Attachment B

S&A Calculation No.91C2672-C018, Revision 0

Fragility Analysis of Station Blackout Diesel Generator
Pilgrim Nuclear Power Sation

Calc No 91C2672-C018

Rev. 0

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of
Station Blackout Diesel Generator
Pilgrim Nuclear Power Station

by

Bahman Lashkari
John W. Reed

Prepared for

Stevenson & Associates

January 1994

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Objective:

This calculation presents the fragility analysis of Station Blackout Diesel Generator for Pilgrim Nuclear Power Station. It utilizes information and data from SRT walkdown, as provided by Stevenson & Associates, and the relevant plant documents and drawings. Results are quantified in terms of the median capacity and logarithmic standard deviations. It is assumed that the reader has a general knowledge of fragility evaluation and methodology.

Approach:

As in any fragility analysis, first the weakest component of the Diesel Generator Components is identified by judgment, document review, and discussion with SRT. Next the credible failure modes for the weak component are examined and deterministic calculations are performed in order to arrive with the controlling failure mode. Finally, based on the controlling failure mode fragility values are developed.

To expedite the calculations, a Mathcad program was written based on the controlling failure mode which was the anchorage system for the 4160 V Switchgear A8.

Section 1.0 presents a general description of the SBO Diesel Generator and its components. Section 2.0 describes the seismic input (reference earthquake) for the seismic PRA (SPRA), Section 3.0 provides the deterministic analysis of Switchgear A8 with emphasis on the structural integrity and anchorage and finally Section 4.0 contains the fragility analysis and results. Section 5.0 gives a listing of references. A copy of some of the photos taken during the SRT walkdown are presented in Appendix A.

Note that relays are outside the scope of this Calculation and are not addressed.

1.0 Station Blackout Diesel Generator

1.1 General Description

Station Blackout Diesel Generator is a backup power source to the main diesel generator. The critical parts of this component consist of the engine and generator enclosed in a trailer building located in the yard, two underground fuel storage tanks, and a 4160 V Switchgear A8.

In order to calculate the fragility parameters for this component need to identify the weakest part. A description of each part and its credible failure modes are discussed in the following sections.

Note that this calculation accounts for the planned modifications and represents the component in its upgraded condition. Planned modifications, as provided by Stevenson and Associates, are discussed in each appropriate section.

1.2 Generator - Engine

The generator-engine is housed in a trailer located in the yard. The trailer is approximately 45' long, 12' wide and 12' high (from base to roof). Based on discussions with SRT who performed a walkdown of this component the credible failure mode is the sliding of the silencer tank on the trailer roof. The silencer tank is supported by three saddles which are not tied together and the tank is held by a strap at each saddle. The plan modification calls for the saddles to be tied together along the axial axis of the tank to prevent weak-axis bending of the saddles and sliding of the silencer.

Based on the review of manufacturer's drawings and according to Mr. Tom Tracy of Stevenson & Associates, the generator-engine is an integral part of the trail building including all peripheral components such as day tank, air receiver, lube oil tank, etc. and therefore can take credit for shipping as a proof test. In other words, during the shipping the unit was subjected to high g's and, therefore, has high capacity.

The trailer is anchored at 16 places to the concrete foundation which is 4'-6" deep as shown on Dwg. C1144, Rev. E0 and M6A4, Rev. E1. A review of the anchorage (Dwg. C1144, Rev. E0) shows the anchor bolts as 1 1/8" ϕ stud bolts with proper sleeve system embedded in concrete with adequate rebars around the sleeve to prevent concrete cone failure. Based on this review the anchorage is judged to have high capacity.

The following documents were reviewed as part of the Generator-Engine evaluation:

- Dwg. M6A116, Rev. E0
- Dwg. C1142, Rev. E0
- Dwg. M6A115, Rev. E0
- Dwg. M6A117, Rev E0
- Dwg. M6A4, Rev. E1
- Dwg. M6A13, Rev. E1
- Dwg. C1144, Rev. E0
- Purchase Order No. 17322-M-SAM-13, Rev. 3
- Dwg. M6A14, Rev. E0
- Dwg. M6A2 Sh. 2, Rev. E0
- Dwg. M6A2 Sh.1, Rev. E0

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1.3 Underground Fuel Storage Tanks & Piping

There are two underground fuel storage tanks each with a 20,000 gallon capacity. The tanks are adequately secured with six straps for each tank where each strap is properly embedded in the concrete slab (1'-0" deep). There are back fill material around the tanks which will act as a passive energy dissipation medium limiting seismic response of the tanks. The straps include turn buckles to ensure snug fit.

The fuel supply and return lines are 2" ϕ steel pipes with a zee layout from the tank to the engine trailer. The pipes are also underground and may be susceptible to relative displacement. However, because of the massive size of the tanks and their anchoring system it is judged that no significant displacement will take place before a peak ground acceleration of about 1.5 to 2.0g. Additionally, the inherent fuel lines flexibility due to geometry minimizes the pipe strain from relative displacements.

Following documents were reviewed as part of the underground Fuel Storage Tanks and Piping:

- Dwg. C1143, Rev. E0
- Dwg. C1145, Rev. E0
- Dwg. for Yard Piping

1.4 4160 V Switchgear A8

The A8 Switchgear is a 4160 V metal-clad switchgear consisting of 3 cubicles and an outside enclosure. It is located outdoor and is provided with a channel base consisting of 6" channels. The cabinet is 91" by 115" in plan at the channel base and is 109" high. Each cubicle consists of sheet metal panels and steel tube frame welded to the enclosure base and roof. The enclosure consists of hinged doors and sheet metal panels.
The enclosure is bolted to a plate which is welded to the channel base. The connection between the cubicles and enclosure consist of series of 3/8" screws along the height of the cabinet. From review of photos (see Appendix A) taken by SRT (see Picture 20) it appears that a total of 12 screws (3 rows each having 4 screws) connect the 1"x1" tubes to the posts.

The channel base is anchored to the concrete foundation at four locations each consisting of a mounting plate (clips) which is secured to the concrete with ½" Φ "J" bolts.

The cabinet houses batteries and battery charger, removable circuit breakers, transformers, bushings and instrumentation.

The batteries are arranged in a step-by-step layout with two side walls constraining the batteries from lateral movements. Each step of batteries provides a support for the upper batteries except for the first row that does not have any support (see Pictures 6 and 7). The plan modification calls for installation of a strap around the first row of the batteries to provide restraint.

Based on the conversation with SRT and Mr. Tom Tracy it was indicated that all components inside the switchgear are adequately attached to the cabinet and since they are (except for the circuit breakers) an integral part of the cabinet, thus can take credit for shipping (i.e., high capacity) and assume that structural failure of these components is not a credible failure mode.

From above discussion and noting that relays are outside the scope of this calculation, emphasis was placed on structural integrity of the switchgear and its anchorage.

For the structural integrity the weak connection was judged to be the ability of 3/8" screws connecting the cubicles to the posts in carrying the shear caused by overturning. For anchorage, the anchor bolts (J-bolts) and the hold down clips are analyzed. Section 3.0 provides a deterministic evaluation of the cabinet for these failure modes; i.e., hold down plates (clips), anchor bolts (J-bolts), and the 3/8" screws (structural integrity).

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1.4 Summary

From previous sections, the weakest element of the Station Blackout (SBO) Diesel Generator was judged to be the 4160 V Switchgear A8 and as such it represents all the components of the SBO Diesel Generator.

A deterministic analysis of this component with emphasis on the structural integrity and anchorage will be performed in order to evaluate the controlling failure mode for the switchgear.

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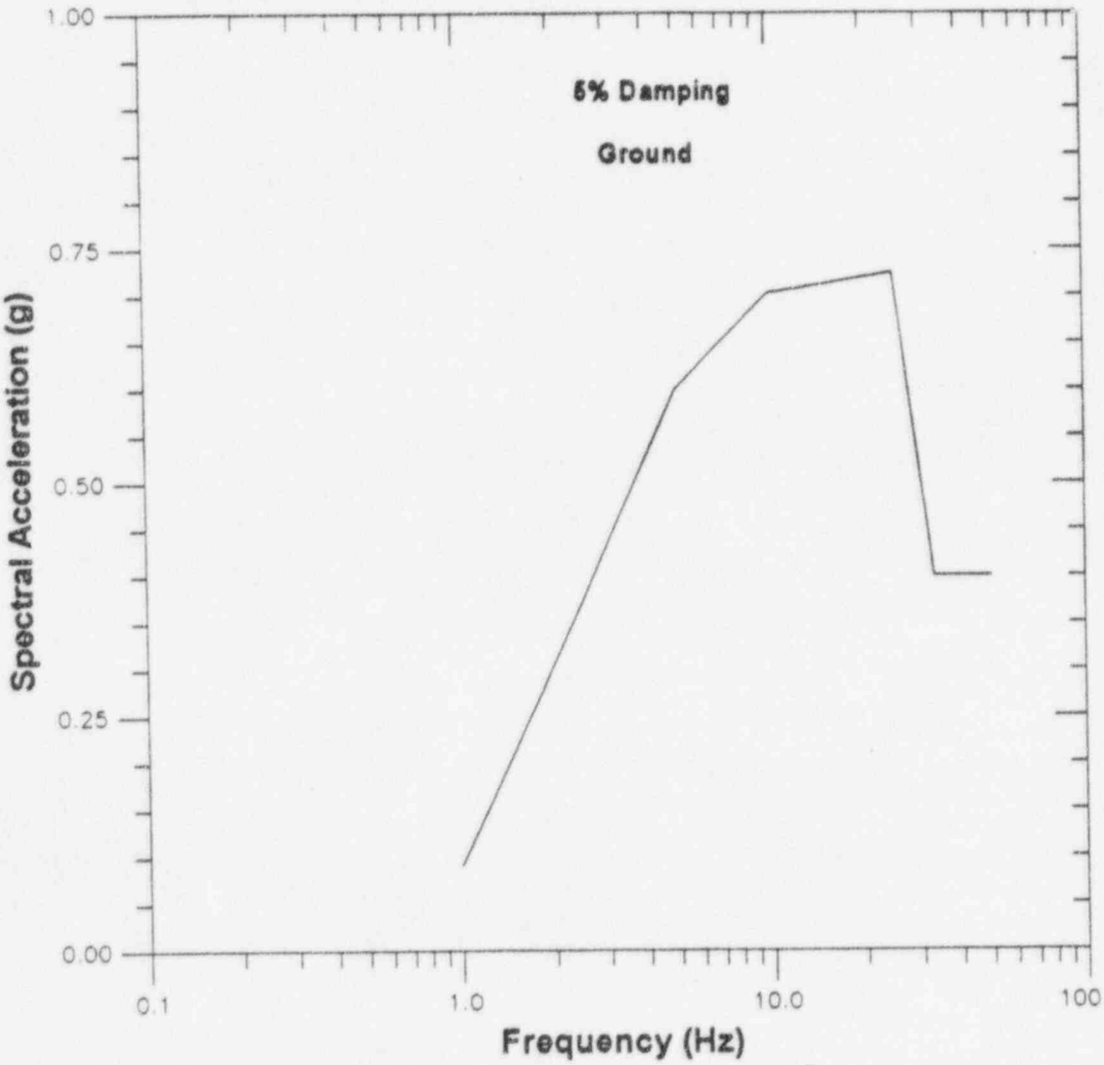
2.0 Seismic Input

The Pilgrim Station SPRA ground spectrum (reference earthquake) for a 5% damping as provided by Stevenson and Associates (Reference 1) is shown below. This ground spectrum which is anchored to 0.4g will be used as the seismic input for both horizontal directions in the analysis. For vertical direction, 2/3 of horizontal seismic input will be used.

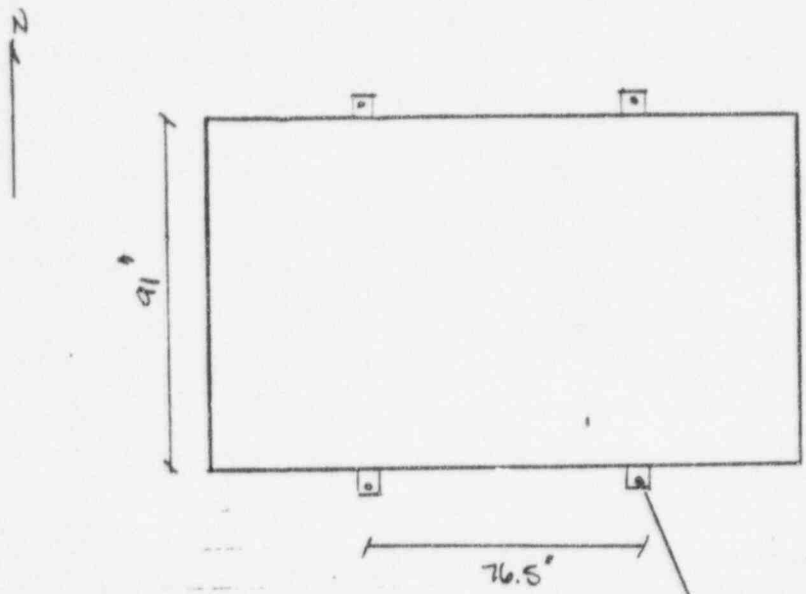


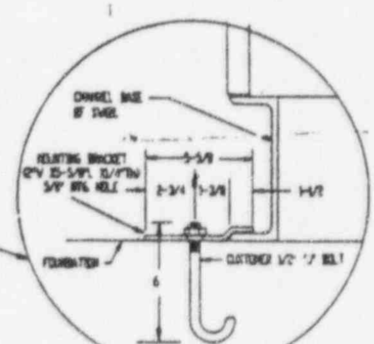
3.0 Deterministic Analysis of 4160V Switchgear A8

A general layout of the switchgear is provided in Attachment 1. The switchgear consist of 3 cubicles and an enclosure welded to the channel base which is anchored to the floor at four places with $1/2"\phi$ J-bolts and hold down clips. Anchorage detail is shown below.

3.1 Anchorage

The enclosure is 91" x 115" in plan and 109" high.





ANCHORING DETAIL
DETAIL 'I'

Note, the actual switchgear orientation is 90° rotated with respect to N-S direction. However, this does not affect the analysis since the seismic input for both N-S and E-W directions is the same

Based on GIP, use a 31 #/ft^3 for weight, thus:

$$\text{weight} = \frac{91 \times 115 \times 109}{12^3} \times 31 = 20464^{\#}$$

However, the switchgear has lots of open space and is not heavily loaded in all places. Therefore, use a weight of 10 kips as a median weight.

$\therefore \quad W = 10$ kips

Assume a 10 Hz fixed-base frequency in both directions with a 5% damping.

3.1.1 Anchorage Model

A detail description of anchorage model is provided in Attachment 2. In essence the model is based on elastic piping theory and accounts for base channel rotation and uplift.

A MathCad program was written, incorporating the analytical model, to expedite the calculations since to solve the governing equations is an iterative procedure. The Math-Cad program listing and results are presented in Attachment 3.

Note that the final results of the MathCad program are based on a hold down plate with a 2.25" width instead of existing 2.0" width as shown on the drawings. In order to have a median PGA capacity of around 1.0g ... required the plate to be 2.25" wide. Also it is assumed that the hold down plate is fully extended over the channel flange and is welded (full penetration) to the channel web.

Additionally, based on a fixed-base frequency of 10 Hz, the final cabinet/anchorage frequency was calculated to be 7.59 Hz in the north-south direction and 9.85 Hz in the east-west direction.

3.1.2 Hold Down Plate (Clip)

The controlling failure mode for the hold down plate is the moment failure at the anchor bolt. Assuming a full plastic section as the ultimate capacity, i.e,

$$M_p = \sigma_y Z$$

where $$Z = \frac{(w_p - d_h) t_p^2}{4}$$

Z = Plate plastic section modulus at bolt hole

w_p = Plate width (assumed to be 2.25")

d_h = Bolt hole diameter

t_p = Plate thickness = 0.25"

σ_y = yield capacity for A36 steel

From the Mathcad run results (see Attachment 3, Page 12) using median values, an earthquake scale factor (, F_{S1}) of 1.67 was calculated.

In the following two sections, the anchor bolts and the structural integrity of the switchgear, will be checked to see if hold down plate is (in fact) the controlling element.

3.1.3 Anchor Bolt

The anchor bolts consist of ½" φ J-bolts, one per each field drum plate. From Math Cad run results (see Attachment 3, Page 11), the acting bolt forces for an earthquake scale factor of 1.67 are:

$$T_E = \text{Tension} = 3.808 \text{ k}$$

$$V_E = \text{Shear} = 2.759 \text{ k}$$

Bolt Failure

Assuming A-307 bolt material then $\check{\sigma}_u = 64$ ksi. Also the tensile bolt area is 0.142 in² for ½" φ bolt. Therefore, the allowable median bolt capacity is:

$$\text{Tension} \quad \check{T}_A = 0.9\check{\sigma}_u A_b = 0.9 \times 64 \times 0.142 = 8.2 \text{ k}$$

$$\text{Shear} \quad \check{V}_A = 0.62\check{\sigma}_u A_b = 0.62 \times 64 \times 0.142 = 5.6 \text{ k}$$

Using a squared interaction equation, calculate the margin factor, F_M, over the 1.67 scale factor.

$$\left(\frac{F_M V_E}{\check{V}_A}\right) + \left(\frac{F_M T_E - T_m}{\check{T}_A}\right)^2 = 1.0$$

where T_m = Compression due to gravity = $\frac{10}{4} = 2.5$ k

Substituting values into the interaction equation

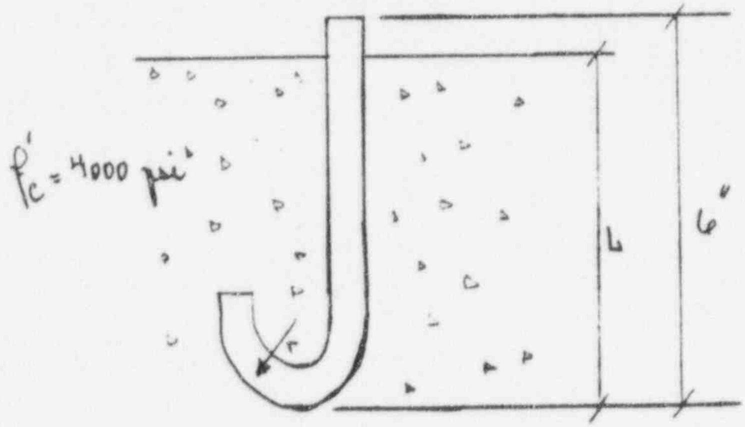
$$\left(\frac{F_M 2.759}{5.6}\right)^2 + \left(\frac{F_M 3.808 - 2.5}{8.2}\right)^2 = 1.0 \Rightarrow F_M = 1.75$$

Therefore the margin factor for the bolt failure is 1.75 over the scale factor of 1.67. Thus the hold down plate controls.

Concrete Failure

Another credible failure mode for J-bolts is the bond (pull out) failure.

$\frac{1}{2}$" Φ J-Bolt



Assuming a projection length of 1" then $L = 6 - 1 = 5$"
From GIP, use $r = 3D$ where D = bolt diameter
From Reference 2, calculate equivalent anchorage length

$$l = (L - r) + \underbrace{8r - 4D}$$

let $r = 3D$; due to hook

$$l = L - 3D + 24D - 4D = L + 17D$$

but $D = 0.5''$,

$$l = 5 + 17 \times 0.5 = 13.5''$$

Assuming a plain bar and noting that median $f'_c = 4000 \times 1.2 = 4800$ psi where factor of 1.2 accounts for concrete aging (Reference 4), the ultimate anchorage bond stress from Reference 2 is $1.9 \times 145.038 = 275.6$ psi (the 145.038 is the units conversion factor). Therefore, the allowable bond force is:

$$\check{T}_A = 275.6 \times \text{bar surface area}$$

But bar surface area $= \pi D \times l = \pi \times 0.5 \times 13.5 = 21.2$ in^2

and

$$\check{T}_A = 275.6 \times 21.2 = 5843 \text{ lbs}$$

Calculate the seismic margin factor, F_M using a linear equation

$$\frac{F_M T_e - T_m}{\check{T}_A} = 1$$

$$\frac{F_M 3.808 - 2.5}{5.843} = 1 \Rightarrow F_M = 2.19$$

And the seismic margin over the 1.67 scale factor is 2.19. Thus, the hold down plate controls.

3.2 Structural Integrity

Structural failure of the switchgear is judged to be the ability of 3/8" screws connecting the cubicles to the posts in carrying the shear caused by overturning. The connection consist of a total of 12 - 3/8" φ screws (3 rows each having 4 screws, one screw per post per row). It should be noted that global shear forces are resisted in both directions by the sheet metal panels. Each cubicle is susceptible to overturning in E-W direction (short direction).

Since there are 3 cubicles, then

$$M_{\text{overturning per cubicle}} = \frac{W}{3} \times Sa_{EW} \times \frac{h}{2}$$

where from Mathcad run results (see Attachment 3, Page 1)

$$Sa_{EW} = 0.70g \quad \text{(corresponding to a 0.4g PGA)}$$

Thus; knowing that $h = 109"$ and $W = 10\ k$

$$M_{\text{overturning}} = \frac{10}{3} \times 0.7 \times \frac{109}{2} = 127.166 \quad k.in$$

Calculate tension-compression forces in each post;

$$T = C = \frac{M_{\text{overturning}}}{2 \times l}$$

where $l = 38.375"$ and the 2 factor is because there are 2 posts.

or $$T = C = \frac{127.166}{2 \times 38.375} = 1.657 \text{ k}$$

Since there are 3 screws for each post (3 rows) the acting shear per screw is

$$V_e = \frac{1.657}{3} = 0.552^{①} \text{ k}$$

Now for 3/8" ϕ screw, assuming A-307 material then $\check{\sigma}_u = 64$ ksi.
Also the tensile bolt area is 0.078 in^2. Therefore, the bolt allowable shear capacity is:

$$\check{V}_A = 0.62\,\check{\sigma}_u\,A_b = 0.62 \times 64 \times 0.078 = 3.095 \text{ k}$$

Calculate earthquake scale factor, F_S, using a linear equation

$$F_S = \frac{3.095}{0.552} = 5.61$$

This F_S value is higher than the 1.67 scale factor calculated for the hold down plate. Thus, the hold down plate controls.

① Note that the 0.552 k shear force per screw assumes equal distribution between the three screws and as such may be a little low for the worst case screw (which is the lowest screw). However, since the F_S is much higher than 1.67 a more detailed analysis is not required.

4.1.1 Strength Factor, F_S

The Strength Factor, F_S, is defined as the elastic scale factor required to scale the reference earthquake to reach the elastic capacity. The reference earthquake is shown in Section 2.0 with a PGA value of 0.4g.

The controlling failure mode is determined to be the bending of the hold down plate due to upward (tensile) forces. This acting upward force, F_e, is given by (see Attachments 2 and 3, specifically Attachment 3 Pages 1 and 12)

$$F_e = 3.006 F_S - 2.5 \quad \text{in kips} \qquad (1)$$

A Mathcad program was written to calculate the allowable (capacity) F_e value based on full plastic moment capacity of the plate. This program utilizes the formulation developed for the plate-bolt anchorage system as discussed in Section 3.1.1, except for using a 2.25" width and full extension to the web of the channel.

Using the structural and geometrical properties of the plate, bolt and channel with a median yield capacity (plate) of 44 ksi (for A36 steel and A307 bolt), from Mathcad Run results (Attachment 3 Page 11) median $\check{F}_e$ is calculated as 2.530 kips. Thus,

$$2.530 = 3.006 \check{F}_S - 2.5 \Rightarrow \boxed{\check{F}_S = 1.673}$$

Note that as part of the Mathcad program a check of the calculated cabinet/anchorage system frequency against the assumed value was made (see Attachment 3, Page 12).

4.1.2 Inelastic Energy Absorption Factor, F_μ

Since moment failure is a ductile mode of failure for which a ductility factor and subsequently F_μ should be calculated.

Let the median plate vertical displacement at ultimate be 20 times the elastic displacement. From Mathcad Run the elastic plate displacement is 0.01275" (See Attachment 3, Page 28, Del_{plate}). Thus;

$$\Delta v_{ultimate} = 20 \times 0.01275 = 0.255"$$

From Math cad Run (see Attachment 3, Page 23), for the ultimate case the F_S was calculated to be

$$F_{S\mu} = 1.92$$

Now to calculate ductility factor need to construct the $F - \Delta$ curve for the plate where Δ is the total horizontal displacement. From before for the elastic (yield) point;

$F_{Se} = 1.67$ and $\Delta_e = 0.18759"$ (See Attachment 3, Page 12. Note that N-S direction is the controlling direction

Need to calculate Δ for the ultimate point. From Mathcad Run (see Attachment 3, Page 23)

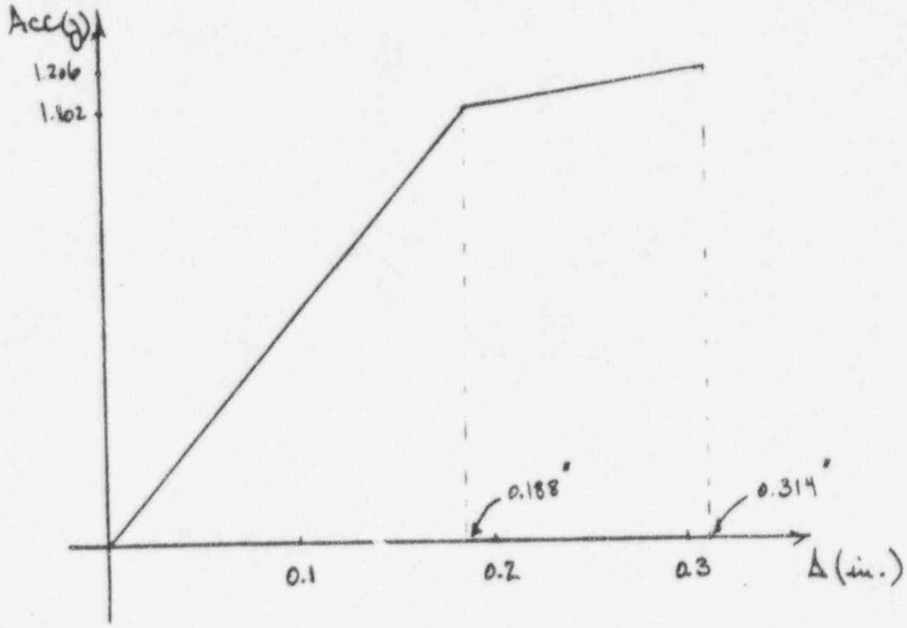
$$\Delta_\mu = 0.31351" \quad \text{and} \quad F_{S\mu} = 1.92$$

Note that the north-south frequency associated with yield case is 7.586 HZ and for the ultimate case is 6.134 Hz. Therefore, the acting accelerations are

$$Acc_{y} = F_{S_{y}} S_{a_{N\text{-}S}(7.586, 5\%)} = 1.67 \times 0.660 = 1.102\ g$$

$$Acc_{\mu} = F_{S_{\mu}} S_{a_{N\text{-}S}(6.134, 5\%)} = 1.92 \times 0.628 = 1.206\ g$$

Plot F-Δ Curve and Calculate ductility factor μ and strain hardening ratio, S:



$$\mu = \frac{0.314}{0.188} = 1.67$$

$$S = \frac{\dfrac{1.206 - 1.102}{0.314 - 0.188}}{\dfrac{1.102}{0.188}} = 0.14$$

A more refined F-Δ Curve and μ and S values are shown on Page 23 of the Attachment 3.

Estimation of the Median Ductility Scale Factor, Fmu

The median ductility scale factor, Fmu, is obtained by several methods (See Reference 1):

The following variables are used in the analysis:

$f_{ns} = 7.5856 \cdot Hz$	The A8 Bus frequency
$b := \beta eta$ $\quad b = 0.05$	Elastic damping
$fk := 2.5 \cdot \frac{1}{sec}$	Response spectrum knuckle frequency (see Attachment 5)
$s = 0.13794$	Strain hardening ration
$mu = 1.67125$	Ductility factor

Riddle-Newmark method for Calculating FuRn

The details of the methodology are given in Reference 1

Lets first calculate Fmu at the peak ground acceleration level (zpa)

The zpa value is:

$zpa := 0.4 \cdot g$

and the spectral acceleration at the fundamental frequency, f, and elastic damping, b, is:

$Sa := SSa\left(f_{ns}, b\right) \qquad Sa = 0.65961 \cdot g$

Fmu is estimated as:

$Fu4 := \frac{Sa}{zpa} \cdot mu^{0.11} \qquad Fu4 = 1.74485$

Then, in the acceleration range of the spectrum, Fmu is estimated as:

$Fu3 := (2.67 \cdot mu - 1.673)^{0.411} \qquad Fu3 = 1.52439$

Finally, in the velocity range of the spectrum, Fu is estimated as:

$Cf := \left(\frac{fk}{f_{ns}}\right) \cdot \left(\frac{fk}{f_{ns}} < 1.0\right) + \left(\frac{fk}{f_{ns}} \ge 1.0\right) \qquad Cf = 0.32957$

$Fu2 := (2.24 \cdot mu - 1.24)^{0.611} \cdot Cf \qquad Fu2 = 0.5774$

The median ductility scale factor, Fu.RN, using the Riddle-Newmark method, is obtained as:

$Fu1 := Fu3\cdot(Fu3<Fu4) + Fu4\cdot(Fu3 \geq Fu4)$ $\qquad Fu1 = 1.52439$

Using the ratio of ultimate static capacity to yield static capacity, R, defined as:

$R := 1 + s\cdot(mu - 1)$ $\qquad R = 1.09259$

$Fu_{RN} := \dfrac{(Fu1\cdot(Fu2<Fu1) + Fu2\cdot(Fu2 \geq Fu1))}{R}$ $\qquad Fu_{RN} = 1.3952$

Modified Riddle-Newmark method for Calculating FuMRN

Because the Riddle-Newmark method does not account for second slope of the force-deformation curve, the effect of second slope is accounted for by modifying the ductility ratio. Therefore, the modified ductility ratio is:

$mu_1 := 0.5 + \dfrac{(mu - 1)\cdot(1 + R) + 1}{2\cdot R^2}$ $\qquad mu_1 = 1.50718$

At the peak zpa level, Fu is:

$Fum4 := \dfrac{Sa}{zpa}\cdot mu_1^{\,0.11}$ $\qquad Fum4 = 1.72513$

In the acceleration range of the spectrum, Fmu is:

$Fum3 := \left(2.67\cdot mu_1 - 1.673\right)^{0.411}$ $\qquad Fum3 = 1.42101$

In the velocity range of the spectrum, Fmu is:

$Fum2 := \left(2.24\cdot mu_1 - 1.24\right)^{0.611}\cdot Cf$ $\qquad Fum2 = 0.52402$

where Cf has been defined previously.

The median ductility scale factor, Fu.MRN, using the modified Riddle-Newmark method is:

$Fum1 := Fum3\cdot(Fum3<Fum4) + Fum4\cdot(Fum3 \geq Fum4)$ $\qquad Fum1 = 1.42101$

$Fu_{MRN} := Fum1\cdot(Fum2<Fum1) + Fum2\cdot(Fum2 \geq Fum1)$ $\qquad Fu_{MRN} = 1.42101$

Effective Riddle-Newmark method for Calculating FuERN

Because the modified Riddle-Newmark method only accounts for second slope of the force-deformation curve, the last correction to perform is to account for ground motion duration.

The factor to account for earthquake duration is:

$CD := 1.0$ set to 1.0 since pinching or fatigue is not a concern (short duration)

and the median ductility scale factor, FuERN, using the effective Riddle-Newmark method is:

$Fu_{ERN} := 1 + CD \cdot (Fu_{MRN} - 1)$ $Fu_{ERN} = 1.42101$

Effective Spectral method for Calculating FuSA

In this method, Fmu is calculated using the effective frequency and damping ratio of the structure. First, the ratio of secant stiffness to elastic stiffness is estimated as:

$Ks_K := \dfrac{(1 + s \cdot (mu - 1))}{mu}$ $Ks_K = 0.65376$

Next, the ratio of the secant frequency to elastic frequency is estimated to be:

$fs_f := \sqrt{Ks_K}$ $fs_f = 0.80855$ Compare to:

$fs := fs_f \cdot f_{ns}$ $fs = 6.13335 \cdot Hz$ $f_{ns\mu} = 6.13377 \cdot Hz$

The ratio of the effective frequency to elastic frequency is calculated as follows (See Reference 1):

$cf := 1.9$ Coefficient to account for short duration motion

$A1 := cf \cdot (1 - fs_f)$

$A := A1 \cdot (A1 \le 0.85) + 0.85 \cdot (A1 > 0.85)$

$fe_f := (1 - A) + A \cdot (fs_f)$ $fe_f = 0.93036$

The effective damping ratio, be, is calculated as follows:

$Cn := 0.15$ Coefficient to account for short duration motion

$bh := Cn \cdot (1 - fs_f)$ Hysteretic energy dissipation damping $bh = 0.02872$

$be := \left(\dfrac{fs_f}{fe_f}\right)^2 \cdot (b + bh)$ $be = 0.05945$

The median ductility scale factor, Fu.SA, is estimated as follows:

[redacted] effective frequency, fe, [redacted]

$fe := fe_f \cdot f_{ns}$ $\qquad$ $fe = 7.05734 \cdot Hz$

$Sa_e := SSa(fe, be)$ $\qquad$ $Sa_e = 0.59505 \cdot g$ $\qquad$ Sa.e is the spectral acceleration at the effective frequency, fe, and the effective damping, be

$$Fu_{SA} := \left(\frac{fe_f}{fs_f}\right)^2 \cdot \frac{Sa}{Sa_e} \qquad Fu_{SA} = 1.46763$$

Final Estimate of the median ductility scale factor

Different methods have been used to estimate the median ductility scale factor. Based on a recent study, the median ductility scale factor, Fu, is taken as the average of the ductility scale factors found using the Effective Riddle-Newmark method, Fu.ERM, and the Effective Spectral Method, Fu.SA.

$$Fmu_{median} := \frac{Fu_{ERN} + Fu_{SA}}{2} \qquad Fmu_{median} = 1.44432$$

$\boxed{\check{F}_{\mu} = 1.44432}$

4.1.3 Calculate β_S

A series of MathCad runs were made in order to quantify the variabilities. The results of these runs are documented in Attachment 4 and are referred to by a specific run number as listed in the first column of the table in Attachment 4.

Material: Assuming a $\beta_u = 0.12$ for A36 ($\breve{\sigma}_y = 44$ ksi), Calculate F_S and F_μ for $\sigma_y(-1\sigma) = 44e^{-0.12} = 39.02$ ksi.

From MathCad Run ② with $\sigma_y = 39.02$ ksi
$F_S = 1.611$ and $F_\mu = 1.421$

Thus,

$$\boxed{\beta_u = \ln \frac{1.673 \times 1.444}{1.611 \times 1.421} = 0.05}$$

F_μ: Assuming that the vertical plate displacement of 10 times the elastic displacement is at -1σ, calculate F_μ associated with this case (-1σ).

From MathCad Run ③ the $F_\mu(-1\sigma) = 1.257$.

Logarithic Standard Deviation of the ~~Ductility Scale Factor~~ F_μ

The variability due to random scatter of time history-computed Fmu versus predicted Fmu values using approximate methods (for example, the spectral averaging method) is:

$$br_{Fmu} := 0.4 \cdot \left[0.06 + 0.03 \cdot \left(Fu_{SA} \cdot R - 1\right)\right] \qquad br_{Fmu} = 0.03124$$

$\boxed{\beta_r = 0.03}$

The uncertainty due to story drift associated with failure is calculated, assuming that the 10*elastic deflection is at 1.0 standard deviation from the mean, to be 1.25673 using the same equation as above. Therefore:

From Math cad Run 3

$$bu_{Fmu} := \frac{\ln\left(\frac{Fmu_{median}}{1.25673}\right)}{1.0} \qquad bu_{Fmu} = 0.13913$$

The uncertainty due to inelastic energy absorption model is:

$$bu_{Fmum} := 0.1 \cdot \left(Fmu_{median} - 1\right) \qquad bu_{Fmum} = 0.04443$$

The combined variability is:

$$b_{Fmu} := \sqrt{br_{Fmu}^{\,2} + bu_{Fmu}^{\,2} + bu_{Fmum}^{\,2}} \qquad b_{Fmu} = 0.14935$$

$\boxed{\beta_u = 0.15}$

Capacity Model : Assume a $\beta_u = 0.15$ for the Capacity model developed for the anchorage system. Thus, use a

$$\boxed{\beta_u = 0.15}$$

Note, $\beta_u = 0.15$ represents model variability on the equipment Capacity.

4.2 Equipment Response Analyses Variables

The equipment response analysis variables consist of:

- Modeling
- Mode Combination
- Earthquake Component Combination
- Ground Motion

Each subject is discussed below.

4.2.1 Ground Motion

Ground motion variabilities include:

- Earthquake Response Spectrum Shape
- Horizontal Direction Peak Response Variability
- Vertical Component Response Variability

4.2.1.1 Earthquake Response Spectrum Shape & Horizontal Direction Peak Response Variab

From site-specific data the response spectrum shape and horizontal direction peak response random variabilities, β_r, were calculated to be 0.29 and 0.27, respectively. Additionally, a $\beta_u = 0.20$ is utilized to account for reference earthquake spectral shape. All median factors are 1.0.

based on a frequency of 7.5 Hz (close to 7.586 Hz) and anchored to PGA (Reference 3).

For response spectrum shape variability with $\beta_r = 0.29$ and $\beta_u = 0.20$, Calculate F_S and F_μ.

$\beta_r = 0.29$

From Mathcad Run (4) (Attachment 4) for S_a shape with $e^{+0.29}$

$$F_S = 1.303 \quad \text{and} \quad F_\mu = 1.484$$

Thus :

$$\boxed{\beta_r = \ln \frac{1.673 \times 1.444}{1.303 \times 1.484} = 0.22}$$

$\beta_u = 0.20$

Ratio the results for $\beta_r = 0.29$

$$\beta_u = \frac{0.20}{0.29} \times 0.22$$

$$\boxed{\beta_u = 0.15}$$

For Horizontal Direction Peak Response with $\beta_r = 0.27$, Calculate F_s and F_μ from Mathcad Run ⑥ (Attachment 4) with $S_{a_{N\text{-}S}}$ increased by $e^{+0.27}$ and $S_{a_{E\text{-}W}}$ decreased by $e^{-0.27}$;

$$F_s = 1.556 \text{ and } F_\mu = 1.456$$

Thus

$$\boxed{\beta_r = \ln \frac{1.673 \times 1.444}{1.556 \times 1.456} = 0.06}$$

4.2.1.2 Vertical Component Response Variability

The vertical component at the ground level is assumed 2/3 of the horizontal component and assuming that there is a 0.01 chance that the factor could be as high as 1.5 leads to a $\beta_r = 0.34$. From Mathcad Run ⑦ (Attachment 4) with S_{a_v} increased by $e^{+0.34}$,

$$F_s = 1.596 \text{ and } F_\mu = 1.468$$

Thus

$$\boxed{\beta_r = \ln \frac{1.673 \times 1.444}{1.596 \times 1.468} = 0.03}$$

4.2.2 Earthquake Component Combination

The 100-40-40 rule is used in this analysis as a median component combination. Calculate F_S and F_μ for a 100-100-100 rule as 3.0 σ (since both non-controlling directions, i.e., east-west and vertical, contribute more than 20% to the total). From MathCad Run ⑧ (Attachment 4) with 100-100-100 rule;

$$F_S = 0.986 \text{ and } F_\mu = 1.510$$

Thus

$$\boxed{\beta_r = \frac{1}{3} \ln \frac{1.673 \times 1.444}{0.986 \times 1.510} = 0.16}$$

The median factor is 1.0

4.2.3 Mode Combination

The randomness in the equipment response, due to the combination of dynamic modes, is a function of the complexity of the equipment. For a simple component such as this switchgear a $\beta_r = 0.05$ is assumed. The median factor is 1.0.

4.2.4 Modeling

The modeling variability consists of following parameters:

- Weight
- Damping
- Frequency
- Bolt Streching Length, ℓ_P
- Shear Effect
- Mode Shape

Each variable is discussed below.

4.2.4.1 Mode Shape

The uncertainty in response due to the effects of mode shape as expressed by β_u is estimated to range between 0.05 to 0.15, depending on the complexity of the equipment. For a simple component such as this switchgear a $\beta_u = 0.05$ is assumed. The median factor is 1.0.

4.2.4.2 Bolt Stretching Length, g_p

In median capacity calculation a g_p value of 2.5 inches was assumed. In order to quantify the variability in response due to this variable will assume that a $g_p = 0.5''$ is at -1σ.

From Mathcad Run (11) (Attachment 4) with $g_p = 0.5''$;

$$F_S = 1.612 \text{ and } F_\mu = 1.345$$

And

$$\boxed{\beta_u = \ln \frac{1.673 \times 1.444}{1.612 \times 1.345} = 0.11}$$

The median factor is 1.0.

4.2.4.3 Weight

For calculating the median capacity, switchgear weight was assumed to be 10 kips. In order to quantify the variability in response due to this variable will assume that a weight of 15 kips is at $+1\sigma$. This also requires the fixed-base frequency to change from 10 Hz to

$$\sqrt{\frac{10}{15}} \times 10 = 8.16 \text{ Hz}.$$

From Mathcad Run (10) (Attachment 4) with a weight of 15 kips and a fixed-base frequency of 8.16 Hz,

$$F_S = 1.552 \text{ and } F_\mu = 1.355$$

And

$$\boxed{\beta_u = \ln \frac{1.673 \times 1.444}{1.552 \times 1.355} = 0.14}$$

The median factor is 1.0

4.2.4.4 Damping

In calculating the median capacity, a 5% damping was assumed for the switchgear. In order to quantify the variability in response due to damping will assume that a 3.5% damping is at -1σ.

From Mathcad Run ⑨ (Attachment 4) with a 3.5% damping:

$$F_S = 1.428 \text{ and } F_\mu = 1.489$$

And

$$\boxed{\beta_u = \ln \frac{1.673 \times 1.444}{1.428 \times 1.489} = 0.13}$$

The median factor is 1.0.

4.2.4.5 Frequency

In calculating the median capacity, the switchgear fixed-base frequency was assumed to be 10 Hz. In order to quantify the variability in response due to frequency will assume that a 15 Hz fixed-base frequency is at $+1\sigma$.

From Mathcad Run (13) (Attachment 4) with a fixed-base frequency of 15 HZ;

$F_s = 1.626$ and $F_\mu = 1.585$

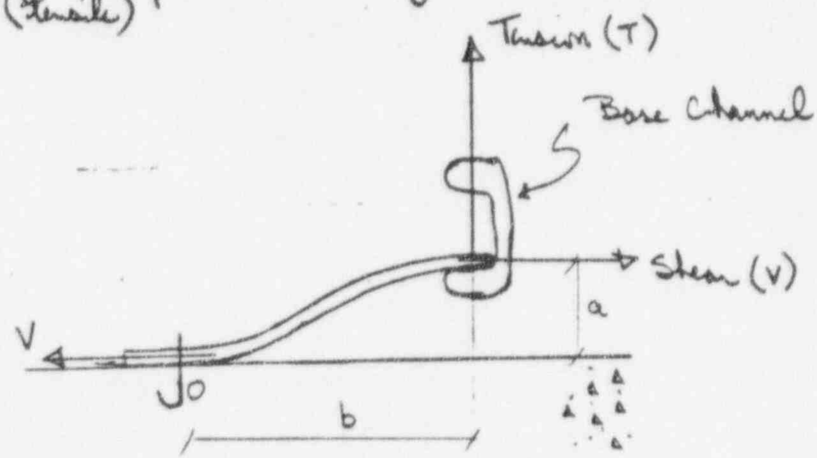
And

$$\boxed{\beta_u = \ln \frac{1.673 \times 1.949}{1.626 \times 1.585} = 0.06}$$

The median factor is 1.0

4.2.4.6 <u>Shear Effect</u>

The idealized model in calculating the median capacity included the acting shear in switchgear which become a tensile force in the hold down plate and in turn counteracts the moment in the plate caused by the upward (tensile) force in switchgear



$M_O = T \times b - V \times a$

In order to quantify the variability in response due to the shear effect, will assume that no shear effect represents -1σ.

From MathCad Run (12) with no shear effect;

$$F_S = 1.197 \text{ and } F_\mu = 1.808$$

And

$$\boxed{\beta_u = \ln \frac{1.673 \times 1.444}{1.197 \times 1.808} = 0.11}$$

4.3 Fragility Results

Utilizing results developed in previous sections and using lognormal model and separation of variables approach, median PGA capacity and combined β_r and β_u values are calculated below. Note that combined β values are obtained by a SRSS combination of the individual values.

Variable	Median	β_r	β_u
Capacity			
Strength	1.67		0.05
Inelastic Energy Absorption	1.44	0.03	0.15
Model	1.0		0.15
Response			
Ground Motion			
Earthquake Response Spectrum Shape	1.0	0.22	0.15
Horizontal Direction Peak Response	1.0	0.06	
Vertical Component Response	1.0	0.03	
Earthquake Component Combination	1.0	0.16	
Mode Combination	1.0	0.05	
Modeling			
Frequency	1.0		0.06
Damping	1.0		0.13
Weight	1.0		0.14
Bolt Stretching Length	1.0		0.11
Shear Effect	1.0		0.11
Mode Shape	1.0		0.05
Combined	2.41	0.29	0.37

For the reference (PRA) earthquake, the PGA is 0.4g; therefore,

Median Peak Ground Acceleration Capacity = $\breve{A}$ (PGA) = 0.4 X 2.41 = 0.96 g

From before, $\beta_r = 0.37$ and $\beta_u = 0.29$ $\Rightarrow$ $\beta_c = 0.47$

And

High Confidence Low Probability of Failure = HCLPF (PGA) = $0.96e^{-1.65(0.37+0.29)}$

HCLPF (PGA) = 0.32 g

Again, it should be noted that the above fragility values are for a hold down plate which is 1/4" thick and 2 1/4" wide and welded (full penetration) to the base channel web.

Also, relays are outside the scope of this calculation and are not addressed.

5.0 References

1. Letter from Tom Tracy of Stevenson & Associates to John Reed of Jack R. Benjamin & Associates transmitting Pilgrim Station SPRA Ground Spectrum, dated November 16, 1993.

2. "Analysis and Design of Structural Connections: Reinforced Concrete and Steel," by M. Holmes and L. Martin, Ellis Horwood Limited

3. "Fragility Evaluation for Nuclear Power Plant Seismic Probabilistic Safety Assessment," A Utility Training Course, R.P. Kennedy and J.W. Reed Presenters, Palo Alto, CA, October 1993.

4. "Statistical Description of Strength of Concrete," Mirza, et al., Journal Structural Division, ASCE, May 1979.

BY ________ DATE ________ PROJECT ________ PAGE ____ OF ____
CHKD. BY ________ DATE ________ SUBJECT ________ JOB NO. ________

Appendix A

SRT Walkdown Photos

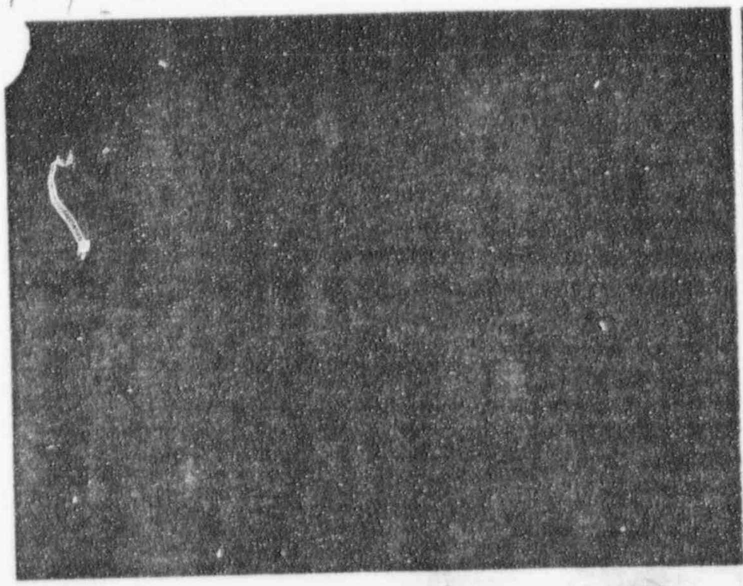
weld for the partition wall

3/8" thru enclosure

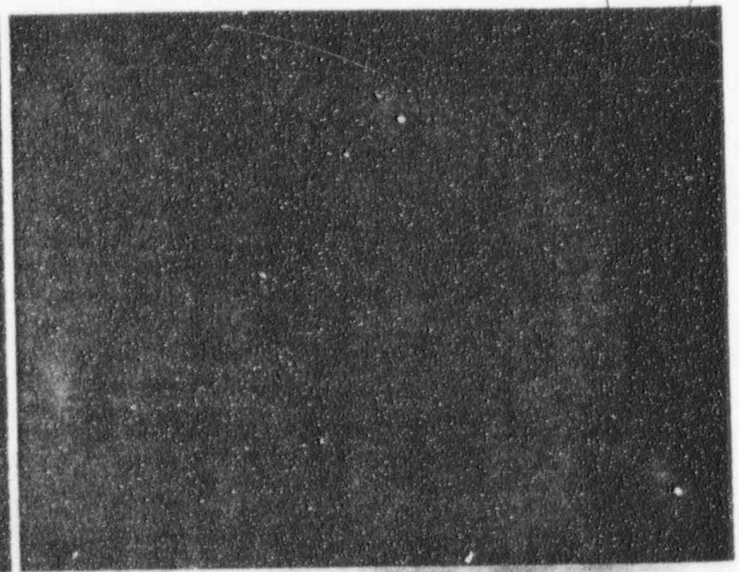
Pictures of Pilgrim Station A8 bus [redacted]

enclosure port.

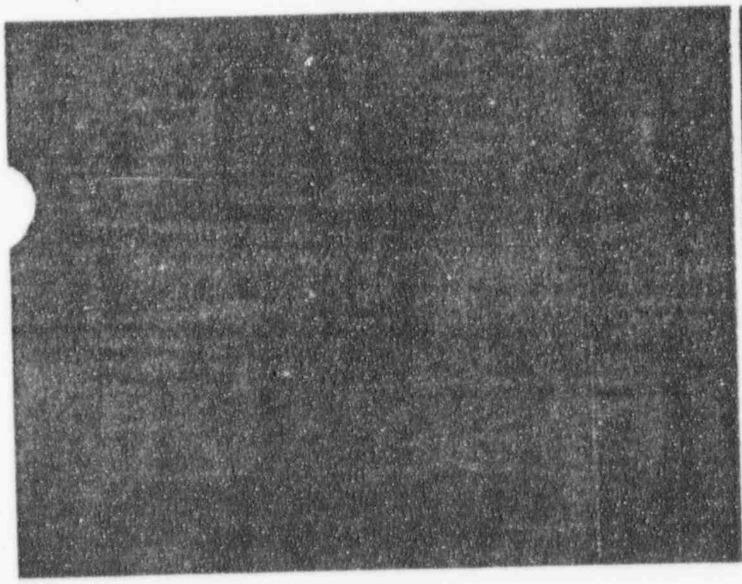
weld for the partition wall



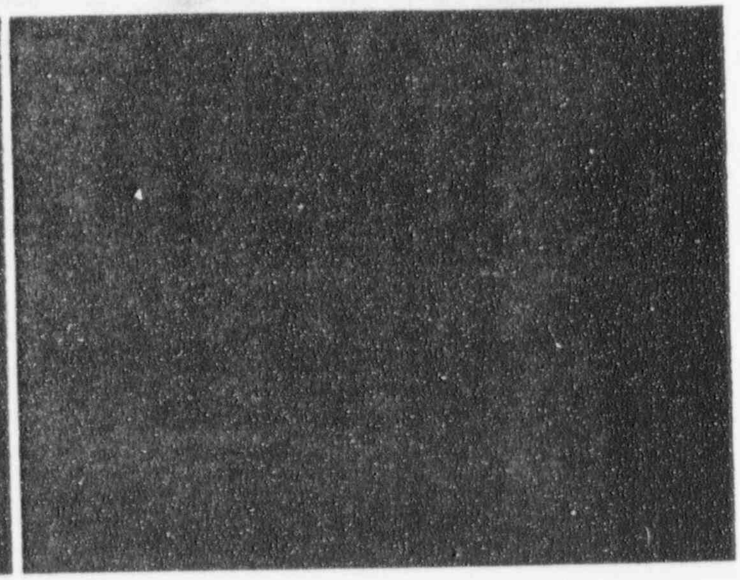
Picture 1



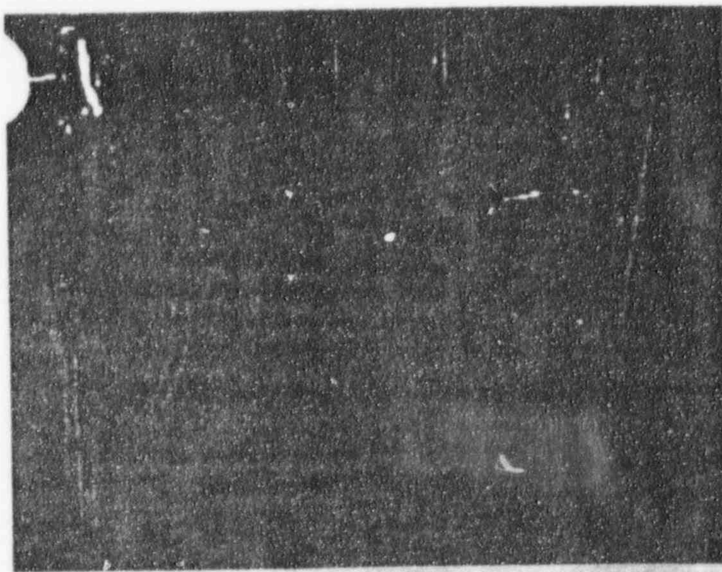
Picture 2



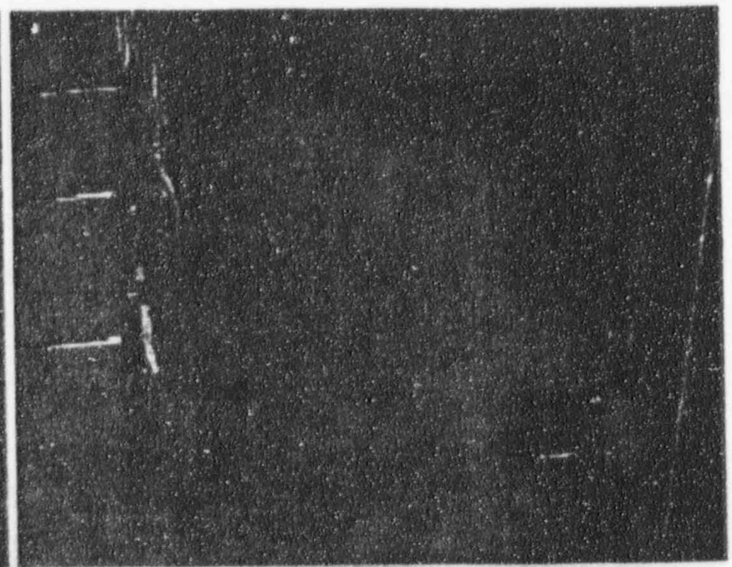
Picture 3



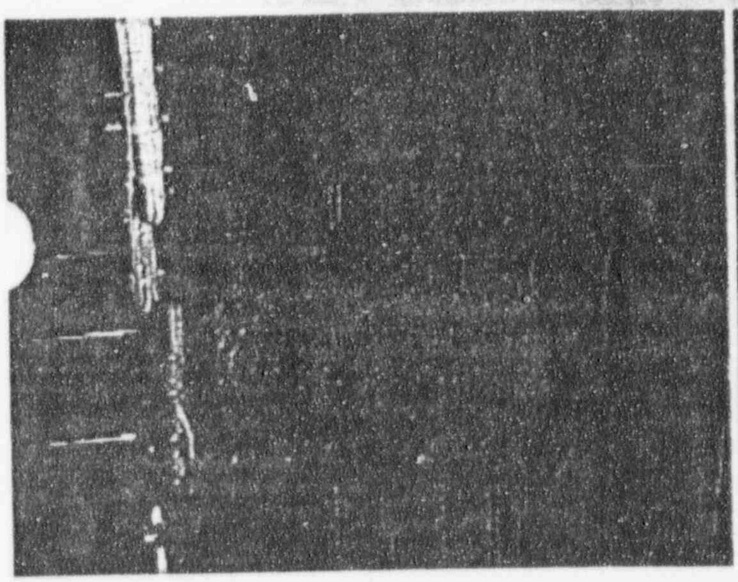
Picture 4



Picture 5



Picture 6



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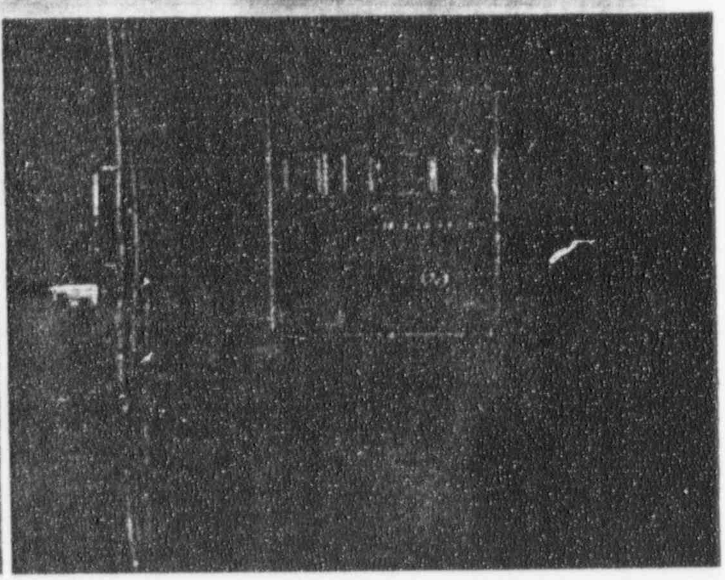

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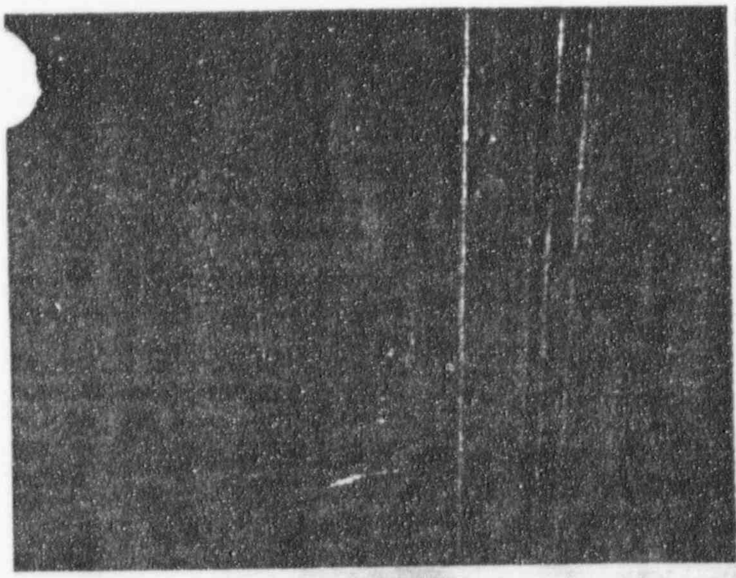

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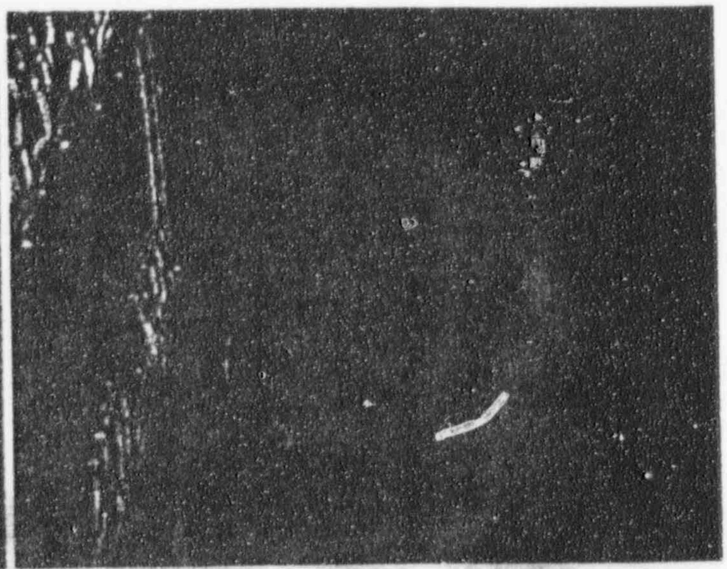
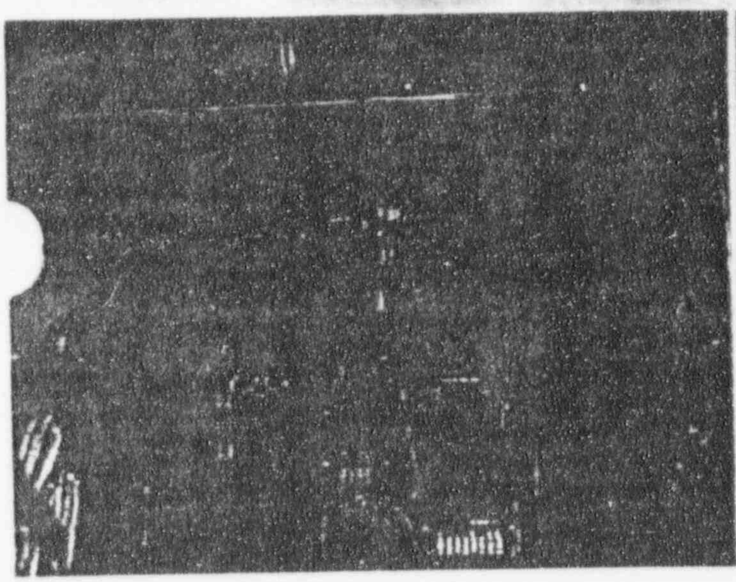
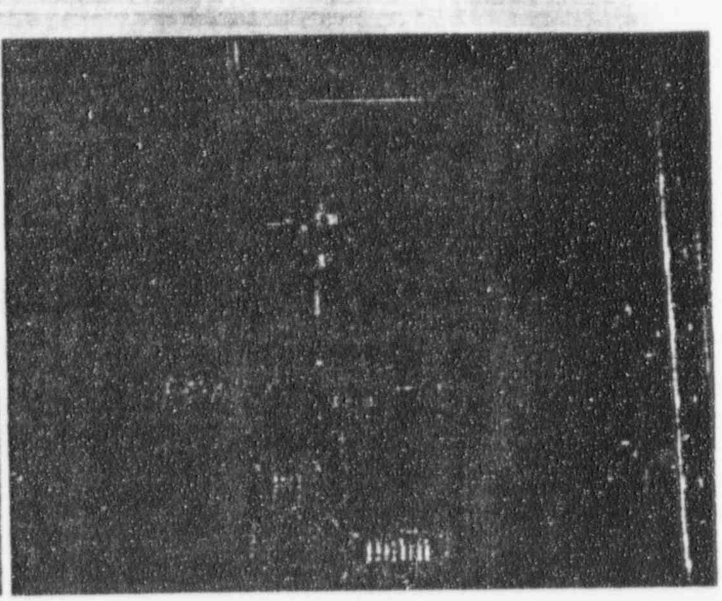


Picture 10

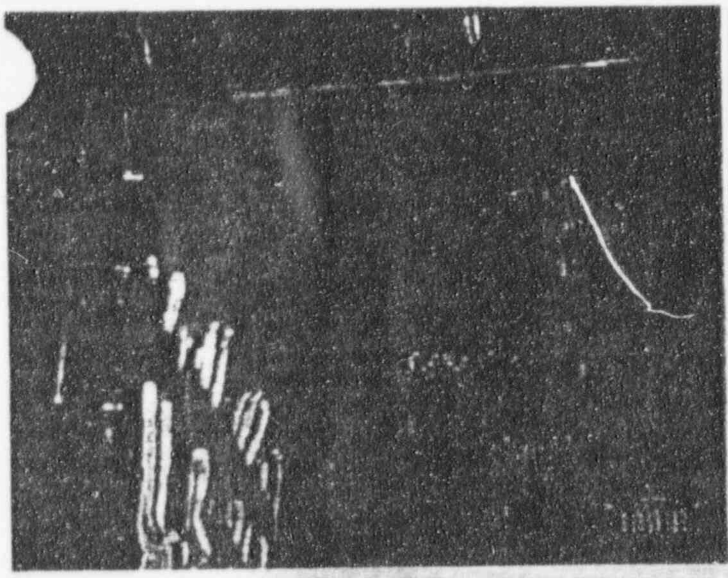


Picture 11

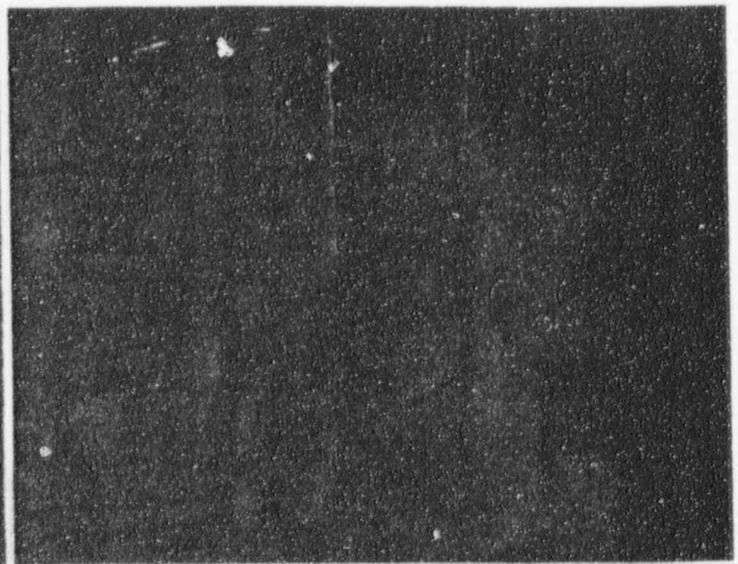


Picture 12

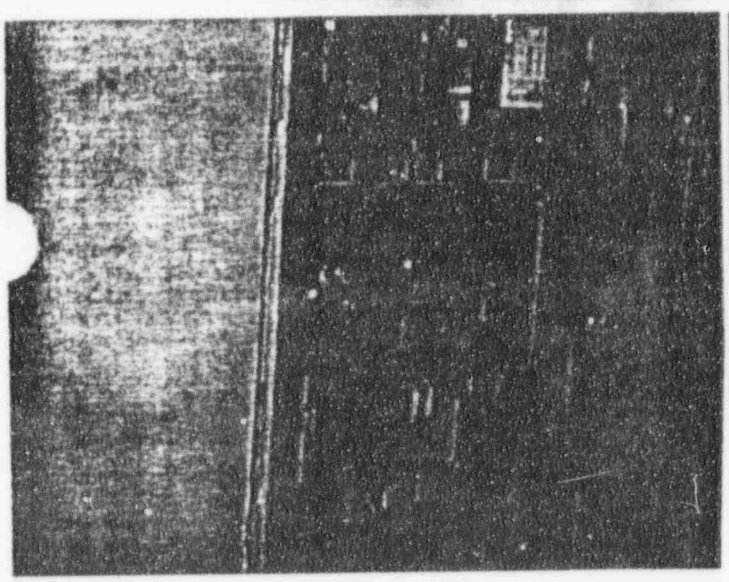
Pictures of Pilgrim Station A8 bus [redacted]



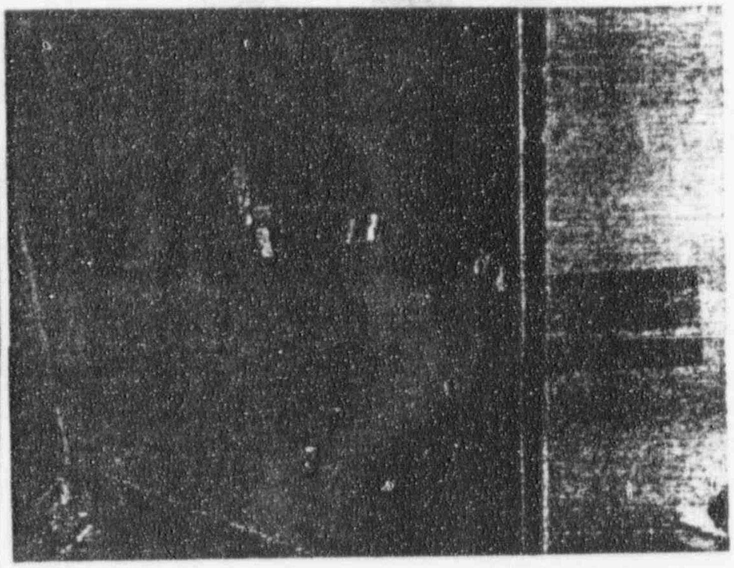
Picture 13



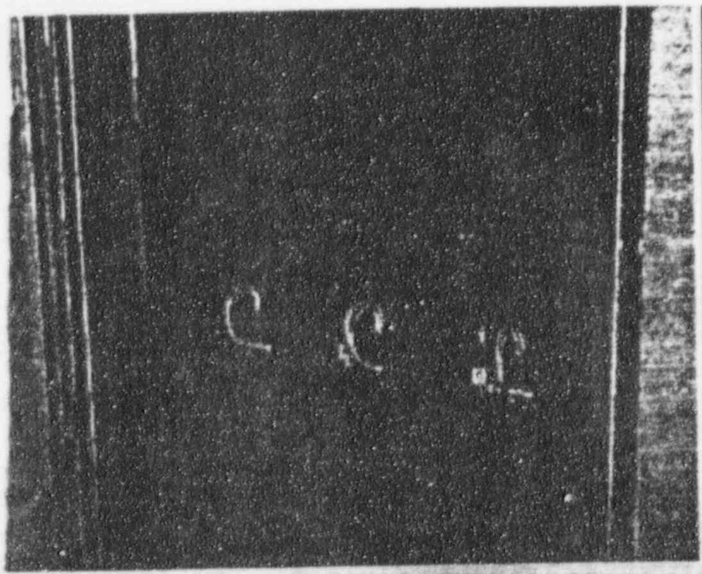
Picture 14



Picture 15



Picture 16



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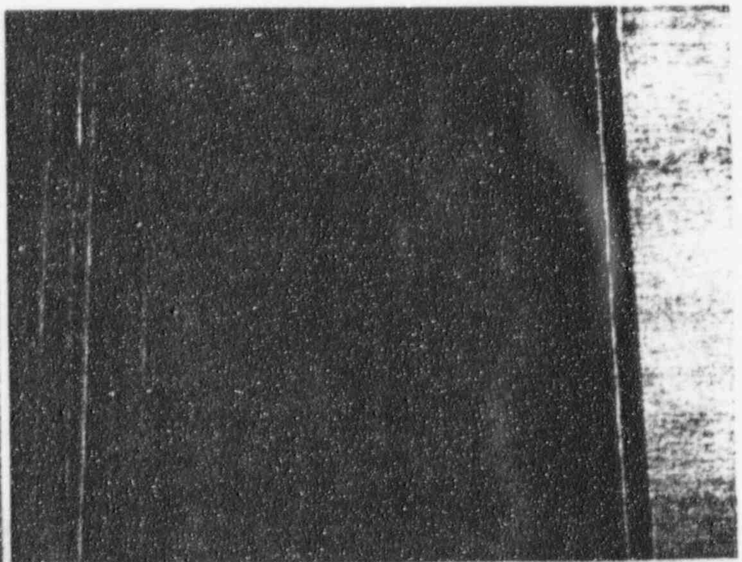

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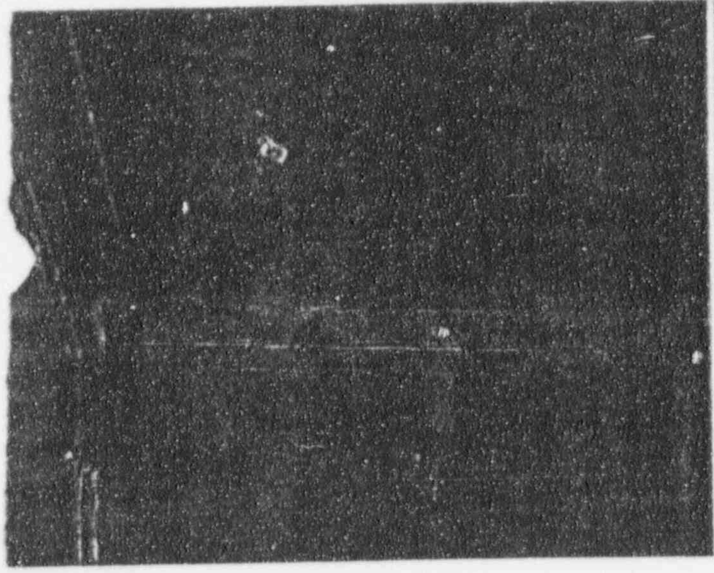
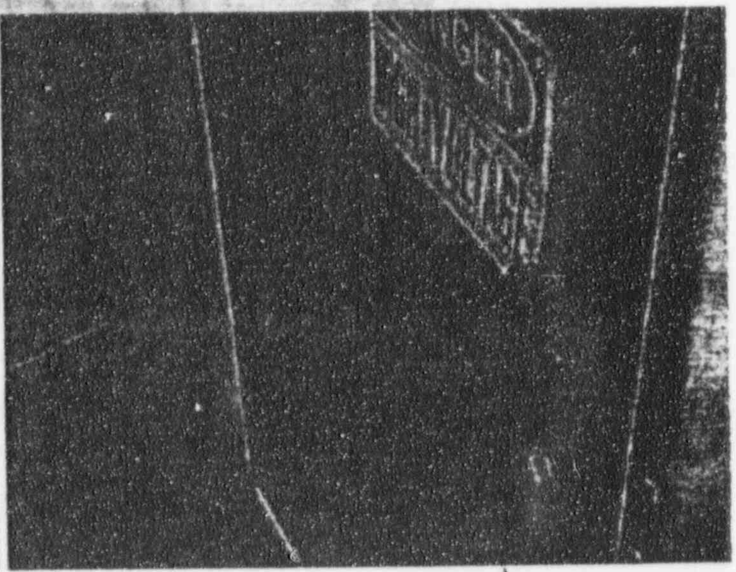
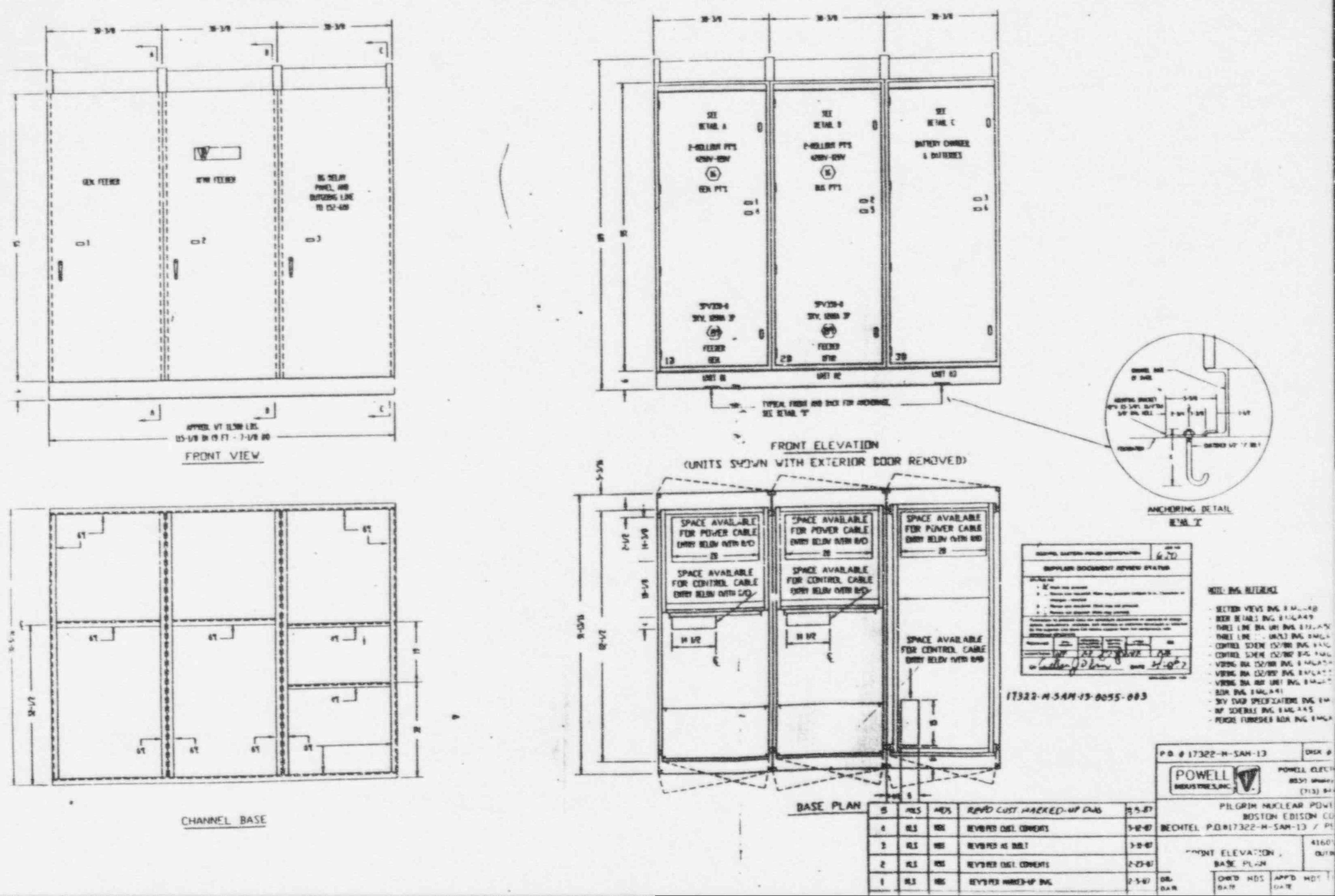


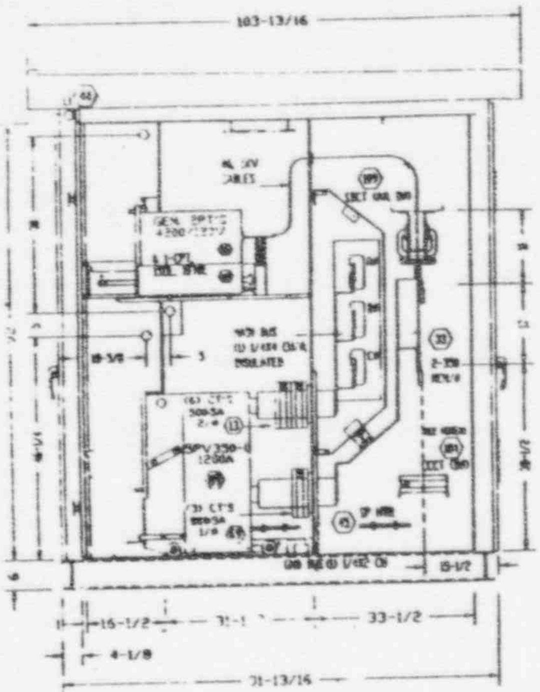
Picture 19



Picture 20

3/8" screw thru the enclosure port


FRONT VIEW
GEN FEEDER
SEE DETAIL A
SEE DETAIL B
SEE DETAIL C
BATTERY CHARGER & BATTERIES
BUS PT'S
GEN PT'S
FEEDER
FRONT ELEVATION
(UNITS SHOWN WITH EXTERIOR DOOR REMOVED)
SPACE AVAILABLE FOR POWER CABLE
SPACE AVAILABLE FOR CONTROL CABLE
ANCHORING DETAIL
CHANNEL BASE
BASE PLAN
17322-M-SAM-13-0055-003
P.O. # 17322-M-SAM-13
POWELL INDUSTRIES, INC.
PILGRIM NUCLEAR POWER
BOSTON EDISON CO
BECHTEL P.O.#17322-M-SAM-13
FRONT ELEVATION, BASE PLAN
5 REV'D CUST MARKED-UP DWG
4 REVISED PER CUST. COMMENTS
3 REVISED PER AS BUILT
2 REVISED PER CUST. COMMENTS
1 REVISED PER MARKED-UP DWG



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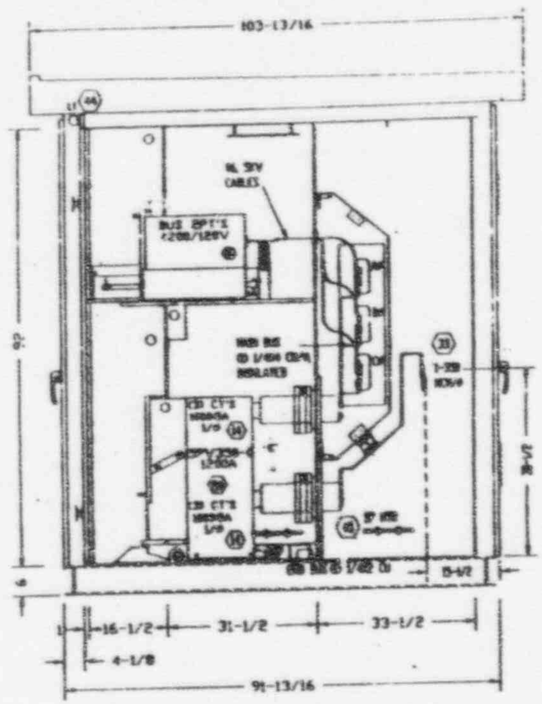
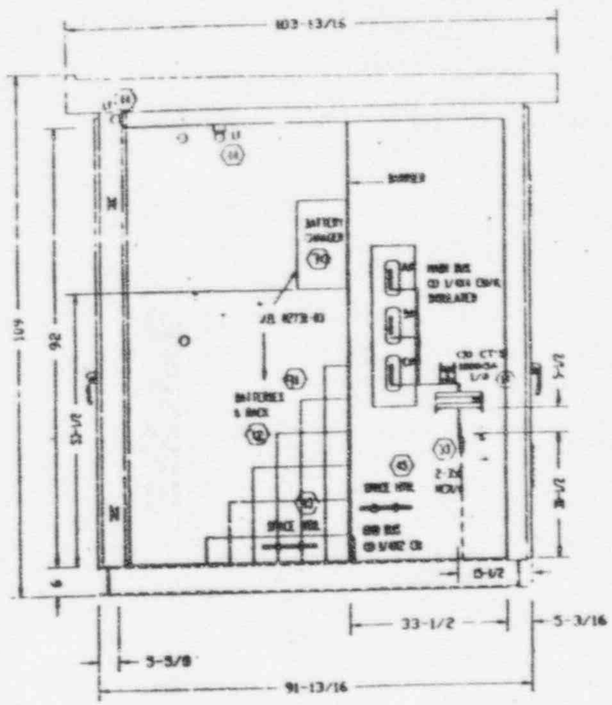
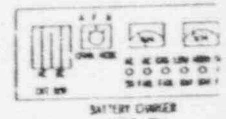
UNIT #1



SECTION "B-B"
UNIT #2



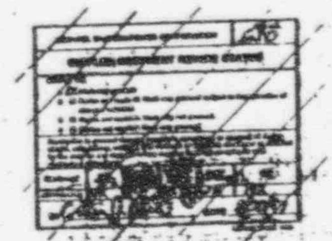
SECTION "C-C"
UNIT #3



BATTERY CHARGER
FRONT VIEW

NOTE:
1- ALL CONNECTIONS TO GROUND (SUCH AS SHIELDING) MUST BE CARRIED THROUGH THE CT AND SOLIDLY GROUNDED ON THE LOAD SIDE OF THE CT.

NOTE: DWG REFERENCE:
- [illegible]
- SKV INSUL SPECIFICATIONS M8A4
- EP SCHEDULE M8A4B
- PENSKE FURNISHED BOM DWG



17322-M-SAM-13-0056-003

S	MLS	MCS	REV'D PER CUST MARKED-UP DWG	83-...
4	MLS	MDS	REV'S PER MARKED-UP DWG	3-13-87
3	MLS	MDS	REV'S PER AS BUILT	3-8-87
2	MLS	MDS	REV'S PER MARKED-UP DWG	
1	MLS	MDS	REV'S PER MARKED-UP DWG	

P.O. # 17322-M-SAM-13
POWELL INDUSTRIES, INC.
PILGRIM NUC...
BOST...
BECHTEL P.O.#17322-M-SA...
SECTION VIEWS

BY ______ DATE ______ PROJECT ______ PAGE ______ OF ______
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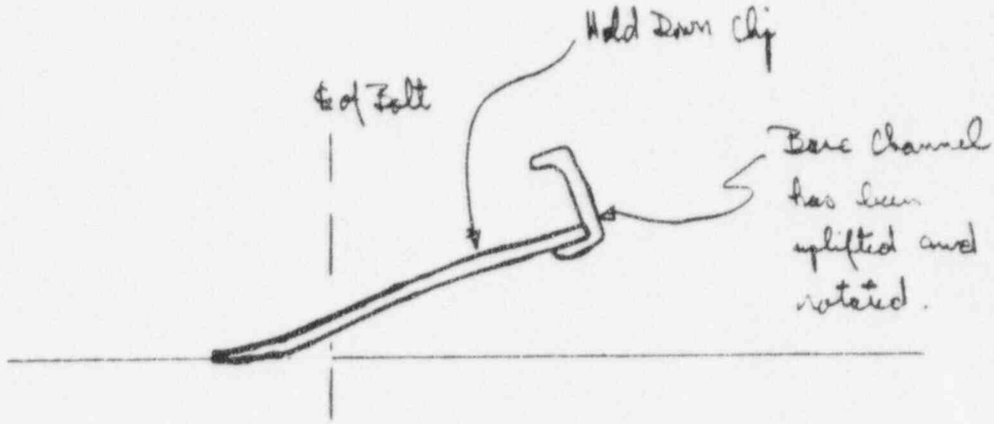
Attachment 2

Anchorage System Model Development

This attachment contains the development of the model for switchgear anchorage. The theory based on elastic prying is that of the one presented in the book by Holmes and Martin entitled "Analysis and Design of Structural Connections," however, the model has been refined to include the flexibility of the channel base, i.e., channel rotation due to acting shear and moment has been considered.



Exaggerated Prying Model

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ANALYSIS AND DESIGN OF STRUCTURAL CONNECTIONS: Reinforced Concrete and Steel

M. HOLMES, B.Sc., D.Sc., C.Eng., F.I.C.E., F.Struct.E.
Professor and Head of Department of Civil Engineering
University of Aston in Birmingham

and

L. H. MARTIN, B.Sc., Ph.D.
Reader in Structural Engineering
Department of Civil Engineering
University of Aston in Birmingham

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New York · Brisbane · Chichester · Toronto

shown graphically in Fig. 4.10. The tensile forces in the bolts adjacent to the tension flange of the beam are approximately equal. The preloaded bolts adjacent to the compression flange eventually resist part of the bending moment. The prying force in the elastic stage of behaviour may be greater than at ultimate load and depends on the number of contact areas.

4.13 ELASTIC THEORY FOR PRYING FORCES

The bolts must be designed to resist the external forces plus the prying forces, and it is therefore necessary to develop a theory to calculate the magnitude of the prying force. For an allowable stress method of design at service load conditions, the prying force Q_{be} is related to the external force F_e, assuming linear elastic behaviour of the components.

The theoretical model in Fig. 4.11 shows an end plate of thickness t_p and of cantilever length $(a_p + b_p)$. The extremity of the plate is in contact with a column flange, for example, and this introduces the prying force Q_{be}. The external force F_e is assumed to be balanced by the prying force Q_{be} and an axial force F_{bt} in the bolt. The axial force in the bolt produces an extension of the bolt of δ_b.

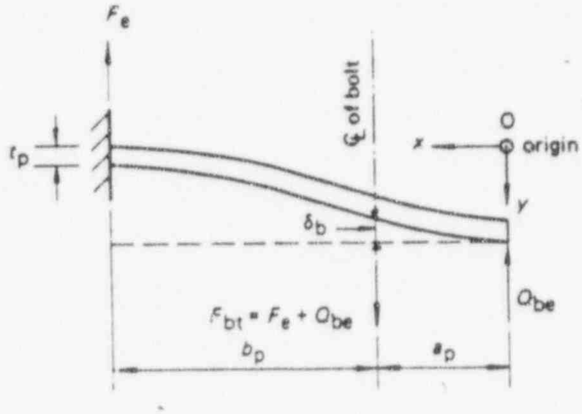


Fig. 4.11 – Forces acting in the elastic stage for prying force theory.

Applying McCaulay's method for the deflection of a beam with the origin of O and the deflection positive downwards.

$$EI\frac{d_2y}{dx^2} = -\,Q_{be}x + (F_e + Q_{be})\,[x - a_p] \quad . \tag{4.29}$$

Integrating

$$EI\frac{dy}{dx} = -\,Q_{be}\frac{x^2}{2} + (F_e + Q_{be})\frac{[x - a_p]^2}{2} + A \quad . \tag{4.30}$$

Integrating

$$EIy = -Q_{be}\frac{x^3}{6} + (F_e + Q_{be})\frac{[x - a_p]^3}{6} + Ax + B \qquad (4.31)$$

when $x = a_p + b_p$, $dy/dx = 0$ and therefore from equation (4.30)

$$0 = -Q_{be}\frac{(a_p + b_p)^2}{2} + (F_e + Q_{be})\frac{b_p^2}{2} + A\ .$$

Rearranging

$$A = \frac{Q_{be}}{2}a_p(a_p + 2\cancel{a_p}b_p) - F_e\frac{b_p^2}{2}$$

but eqn 4.31 is only applicable for $x \geq a_p$

when $x = 0$, $y = 0$, and therefore from equation (4.31) $B = 0$ when $x = a_p$ the extension of the bolt $y = -\delta_b$, and from equation (4.31)

$$-EI\delta_b = -Q_{be}\frac{a_p^3}{\cancel{3}\,6} + a_p\left\{Q_{be}\frac{a_p}{2}(a_p + 2b_p) - F_e\frac{b_p^2}{2}\right\}.$$

Rearranging

$$Q_{be} = \frac{F_e - 2EI\,\delta_b/a_p b_p^2}{2(a_p/b_p) + (2/3)\,(a_p/b_p)^2}\ . \qquad (4.32)$$

The extension of the bolt originally preloaded with a force F_{bs} is

$$\delta_b = (F_{bt} - F_{bs})\,g_p/A_b E_b = (F_e + Q_{be} - F_{bs})\,g_p/A_b E_b\ . \qquad (4.33)$$

Substuting equation (4.33) in (4.32) and rearranging

$$Q_{be} = \frac{F_e(1 - k_e) + k_e F_{bs}}{2(a_p/b_p) + (2/3)\,(a_p/b_p)^2 + k_e} \qquad (4.34)$$

where $\quad k_e = E_p w_p t_p^3 g_p / 6 a_p b_p^2 A_b E_b\ .$ (4.35)

This form of the equation is applicable in the linear elastic range of behaviour and relates the prying force Q_{be} to the external applied force F_e.

4.14 ULTIMATE LOAD THEORY FOR PRYING FORCES

At ultimate load when the bolt fractures it is preferable to relate the prying force Q_{bu} to the ultimate tensile strength of the bolt F_{bu}. Equation (4.34) can be expressed in terms of the bolt force and strain in the bolt, when combined with equation (4.28). At ultimate load the equation would be

Q_{bu} =

where F_p =

The strain in
[4.20], and the s
values of ϵ_{bu} and
thickness plus was

The width of
defined and is al
column connectic
tions in a colum
theory [4.22], a
whichever is the l

4.15 CONNECTI

Some connection
diagram is gener
beam, and colum
are applied to th
stiff bearing nea
force H are resis
fasteners, e.g. bo

Redo Holmes & Martin derivation correction to possible error in equation range (ie x=0 y=0)

$a_p \le x \le a_p + b_p$	$0 \le x \le a_p$
$EI\dfrac{d^2y_1}{dx^2} = -Q_{be}x + (F_c + Q_{be})(x-a_p)$	$EI\dfrac{d^2y_2}{dx^2} = -Q_{be}x$
$EI\dfrac{dy_1}{dx} = -Q_{be}\dfrac{x^2}{2} + (F_c+Q_{be})\dfrac{(x-a_p)^2}{2} + A$	$EI\dfrac{dy_2}{dx} = -Q_{be}\dfrac{x^2}{2} + C$
$EIy_1 = -Q_{be}\dfrac{x^3}{6} + (F_c+Q_{be})\dfrac{(x-a_p)^3}{6} + Ax + B$	$EIy_2 = -Q_{be}\dfrac{x^3}{6} + Cx + D$

at $x = a_p \quad y_1 = y_2 \quad y_1' = y_2'$

$$-Q_{be}\frac{a_p^2}{2} + A = -Q_{be}\frac{a_p^2}{2} + C \implies A = C$$

$$-Q_{be}\frac{a_p^3}{6} + Aa_p + B = -Q_{be}\frac{a_p^3}{6} + Aa_p + D \implies B = D$$

Now at $x = a_p + b_b$ $\frac{dy_1}{dx} = 0$

$$0 = -Q_{be}\frac{(a_p+b_b)^2}{2} + (F_e + Q_{be})\frac{a_p^2}{2} + A$$

$$A = \frac{Q_{be}}{2}a_p(a_p + 2b_p) - F_e\frac{b_p^2}{2}$$

& at $x = 0$ $y_2 = 0$ $\Rightarrow D = 0$ & $B = 0$

(hence no error & rest of Holman & Martin Derivation looks ok)

Now consider case where fixed end is allowed to rotate slightly (the channel is not fixed) & define a moment at end equal to M_e, where

$$0 \le M_e \le M_{e\,fixed} \quad (\text{ie for } \frac{dy_1}{dx} = 0 \text{ case})$$

so:

$$M_e = -Q_{be}(a_p + b_p) + (F_e + Q_{be})\,b_p$$

as before: $A = C$ & $B = D = 0$

In order to follow Homer & Martin derivation it is better to assume Θ_e corresponding to M_e

Thus at $x = a_p + b_b$ $\frac{dy_1}{dx} = \Theta_e$

$$EI\,\Theta_e = -Q_{be}\frac{(a_p+b_b)^2}{2} + (F_e + Q_{be})\frac{b_p^2}{2} + A$$

$$A = EI\Theta_e + \frac{Q_{be}}{2}a_p(a_p + 2b_p) - F_e\frac{b_p^2}{2}$$

& at $x = a_p$ the extension of bolt $y = -\delta_b$

$$-EI\delta_b = -Q_{be}\frac{a_p^3}{6} + a_p\left\{EI\Theta_e + \frac{Q_{be}}{2}a_p(a_p+2b_p) - F_e\frac{b_p^2}{2}\right\}$$

$$Q_{be}\left[\frac{a_p^2}{2}(a_p+2b_p) - \frac{a_p^3}{6}\right] = a_p\left(F_e\frac{b_p^2}{2} - EI\Theta_e - \frac{EI\delta_b}{a_p}\right)$$

$\frac{2}{a_p b_p^2}$

$$Q_{be} = \frac{F_e - \frac{2EI\Theta_e}{b_p^2} - \frac{2EI\delta_b}{a_p b_p^2}}{\frac{a_p^2}{b_p^2} + \frac{2a_p}{b_p} - \frac{1}{3}\frac{a_p^2}{b_p^2}}$$

$$Q_{be} = \frac{F_e - \frac{2EI}{b_p^2}\left(\Theta_e + \frac{\delta_b}{a_p}\right)}{2(a_p/b_p) + \frac{2}{3}(a_b/b_p)^2}$$

Now for a bolt w/o prestress

$$\delta_b = \frac{(F_e + Q_{be})\,g_p}{A_b E_b}$$

substituting in for δ_b

$$Q_{be} = \frac{F_e - \frac{2EI}{b_p^2}\left[\theta_e + \frac{(F_e + Q_{be})g_p}{A_b E_b a_p}\right]}{\underbrace{2(a_p/b_p) + (2/3)(a_p/b_p)^2}_{K}}$$

$$Q_{be}\left[1 + \frac{2EI\,g_p}{K\,b_p^2 A_b E_b a_p}\right] = \frac{F_e - \frac{2EI}{b_p^2}\left[\theta_e + \frac{F_e g_p}{A_b E_b a_p}\right]}{K}$$

$$Q_{be} = \frac{F_e\left[1 - \frac{2EI\,g_p}{b_p^2 A_b E_b a_p}\right] - \frac{2EI\theta_e}{b_p^2}}{K + \frac{2EI\,g_p}{b_p^2 A_b E_b a_p}}$$

Now $I = \frac{t_p^3 w_p}{12}$

$$\frac{2EI g_p}{b_p^2 A_b E_b a_p} = \frac{E\, w_p t_p^3 g_p}{6\, a_p b_p^2 A_b E_b} = K_e$$

$$Q_{be} = \frac{F_e(1 - K_e) - \frac{2EI\theta_e}{b_p^2}}{\underbrace{2(a_p/b_p) + (2/3)(a_p/b_p)^2}_{K} + K_e}$$

note it is clear that releasing the moment at the support corresponds to a negative Θ_e since Q_{be} will then increase until Θ_e reaches maximum when moment $M_e = 0$

find Θ_e when $M_e = 0$

$$Q_{be}(a_p + b_p) - (F_e + Q_{be})\,b_p = 0$$

$$Q_{be}(a_p + b_p - b_p) = F_e b_p$$

$$Q_{be} = F_e \cdot \frac{b_p}{a_p}$$

substituting into Q_{be} on p. 4 & solving for Θ_e

$$\frac{F_e(1-K_e) - \dfrac{2EI\Theta_e}{b_p^2}}{K + K_e} = F_e \cdot \frac{b_p}{a_p}$$

$$\frac{2EI\Theta_e}{b_p^2} = F_e(1-K_e) - (K+K_e)F_e\frac{b_p}{a_p}$$

$$= F_e\left\{\left[1 - K_e\left(1+\frac{b_p}{a_p}\right)\right] - K\frac{b_p}{a_p}\right\}$$

$$\Theta_e = \frac{F_e\left\{\left[1 - K_e\left(1+\frac{b_p}{a_p}\right)\right] - K\frac{b_p}{a_p}\right\}}{2EI/b_p^2}$$

Find the limiting value of Θ_e & F_e when Q_{be} is fixed to value corresponding to plastic section at bolt hole and the moment at the channel is zero

From pg 5 with $Q_{be} = F_e \frac{b_p}{a_p}$

$$\Theta_e = \frac{\frac{a_p}{b_p} Q_{be} \left\{ \left[1 - k_e \left(1 + \frac{b_p}{a_p} \right) \right] - k \frac{b_p}{a_p} \right\}}{2EI/b_p^2}$$

From last eqn on pg 5

$$F_e = \frac{Q_{be}(k_e + k) + \frac{2EI\Theta_e}{b_p^2}}{1 - k_e}$$

or just

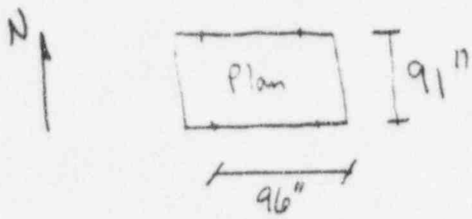
$$F_e = Q_{be} \cdot \frac{a_p}{b_p}$$

BY [illegible] DATE 11/22/93 PROJECT [illegible] PAGE 1 OF
CHKD. BY DATE SUBJECT [illegible] JOB NO. 225-[illegible]

Develop relationship between Fe & V on anchor size (Since V will reduce moment in plate which will increase tension capacity of bolt)

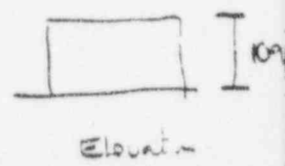
Assume bolts share 1/4 shear (no friction)

Use Frequency @ 7.5 Hz (check later)



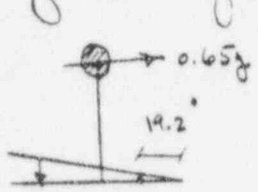
$S_a = 0.65g$ Horiz

$S_a = \frac{2}{3}(.40g)g$ Vert



Tension Forces ($Wt = 10^k$)

Rocking about edges



$$P_{N-S} = \frac{(0.65)(10)(109/2)}{(91)(2)} = 1.95^k$$

$$P_{E-W} = \frac{(0.65)(10)(109/2)}{2\left(96 + \frac{19.2}{96} \times 19.2\right)} = 1.77^k$$

$$P_{vert} = (.27)(10)(1/4) = 0.68^k$$

N-S direction Controls

$$Fe = [\underbrace{1.95 + 0.4(1.77 + 0.68)}_{2.93^k}] FS - \frac{10}{4}k$$

Shear Force on bolt (use only shear in N-S, i.e. principal direction)

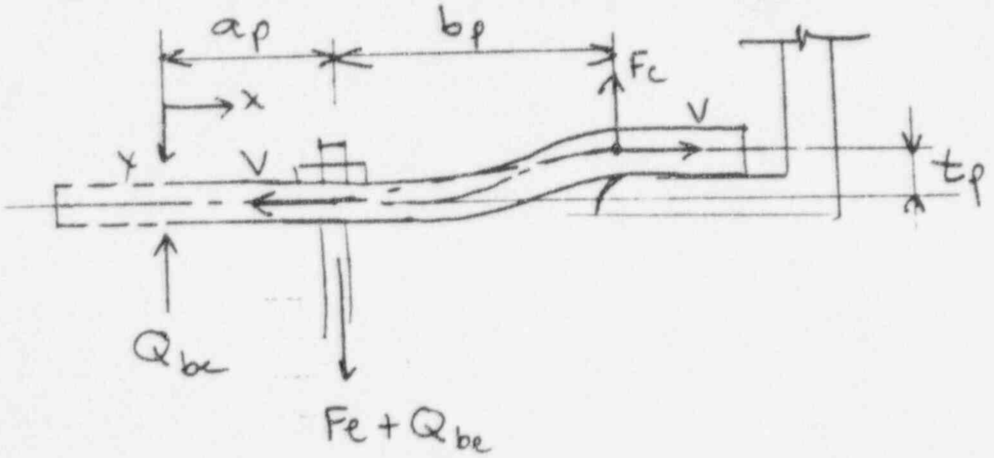
$$V = \frac{10}{4}(.65)\ FS = 1.63\ k\ FS$$

Combining eqns

$$F_e = 2.93\ \frac{V}{1.63} - 2.5\ k$$

$$V = 0.556\ F_e + 1.39\ k \qquad (FS > 0.85)$$

Moment to Add to Formulation (caused by V)



Assume linear transition on moment between a_p & $a_p + b_p$ (see red line)

$$\Delta \text{Moment} = -V \frac{t_p'}{b_p}(x - a_p) \qquad (x > a_p)$$

$$t_p' = t_p + y_{max}$$

In formulation just replace Fe with

$$Fe - \frac{V t_p'}{b_p}$$

where $V = 0.556\,Fe + 1.39\,k$

so replace with

$$Fe\left(1 - 0.556\frac{t_p'}{b_p}\right) - \frac{t_p'}{b_p}(1.39\,k)$$

So old definition of $Fe(a_p)$ for case zero moment at end:

$$Fe(a_p) = \frac{a_p}{b_p} Q_{be}(a_p)$$ is replaced with

$$Fe(a_p) = \frac{1}{1 - 0.556\frac{t_p'}{b_p}}\left[\frac{a_p}{b_p} Q_{be}(a_p) + \frac{t_p'}{b_p}(1.39\,k)\right]$$

Redo Formulation including shear term

$a_p \le x \le a_p + b_p$

$$EI\frac{d^2y_1}{dx^2} = -Q_{be}x + [(\psi F_e - \zeta) + Q_{be}](x-a_p)$$

$$EI\frac{dy_1}{dx} = -Q_{be}\frac{x^2}{2} + [(\psi F_e - \zeta) + Q_{be}]\frac{(x-a_p)^2}{2} + A$$

$$EI y_1 = -Q_{be}\frac{x^3}{6} + [(\psi F_e - \zeta) + Q_{be}]\frac{(x-a_p)^3}{6} + Ax + B$$

$0 \le x \le a_p$

$$EI\frac{d^2y_2}{dx^2} = -Q_{be}x$$

$$EI\frac{dy_2}{dx} = -Q_{be}\frac{x^2}{2} + C$$

$$EI y_2 = -Q_{be}\frac{x^3}{6} + Cx + D$$

where $\psi = 1 - 0.556\frac{t_p'}{b_p}$

$\zeta = \frac{t_p'}{b_p}(1.39k)$

at $x = a_p$ $y_1 = y_2$, $y_1' = y_2'$

$\Downarrow$ $\Downarrow$

$B = 0$ $A = C$

Now at $x = a_p + b_p \quad \frac{dy}{dx} = 0$ (since case of full restraint from channel)

$$0 = -Q_{be}\frac{(a_p+b_p)^2}{2} + \left[(4F_e-3)+Q_{be}\right]\frac{b_p^2}{2} + A$$

$$A = \frac{Q_{be}}{2}a_p(a_p+2b_p) - (4F_e-3)\frac{b_p^2}{2}$$

& at $x=0 \quad y_2 = 0 \Rightarrow D = 0$ & $B = 0$

Now consider case where fixed end is allowed to rotate slightly (the channel is not fixed) & define a moment at end equal to M_e, where

$$0 \le M_e \le M_{e_{Fixed}} \quad (\text{ie for } \frac{dy_2}{dx} = 0 \text{ case})$$

So $M_e = -Q_{be}(a_p+b_p) + (F_e + Q_{be})b_p - V t_p$

where $V = .556 F_e + 1.39^k$

hence $M_e = -Q_{be}(a_p+b_p) + \left[(4F_e-3)+Q_{be}\right]b_p$

as before $A = C$ & $B = D = 0$

In order to follow Nomr & Martin derivation it is better to assume G_e corresponding

Thus at $x = a_p + b_p$ $\quad \frac{dy}{dx} = \theta_e$

$$EI\theta_e = -Q_{be}\frac{(a_p+b_p)^2}{2} + [(\psi F_e - 3) - Q_{be}]\frac{b_p^2}{2} + A$$

$$A = EI\theta_e + \frac{Q_{be}}{2}a_p(a_p+2b_p) - (\psi F_e - 3)\frac{b_p^2}{2}$$

& at $x = a_p$ the extension of bolt $y = -\delta_b$

$$-EI\delta_b = -Q_{be}\frac{a_p^3}{6} + a_p\left\{EI\theta_e + \frac{Q_{be}}{2}a_p(a_p+2b_p) - (\psi F_e - 3)\frac{b_p^2}{2}\right\}$$

$$\frac{2}{a_p b_p^2}$$

$$Q_{be}\left[\frac{a_p^2}{2}(a_p+2b_p) - \frac{a_p^3}{6}\right] = a_p\left[(\psi F_e - 3)\frac{b_p^2}{2} - EI\theta_e - EI\frac{\delta_b}{a_p}\right]$$

$$Q_{be} = \frac{(\psi F_e - 3) - \frac{2EI\theta_e}{b_p^2} - \frac{2EI\delta_b}{a_p b_p^2}}{\frac{a_p^2}{b_p^2} - \frac{2a_p}{b_p} - \frac{1}{3}\frac{a_p^2}{b_p^2}}$$

$$Q_{be} = \frac{(\psi F_e - 3) - \frac{2EI}{b_p^2}\left(\theta_e + \frac{\delta_b}{a_p}\right)}{\underbrace{2(a_p/b_p) + \frac{2}{3}(a_p/b_p)^2}_{K}}$$

For a bolt w/o prestress

$$\delta_b = \frac{(F_e + Q_{be})g_p}{A_b E_b}$$

Substitute for S_b

$$Q_{be} = -\frac{(4F_e - 3) - \frac{2EI}{b_p^2}\left[\Theta_e + \frac{(F_e + Q_{be})g_p}{A_b E_p a_p}\right]}{\underbrace{2(a_p/b_p) + 2/3\,(a_p/b_p)^2}_{K}}$$

$$Q_{be}\left[1 + \frac{2EIg_p}{Kb_p^2 A_b E_b a_p}\right] = \frac{(4F_e - 3) - \frac{2EI}{b_p^2}\left[\Theta_e + \frac{F_e g_p}{A_b E_p a_p}\right]}{K}$$

$$Q_{be} = \frac{F_e\left[4 - \frac{2EIg_p}{b_p^2 A_b E_p a_p}\right] - \frac{2EI\Theta_e}{b_p^2} - 3}{K + \frac{2EIg_p}{b_p^2 A_b E_b a_p}}$$

Now $I = \frac{t_p^3 w_p}{12}$

$$\frac{2EIg_p}{b_p^2 A_b E_p a_p} = \frac{E w_p t_p^3 g_p}{6 a_p b_p^2 A_b E_b} = K_e$$

$$Q_{be} = \frac{F_e(4 - K_e) - \frac{2EI\Theta_e}{b_p^2} - 3}{K + K_e}$$

Note it is clear that relieving the moment at the support corresponds to a negative Θ_e since Q_{be} will then increase until Θ_e reaches maximum when moment $M_e = 0$

Find Θ_e when $M_e = 0$ (from p. 2)

$$M_e = +Q_{be}\,(a_p + b_p) - \left[(\psi F_e - 3) + Q_{be}\right] b_p = 0$$

$$Q_{be}\left(a_p + \cancel{b_p} - \cancel{b_p}\right) = (\psi F_e - 3)\, b_p$$

$$Q_{be} = (\psi F_e - 3)\frac{b_p}{a_p}$$

Substituting into Q_{be} on p. 4 & Solving for Θ_e

$$\frac{F_e(\Psi - k_e) - \dfrac{2EI\Theta_e}{b_p^2} - 3}{k + k_e} = (\psi F_e - 3)\frac{b_p}{a_p}$$

$$\frac{2EI\Theta_e}{b_p^2} = F_e(\Psi - k_e) - (k + k_e)(\psi F_e - 3)\frac{b_p}{a_p} - 3$$

$$\Theta_e = \frac{F_e\left\{\Psi - k_e\left(1 + \psi\frac{b_p}{a_p}\right) - k\psi\frac{b_p}{a_p}\right\} - 3\left[1 - (k + k_e)\frac{b_p}{a_p}\right]}{2EI/b_p^2}$$

To Find relationship between Fe & Q_{be} when moment at channel flange is zero use $M_e = 0$ from p 2

$$Q_{be}(a_p + b_p) = \left[(4Fe - 3) + Q_{be}\right] b_p$$

$$4 Fe\, b_p = Q_{be}(a_p + \cancel{b_p} - \cancel{b_p}) + 3 b_p$$

$$Fe = \left[Q_{be}\frac{a_p}{b_p} + 3\right]\frac{1}{4}$$

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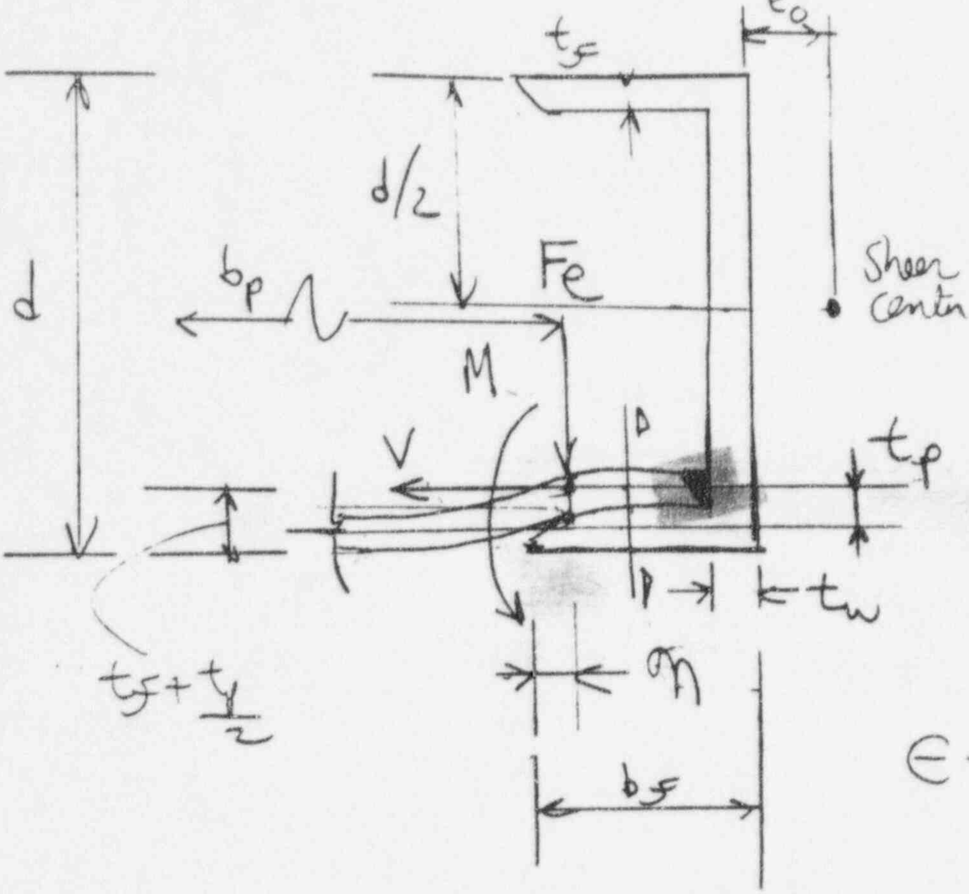
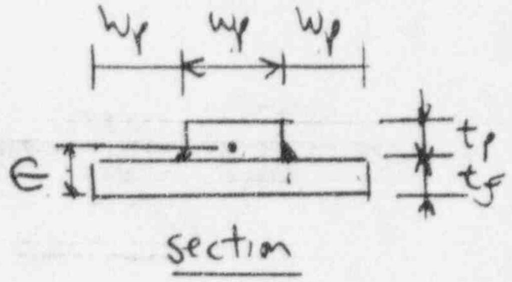
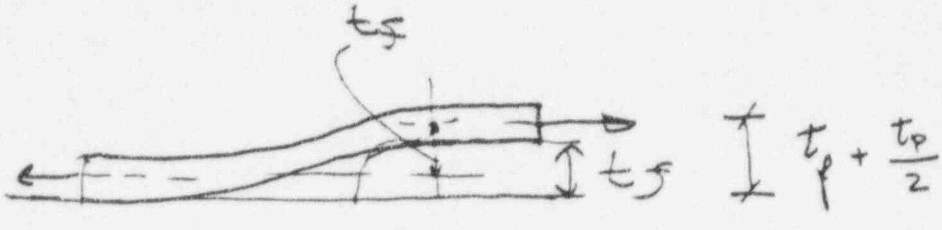




$$\in = \frac{3 t_f w_p \cdot \frac{t_f}{2} + w_p \cdot t_p \left(t_f + \frac{t_p}{2} \right)}{3 t_f w_p + w_p \cdot t_p}$$



Will assume force is acting at the middle of the plate.

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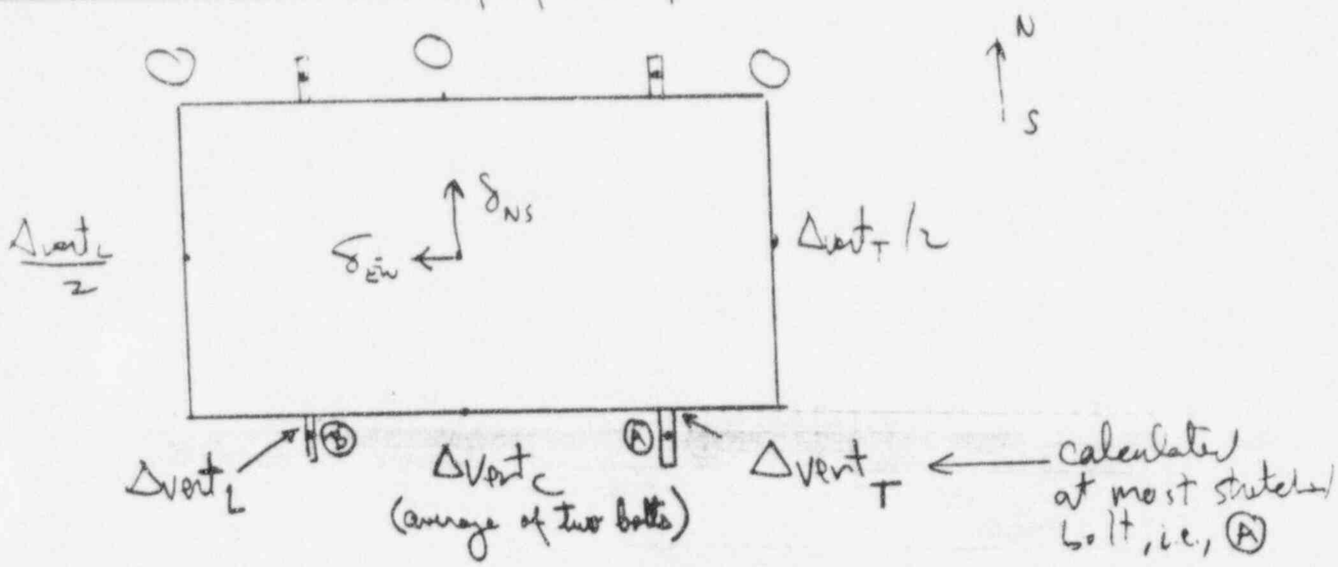
Revise Calculation For frequency of cabinet in N/S & E/W Direction



δ_{NS}

Assume (Approx) $\Delta vert_L$ in proportion to tension force

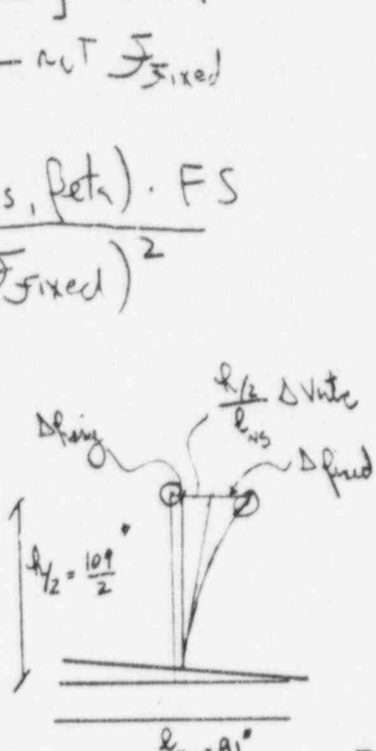
$$\Delta vert_L = \frac{\overbrace{P_{NS} - 0.4\,P_{EW} + 0.4\,P_{vert}}^{\text{force in Bolt B}}}{\underbrace{P_{NS} + 0.4(P_{EW} + P_{vert})}_{\text{force in Bolt A} \; = \; 4_1}} \cdot \Delta vert_T$$

$$\Delta vert_C = \frac{\Delta vert_L + \Delta vert_T}{2} = \left[\frac{P_{NS} + 0.4\,P_{vert}}{4_1}\right] \Delta vert_T$$

not F_{fixed}

$$\delta_{NS} = \left| \Delta_{horiz} + \frac{h/2}{l_{NS}} \Delta vert_C \right| + \frac{SS_a(F_{NS}, \beta eta) \cdot FS}{(2\pi F_{fixed})^2}$$

$$F_{NS_{NEW}} = \frac{1}{2\pi}\sqrt{\frac{SS_a(f_{NS}, \beta eta) FS}{\delta_{NS}}}$$



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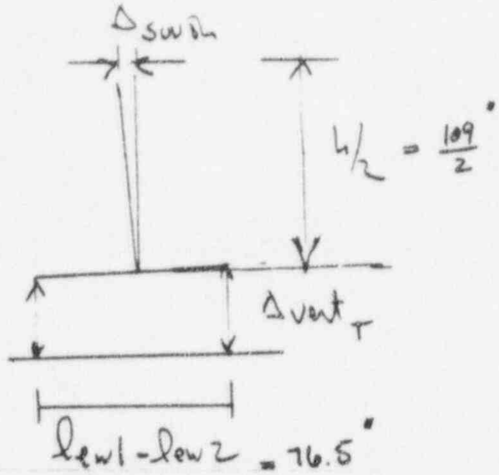
$\underline{\delta_{EW}}$

$\Delta_{horiz} \approx 0$



$$\Delta_{south} = \left(\Delta vert_T - \Delta vert_L\right) \frac{h/2}{(l_{ew1} - l_{ew2})}$$

$$\Delta_{south} = \frac{0.4 P_{ew}}{\psi_1} \frac{h}{(l_{ew1} - l_{ew2})}$$

but $\Delta_{north} = 0$

$$\text{so} \quad \delta_{EW} = \left| \underbrace{\frac{1}{2} \frac{0.4 P_{ew}}{\psi_1} \frac{h}{(l_{ew1} - l_{ew2})}}_{\frac{\Delta_{south} + \Delta_{North}}{2}} \right| + \frac{SS_a(f_{EW}, \beta eta_a) \cdot FS}{(2\pi f_{fixed})^2}$$

$$f_{EW_{new}} = \frac{1}{2\pi} \sqrt{\frac{SS_a(f_{EW}, \beta eta_a)\, FS}{\delta_{EW}}}$$

Attachment 3

Mathcad Program Listing and Results

$kip := 1000 \cdot lbf$
$ksi := 1000 \cdot psi$

$f_{fixed} := 10 \cdot Hz$ Fixed-base frequency of cabinet

$f_{ns} = 7.5856 \cdot Hz$ $f_{ew} = 9.848 \cdot Hz$ $\beta eta := 0.05$ Frequency and damping of the entire cabinet

$$SA := \begin{bmatrix} 0.375290 \\ 0.60 \\ 0.702350 \\ 0.723530 \end{bmatrix} \cdot g \qquad F := \begin{bmatrix} 2.5 \\ 5 \\ 10 \\ 25 \end{bmatrix} \cdot Hz$$ Ground input response spectrum

$$SSa(f,\beta) := \exp\left(\text{linterp}\left(\overrightarrow{\ln\left(\frac{F}{Hz}\right)}, \overrightarrow{\ln\left(\frac{SA}{g}\right)}, \ln\left(\frac{f}{Hz}\right)\right)\right) \cdot g \cdot \left(\frac{\sqrt{0.05}}{\sqrt{\beta}}\right)$$ Values of Sa at different frequencies and damping values

$Wt := 10 \cdot kip$ Weight of cabinet

Spectral acceleration in n/s direction

$Sa_{ns} := SSa(f_{ns}, \beta eta)$

$Sa_{ns} = 0.65961 \cdot g$

Spectral acceleration in e/w direction

$Sa_{ew} := SSa(f_{ew}, \beta eta)$

$\boxed{Sa_{ew} = 0.69991 \cdot g}$

Spectral acceleration in vertical direction

$Sa_{vert} := \left(\frac{2}{3} \cdot 0.40\right) \cdot g$

$Sa_{vert} = 0.26667 \cdot g$

$h := 109 \cdot in$ Height ground to top of cabinet

$l_{ns} := 91 \cdot in$ Base lever arm for north/south direction axial force at support

$l_{ew1} := 96 \cdot in$ Base lever arm for east/west direction axial force at support at first bolt

$l_{ew2} := 19.2 \cdot in$ Base lever arm for east/west direction axial force at support at second bolt

$$P_{ns} := \frac{\frac{Sa_{ns}}{g} \cdot Wt \cdot \left(\frac{h}{2}\right)}{l_{ns} \cdot 2} \qquad P_{ew} := \frac{\frac{Sa_{ew}}{g} \cdot Wt \cdot \left(\frac{h}{2}\right)}{2 \cdot \left(l_{ew1} + \frac{l_{ew2}^2}{l_{ew1}}\right)} \qquad P_{vert} := \frac{\frac{Sa_{vert}}{g} \cdot Wt}{4}$$

$P_{ns} = 1.97519 \cdot kip$ $P_{ew} = 1.91031 \cdot kip$ $P_{vert} = 0.66667 \cdot kip$

$\Psi_1 := P_{ns} + 0.4 \cdot (P_{ew} + P_{vert})$ $\Psi_2 := \frac{Wt}{4}$ $\Psi_3 := \frac{Wt}{4} \cdot \frac{Sa_{ns}}{g}$

$\Psi_1 = 3.00598 \cdot kip$ $\Psi_2 = 2.5 \cdot kip$ $\Psi_3 = 1.64901 \cdot kip$

$c_1 := \frac{\Psi_3}{\Psi_1}$ $c_1 = 0.54858$

$c_2 := \frac{\Psi_2 \cdot \Psi_3}{\Psi_1}$ $c_2 = 1.37144 \cdot kip$

STOP1

ELASTIC POINT ANALYSIS

This program determines the allowable capacity of the plate bolt anchorage system for the 4160V Swithchgear A8 at the Pilgrim Station

Plate and steel properties

$E := 29 \cdot 10^6 \cdot psi$	Modulus of elasticity for steel
$G := \dfrac{E}{2 \cdot (1 + 0.3)}$	Modulus of rigidity for steel
$t_p := 0.25 \cdot in$	Thickness hold down plate
$\boxed{w_p := 2.25 \cdot in}$	Width hold down plate
$d_h := 0.625 \cdot in$	Diameter bolt hole in plate
$a_p := 2.75 \cdot in$	Length plate from end to bolt (maximum value)
$b_p := 1.375 \cdot in$	Length plate from bolt to support
$\sigma_{yp} := 44 \cdot ksi$	Median yield capacity plate

Bolt properties

$E_b := 29 \cdot 10^6 \cdot psi$	Modulus of elasticity for bolt
$A_b := 0.196 \cdot in^2$	Area bolt (Gross area for stretching)
$g_p := 2.5 \cdot in$	Effective length bolt for stretching

Channel properties C6x8.2 (AISC 9th Ed.)

$d := 6.00 \cdot in$	Height of channel
$b_f := 1.92 \cdot in$	Width channel flange
$t_f := 0.343 \cdot in$	Thickness channel flange
$t_w := 0.200 \cdot in$	Thickness channel web
$e_o := 0.599 \cdot in$	Distance outside edge channel web to shear center
$K := 0.08 \cdot in^4$	Torsion constant
$C_w := 4.72 \cdot in^6$	Warping constant
$I_x := 13.1 \cdot in^4$	Moment of inertia about x-axis
$I_y := 0.692 \cdot in^4$	Moment of inertia about y-axis
$L := 38.375 \cdot in$	Channel span between assumed supports
$a := \dfrac{L}{2}$	Distance of anchorage to support along channel
$\eta := 0.25 \cdot in$	Distance from edge of flange where point of contact between plate and channel flange occurs

General properties

$c_1 = 0.54858$

$c_2 = 1.37144 \cdot kip$

c_1 and c_2 are used for the case where the shear force on the bolt is proportional to the the F_e on the anchorage:

$$V = c_1 \cdot F_e + c_2$$

$t_{pp} := t_f + 1 \cdot \left(\left|\Delta assumed_{el}\right|\right)$

Offset in plate which causes moment in plate from shear force on the bolt (equal to thickness of channel flange plus deflection upward)

Calculated properties

$\Psi := 1 - c_1 \cdot \dfrac{t_{pp}}{b_p}$ $\quad \Psi = 0.85807$ Variable reduction factor on the force Fe due to shear on bolt

$\zeta := \dfrac{t_{pp}}{b_p} \cdot c_2$ $\quad \zeta = 0.35482 \cdot kip$ Constant reduction factor on the force Fe due to shear on the bolt

$I := \dfrac{t_p^{\,3} \cdot w_p}{12}$ $\quad I = 0.00293 \cdot in^4$ Moment of inertia plate

$Z_h := \dfrac{(w_p - d_h) \cdot t_p^{\,2}}{4}$ $\quad Z_h = 0.02539 \cdot in^3$ Plastic section modulus plate at hole

$M_h := Z_h \cdot \sigma_{yp}$ $\quad M_h = 1.11719 \cdot kip \cdot in$ Plastic moment capacity plate at hole

CASE 1 - MOMENT BETWEEN PLATE AND CHANNEL FLANGE IS ZERO

This calculation finds the value of a_p such that the slope at the end of the plate is zero and the moment at the attachment to the channel (i.e., at $x = a_p + b_p$) is also zero. This is the limiting case where the channel offers no resistence to rotation

See derivation for definition of terms and theoretical basis (Reed 11/22/93)

$Q_{be}(a_p) := \dfrac{M_h}{a_p}$ Allowable prying force corresponding to plastic moment capacity plate at hole

$$k_e(a_p) := \frac{2 \cdot E \cdot I \cdot g_p}{b_p^{\,2} \cdot A_b \cdot E_b \cdot a_p}$$

$$k(a_p) := 2 \cdot \frac{a_p}{b_p} + \frac{2}{3} \cdot \left(\frac{a_p}{b_p}\right)^2$$

$$F_e(a_p) := \frac{1}{\Psi} \cdot \left(\frac{a_p}{b_p} \cdot Q_{be}(a_p) + \zeta\right)$$

Constrain A to equal zero, which corresponds to zero slope at the end of the plate

Find a_p

$$a_p := \mathrm{root}\left[\left[\frac{Q_{be}(a_p)}{2}\cdot a_p\cdot(a_p+2\cdot b_p)-(F_e(a_p)\cdot\Psi-\zeta)\cdot\frac{b_p^{\,2}}{2}\right]\ldots + \left[\frac{F_e(a_p)\cdot\left[\left[\Psi-k_e(a_p)\cdot\left(1+\Psi\cdot\frac{b_p}{a_p}\right)\right]-k(a_p)\cdot\Psi\cdot\frac{b_p}{a_p}\right]-\zeta\cdot\left[1-(k(a_p)+k_e(a_p))\cdot\frac{b_p}{a_p}\right]}{\frac{2}{b_p^{\,2}}}\right], a_p\right]$$

$a_p = 0.75511\cdot in$

Find limiting value of θ_e

$$\theta_e := \frac{F_e(a_p)\cdot\left[\left[\Psi-k_e(a_p)\cdot\left(1+\Psi\cdot\frac{b_p}{a_p}\right)\right]-k(a_p)\cdot\Psi\cdot\frac{b_p}{a_p}\right]-\zeta\cdot\left[1-(k(a_p)+k_e(a_p))\cdot\frac{b_p}{a_p}\right]}{\frac{2\cdot E\cdot I}{b_p^{\,2}}}$$

$\theta_e = -0.014004148$

Tension force on anchorage at edge of channel flange $\quad F_e(a_p) = 1.3604\cdot kip$

Prying action force on plate $\quad Q_{be}(a_p) = 1.4795\cdot kip$

Bolt force $\quad F_b(a_p) := F_e(a_p) + Q_{be}(a_p) \quad F_b(a_p) = 2.8399\cdot kip$

Shear force $\quad V_{bolt}(a_p) := c_1\cdot F_e(a_p) + c_2 \quad V_{bolt}(a_p) = 2.11773\cdot kip$

Plot deflected shape, slope, shear and moment of the plate

$$A(a_p) := \left[\frac{Q_{be}(a_p)}{2}\cdot a_p\cdot(a_p+2\cdot b_p)-(F_e(a_p)\cdot\Psi-\zeta)\cdot\frac{b_p^{\,2}}{2}\right]+E\cdot I\cdot\theta_e$$

$$y(x,a_p) := \frac{1}{E\cdot I}\cdot\left[\left(-Q_{be}(a_p)\cdot\frac{x^3}{6}+A(a_p)\cdot x\right)+(F_e(a_p)\cdot\Psi-\zeta+Q_{be}(a_p))\cdot\frac{(x-a_p)^3}{6}\cdot(x>a_p)\right]$$

$$r(x,a_p) := \frac{1}{E\cdot I}\cdot\left[\left(-Q_{be}(a_p)\cdot\frac{x^2}{2}+A(a_p)\right)+(F_e(a_p)\cdot\Psi-\zeta+Q_{be}(a_p))\cdot\frac{(x-a_p)^2}{2}\cdot(x>a_p)\right]$$

$$V(x,a_p) := -Q_{be}(a_p)+(F_e(a_p)+Q_{be}(a_p))\cdot(x>a_p)$$

$$M(x,a_p) := -Q_{be}(a_p)\cdot x+\left[(F_e(a_p)\cdot\Psi-\zeta)+Q_{be}(a_p)\right]\cdot(x-a_p)\cdot(x>a_p)$$

$$P_{bolt}(a_p) := \frac{-y(a_p,a_p)\cdot A_b\cdot E_b}{g_p} \qquad P_{bolt}(a_p) = 2.8399\cdot kip$$

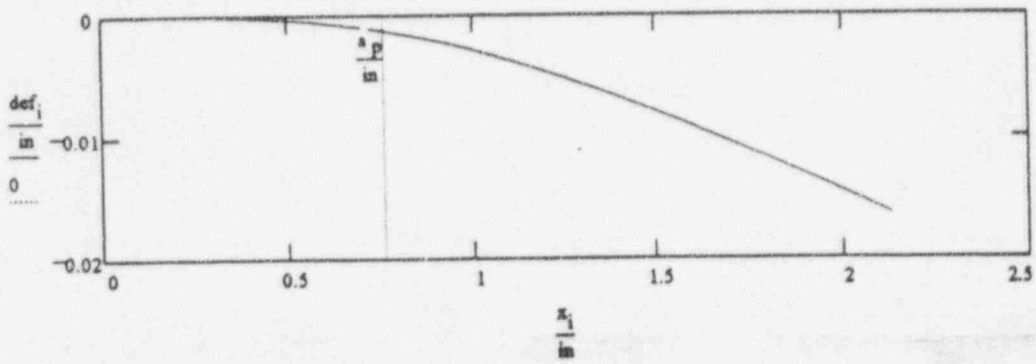
$n := 100 \qquad \Delta x := \dfrac{a_p + b_p}{n} \qquad \Delta x = 0.0213 \cdot in$

$i := 1 .. n + 1$

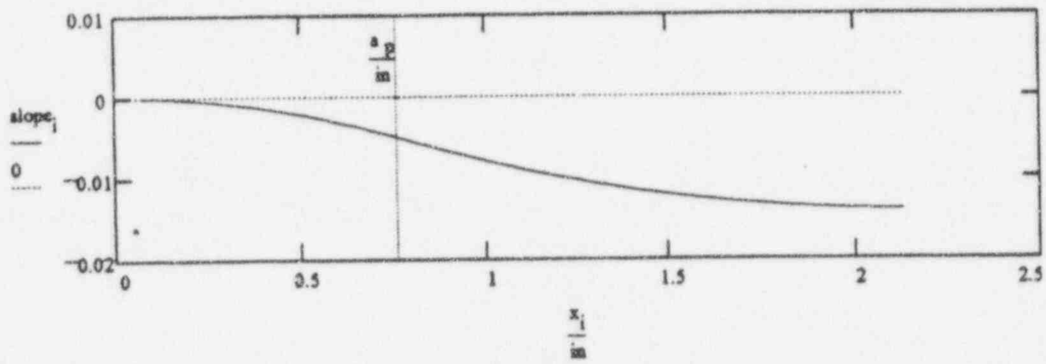
$x_i := (i - 1) \cdot \Delta x$

$def_i := y(x_i, a_p) \qquad slope_i := r(x_i, a_p) \qquad shear_i := V(x_i, a_p) \quad mom_i := M(x_i, a_p)$

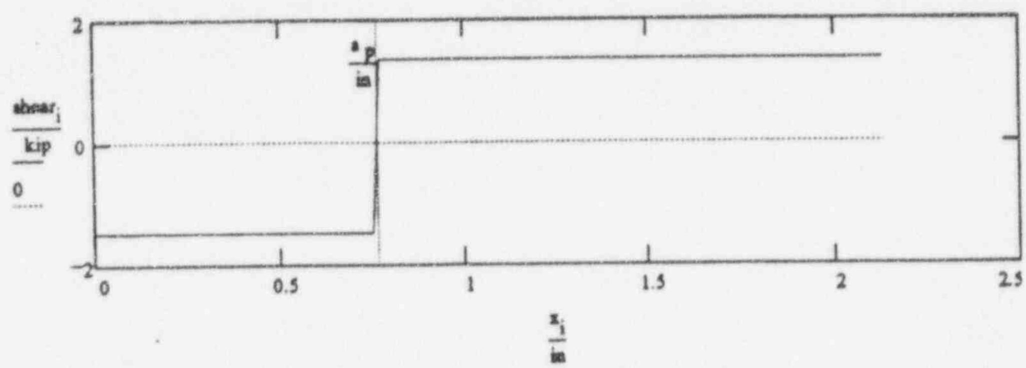
$y(0 \cdot in, a_p) = 0 \cdot in \qquad y(a_p, a_p) = -0.00125 \cdot in \qquad y(a_p + b_p, a_p) = -0.01636 \cdot in$



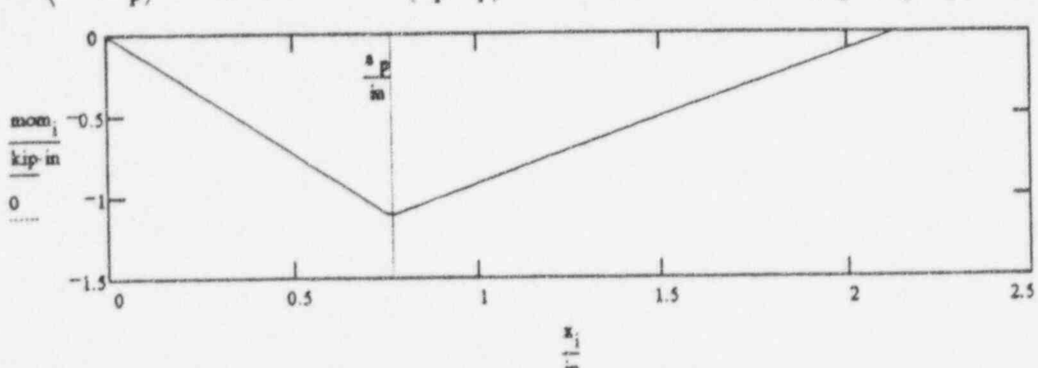
$r(0 \cdot in, a_p) = 7.067209259 \cdot 10^{-7} \quad r(a_p, a_p) = -0.004963919 \qquad r(a_p + b_p, a_p) = -0.014004148$



$V(0 \cdot in, a_p) = -1.4795 \cdot kip \qquad P_{bolt}(a_p) = 2.8399 \cdot kip \qquad V(a_p + b_p, a_p) = 1.3604 \cdot kip$



$M(0 \cdot in, a_p) = 0 \cdot kip \cdot in \qquad M(a_p, a_p) = -1.11719 \cdot kip \cdot in \qquad M(a_p + b_p, a_p) = 0 \cdot kip \cdot in$



CASE 2 - ROTATION AT CONNECTION BETWEEN PLATE AND CHANNEL FLANGE IS SET TO: θ_e

For this case a value of θ_e between zero and the limiting value $\theta_e = -0.014$ is assumed and the plate response is calculated. Again, the slope at the end of the plate is set equal to zero.

$\theta_e := \theta assumed_{el}$ Assumed value of θ_e

$$F_e(a_p) := \frac{Q_{be}(a_p)\cdot(k(a_p)+k_e(a_p)) + \frac{2\cdot E\cdot I\cdot\theta_e}{b_p^2} + \zeta}{\Psi - k_e(a_p)}$$

$$A(a_p) := \left[\frac{Q_{be}(a_p)}{2}\cdot a_p\cdot(a_p+2\cdot b_p) - (F_e(a_p)\cdot\Psi-\zeta)\cdot\frac{b_p^2}{2}\right] + E\cdot I\cdot\theta_e$$

$$y(x,a_p) := \frac{1}{E\cdot I}\left[\left(-Q_{be}(a_p)\cdot\frac{x^3}{6} + A(a_p)\cdot x\right) + (F_e(a_p)\cdot\Psi - \zeta + Q_{be}(a_p))\cdot\frac{(x-a_p)^3}{6}\cdot(x>a_p)\right]$$

$$r(x,a_p) := \frac{1}{E\cdot I}\left[\left(-Q_{be}(a_p)\cdot\frac{x^2}{2} + A(a_p)\right) + (F_e(a_p)\cdot\Psi - \zeta + Q_{be}(a_p))\cdot\frac{(x-a_p)^2}{2}\cdot(x>a_p)\right]$$

$$V(x,a_p) := -Q_{be}(a_p) + (F_e(a_p) + Q_{be}(a_p))\cdot(x>a_p)$$

$$M(x,a_p) := -Q_{be}(a_p)\cdot x + \left[(F_e(a_p)\cdot\Psi-\zeta) + Q_{be}(a_p)\right]\cdot(x-a_p)\cdot(x>a_p)$$

$$V_{bolt}(a_p) := c_1\cdot F_e(a_p) + c_2$$

Given

Constrain slope to equal zero at end of plate

$$r(0\cdot in, a_p) = 0$$

$a_p := find(a_p)$ $a_p = 0.87422\cdot in$

$F_e(a_p) = 2.53015\cdot kip$

$Q_{be}(a_p) = 1.27793\cdot kip$

$F_b(a_p) := F_e(a_p) + Q_{be}(a_p)$ $F_b(a_p) = 3.80808\cdot kip$

$$P_{bolt}(a_p) := \frac{-y(a_p,a_p)\cdot A_b\cdot E_b}{g_p}$$

$P_{bolt}(a_p) = 3.80808\cdot kip$

$V_{bolt}(a_p) = 2.75942\cdot kip$

Attachment 4

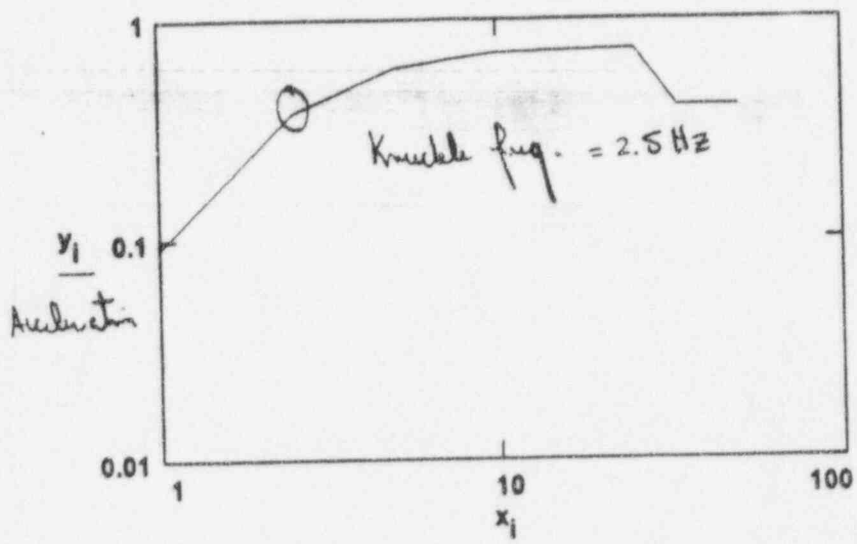
MathCad Runs Results

Determination of Knuckle Freq.

$$x := \begin{bmatrix} 1 \\ 2.5 \\ 5 \\ 10 \\ 25 \\ 33 \\ 50 \end{bmatrix} \qquad y := \begin{bmatrix} 0.091 \\ 0.375 \\ 0.6 \\ 0.702 \\ 0.724 \\ 0.4 \\ 0.4 \end{bmatrix}$$

$i := 1 .. 7$

RLE response Spectra

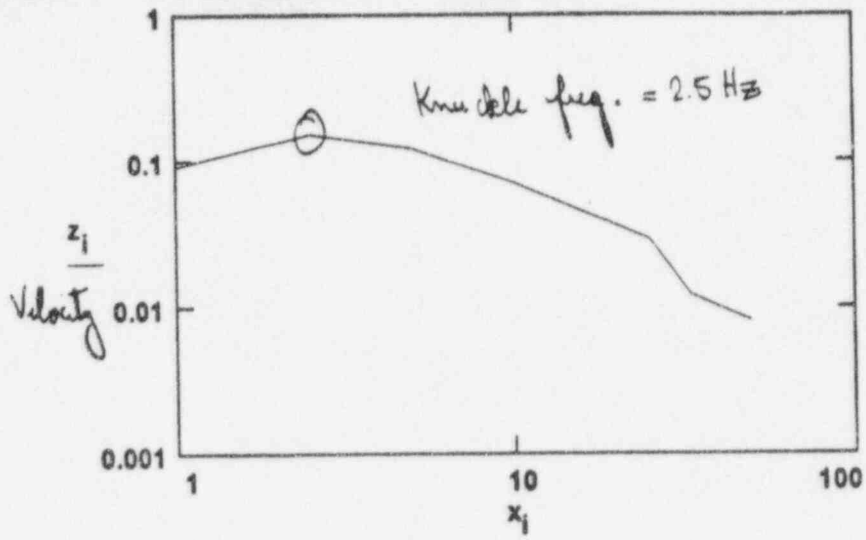


$i := 1..7$

$$x := \begin{bmatrix} 1 \\ 2.5 \\ 5 \\ 10 \\ 25 \\ 33 \\ 50 \end{bmatrix} \qquad y := \begin{bmatrix} 0.091 \\ 0.375 \\ 0.6 \\ 0.702 \\ 0.724 \\ 0.4 \\ 0.4 \end{bmatrix}$$

$$z_i := \frac{y_i}{x_i}$$

RLE Velocity Response



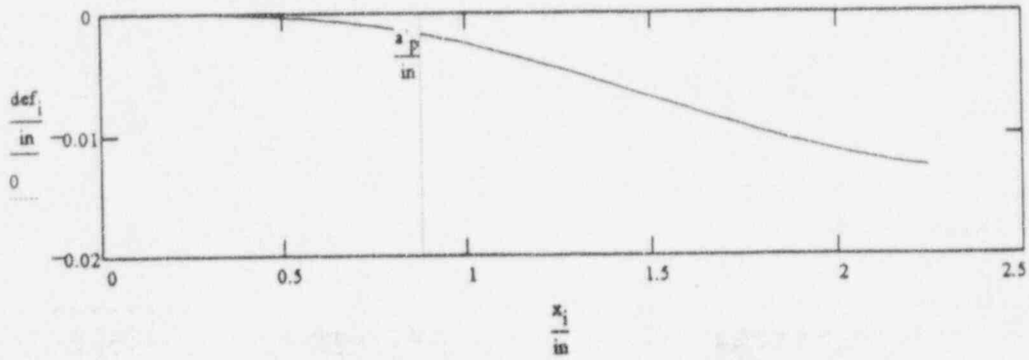
$n := 100 \qquad \Delta x := \dfrac{a_p + b_p}{n} \qquad \Delta x = 0.02249 \cdot in$

$i := 1 .. n + 1$

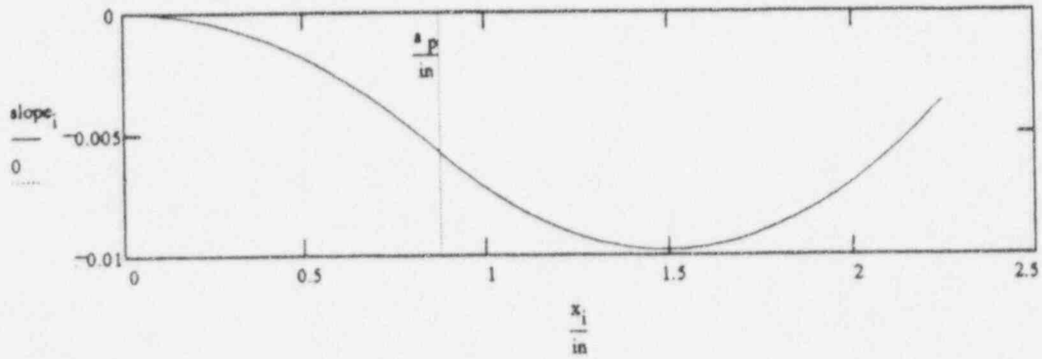
$x_i := (i - 1) \cdot \Delta x$

$def_i := y(x_i, a_p) \qquad slope_i := r(x_i, a_p) \qquad shear_i := V(x_i, a_p) \quad mom_i := M(x_i, a_p)$

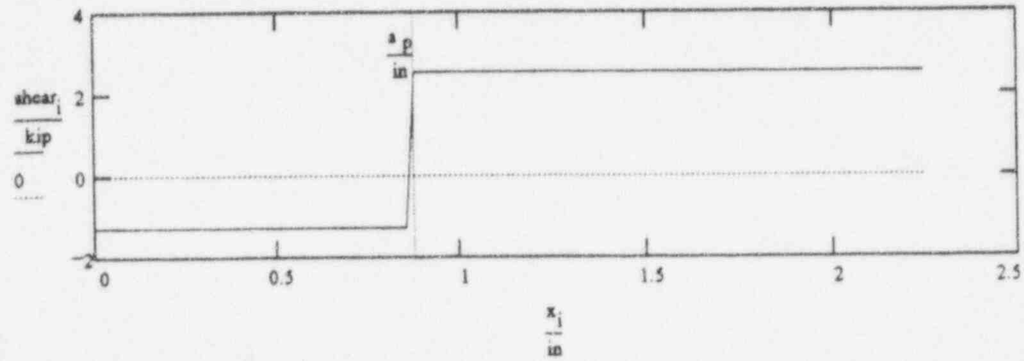
$y(0 \cdot in, a_p) = 0 \cdot in \qquad y(a_p, a_p) = -0.0016749 \cdot in \qquad y(a_p + b_p, a_p) = -0.01275 \cdot in$



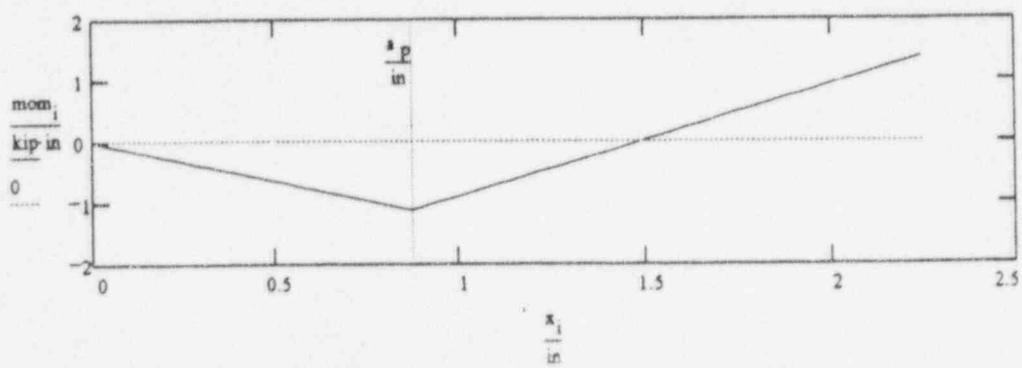
$r(0 \cdot in, a_p) = 0 \qquad r(a_p, a_p) = -0.005747713 \qquad r(a_p + b_p, a_p) = -0.00362$



$V(0 \cdot in, a_p) = -1.27793 \cdot kip \qquad P_{bolt}(a_p) = 3.80808 \cdot kip \qquad V(a_p + b_p, a_p) = 2.53015 \cdot kip$



$M(0 \cdot in, a_p) = 0 \cdot kip \cdot in \qquad M(a_p, a_p) = -1.11719 \cdot kip \cdot in \qquad M(a_p + b_p, a_p) = 1.38013 \cdot kip \cdot in$



8

Calculate rotation of channel due applied shear and moment and compare to assumed rotation. Also, calculate the total deflection of anchorage system and channel.

Rotation and deflection of Channel Flange

Assume moment of inertia is equal to sum of:

1. Plate and channel flange composite for plate width w_p

$$I_{fp} := \frac{w_p \cdot (t_p + t_f)^3}{12} \qquad I_{fp} = 0.0391 \cdot in^4$$

2. Channel flange width equal to twice distance from edge of flange to inside web surface

$$I_f := \frac{2 \cdot (b_f - t_w) \cdot t_f^3}{12} \qquad I_f = 0.01157 \cdot in^4$$

Distance from ground to center of area of plate channel flange composite

$$A_{fp} := w_p \cdot (t_p + t_f) \qquad A_f := 2 \cdot \left[(b_f - t_w) \cdot t_f \right]$$

$$\varepsilon := \frac{A_{fp} \cdot \frac{t_p + t_f}{2} + A_f \cdot \frac{t_f}{2}}{A_{fp} + A_f} \qquad \varepsilon = 0.23784 \cdot in$$

Total moment of inertia

$$I_t := I_{fp} + I_f + A_{fp} \cdot \left(\varepsilon - \frac{t_p + t_f}{2} \right)^2 + A_f \left(\varepsilon - \frac{t_f}{2} \right)^2 \qquad I_t = 0.06045 \cdot in^4$$

Calculate rotation and vertical deflection of flange/plate composite at edge of flange

$$\theta_1 := \frac{-1}{E \cdot I_t} \cdot \left[\frac{F_e(a_p) \cdot (b_f - t_w - \eta)^2}{2} \ldots \atop + \left[M(a_p + b_p, a_p) + V_{bolt}(a_p) \cdot \left(t_f + \frac{t_p}{2} - \varepsilon \right) \right] \cdot (b_f - t_w - \eta) \right] \qquad \theta_1 = -0.00325$$

$$\Delta vert_1 := \frac{-1}{E \cdot I_t} \cdot \left[\frac{F_e(a_p) \cdot (b_f - t_w - \eta)^3}{3} \ldots \atop + \frac{\left[M(a_p + b_p, a_p) + V_{bolt}(a_p) \cdot \left(t_f + \frac{t_p}{2} - \varepsilon \right) \right] \cdot (b_f - t_w - \eta)^2}{2} \right] \qquad \Delta vert_1 = -0.00277 \cdot in$$

$$V_{bolt}(a_p) = 2.75942 \cdot kip$$

Rotation of channel due to twisting and deflection at edge of flange .

Torsion moment at shear center

$$T_0 := \left[M\left(a_p + b_p, a_p\right) + F_e\left(a_p\right)\cdot\left(b_f - \eta + e_o\right)\right] - V_{bolt}\left(a_p\right)\cdot\left(\frac{d}{2} - t_f - \frac{t_p}{2}\right)$$

$$T_0 = 0.13417 \cdot kip \cdot in$$

At distance "a" from the exterior channel intersection to anchorage

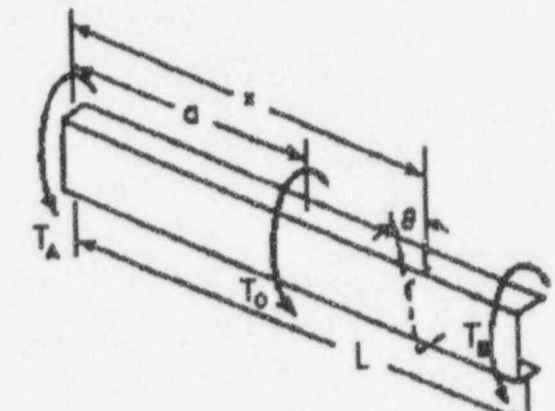
$$a = 19.1875 \cdot in$$

Rotation of channel

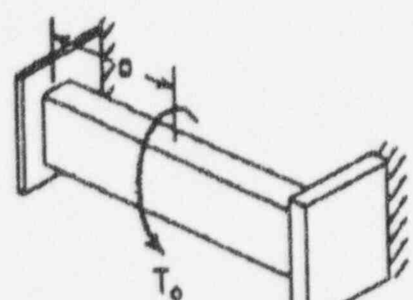
The following formulation is taken from Roark 6th Ed (as given in Mathcad Roark's 1 Handbook)

This file corresponds to Table 21, Case 1, and Table 22, Cases 1e-1g, in ***Roark's Formulas for Stress and Strain.***

Concentrated intermediate torque



Case f **Left end free to warp but not twist, right end fixed (no twist or warp)**



NOTE THAT THE "LEFT" SUPPORT CORRESPONDS TO EXTERIOR CONNECTION AND THE "RIGHT" SUPPORT CORRESPONDS TO AN INTERIOR CONNECTION ON THE A SWITCHGEAR CHANNEL BASE FRA

$$\beta = \left(\frac{K \cdot G}{C_w \cdot E}\right)^{\frac{1}{2}} \qquad \beta = 0.08074 \cdot \frac{1}{in}$$

$$F_1(x) = \cosh(\beta \cdot x)$$

$$F_2(x) = \sinh(\beta \cdot x)$$

$$F_3(x) = \cosh(\beta \cdot x) - 1$$

$$F_4(x) = \sinh(\beta \cdot x) - \beta \cdot x$$

$$F_{a1}(x) = (x>a) \cdot \cosh(\beta \cdot (x - a))$$

$$F_{a3}(x) = (x>a) \cdot (\cosh(\beta \cdot (x - a)) - 1)$$

$$F_{a4}(x) = (x>a) \cdot \sinh(\beta \cdot (x - a)) - (x>a) \cdot (x - a) \cdot \beta$$

$$C_1 = \cosh(\beta \cdot L)$$

$$C_2 = \sinh(\beta \cdot L)$$

$$C_3 = \cosh(\beta \cdot L) - 1$$

$$C_4 = \sinh(\beta \cdot L) - \beta \cdot L$$

$$C_{a3} = \cosh(\beta \cdot (L - a)) - 1$$

$$C_{a4} = \sinh(\beta \cdot (L - a)) - \beta \cdot (L - a)$$

$$I = \begin{bmatrix} 0 \\ \frac{T_0}{C_w \cdot E \cdot \beta^2} \cdot \left(\frac{C_3 \cdot C_{a4} - C_4 \cdot C_{a3}}{C_1 \cdot C_4 - C_2 \cdot C_3}\right) \cdot \frac{ft}{deg} \\ 0 \\ -T_0 \cdot \left(\frac{C_1 \cdot C_{a4} - C_2 \cdot C_{a3}}{C_1 \cdot C_4 - C_2 \cdot C_3}\right) \cdot \frac{1}{lbf \cdot in} \end{bmatrix}$$

Vector of end constraints for this case. Recall:

$$I = \begin{bmatrix} \theta_A \\ \theta'_A \\ \theta''_A \\ T_A \end{bmatrix} \qquad I = \begin{bmatrix} 0 \\ 0.02022 \\ 0 \\ -48.22618 \end{bmatrix}$$

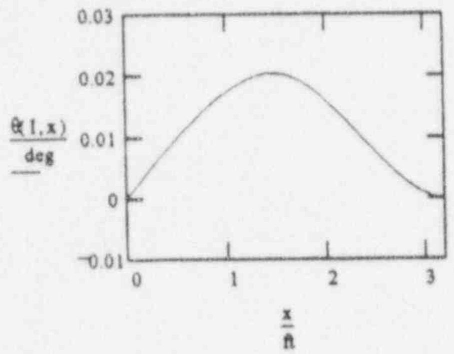
$$\theta(I,x) = I_1 \cdot deg + \frac{I_2 \cdot \frac{deg}{ft}}{\beta} \cdot F_2(x) + \frac{I_3 \cdot \frac{deg}{ft^2}}{\beta^2} \cdot F_3(x) + \frac{I_4 \cdot lbf \cdot in}{C_w \cdot E \cdot \beta^3} \cdot F_4(x) + \frac{T_0}{C_w \cdot E \cdot \beta^3} \cdot F_{a4}(x)$$

Rotation at point of anchorage attachment to channel

$$\theta_2 = -\theta(I, a) \qquad \theta_2 = -3.51099 \cdot 10^{-4} \cdot rad$$

Range of x-values: $x := 0\,ft, \frac{L}{100} .. L$

Torsion Pinned end — Torsion Fixed end



Angle of twist, θ

$\theta(I, 0\cdot ft) = 0 \cdot rad$

$\theta(I, a) = 3.51099 \cdot 10^{-4} \cdot rad$

$\theta(I, L) = 0 \cdot rad$

Vertical deflection of channel at contact point between plate and flange due to channel rotation

$\Delta vert_2 := \theta_2 \cdot \left[\left[\left(b_f + e_o\right) - \eta\right]\right]$ $\qquad \Delta vert_2 = -7.96644 \cdot 10^{-4} \cdot in$

Vertical deflection of channel between cross channels - assume fixed/pinned span as effective boundary conditions

$$\Delta vert_3 := \frac{-7 \cdot F_e(a_p) \cdot L^3}{768 \cdot E \cdot I_x} \qquad \Delta vert_3 = -0.00343 \cdot in$$

Horizontal deflection of channel between cross channels - assume fixed/pinned span as effective boundary conditions

$$\Delta horiz := \frac{-7 \cdot V_{bolt}(a_p) \cdot L^3}{768 \cdot E \cdot I_y} \qquad \Delta horiz = -0.07083 \cdot in$$

Total rotation from channel support system and deflection from concrete to switchgear

$\theta_{Tel} := \theta_1 + \theta_2$ $\qquad \theta_{Tel} = -0.0036003 \cdot rad$

Compare to calculated rotation at end of plate (connection to channel flange) $\qquad r(a_p + b_p, a_p) = -0.00362 \cdot rad$

Total displacement

$\Delta horiz = -0.07083 \cdot in$

$$\Delta vert_T := \Delta vert_1 + \Delta vert_2 + \Delta vert_3 \ldots + y(a_p + b_p, a_p) \qquad \Delta vert_T = -0.01974 \cdot in$$

Cantilever flange $\quad \Delta vert_1 = -0.00277 \cdot in$

Channel twisting $\quad \Delta vert_2 = -7.96644 \cdot 10^{-4} \cdot in$

Channel bending $\quad \Delta vert_3 = -0.00343 \cdot in$

Plate bending $\quad y(a_p + b_p, a_p) = -0.01275 \cdot in$ $\qquad \Delta el_{plate} := y(a_p + b_p, a_p)$

Force at ends of plate $\qquad Q_{be}(a_p) = 1.27793 \cdot kip$ $\quad \boxed{F_e(a_p) = 2.53015 \cdot kip}$

Force in bolt $\qquad \boxed{P_{bolt}(a_p) = 3.80808 \cdot kip \quad V_{bolt}(a_p) = 2.75942 \cdot kip}$

$\boxed{\Delta bar_{el} := y(a_p + b_p, a_p)}$

Calculate scale factor

$$FS := \frac{F_e(a_p) + \Psi_2}{\Psi_1}$$

$\boxed{FS = 1.67338}$

Calculate frequency of cabinet in East/West direction

$$\Delta_{el} := \left| \frac{h}{(l_{ew1} - l_{ew2})} \cdot \left(\frac{0.4 \cdot P_{ew}}{\Psi_1} \right) \cdot \frac{\Delta vert_T}{2} \right| + \frac{SSa(f_{ew}, \beta eta) \cdot FS}{(2 \cdot \pi \cdot f_{fixed})^2}$$

$\Delta_{el} = 0.1181 \cdot in$

$$SpAcc_{el} := SSa(f_{ew}, \beta eta) \cdot FS$$

$SpAcc_{el} = 1.17121 \cdot g$

$$f\_new_{ew} := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{SpAcc_{el}}{\Delta_{el}}}$$

$f\_new_{ew} = 9.84806 \cdot Hz$ Calculated frequency

$f_{ew} = 9.848 \cdot Hz$ Assumed frequency

Calculate frequency of cabinet in North/South direction

$$\Delta_{el} := \left| \Delta horiz + \frac{\frac{h}{2}}{l_{ns}} \cdot \left(\frac{P_{ns} + 0.4 \cdot P_{vert}}{\Psi_1} \right) \cdot \Delta vert_T \right| + \frac{SSa(f_{ns}, \beta eta) \cdot FS}{(2 \cdot \pi \cdot f_{fixed})^2}$$

$\boxed{\Delta_{el} = 0.18759 \cdot in}$

$$SpAcc_{el} := SSa(f_{ns}, \beta eta) \cdot FS$$

$SpAcc_{el} = 1.10377 \cdot g$

$$f\_new_{ns} := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{SpAcc_{el}}{\Delta_{el}}}$$

$f\_new_{ns} = 7.58572 \cdot Hz$ Calculated frequency

$f_{ns} = 7.5856 \cdot Hz$ Assumed frequency

STOP3

SECANT POINT ANALYSIS

Spectral acceleration in n/s direction

$$Sa_{ns} := SSa\left(f_{ns\mu}, \beta eta\right)$$

$$Sa_{ns} = 0.62852 \cdot g$$

Spectral acceleration in e/w direction

$$Sa_{ew} := SSa\left(f_{ew\mu}, \beta eta\right)$$

$$Sa_{ew} = 0.67716 \cdot g$$

Spectral acceleration in vertical direction

$$Sa_{vert} := \left(\frac{2}{3} \cdot 0.40\right) \cdot g$$

$$Sa_{vert} = 0.26667 \cdot g$$

$$P_{ns} := \frac{\frac{Sa_{ns}}{g} \cdot Wt \cdot \left(\frac{h}{2}\right)}{l_{ns} \cdot 2}$$

$$P_{ew} := \frac{\frac{Sa_{ew}}{g} \cdot Wt \cdot \left(\frac{h}{2}\right)}{2 \cdot \left(l_{ew1} + \frac{l_{ew2}^{2}}{l_{ew1}}\right)}$$

$$P_{vert} := \frac{\frac{Sa_{vert}}{g} \cdot Wt}{4}$$

$$P_{ns} = 1.88211 \cdot kip \qquad P_{ew} = 1.84823 \cdot kip \qquad P_{vert} = 0.66667 \cdot kip$$

$$\Psi_1 := P_{ns} + 0.4 \cdot \left(P_{ew} + P_{vert}\right) \qquad \Psi_2 := \frac{Wt}{4} \qquad \Psi_3 := \frac{Wt}{4} \cdot \frac{Sa_{ns}}{g}$$

$$\Psi_1 = 2.88807 \cdot kip \qquad \Psi_2 = 2.5 \cdot kip \qquad \Psi_3 = 1.5713 \cdot kip$$

$$c_1 := \frac{\Psi_3}{\Psi_1} \qquad c_1 = 0.54407$$

$$c_2 := \frac{\Psi_2 \cdot \Psi_3}{\Psi_1} \qquad c_2 = 1.36017 \cdot kip$$

General properties

$c_1 = 0.54407$

$c_2 = 1.36017 \cdot kip$

c_1 and c_2 are used for the case where the shear force on the bolt is proportional to the the F_e on the anchorage:

$$V = c_1 F_e + c_2$$

$t_{pp} := t_f + 20 \cdot \left(\left| \Delta bar_{el} \right| \right)$

Offset in plate which causes moment in plate from shear force on the bolt (equal to thickness of channel flange plus deflection upward)

Calculated properties

$\Psi := 1 - c_1 \cdot \dfrac{t_{pp}}{b_p}$ $\qquad \Psi = 0.76341$ $\qquad$ Variable reduction factor on the force Fe due to shear on bolt

$\zeta := \dfrac{t_{pp}}{b_p} \cdot c_2$ $\qquad \zeta = 0.59147 \cdot kip$ $\qquad$ **Constant reduction factor on the force Fe due to shear on the bolt**

$I := \dfrac{t_p^{\,3} \cdot w_p}{12}$ $\qquad I = 0.00293 \cdot in^4$ $\qquad$ Moment of inertia plate

$Z_h := \dfrac{\left(w_p - d_h\right) \cdot t_p^{\,2}}{4}$ $\qquad Z_h = 0.02539 \cdot in^3$ $\qquad$ Plastic section modulus plate at hole

$M_h := Z_h \cdot \sigma_{yp}$ $\qquad M_h = 1.11719 \cdot kip \cdot in$ $\qquad$ Plastic moment capacity plate at hole

CASE 1 - MOMENT BETWEEN PLATE AND CHANNEL FLANGE IS ZERO

This calculation finds the value of a_p such that the slope at the end of the plate is zero and the moment at the attachment to the channel (i.e., at $x = a_p + b_p$) is also zero. This is the limiting case where the channel offers no resistence to rotation

See derivation for definition of terms and theoretical basis (Reed 11/22/93)

$Q_{be}(a_p) := \dfrac{M_h}{a_p}$ $\qquad$ Allowable prying force corresponding to plastic moment capacity plate at hole

$$k_e(a_p) := \frac{2 \cdot E \cdot I \cdot g_p}{b_p^{\,2} \cdot A_b \cdot E_b \cdot a_p}$$

$$k(a_p) := 2 \cdot \frac{a_p}{b_p} + \frac{2}{3} \cdot \left(\frac{a_p}{b_p} \right)^2$$

$$F_e(a_p) := \frac{1}{\Psi} \cdot \left(\frac{a_p}{b_p} \cdot Q_{be}(a_p) + \zeta \right)$$

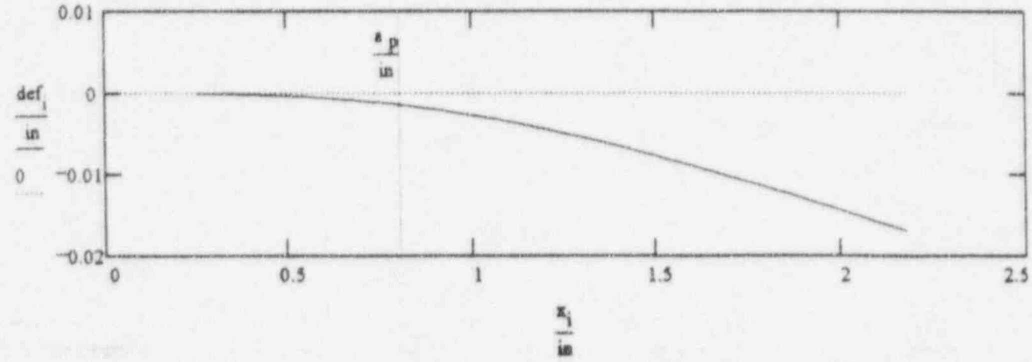
$n := 100 \qquad \Delta x := \dfrac{a_p + b_p}{n} \qquad \Delta x = 0.0218 \cdot in$

$i := 1 .. n + 1$

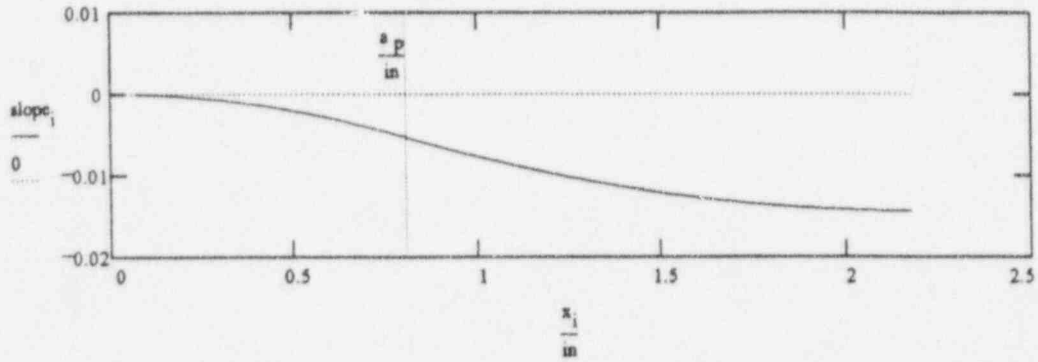
$x_i := (i - 1) \cdot \Delta x$

$def_i := y(x_i, a_p) \qquad slope_i := r(x_i, a_p) \qquad shear_i := V(x_i, a_p) \quad mom_i := M(x_i, a_p)$

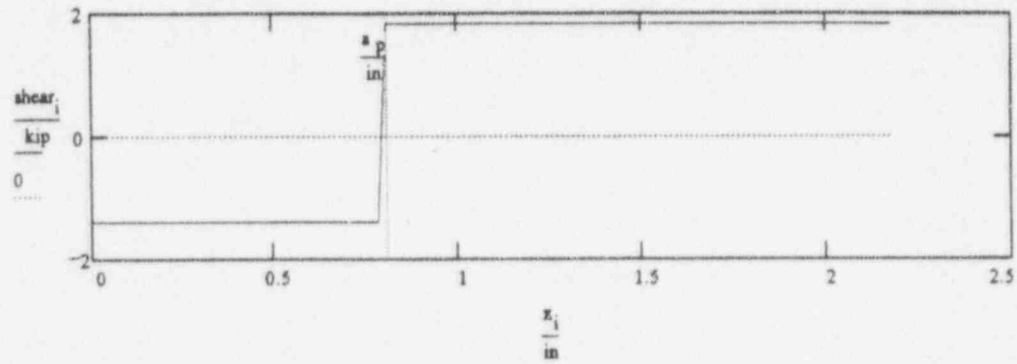
$y(0 \cdot in, a_p) = 0 \cdot in \qquad y(a_p, a_p) = -0.00142 \cdot in \qquad y(a_p + b_p, a_p) = -0.01698 \cdot in$



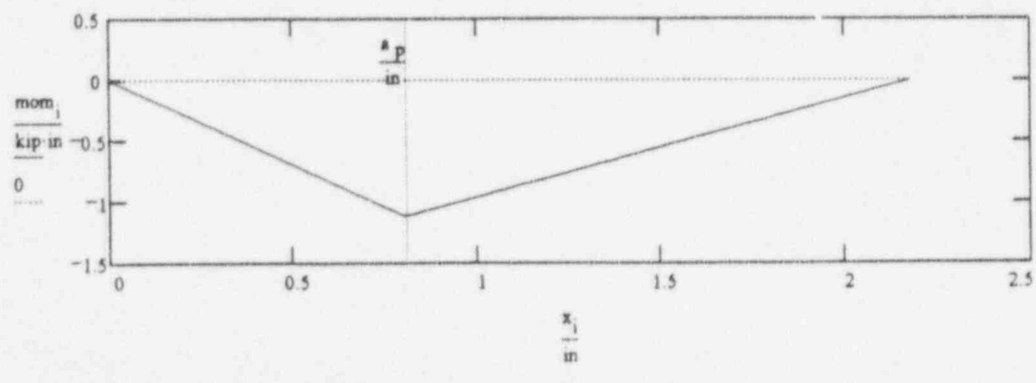
$r(0 \cdot in, a_p) = 1.465303015 \cdot 10^{-6} \quad r(a_p, a_p) = -0.005291601 \qquad r(a_p + b_p, a_p) = -0.014331831$



$V(0 \cdot in, a_p) = -1.3877 \cdot kip \qquad P_{bolt}(a_p) = 3.22678 \cdot kip \qquad V(a_p + b_p, a_p) = 1.83908 \cdot kip$



$M(0 \cdot in, a_p) = 0 \cdot kip \cdot in \qquad M(a_p, a_p) = -1.11719 \cdot kip \cdot in \qquad M(a_p + b_p, a_p) = 0 \cdot kip \cdot in$



CASE 2 - ROTATION AT CONNECTION BETWEEN PLATE AND CHANNEL FLANGE IS SET TO θ_e

For thIS case a value of θ_e between zero and the limiting value $\theta_e = -0.01433$ is assumed and the plate response is calculated. Again, the slope at the end of the plate is set equal to zero.

$\theta_e := \theta assumed_{\mu}$ Assumed value of θ_e

$$F_e(a_p) := \frac{Q_{be}(a_p)\cdot\left(k(a_p)+k_e(a_p)\right)+\dfrac{2\cdot E\cdot I\cdot\theta_e}{b_p^{\,2}}+\zeta}{\Psi-k_e(a_p)}$$

$$A(a_p) := \left[\frac{Q_{be}(a_p)}{2}\cdot a_p\cdot(a_p+2\cdot b_p)-\left(F_e(a_p)\cdot\Psi-\zeta\right)\cdot\frac{b_p^{\,2}}{2}\right]+E\cdot I\cdot\theta_e$$

$$y(x,a_p) := \frac{1}{E\cdot I}\left[\left(-Q_{be}(a_p)\cdot\frac{x^3}{6}+A(a_p)\cdot x\right)+\left(F_e(a_p)\cdot\Psi-\zeta+Q_{be}(a_p)\right)\cdot\frac{(x-a_p)^3}{6}\cdot(x>a_p)\right]$$

$$r(x,a_p) := \frac{1}{E\cdot I}\left[\left(-Q_{be}(a_p)\cdot\frac{x^2}{2}+A(a_p)\right)+\left(F_e(a_p)\cdot\Psi-\zeta+Q_{be}(a_p)\right)\cdot\frac{(x-a_p)^2}{2}\cdot(x>a_p)\right]$$

$$V(x,a_p) := -Q_{be}(a_p)+\left(F_e(a_p)+Q_{be}(a_p)\right)\cdot(x>a_p)$$

$$M(x,a_p) := -Q_{be}(a_p)\cdot x+\left[\left(F_e(a_p)\cdot\Psi-\zeta\right)+Q_{be}(a_p)\right]\cdot(x-a_p)\cdot(x>a_p)$$

$$V_{bolt}(a_p) := c_1\cdot F_e(a_p)+c_2$$

Given

Constrain slope to equal zero at end of plate

$$r(0\cdot in, a_p) = 0$$

$a_p := \mathrm{find}(a_p)$ $a_p = 0.92366\cdot in$

$F_e(a_p) = 3.04146\cdot kip$

$Q_{be}(a_p) = 1.20953\cdot kip$

$F_b(a_p) := F_e(a_p)+Q_{be}(a_p)$ $F_b(a_p) = 4.25099\cdot kip$

$$P_{bolt}(a_p) := \frac{-y(a_p,a_p)\cdot A_b\cdot E_b}{g_p}$$

$P_{bolt}(a_p) = 4.25099\cdot kip$

$V_{bolt}(a_p) = 3.01492\cdot kip$

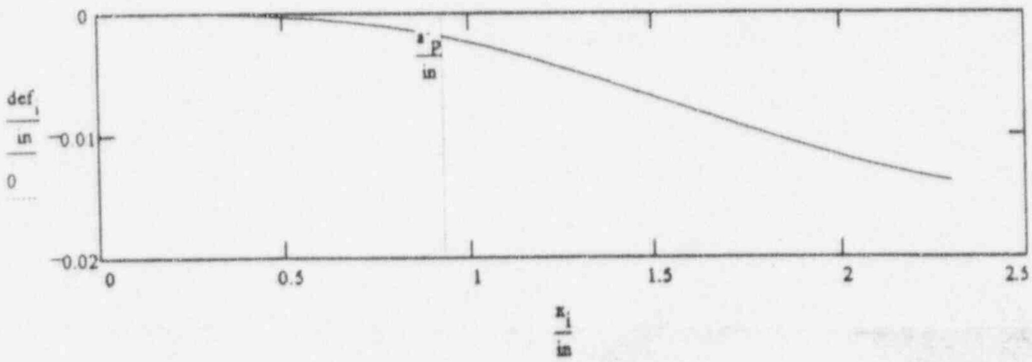
$n := 100 \qquad \Delta x := \frac{a_p + b_p}{n} \qquad \Delta x = 0.02299 \cdot in$

$i := 1 .. n+1$

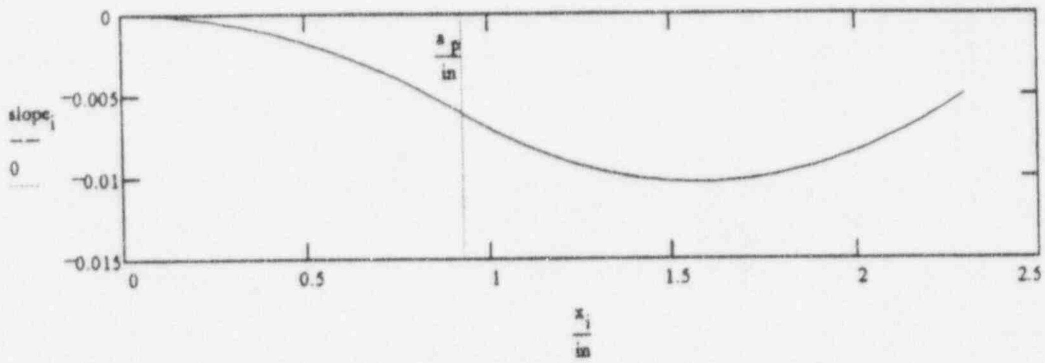
$x_i := (i-1) \cdot \Delta x$

$def_i := y(x_i, a_p) \qquad slope_i := r(x_i, a_p) \qquad shear_i := V(x_i, a_p) \quad mom_i := M(x_i, a_p)$

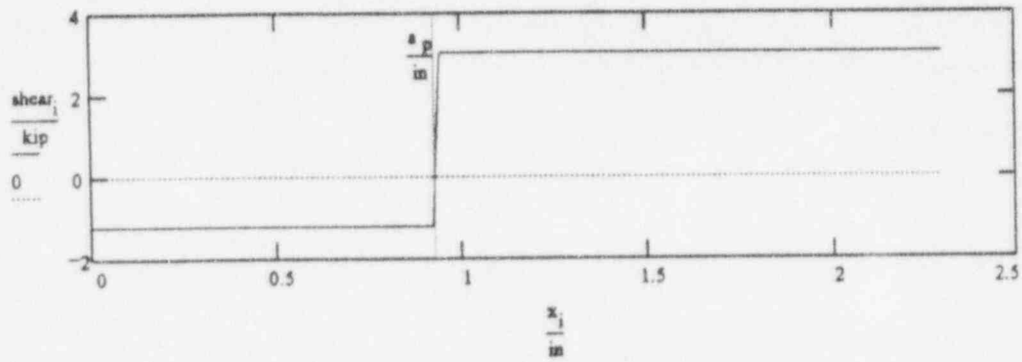
$y(0 \cdot in, a_p) = 0 \cdot in \qquad y(a_p, a_p) = -0.0018697 \cdot in \qquad y(a_p + b_p, a_p) = -0.01383 \cdot in$



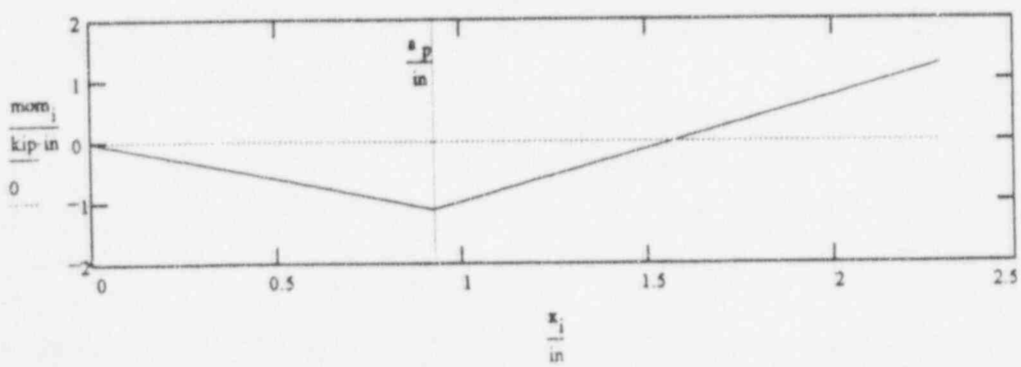
$r(0 \cdot in, a_p) = 0 \qquad r(a_p, a_p) = -0.00607270 9 \qquad r(a_p + b_p, a_p) = -0.0049$



$V(0 \cdot in, a_p) = -1.20953 \cdot kip \qquad P_{bolt}(a_p) = 4.25099 \cdot kip \qquad V(a_p + b_p, a_p) = 3.04146 \cdot kip$



$M(0 \cdot in, a_p) = 0 \cdot kip \cdot in \qquad M(a_p, a_p) = -1.11719 \cdot kip \cdot in \qquad M(a_p + b_p, a_p) = 1.26212 \cdot kip \cdot in$



Calculate rotation of channel due applied shear and moment and compare to assumed rotation. Also, calculate the total deflection of anchorage system and channel.

Rotation and deflection of Channel Flange

Assume moment of inertia is equal to sum of:

1. Plate and channel flange composite for plate width w_p

$$I_{fp} := \frac{w_p \cdot (t_p + t_f)^3}{12} \qquad I_{fp} = 0.0391 \cdot in^4$$

2. Channel flange width equal to twice distance from edge of flange to inside web surface

$$I_f := \frac{2 \cdot (b_f - t_w) \cdot t_f^3}{12} \qquad I_f = 0.01157 \cdot in^4$$

Distance from ground to center of area of plate channel flange composite

$$A_{fp} := w_p \cdot (t_p + t_f) \qquad A_f := 2 \cdot \left[(b_f - t_w) \cdot t_f \right]$$

$$\varepsilon := \frac{A_{fp} \cdot \frac{t_p + t_f}{2} + A_f \cdot \frac{t_f}{2}}{A_{fp} + A_f} \qquad \varepsilon = 0.23784 \cdot in$$

Total moment of inertia

$$I_t := I_{fp} + I_f + A_{fp} \cdot \left(\varepsilon - \frac{t_p + t_f}{2} \right)^2 + A_f \cdot \left(\varepsilon - \frac{t_f}{2} \right)^2 \qquad I_t = 0.06045 \cdot in^4$$

Calculate rotation and vertical deflection of flange/plate composite at edge of flange

$$\theta_1 := \frac{-1}{E \cdot I_t} \cdot \left[\frac{F_e(a_p) \cdot (b_f - t_w - \eta)^2}{2} \ldots \atop + \left[M(a_p + b_p, a_p) + V_{bolt}(a_p) \cdot \left(t_f + \frac{t_p}{2} - \varepsilon \right) \right] \cdot (b_f - t_w - \eta) \right] \qquad \theta_1 = -0.00351$$

$$\Delta vert_1 := \frac{-1}{E \cdot I_t} \cdot \left[\frac{F_e(a_p) \cdot (b_f - t_w - \eta)^3}{3} \ldots \atop + \frac{\left[M(a_p + b_p, a_p) + V_{bolt}(a_p) \cdot \left(t_f + \frac{t_p}{2} - \varepsilon \right) \right] \cdot (b_f - t_w - \eta)^2}{2} \right] \qquad \Delta vert_1 = -0.00304 \cdot in$$

$$V_{bolt}(a_p) = 3.01492 \cdot kip$$

Rotation of channel due to twisting and deflection at edge of flange .

Torsion moment at shear center

$$T_0 = \left[M\left(a_p + b_p, a_p\right) + F_e\left(a_p\right)\cdot\left(b_f - \eta + e_o\right)\right] - V_{bolt}\left(a_p\right)\cdot\left(\frac{d}{2} - t_f - \frac{t_p}{2}\right)$$

$T_0 = 0.5294 \cdot \text{kip} \cdot \text{in}$

At distance "a" from the exterior channel intersection to anchorage

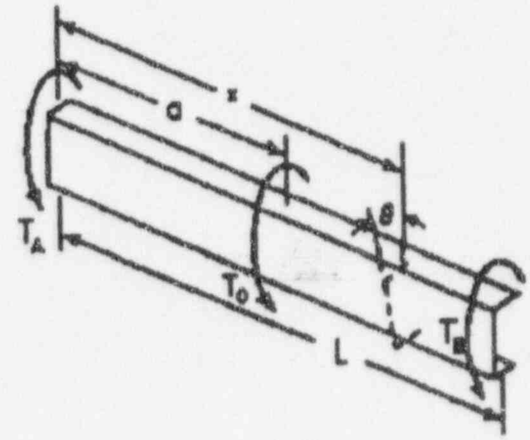
$a = 19.1875 \cdot \text{in}$

Rotation of channel

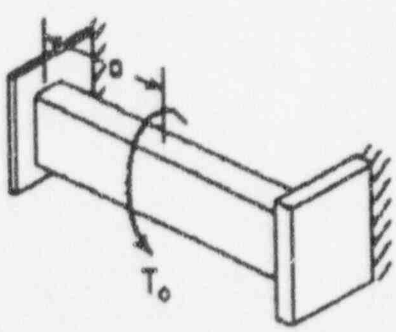
The following formulation is taken from Roark 6th Ed (as given in Mathcad Roark's 1 Handbook)

This file corresponds to Table 21, Case 1, and Table 22, Cases 1e-1g, in ***Roark's Formulas for Stress and Strain.***

Concentrated intermediate torque



Case f **Left end free to warp but not twist, right end fixed (no twist or warp)**



NOTE THAT THE "LEFT" SUPPORT CORRESPONDS TO EXTERIOR CONNECTION AND THE "RIGHT" SUPPORT CORRESPONDS TO AN INTERIOR CONNECTION ON THE A8 SWITCHGEAR CHANNEL BASE FRAME

$$\beta := \left(\frac{K \cdot G}{C_w \cdot E}\right)^{\frac{1}{2}} \qquad \beta = 0.08074 \cdot \frac{1}{in}$$

$$F_1(x) := \cosh(\beta \cdot x)$$

$$F_2(x) := \sinh(\beta \cdot x)$$

$$F_3(x) := \cosh(\beta \cdot x) - 1$$

$$F_4(x) := \sinh(\beta \cdot x) - \beta \cdot x$$

$$F_{a1}(x) := (x>a) \cdot \cosh(\beta \cdot (x-a))$$

$$F_{a3}(x) := (x>a) \cdot (\cosh(\beta \cdot (x-a)) - 1)$$

$$F_{a4}(x) := (x>a) \cdot \sinh(\beta \cdot (x-a)) - (x>a) \cdot (x-a) \cdot \beta$$

$$C_1 := \cosh(\beta \cdot L)$$

$$C_2 := \sinh(\beta \cdot L)$$

$$C_3 := \cosh(\beta \cdot L) - 1$$

$$C_4 := \sinh(\beta \cdot L) - \beta \cdot L$$

$$C_{a3} := \cosh(\beta \cdot (L-a)) - 1$$

$$C_{a4} := \sinh(\beta \cdot (L-a)) - \beta \cdot (L-a)$$

$$I := \begin{bmatrix} 0 \\ \frac{T_0}{C_w \cdot E \cdot \beta^2} \cdot \left(\frac{C_3 \cdot C_{a4} - C_4 \cdot C_{a3}}{C_1 \cdot C_4 - C_2 \cdot C_3}\right) \cdot \frac{ft}{deg} \\ 0 \\ -T_0 \cdot \left(\frac{C_1 \cdot C_{a4} - C_2 \cdot C_{a3}}{C_1 \cdot C_4 - C_2 \cdot C_3}\right) \cdot \frac{1}{lbf \cdot in} \end{bmatrix}$$

Vector of end constraints for this case. Recall:

$$I \equiv \begin{bmatrix} \theta_A \\ \theta'_A \\ \theta''_A \\ T_A \end{bmatrix} \qquad I = \begin{bmatrix} 0 \\ 0.07977 \\ 0 \\ -190.28083 \end{bmatrix}$$

$$\theta(I,x) := I_1 \cdot deg + \frac{I_2 \cdot \frac{deg}{ft}}{\beta} \cdot F_2(x) + \frac{I_3 \cdot \frac{deg}{ft^2}}{\beta^2} \cdot F_3(x) + \frac{I_4 \cdot lbf \cdot in}{C_w \cdot E \cdot \beta^3} \cdot F_4(x) + \frac{T_0}{C_w \cdot E \cdot \beta^3} \cdot F_{a4}(x)$$

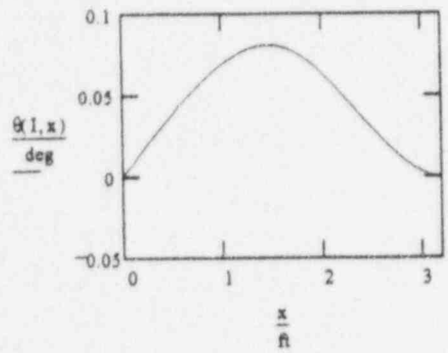
Rotation at point of anchorage attachment to channel

$$\theta_2 := -\theta(I,a) \qquad \theta_2 = -0.00139 \cdot rad$$

Range of x-values: $x := 0 \cdot ft, \frac{L}{100} .. L$

Torsion Pinned end

Torsion Fixed end



Angle of twist, θ

$\theta(I, 0 \cdot ft) = 0 \cdot rad$

$\theta(I, a) = 0.00139 \cdot rad$

$\theta(I, L) = 0 \cdot rad$

Vertical deflection of channel at contact point between plate and flange due to channel rotation

$$\Delta vert_2 := \theta_2 \cdot \left[\left[\left(b_f + e_o\right) - \eta\right]\right] \qquad \Delta vert_2 = -0.00314 \cdot in$$

Vertical deflection of channel between cross channels - assume fixed/pinned span as effective boundary conditions

$$\Delta vert_3 := \frac{-7 \cdot F_e(a_p) \cdot L^3}{768 \cdot E \cdot I_x} \qquad \Delta vert_3 = -0.00412 \cdot in$$

Horizontal deflection of channel between cross channels - assume fixed/pinned span as effective boundary conditions

$$\Delta horiz := \frac{-7 \cdot V_{bolt}(a_p) \cdot L^3}{768 \cdot E \cdot I_y} \qquad \Delta horiz = -0.07738 \cdot in$$

Total rotation from channel support system and deflection from concrete to switchgear

$$\theta_{T\mu} := \theta_1 + \theta_2 \qquad \theta_{T\mu} = -0.0049 \cdot rad$$

Compare to calculated rotation at end of plate (connection to channel flange) $r(a_p + b_p, a_p) = -0.0049 \cdot rad$

Total displacement

$\Delta horiz = -0.07738 \cdot in$

$$\Delta vert_T := \Delta vert_1 + \Delta vert_2 + \Delta vert_3 ... + \left(t_f - t_{pp}\right) \qquad \Delta vert_T = -0.26523 \cdot in$$

Cantilever flange $\Delta vert_1 = -0.00304 \cdot in$

Channel twisting $\Delta vert_2 = -0.00314 \cdot in$

Channel bending $\Delta vert_3 = -0.00412 \cdot in$

Plate bending $t_f - t_{pp} = -0.25493 \cdot in$

Force at ends of plate $Q_{be}(a_p) = 1.20953 \cdot kip$ $F_e(a_p) = 3.04146 \cdot kip$

Force in bolt $P_{bolt}(a_p) = 4.25099 \cdot kip$

$V_{bolt}(a_p) = 3.01492 \cdot kip$

STOP4

Calculate scale factor

$$FSS := \frac{F_e(a_p) + \Psi_2}{\Psi_1}$$ $\boxed{FSS = 1.91874}$

Calculate frequency of cabinet in East/West direction

$$\Delta_\mu := \left|\frac{h}{(l_{ew1} - l_{ew2})}\cdot\left(\frac{0.4\cdot P_{ew}}{\Psi_1}\right)\cdot\frac{\Delta vert_T}{2}\right| + \frac{SSa(f_{ew\mu}, \beta eta)\cdot FSS}{(2\cdot\pi\cdot f_{fixed})^2}$$ $\Delta_\mu = 0.17525 \cdot in$

$SpAcc_\mu := SSa(f_{ew\mu}, \beta eta)\cdot FSS$ $SpAcc_\mu = 1.2993 \cdot g$

$$f\_new\mu_{ew} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{SpAcc_\mu}{\Delta_\mu}}$$ $f\_new\mu_{ew} = 8.51512 \cdot Hz$ Calculated frequency

$f_{ew\mu} = 8.51538 \cdot Hz$ Assumed frequency

Calculate frequency of cabinet in North/South direction

$$\Delta_\mu := \left|\Delta horiz + \frac{\frac{h}{2}}{l_{ns}}\cdot\left(\frac{P_{ns} + 0.4\cdot P_{vert}}{\Psi_1}\right)\cdot\Delta vert_T\right| + \frac{SSa(f_{ns\mu}, \beta eta)\cdot FSS}{(2\cdot\pi\cdot f_{fixed})^2}$$ $\boxed{\Delta_\mu = 0.31351 \cdot in}$

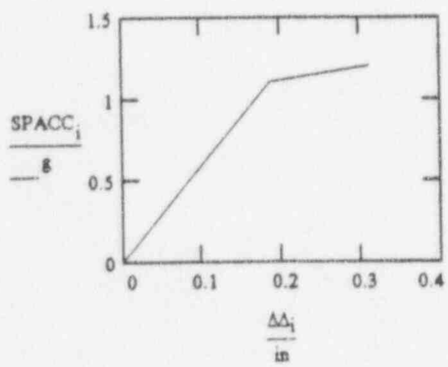
$SpAcc_\mu := SSa(f_{ns\mu}, \beta eta)\cdot FSS$ $SpAcc_\mu = 1.20597 \cdot g$

$$f\_new\mu_{ns} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{SpAcc_\mu}{\Delta_\mu}}$$ $f\_new\mu_{ns} = 6.13345 \cdot Hz$ Calculated frequency

$f_{ns\mu} = 6.13377 \cdot Hz$ Assumed frequency

$$SPACC := \begin{bmatrix} 0\cdot g \\ SpAcc_{el} \\ SpAcc_\mu \end{bmatrix} \qquad \Delta\Delta := \begin{bmatrix} 0\cdot in \\ \Delta_{el} \\ \Delta_\mu \end{bmatrix}$$

$i := 1..3$ Force/Deflection Diagram



$$SPACC = \begin{pmatrix} 0 \\ 1.10377 \\ 1.20597 \end{pmatrix}\cdot g \qquad \Delta\Delta = \begin{pmatrix} 0 \\ 0.18759 \\ 0.31351 \end{pmatrix}\cdot in$$

Calculate ductility factor and slope

$$mu := \frac{\Delta_\mu}{\Delta_{el}}$$ $mu = 1.67125$

$$s := \frac{\dfrac{SpAcc_\mu - SpAcc_{el}}{\Delta_\mu - \Delta_{el}}}{\left(\dfrac{SpAcc_{el}}{\Delta_{el}}\right)}$$ $s = 0.13794$

STOP6

Estimation of the Median Ductility Scale Factor, Fmu

The median ductility scale factor, Fmu, is obtained by several methods (See Reference 1):

The following variables are used in the analysis:

$f_{ns} = 7.5856 \cdot Hz$ The A8 Bus frequency

$b := \beta eta$ $b = 0.05$ Elastic damping

$fk := 2.5 \cdot \frac{1}{sec}$ Response spectrum knuckle frequency

$s = 0.13794$ Strain hardening ration

$mu = 1.67125$ Ductility factor

Riddle-Newmark method for Calculating FuRn

The details of the methodology are given in Reference 1

Lets first calculate Fmu at the peak ground acceleration level (zpa)

The zpa value is:

$zpa := 0.4 \cdot g$

and the spectral acceleration at the fundamental frequency, f, and elastic damping, b, is:

$Sa := SSa\left(f_{ns}, b\right)$ $Sa = 0.65961 \cdot g$

Fmu is estimated as:

$Fu4 := \frac{Sa}{zpa} \cdot mu^{0.11}$ $Fu4 = 1.74485$

Then, in the acceleration range of the spectrum, Fmu is estimated as:

$Fu3 := (2.67 \cdot mu - 1.673)^{0.411}$ $Fu3 = 1.52439$

Finally, in the velocity range of the spectrum, Fu is estimated as

$Cf := \left(\frac{fk}{f_{ns}}\right) \cdot \left(\frac{fk}{f_{ns}} < 1.0\right) + \left(\frac{fk}{f_{ns}} \geq 1.0\right)$ $Cf = 0.32957$

$Fu2 := (2.24 \cdot mu - 1.24)^{0.611} \cdot Cf$ $Fu2 = 0.5774$

The median ductility scale factor, Fu.RN, using the Riddle-Newmark method, is obtained as:

$Fu1 := Fu3\cdot(Fu3<Fu4) + Fu4\cdot(Fu3 \ge Fu4)$ $\quad Fu1 = 1.52439$

Using the ratio of ultimate static capacity to yield static capacity, R, defined as:

$R := 1 + s\cdot(mu - 1)$ $\quad R = 1.09259$

$$Fu_{RN} := \frac{(Fu1\cdot(Fu2<Fu1) + Fu2\cdot(Fu2 \ge Fu1))}{R} \qquad Fu_{RN} = 1.3952$$

Modified Riddle-Newmark method for Calculating FuMRN

Because the Riddle-Newmark method does not account for second slope of the force-deformation curve, the effect of second slope is accounted for by modifying the ductility ratio. Therefore, the modified ductility ratio is:

$$mu_1 := 0.5 + \frac{(mu - 1)\cdot(1 + R) + 1}{2\cdot R^2} \qquad mu_1 = 1.50718$$

At the peak zpa level, Fu is:

$$Fum4 := \frac{Sa}{zpa}\cdot mu_1^{\,0.11} \qquad Fum4 = 1.72513$$

In the acceleration range of the spectrum, Fmu is:

$$Fum3 := \left(2.67\cdot mu_1 - 1.673\right)^{0.411} \qquad Fum3 = 1.42101$$

In the velocity range of the spectrum, Fmu is:

$$Fum2 := \left(2.24\cdot mu_1 - 1.24\right)^{0.611}\cdot Cf \qquad Fum2 = 0.52402$$

where Cf has been defined previously.

The median ductility scale factor, Fu.MRN, using the modified Riddle-Newmark method is:

$Fum1 := Fum3\cdot(Fum3<Fum4) + Fum4\cdot(Fum3 \ge Fum4)$ $\quad Fum1 = 1.42101$

$Fu_{MRN} := Fum1\cdot(Fum2<Fum1) + Fum2\cdot(Fum2 \ge Fum1)$ $\quad Fu_{MRN} = 1.42101$

Effective Riddle-Newmark method for Calculating FuERN

Because the modified Riddle-Newmark method only accounts for second slope of the force-deformation curve, the last correction to perform is to account for ground motion duration.

The factor to account for earthquake duration is:

$CD := 1.0$

and the median ductility scale factor, FuERN, using the effective Riddle-Newmark method is:

$Fu_{ERN} := 1 + CD \cdot (Fu_{MRN} - 1)$ $\qquad Fu_{ERN} = 1.42101$

Effective Spectral method for Calculating FuSA

In this method, Fmu is calculated using the effective frequency and damping ratio of the structure. First, the ratio of secant stiffness to elastic stiffness is estimated as:

$Ks_K := \dfrac{(1 + s \cdot (mu - 1))}{mu}$ $\qquad Ks_K = 0.65376$

Next, the ratio of the secant frequency to elastic frequency is estimated to be:

$fs_f := \sqrt{Ks_K}$ $\qquad fs_f = 0.80855$ $\qquad$ Compare to:

$fs := fs_f f_{ns}$ $\qquad fs = 6.13335 \cdot Hz$ $\qquad f_{ns\mu} = 6.13377 \cdot Hz$

The ratio of the effective frequency to elastic frequency is calculated as follows (See Reference 1):

$cf := 1.9$ $\qquad$ Coefficient to account for short duration motion

$A1 := cf \cdot (1 - fs_f)$

$A := A1 \cdot (A1 \le 0.85) + 0.85 \cdot (A1 > 0.85)$

$fe_f := (1 - A) + A \cdot (fs_f)$ $\qquad fe_f = 0.93036$

The effective damping ratio, be, is calculated as follows:

$Cn := 0.15$ $\qquad$ Coefficient to account for short duration motion

$bh := Cn \cdot (1 - fs_f)$ $\qquad$ Hysteretic energy dissipation damping $\qquad bh = 0.02872$

$be := \left(\dfrac{fs_f}{fe_f}\right)^2 \cdot (b + bh)$ $\qquad be = 0.05945$

The median ductility scale factor, Fu.SA, is estimated as follows:

~~Spectral acceleration at~~ effective frequency, fe, ~~and 5% damping~~

$fe := fe_f \cdot f_{ns}$ $\qquad$ $fe = 7.05734 \cdot Hz$

$Sa_e := SSa(fe, be)$ $\qquad$ $Sa_e = 0.59505 \cdot g$ $\qquad$ Sa.e is the spectral acceleration at the effective frequency, fe, and the effective damping, be

$$Fu_{SA} := \left(\frac{fe_f}{fs_f}\right)^2 \cdot \frac{Sa}{Sa_e} \qquad Fu_{SA} = 1.46763$$

Final Estimate of the median ductility scale factor

Different methods have been used to estimate the median ductility scale factor. Based on a recent study, the median ductility scale factor, Fu, is taken as the average of the ductility scale factors found using the Effective Riddle-Newmark method, Fu.ERM, and the Effective Spectral Method, Fu.SA.

$$Fmu_{median} := \frac{Fu_{ERN} + Fu_{SA}}{2} \qquad Fmu_{median} = 1.44432$$

final factors

Assumed values | Calculated values

Elastic point

$f_{ns} \equiv 7.5856 \cdot Hz$ | $f\_new_{ns} = 7.58572 \cdot Hz$

$FS = 1.67338$

$f_{ew} \equiv 9.848 \cdot Hz$ | $f\_new_{ew} = 9.84806 \cdot Hz$

$Fmu_{median} = 1.44432$

$\Delta assumed_{el} \equiv -0.01274 \cdot in$ | $\Delta el_{plate} = -0.01275 \cdot in$

$\theta assumed_{el} \equiv -0.00362$ | $\theta_{Tel} = -0.0036$

$FS \cdot Fmu_{median} = 2.416897743$

Secant point

$f_{ns\mu} \equiv 6.13377 \cdot Hz$ | $f\_new\mu_{ns} = 6.13345 \cdot Hz$

$f_{ew\mu} \equiv 8.51538 \cdot Hz$ | $f\_new\mu_{ew} = 8.51512 \cdot Hz$

$\theta assumed_{\mu} \equiv -0.00490$ | $\theta_{T\mu} = -0.0049$

$$BETA := \frac{1}{1} \cdot \left| \ln\left(\frac{FS \cdot Fmu_{median}}{2.416897743}\right) \right|$$

$BETA = 2.57973 \cdot 10^{-11}$

Logarithmic Standard Deviation of the Ductility Scale Factor

The variability due to random scatter of time history-computed Fmu versus predicted Fmu values using approximate methods (for example, the spectral averaging method) is:

$$br_{Fmu} := 0.4 \cdot \left[0.06 + 0.03 \cdot \left(Fu_{SA} \cdot R - 1\right)\right] \qquad br_{Fmu} = 0.03124$$

The uncertainty due to story drift associated with failure is calculated, assuming that the 10*elastic deflection is at 1.0 standard deviation from the mean, to be 1.25673 using the same equation as above. Therefore:

$$bu_{Fmu} := \frac{\ln\left(\frac{Fmu_{median}}{1.25673}\right)}{1.0} \qquad bu_{Fmu} = 0.13913$$

The uncertainty due to inelastic energy absorption model is:

$$bu_{Fmum} := 0.1 \cdot \left(Fmu_{median} - 1\right) \qquad bu_{Fmum} = 0.04443$$

The combined variability is:

$$b_{Fmu} := \sqrt{br_{Fmu}^{\;2} + bu_{Fmu}^{\;2} + bu_{Fmum}^{\;2}} \qquad b_{Fmu} = 0.14935$$

Attachment 5

Knuckle Frequency

BY ______ DATE 12/17/93 PROJECT ______ PAGE 1 OF ______
CHKD. BY ______ DATE ______ SUBJECT ______ JOB NO ______

Sensitivity Analysis for Beta (β) Determination

Plate width = 2.25"

Run	Elastic Limit (Hz) f_{NS}	Elastic Limit (Hz) f_{EW}	Elastic Limit (in) Δassumed el	Elastic Limit Θassumed el	Secant Point (Hz) f_{NSP}	Secant Point (Hz) f_{EWP}	Secant Point Θassumed p	FS	F_μ	FS·F_μ	β
(1) Base Case	7.5856	9.8480	0.01274	-0.00362	6.1338	8.5154	.00490	1.673	1.444	2.417	—
(2) σ_y = 39.02 Ksi (-1σ)	7.6119	9.8698	-0.01118	-0.00273	6.2313	8.6131	-0.00381	1.611	1.421	2.289	0.05
(3) F_μ: Δ=10elastic (-1σ)	7.5856	9.8480	-0.01274	-0.00362	6.7467	9.1106	-0.00420	1.673	1.257	2.103	0.15 (1)
(4) S_a Shape: $e^{+.24}$ (+1σ)	7.1464	9.8036	-0.01270	-0.00346	5.6257	8.1646	-0.00475	1.303	1.484	1.932	0.22
(5) S_a Shape $e^{+.20}$ (+1σ)											0.15 (3)
(6) S_a Direction $e^{\pm.27}$ (+1σ)	7.1764	9.908	-0.01243	-0.00239	5.7650	8.9061	-0.00377	1.556	1.456	2.265	0.06
(7) Vertical $e^{+.34}$ (+1σ)	7.5676	9.8404	-0.01281	-0.00389	6.6453	8.4850	-0.00517	1.598	1.468	2.343	0.03
(8) 100-100-100 (+3.0σ)	7.4969	9.7757	-0.01335	-0.00610	5.8665	8.2417	-0.00705	0.986	1.510	1.489	0.16
(9) Damping 3.5% (-1σ)	7.5429	9.8512	-0.01271	-0.00349	6.170	8.5280	-0.00478	1.428	1.489	2.126	0.13
(10) Wt (15%) + f_{veg} (2) (+1σ)	6.2610	8.0970	-0.01225	-0.00160	5.3016	7.2460	-0.00325	1.552	1.355	2.102	0.14
(11) Bolt Stretch: g_p = 0.5" (-1σ)	7.6454	9.898	-0.00832	-0.00239	6.5141	8.8893	-0.00312	1.612	1.345	2.167	0.11
(12) Shear = 0 (-1σ)	9.1205	9.6343	-0.01420	-0.00965	6.648	7.7115	-0.00963	1.197	1.808	2.164	0.11
(13) frequency f_{freq} = 15Hz (+1σ)	9.2038	14.5092	-0.01272	-0.00351	6.9035	11.6216	-0.00490	1.626	1.585	2.578	0.06

(1) Include all variability ie. $\beta_r = 0.03$ $\beta_u = 0.15$

(2) Reduced fixed-base frequency to $\sqrt{10/15}\cdot 10$ Hz

(3) $\frac{0.20}{0.24}\cdot 0.224 = 0.15$