it necessary to introduce a branch cut in the x plane from $x = 0$ to infinity. In the plane with the branch cut, the integrand of (137) is single valued so that the theory of analytic functions may be applied in this plane. As branch cut, the negative part of the real axis is chosen. This means that the argument of x is restricted to the interval

$$-\pi < \arg(x) \leqslant \pi \quad (138)$$

The zeros of $x + (\psi + \rho)x^{1/2} - \rho^2$ are $x = x_1$ and $x = x_2$ where

$$(x_1)^{1/2} = \tfrac{1}{2}\{[(\psi+\rho)^2 + 4\rho^2]^{1/2} - (\psi+\rho)\}$$
$$(x_2)^{1/2} = -\tfrac{1}{2}\{[(\psi+\rho)^2 + 4\rho^2]^{1/2} + (\psi+\rho)\} \quad (139)$$

and since $0 \leqslant \rho < \infty$ and $0 \leqslant \psi < \infty$, it follows that

$$0 \leqslant (x_1)^{1/2} \leqslant \rho, \qquad -\infty < (x_2)^{1/2} \leqslant 0$$

This means that $(x_2)^{1/2} = a\exp(i\pi)$ with $0 < a < \infty$ so that $x_2 = a^2\exp(2i\pi)$. The argument of x_2 appears to be 2π and since restriction is made to values of $\arg(x)$ in the interval $(-\pi, +\pi)$, it follows that x_2 is not a pole in the relevant plane. The singularity $x = x_1$, however, lies on the positive part of the real axis, $\arg(x_1) = 0$, hence $x = x_1$ is indeed a pole to be taken into account.

Now consider the integral

$$I_{AB} = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\{(x^{1/2} + \rho)\exp(-\zeta x^{1/2}) - \rho\exp(-\zeta\rho)\}\exp(\tau x)}{(x - \rho^2)(x - 1)[x - (\phi - \rho)x^{1/2} - \rho^2]}\,dx \qquad (140)$$

In Fig. 6 a contour in the complex x plane is shown. It is supposed

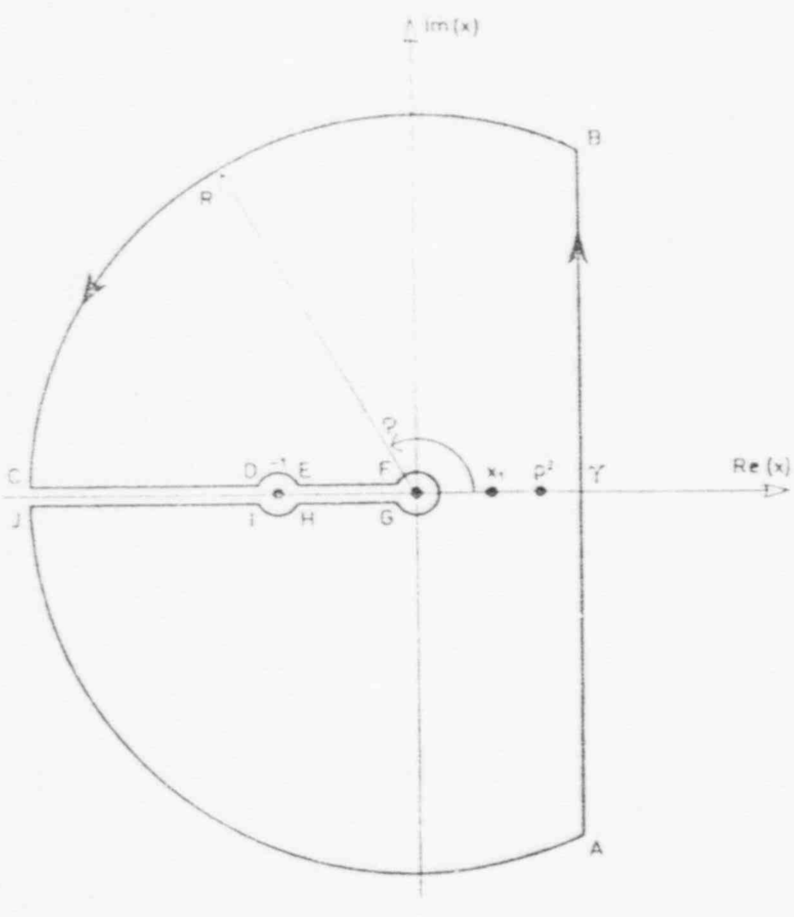

Fig. 6. Contour in the complex x plane.

that the radius R of the circle segments is very large. The closed contour is an extension of the integration path AB, hence

$$I_{AB} = I - I_{BC} - I_{CD} - I_{DE} - I_{EF} - I_{FG} - I_{GH} - I_{HI} - I_{IJ} - I_{JA} \qquad (141)$$

where I denotes the contour integral. Since the contour does not cross the branch cut and the integrand is a single-valued analytic function inside the contour except in the simple poles $x = x_1$ and $x = \rho^2$, it now follows from the residue theorem (Titchmarsh, 1939) that

$$I = \sum \text{ residues in } \; x = x_1 \; \text{ and } \; x = \rho^2$$

Evaluation of these residues gives

$$I = \frac{\exp(-\zeta\rho)\exp(\tau\rho^2)}{(\rho^2 - 1)(\rho - \phi)} + \frac{2x_1^{1/2}\{(x_1^{1/2} + \rho)\exp(-\zeta x_1^{1/2}) - \rho\exp(-\zeta\rho)\}\exp(\tau x_1)}{(\rho^2 - x_1)(1 - x_1)(2x_1^{1/2} - \phi - \rho)} \qquad (142)$$

The integral along AB will be known if the integrals along the parts $BCDEFGHIJA$ are known. It can be shown that

$$\lim_{R\to\infty} (I_{BC} + I_{JA}) = 0 \qquad (143)$$

The proof of the vanishing of this integral, which will not be given in detail, makes use of the fact that along the parts BC and JA of the contour

$$|\exp(-\zeta x^{1/2})| \leqslant 1 \qquad \text{and} \qquad |\exp(\tau x)| \leqslant \exp(\tau\gamma)$$

Hence the numerator tends to infinity of order $R^{1/2}$ when $R \to \infty$. Since the denominator tends to infinity of order R^3 and the length of the path of integration is of order R, it follows that the integral tends to zero of order $R^{-3/2}$ when $R \to \infty$. The contribution of the integral along the small circle FG vanishes because the integrand is bounded in $x = 0$ and the length of the integration path tends to zero,

$$I_{FG} = 0 \qquad (144)$$

The integrals along DE and HI can be evaluated by putting

$$x = \exp(i\pi) + a\exp(i\varphi) \qquad (0 < \varphi < \pi)$$

and

$$x = \exp(-i\pi) + a\exp(i\varphi) \qquad (-\pi < \varphi < 0)$$

respectively, and then passing into the limit $a \to 0$. This gives

$$I_{DE} - I_{HI} = \frac{\{2(\rho^3 - \psi)\cos\zeta - (2\rho^2 - \rho\psi - 1)\sin\zeta - \rho(1-\rho^2)\exp(-\zeta\rho)\}\exp(-\tau)}{(1-\rho^2)[(1-\rho^2)^2 + (\psi+\rho)^2]} \tag{145}$$

Finally, the integrals along $CD + EF$ and $GH + IJ$ are obtained by putting $x = w\exp(i\pi)$ and $x = w\exp(-i\pi)$, respectively. This gives

$$I_{CD+EF+GH+IJ} = -\pi^{-1}\int_0^\infty \{[(2\rho^2 - \rho\psi - w)\, w^{1/2}\cos(\zeta w^{1/2}) - (\rho^3 - w\psi)\sin(\zeta w^{1/2}) + \rho(\psi - \rho)\, w^{1/2}\exp(-\zeta\rho)] \times \exp(-\tau w)\}\{(w + \rho^2)(1 - w)[(w + \rho^2)^2 + w(\psi - \rho)^2]\}^{-1}\, dw \tag{146}$$

where the $\int$ indicates that the integral is a Cauchy principal value. With (142)–(146), Eq. (141) now gives

$$\begin{aligned} I_{AB} = {} & \exp(-\zeta\rho)\exp(\tau\rho^2)\{(\rho^2 - 1)(\psi + \rho)\}^{-1} \\ & - 2x_1^{1/2}\{(x_1^{1/2} - \rho)\exp(-\zeta x_1^{1/2}) - \rho\exp(-\zeta\rho)\} \\ & \times \exp(\tau x_1)\{(\rho^2 - x_1)(1 - x_1)(2x_1^{1/2} - \psi - \rho)\}^{-1} \\ & - \{2(\rho^3 - \psi)\cos\zeta - (2\rho^2 - \rho\psi - 1)\sin\zeta - \rho(1 - \rho^2)\exp(-\zeta\rho)\} \\ & \times \exp(-\tau)\{(\rho^2 - 1)[(\rho^2 - 1)^2 + (\psi - \rho)^2]\}^{-1} \\ & - \pi^{-1}\int_0^\infty \{-(2\rho^2 - \rho\psi - w)\, w^{1/2}\cos(\zeta w^{1/2}) \\ & - (\rho^3 - w\psi)\sin(\zeta w^{1/2}) - \rho(\psi - \rho)\, w^{1/2}\exp(-\zeta\rho)\}\exp(-\tau w) \\ & \times \{(w - \rho^2)(1 - w)[(w - \rho^2)^2 + w(\psi - \rho)^2]\}^{-1}\, dw \end{aligned} \tag{147}$$

where $x_1^{1/2} = \frac{1}{2}\{[(\psi + \rho)^2 - 4\rho^2]^{1/2} - (\psi + \rho)\}$.

No explicit solution of the integral in Eq. (147) is known to the author. Comparison of the two equivalent expressions (140) and (147) for I_{AB} shows that in both expressions a complicated integral appears. However, the integral in (140) is slowly convergent, whereas the integral in (147) is rapidly convergent, especially for large values of the time parameter τ, because of the factor $\exp(-\tau w)$. Hence, for numerical evaluation, (147) is to be preferred. Actually, in this case the above procedure of converting the path of integration is justified only because it yields an integral more suitable for numerical evaluation.

The formula for the pore water pressure is, with (137) and (140),

$$\sigma/P = -\psi\int_0^\infty \rho J_0(\eta\rho)\exp(-\tau\rho^2)\, I_{AB}\, d\rho \tag{148}$$

This integral is also rapidly convergent, especially for large values of τ. In Fig. 7 numerical results are shown as a function of time for the special

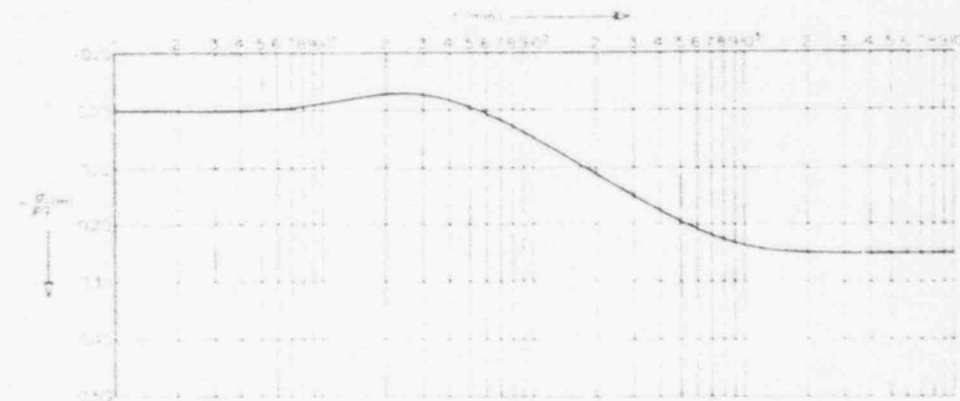

FIG. 7. Theoretical results for the drawdown in the lower layer of a two-storied leaky artesian aquifer during a pumping test in the upper layer.

case $\psi = B/L = 0.35$, $\zeta = -z/B = 0.200$, $\rho = r/B = 0.025$. Numerical values were obtained from Eq. (135) for small values of the time parameter and from Eqs. (147) and (148) for large values of the time parameter. In order to plot the drawdown $-\sigma/\rho g$ as a function of time, the definitions of P and τ

$$P = Q_0\rho g/(2\pi K_1 b_1), \qquad \tau = ct/B^2$$

were used. Since the consolidation coefficient c of the lower layer of the aquifer was set equal to the consolidation coefficient c' of the higher layer, its value is given by

$$c = c' = K_1 b_1/[\rho g b_1(\alpha' + n\beta')]$$

Numerical values for $K_1 b_1$, Q_0, B, and $\rho g b_1(\alpha' + n\beta')$ were chosen in accordance with the values used for Fig. 3, i.e.,

$$Q_0 = 1.74 \times 10^{-2}\ \mathrm{m^3/sec}, \qquad K_1 b_1 = 0.30 \times 10^{-2}\ \mathrm{m^2/sec}$$

$$B = 400\ \mathrm{m}, \qquad \rho g b_1(\alpha' + n\beta') = 5.6 \times 10^{-4}$$

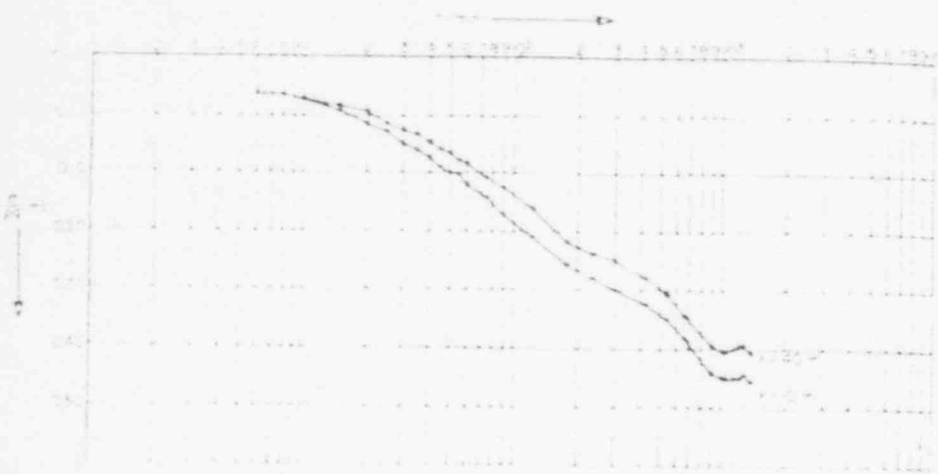

FIG. 8. Experimental results for the drawdown in the lower layer of a two-storied leaky artesian aquifer during a pumping test in the upper layer.

In Fig. 8, some experimental results obtained during the pumping test in the polder Vierbannen, already mentioned in Section 4, are represented graphically. It appears that the theoretical curves of Fig. 7 and the experimental data of Fig. 8 show similar tendencies. A closer fit might be obtained by choosing different values for the soil constants. However, the purpose of the present study is to give a qualitative explanation of the fact that the pore water pressures in the lower layer of a two-storied aquifer initially may increase during a pumping test in the higher layer, and not to describe a method for the determination of the soil constants. Indeed, both the theoretical and experimental curves show this effect, and the orders of magnitude correspond to each other.

7. Conclusion

The reader looking for the definition of the storage coefficient in this study, which according to its title deals with elastic storage of aquifers, may be disappointed not to find it. Although the author is aware that for many practical purposes the simple differential equation, in connection to which this storage coefficient usually appears, may give satisfactory results, he wants to underscore that on theoretical grounds the more complicated Biot theory based on the theory of elasticity has to be preferred. Therefore, in the present work the use

of the words "storage coefficient" has been avoided carefully, even in cases where the Biot theory reduces to the simpler Jacob theory.

To support the theory advocated in this paper, experimental evidence was produced indicating that a remarkable effect occurring in two-storied aquifers (or more generally, in nonhomogeneous aquifers), called the *Noordbergum effect*, can be explained qualitatively by this theory.

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