PLG-0411

UPDATED PREDICTION OF THE FREQUENCY OF AIRCRAFT CRASHES AT THE THREE MILE ISLAND UNIT 1 SITE

Prepared for
GPU NUCLEAR CORPORATION
Parsippany, New Jersey
April 1985

Pickard, Lowe and Garrick, Inc.
Engineers • Applied Scientists • Management Consultants
Newport Beach, CA Washington, DC

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1. INTRODUCTION AND SUMMARY

This report presents an analysis performed to determine the annual frequency of crash by a heavy aircraft during landing or takeoff operation at Harrisburg International Airport (HIA) into the Three Mile Island Unit 1 (TMI-1) nuclear power plant. The scope of the present analysis is limited to updating the results of an earlier analysis (References 1, 2, and 3), using 1984 statistics on aircraft movements at HIA and updated national aerial crash data. Calculation of the hit frequencies is based on the breakdown of aircraft movements into scheduled takeoffs, nonscheduled takeoffs, scheduled landings, and nonscheduled landings. Table 1-1 presents the mean values of the results of this study for various types of aircraft movements. The results are also compared with those obtained in the previous study (Reference 3). Figure 1-1 provides a graphical representation of the main characteristics of the distribution of the total annual hit frequency and the various contributions to it. Finally, the cumulative distribution of the total hit frequency, based on the results of this study as well as the previous study are shown in Figure 1-2.

As can be seen from Table 1-1, the nonscheduled operations contribute significantly to the total hit frequency. The total mean hit frequency of 3.51×10^{-8} per year calculated in this study is slightly more than a factor of 5 larger than the frequency presented in Reference 3 based on 1978 statistics. The increase is essentially due to two factors: (1) use of a target area that was a factor of 2 larger than one used in the previous study and, (2) increase in the number of operations, especially nonscheduled landings and takeoffs at HIA. Both of these factors are calculated based on conservative assumptions as discussed in more detail in this report. Also calculated were the hit frequencies based on all takeoffs and all landings without breakdown by scheduled or nonscheduled operations. The total mean hit frequency obtained in this manner is 6.53×10^{-9} per year, which underestimates the hit frequency by a factor of 5 due to the combined crash frequencies being biased, nonconservatively, toward scheduled crash frequencies. In light of this observation, the prediction of the hit frequency is based on the detailed breakdown by scheduled and nonscheduled operations.

The following sections describe how these results were obtained. The basic analytical model used in the analysis is described in Section 2, followed by the basic data collected for estimating the model parameters in Section 3. Section 4 provides a detailed discussion on the quantification of various components of the model.

TABLE 1-1. MEAN ANNUAL HIT FREQUENCY RESULTS FOR VARIOUS TYPES AND MODES OF AIRCRAFT OPERATION (10^{-9} CRASHES PER YEAR)*

Type of Operation	Mode of Operation		Total
	Landing	Takeoff	
Scheduled	1.20 (0.50)	0.08 (0.03)	1.28 (0.53)
Nonscheduled	22.3 (4.00)	11.5 (2.10)	33.8 (6.10)
Total	23.5 (4.50)	11.5 (2.13)	35.1 (6.63)

*Numbers in parentheses are the results of the previous study (Reference 3).

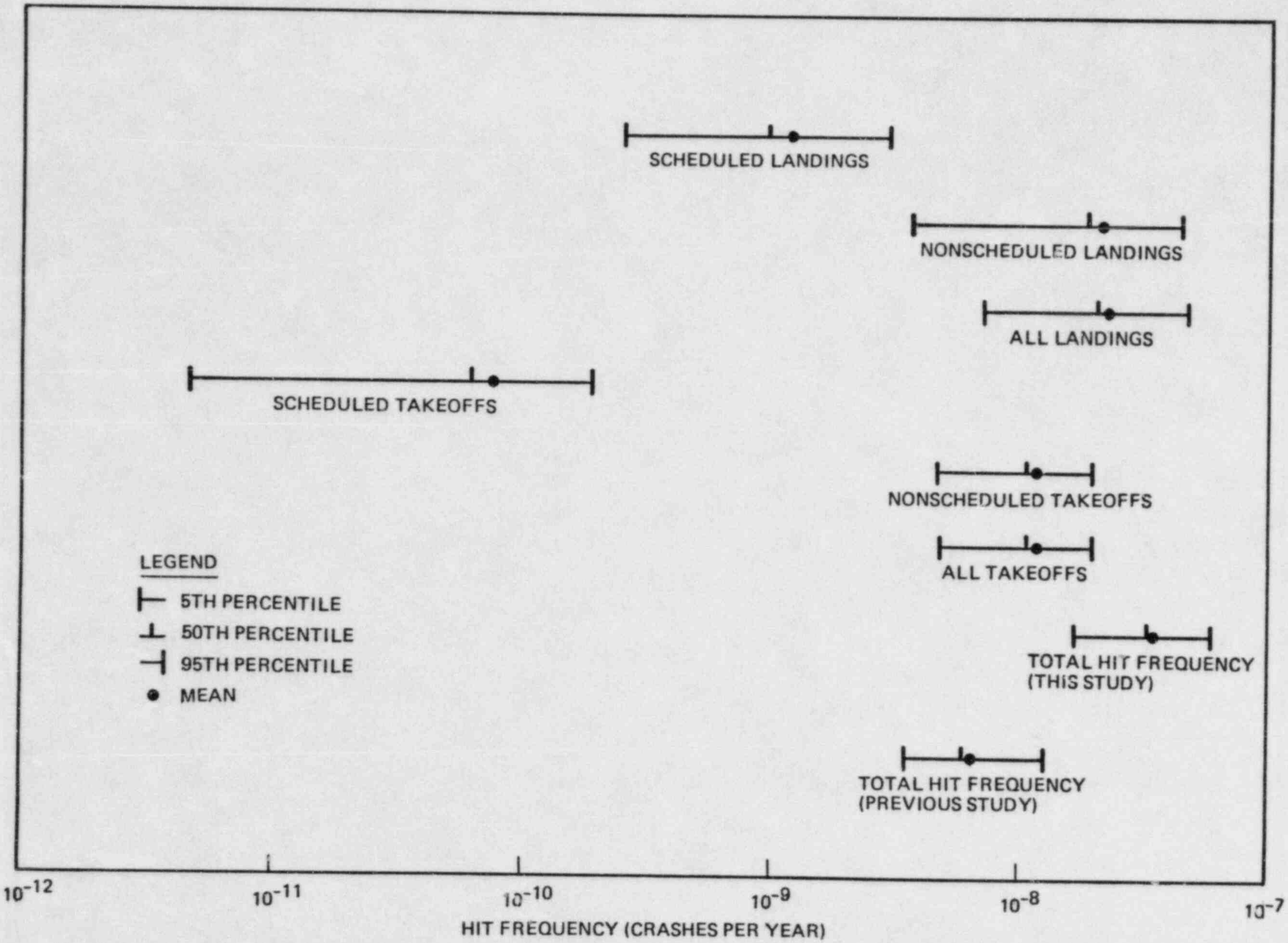

FIGURE 1-1. BASIC CHARACTERISTICS OF THE DISTRIBUTION OF HIT FREQUENCY BASED ON VARIOUS TYPES OF AIRCRAFT OPERATIONS

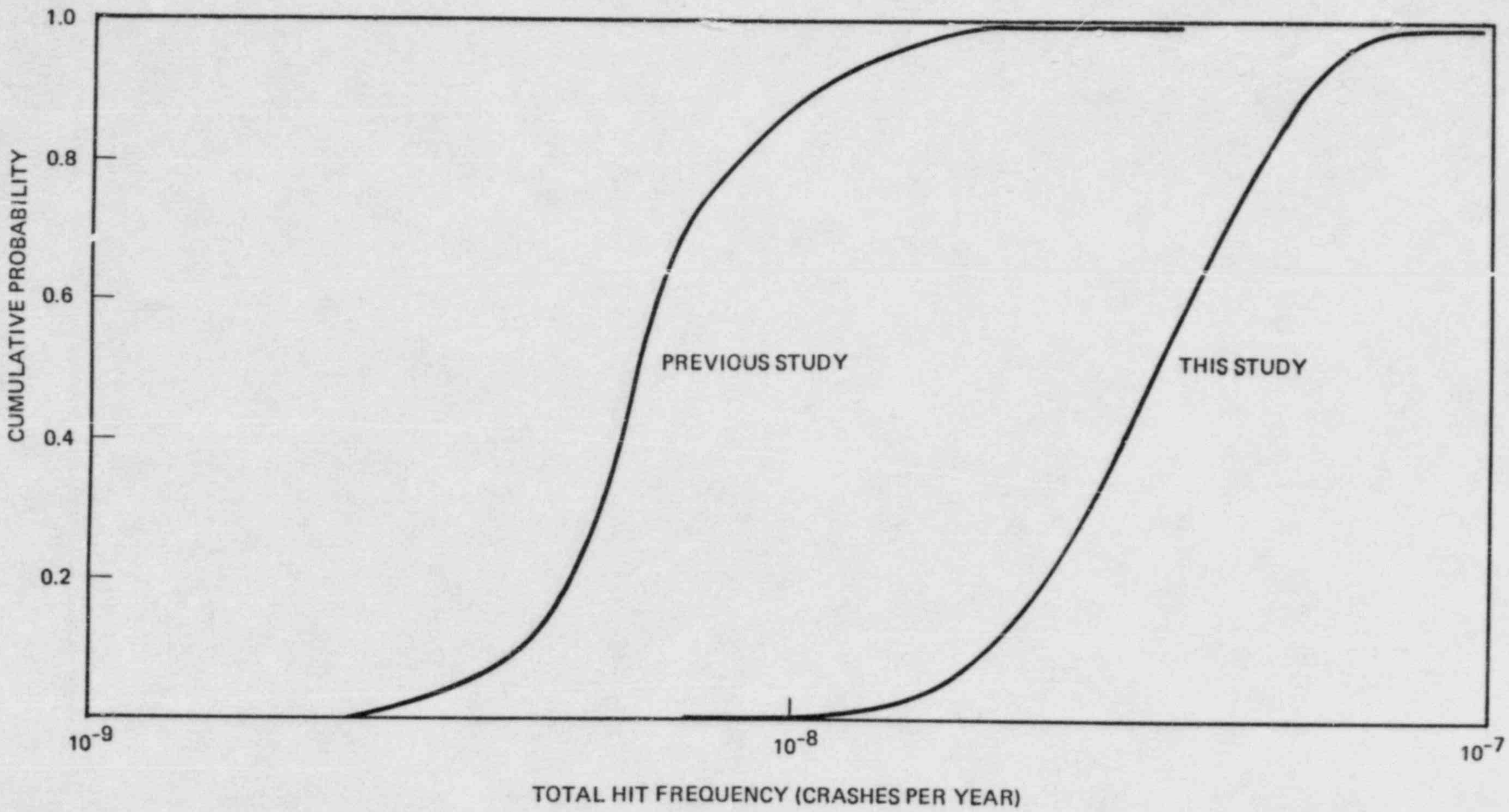

FIGURE 1-2. CUMULATIVE DISTRIBUTION OF THE TOTAL HIT FREQUENCY

crash frequency based on Equation (2.1). Results are also provided for landings and takeoffs without breakdown into scheduled and nonscheduled operations.

2. THE PLANT/AIRPORT GEOMETRY AND THE BASIC ANALYTICAL MODEL

The TMI plant site is located approximately at a radius of 2.7 miles and 34° off the centerline from the southwest end of Runway 13/31 of the Harrisburg, Pennsylvania, airport - as shown in Figure 2-1. The landing strip is called Runway 31 when used in the northwest direction and Runway 13 when used in the southeast direction. The threat to the TMI site is from operations at the south end of this strip; that is, from landings taking place in the northwest direction (Runway 31) and takeoffs in the southeast direction (Runway 13). Of the operations on this strip, 70% use Runway 31 and 30% use Runway 13. The number of landings and takeoffs are approximately equal on each runway. Thus, if N is the number of operations per year on the strip, then

$.35N$ = number of landings at south end $\equiv N_L$
$.15N$ = number of takeoffs at south end $\equiv N_T$

The aircraft hit frequency into TMI-1 is calculated by using the algorithm

$$f = f_{SL} + f_{ST} + f_{NL} + f_{NT} \qquad (2.1)$$

where f is the annual frequency of aircraft crashes into TMI-1 by heavy aircraft using HIA, and f_{SL}, f_{ST}, f_{NL}, and f_{NT} are contributors to that frequency from scheduled landings, scheduled takeoffs, nonscheduled landings, and nonscheduled takeoffs. These frequencies are calculated as follows:

$$f_{SL} = N_{SL}\, C_{SL}\, S_L\,(r,\theta)\, A_L \qquad (2.2)$$

$$f_{ST} = N_{ST}\, C_{ST}\, S_T\,(r,\theta)\, A_T \qquad (2.3)$$

$$f_{NL} = N_{NL}\, C_{NL}\, S_L\,(r,\theta)\, A_L \qquad (2.4)$$

$$f_{NT} = N_{NT}\, C_{NT}\, S_T\,(r,\theta)\, A_T \qquad (2.5)$$

where

N_{ST} and N_{NT} = the annual number of large scheduled and nonscheduled aircraft, respectively, taking off on TMI-1 end of the runway; i.e., using HIA runway 13.

N_{SL} and N_{NL} = the annual number of large scheduled and nonscheduled aircraft, respectively, landing on the TMI-1 end of the runway; i.e., using HIA runway 31.

A_L, A_T = the effective target area of the plant upon landing and takeoff, respectively.

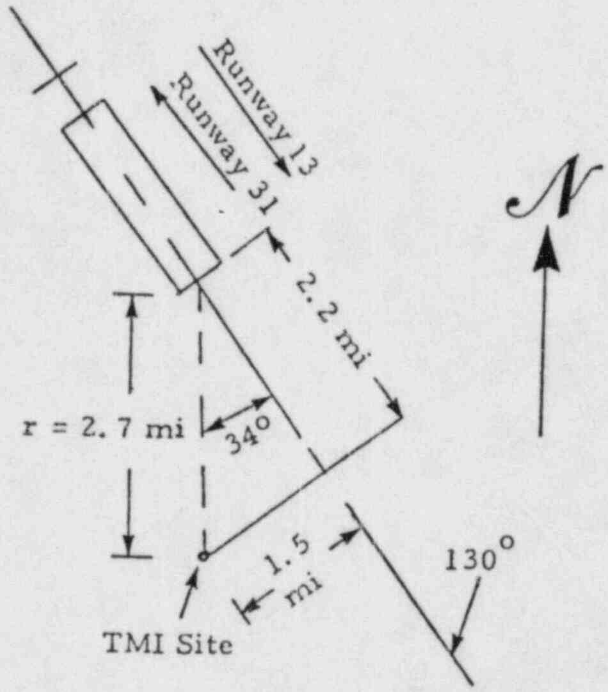

FIGURE 2-1. LOCATION OF TMI SITE WITH RESPECT TO HARRISBURG AIRPORT

C_{SL}, C_{NL}, C_{ST}, and C_{NT} = the applicable accident rate of scheduled landing, nonscheduled landing, scheduled takeoff, and nonscheduled takeoff.

and finally

$S_L(r,\theta)$ = frequency, per unit area, of the crash occurring at coordinates r,θ from end of runway, given that the crash is on landing.

$S_T(r,\theta)$ = frequency, per unit area, of the crash occurring at r,θ, given the crash is on takeoff.

A visual aid to understanding the physical meaning of these spatial distributions is provided in Figure 2-2. It is assumed that $S_L(r,\theta)$ and $S_T(r,\theta)$ are separable into radial and angular components.

More explicitly, let

$R_L(r) \equiv$ the fraction of landing crashes that occur at radius r or greater.

$\Theta_L(\theta) \equiv$ the fraction of landing crashes that occur at angle θ or greater.

Then

$$S_L(r,\theta) = \left[\frac{d}{dr} R_L(r)\right]\left(\frac{360}{2\pi r}\right)\left[\frac{d}{d\theta}\Theta_L(\theta)\right]\left(\frac{1}{2}\right) \qquad (2.6)$$

where θ is measured in degrees, r in miles, and S_L in fraction per square mile.

Similarly, for takeoffs

$$S_T(r,\theta) = \left[\frac{d}{dr} R_T(r)\right]\left(\frac{360}{2\pi r}\right)\left[\frac{d}{d\theta}\Theta_T(\theta)\right]\left(\frac{1}{2}\right) \qquad (2.7)$$

The final 1/2 in these formulae corrects for the fact that in calculating the function Θ we will lump both positive and negative values of θ together--thus, in effect, treating all accidents as if they occurred on the TMI side of the runway.

The issue of separability of $S(r,\theta)$ has been discussed in Reference 4. The conclusion was that the assumption of separability does not introduce any significant error in terms assessing the spatial distribution.

In this analysis, following the method presented in Reference 1, uncertainty distributions are assessed for all the frequencies using Bayesian techniques. The final results are presented in probability distribution form for the frequency of crash for each of the four categories represented by Equations (2.2) through (2.5) and the total

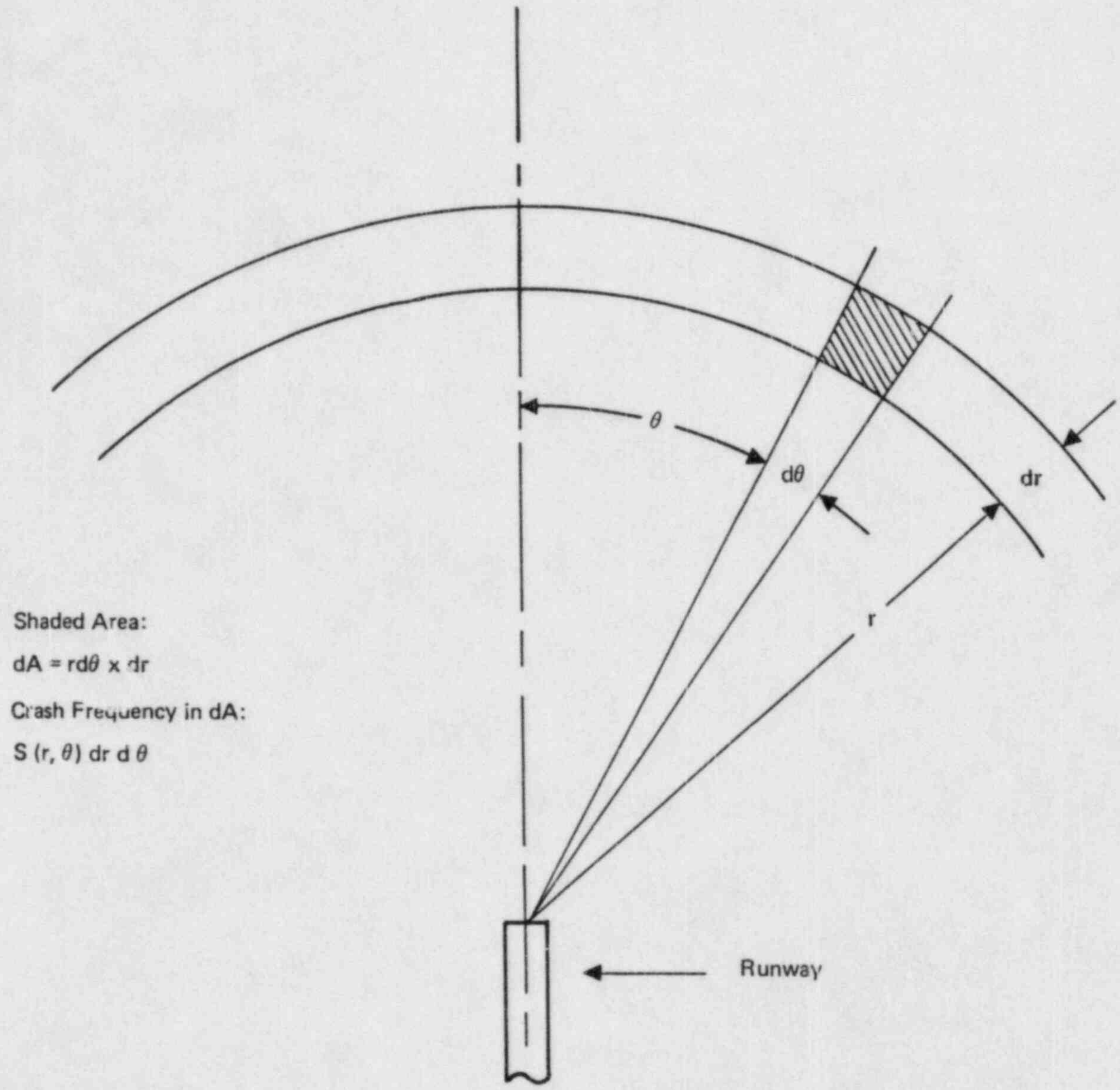

FIGURE 2-2. REPRESENTATION OF SPATIAL CRASH FREQUENCY DISTRIBUTION

3. AIRCRAFT MOVEMENT AND NATIONAL CRASH DATA

The data needed to quantify the various parameters of the model are presented in this section. These data include the number of aircraft movements at Harrisburg International Airport and the national aerial crash statistics.

3.1 NUMBER OF AIRCRAFT MOVEMENTS AT HARRISBURG INTERNATIONAL AIRPORT

Table 3-1 provides the total number of aircraft movements (landings and takeoffs) for various categories of operation for the period 1980 through 1984 (Reference 5). In this analysis, we are concerned with the number of heavy aircraft movements; i.e., aircraft weighing 200,000 pounds or more. A conservative estimate puts the number of such operations at less than 1% of the total operations (References 5 and 6). For instance, for the year 1984, this number is estimated to be less than 1,411.

In order to estimate the number of movements of heavy aircraft in the scheduled and nonscheduled categories, we first observe that Air Taxi and General Aviation aircraft, by definition, do not include heavy aircraft. The total number of movements excluding these two categories for the year 1984 was 26,684. A total of 8,549 of these operations were scheduled. Therefore, the fraction of scheduled operations is 0.32. The fraction of nonscheduled operations (including military) is then 0.68. Therefore, the breakdown of heavy aircraft movements based on these percentages is

Scheduled: $N_S = (0.32)(1{,}411) = 452$

Nonscheduled: $N_N = (0.68)(1{,}411) = 959$ (3.1)

From our earlier discussion regarding the use of runways at the airport, we calculate the following values for the number of scheduled and nonscheduled landings and takeoffs in the TMI-1 direction of the runways.

$$N_{SL} = (0.35)N_S = 158$$

$$N_{ST} = (0.15)N_S = 68$$

$$N_{NL} = (0.35)N_N = 335$$

$$N_{NT} = (0.15)N_N = 144 \quad (3.2)$$

3.2 NATIONAL AERIAL CRASH STATISTICS

Table 3-2 lists U.S. air carrier landing and takeoff accidents in the contiguous U.S. involving destruction of the aircraft for the years 1956 to 1982. The data for the years 1956 to 1977 were taken from Reference 2. The additional data for the years 1978 to 1982 were obtained from the National Transportation Safety Board (NTSB) computerized briefs of accidents and the detailed accident reports

TABLE 3-1. AIRCRAFT OPERATIONS AT HARRISBURG INTERNATIONAL AIRPORT
(198[illegible]

Type of Operation	Total Number of Aircraft Movements (Takeoffs and Landings)				
	1980	1981	1982	1983	1984
Commercial, Scheduled	8,227	6,954	6,268	6,747	8,549
Commercial, Nonscheduled	1,422	356	690	233	157
Air Taxi	23,010	20,135	22,752	22,437	29,724
Military	12,514	11,552	12,231	12,857	17,978
General Aviation	67,525	60,347	62,732	67,189	84,693
Total	112,698	99,344	104,673	109,463	141,101
Estimated Number of Heavy Aircraft Operations*	1,127	993	1,047	1,095	1,411

*Approximately 1% of the total number of aircraft movements.

TABLE 3-2. LISTING OF U. S. AIR CARRIER LANDING AND TAKEOFF ACCIDENTS IN THE CONTIGUOUS U. S., INVOLVING DESTRUCTION OF THE AIRCRAFT (1956 - 1982)

Sheet 1 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1956							
2/17	Owensboro, KY	L	M-404	O	SP	0	0
4/1	Pittsburg, PA	T	M-404	F	SP	0	0
4/2	Seattle, WA	T	B-377	F	SP	4.7	0
11/14	Las Vegas, NV	L	M-404	O	SP	0	0
1957							
1/6	Tulsa, OK	L	CV-240	F	SP	3.5	0
2/1	Rikers Island, NY	T	DC-6	F	SP	0.9	47
9/15	New Bedford, MA	L	DC-3	F	SP	0.8	6
1958							
2/13	Palm Springs, CA	T	CV-240	O	SP	4.0	0
3/25	Miami, FL	T	DC-7	F	SP	3.1	26
4/6	Freeland, MI	L	Viscount	F	SP	0.4	0
6/4	Martinsburg, WV	L	DC-3	F	Training	0.3	90
8/15	Nantucket, MA	L	CV-240	F	SP	0.3	22
8/28	Minneapolis, MN	T	DC-6	O	SP	0.6	0
11/10	New York, NY	T	L-1049	O	Training	0	0

NOTE: Footnotes and legend appear on the last sheet of this table.

TABLE 3-2 (continued)

Sheet 2 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1959							
2/3	New York, NY	L	L-188	F	SP	0.8	0
2/20	San Francisco, CA	L	DC-7	O	NS/C	0	0
3/15	Chicago, IL	L	CV-240	O	SC	1.2	28
5/12	Charleston, WV	L	L-1049	F	SP	0	0
8/15	Calverton, NY	L	B-707	F	Training	3.0	13
9/2	Abilene, TX	L	C-46	F	NS/C	0	0
11/24	Chicago, IL	L	L-1049	F	SC	0.2	0
12/1	Williamsport, PA	L	M-202	F	SP	1.4	90
10/26	Santa Maria, CA	T	DC-3	F	SP	1.5	NA
1960							
5/23	Atlanta, GA	T	CV-880	F	Training	0	0
9/14	New York, NY	L	L-188	O	SP	0	0
10/4	Boston, MA	T	L-188	F	SP	1.0	20
10/29	Toledo, OH	T	C-46	F	NS/P	1.1	4
1961							
7/11	Denver, CO	L	DC-8	F	SP	0	0
9/17	Chicago, IL	T	L-188	F	SP	0.8	90
11/8	Richmond, VA	L	L-1049	F	NS/P	1.1	26

NOTE: Footnotes and legend appear on the last sheet of this table.

TABLE 3-2 (continued)

Sheet 3 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1962							
3/1	Jamaica Bay, NY	T	B-707	F	SP	2.7	90
4/18	Dallas, TX	T	DC-3	F	Test	0	0
7/8	Amarillo, TX	T	V-812	0	SP	1.2	21
8/22	Wilmington, NC	L	M-404	0	Training	0	0
11/30	New York, NY	L	DC-7	F	SP	0.8	9
12/14	Hollywood, CA	L	L-1049	F	SC	1.5	0
12/21	Grand Island, NE	L	CV-340	0	SP	0.8	0
1963							
1/29	Kansas City, MO	L	V-812	F	SP	0	0
2/3	San Francisco, CA	L	L-1049	F	SC	0	0
2/16	Puyallup, WA	L	C-46	0	NS/C	0.5	0
5/28	Manhattan, KS	L	L-1049	0	NS/P	0.1	0
7/2	Rochester, NY	T	M-404	F	SP	0	0
11/29	Morgantown, WV	L	DC-3	F	Ferry	2.5	18
1964							
3/10	Boston, MA	L	DC-4	F	SC	1.3	0
3/12	Miles City, MT	L	DC-3	F	SP	1.9	0
11/20	Detroit, MI	T	C-46	0	NS/C	0.4	0
12/24	San Francisco, CA	T	L-1049	F	SC	4.3	31
12/30	Detroit, MI	L	C-46	F	NS/C	2.3	13

NOTE: Footnotes and legend appear on the last sheet of this table.

TABLE 3-2 (continued)

Sheet 4 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1965							
4/16	Las Vegas, NV	T	F-27	0	Training	0	0
5/18	Knob Knoster, MO	L	DC-6	0	NS/C	0.8	10
7/23	Montoursville, PA	T	CV-440	0	SP	2.8	45
9/13	Kansas City, MO	T	CV-880	0	Training	0.2	27
11/8	Constance, KY	L	B-727	F	SP	2.0	0
11/11	Salt Lake City, UT	L	B-727	F	SP	0.1	0
1966							
3/21	Norfolk, VA	L	CL-44	0	SC	0	0
4/22	Ardmore, OK	L	L-188	F	NS/P	2.3	90
7/28	Newark, NJ	T	C-46	0	NS/C	1.1	90
11/20	New Bern, NC	L	M-404	F	SP	4.0	9
1967							
1/31	San Antonio, TX	L	DC-6	F	NS/C	4.5	0
3/30	Kenner, LA	L	DC-8	F	Training	0.4	27
11/6	Erlanger, KY	T	B-707	F	SP	0	0
11/20	Constance, KY	L	CV-880	F	SP	1.8	3
12/21	Denver, CO	T	DC-3	F	NS/C	0	0

NOTE: Footnotes and legend appear on the last sheet of this table.

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TABLE 3-2 (continued)

Sheet 5 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1968							
1/1	Oxford, MS	L	M-404	O	Ferry	0	0
3/21	Chicago, IL	T	B-727	O	SC	0	0
4/28	Atlantic City, NJ	L	DC-8	O	Training	0	0
8/10	Charleston, WV	L	F-227	F	SP	0	0
9/27	Cherry Point, NC	L	DC-7	O	NS/C	0.4	17
12/24	Bradford, PA	L	CV-580	F	SP	2.8	8
12/27	Sioux City, IA	T	DC-9	O	SP	0	0
12/27	Chicago, IL	L	CV-580	F	SP	0.3	86
1969							
1/6	Bradford, PA	L	CV-440	F	SP	5.0	0
7/15	Jamaica, NY	T	DHC-6	F	SP	0	0
7/26	Pomona, NJ	L	B-707	F	Training	0	0
10/11	Stockton, CA	T	DC-8	O	Training	0	0
1970							
8/24	Hill AFB, UT	T	L-188	O	NS/C	0	0
9/8	Jamaica, NY	T	DC-8	F	Ferry	0	0
10/10	Wrightstown, NJ	L	GA-382	F	NS/C	1.0	0
11/14	Huntington, WV	L	DC-9	F	NS/P	1.1	0

NOTE: Footnotes and legend appear on the last sheet of this table.

TABLE 3-2 (continued)

Sheet 6 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1971							
3/31	Ontario, CA	L	B-720	F	Training	0	0
6/7	New Haven, CN	L	CV-580	F	SP	0.9	6
1972							
3/3	Albany, NY	L	F-227	F	SP	3.8	0
5/18	Ft. Lauderdale, FL	L	DC-9	O	SP	0	0
5/30	Ft. Worth, TX	L	DC-9	F	Training	0	0
12/8	Chicago, IL	L	B-737	F	SP	1.8	10
12/20	Chicago, IL	T	DC-9	F	SP	0	0
1973							
7/23	St. Louis, MO	L	F-227	F	SP	2.6	4
7/31	Boston, MA	L	DC-9	F	SP	0.6	4
11/3	Boston, MA	L	B-707	F	SC	0	0
11/27	Akron, OH	L	DC-9	O	SP	0	0
1974							
1/16	Los Angeles, CA	L	B-707	O	SP	0	0
9/11	Charlotte, NC	L	DC-9	F	SP	3.4	0

NOTE: Footnotes and legend appear on the last sheet of this table.

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TABLE 3-2 (continued)

Sheet 7 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1975							
6/24	Jamaica, NY	L	B-727	F	SP	0	0
11/12	Jamaica, NY	T	DC-10	O	NS/P	0	0
1976							
2/8	Van Nuys, CA	T	DC-6	F	Ferry	1.5	0
6/23	Philadelphia, PA	L	DC-9	O	SP	0	0
1977							
7/6	St. Louis, MO	T	L-188	F	NS/C	0	0
1978							
03/1	Los Angeles, CA	T	DC-10	F	SP	0.1	0
9/25	San Diego, CA	L	B-727	F	SP	3.5	28
1979							
2/9	Miami, FL	T	DC-9	O	Training	0.15	30
1/5	Amiat, AK	L	188A	O	NS/CTR	0	0
5/25	Chicago, IL	T	DC-10	F	SP	0.87	17
6/22	Daggett, CA	T	DC-7	F	M	1.0	20
5/15	Mesa, AZ	T	C-54D	O	Test	0**	0
11/19	McCormick, SC	L	C-54D	F	M	2.5	35

NOTE: Footnotes and legend appear on the last sheet of this table.

TABLE 3-2 (continued)

Sheet 8 of 8

Date	Location	Phase	Aircraft	Injury	Type Operation	Hit Location*	
						r (miles)	θ (degrees)
1980							
6/19	Atlanta, GA	L	SUD AVN SE-210	0	Cargo Service	0	0
11/28	Pecos, TX	T	DC-7B	F	M	†	†
6/22	Columbus, IN	T	1049-H	F	Ferry	0.87	25
1981							
2/17	Santa Ana, CA	L	B-737	F	SP	0	0
1982							
3/13	Glendale, AZ	L	KC-135A	F	Military	3.5	0
1/23	Boston, MA	L	DC-10-30	F	SP	0	0

*Hit location: r = radial distance of the hit to the end of the runway in use. θ is the angle to the runway centerline. r = 0 is considered if the hit occurred within 0.05 mile of the runway, and θ = 0 is considered if the hit occurred within 200 feet of the extended runway center-line. Note that we do not distinguish between a positive or negative angle (θ).
**This plane ran off the runway after aborted takeoff. The radial distance would be 0.25 mile (1,300 feet) if final resting place is considered.
†Sufficient information unavailable to determine r or θ.

LEGEND:

Phase: L = landing; T = takeoff.
Injury: F = one or more occupant fatalities; 0 = none.
Type operation: SC = scheduled cargo; SP = scheduled passenger; NS/C = nonscheduled cargo; NS/CTR = nonscheduled charter; NS/P = nonscheduled passenger; M = smuggling.

available from NTSB. Detailed reports for accidents beyond 1982 were not available at the time of this analysis. Table 3-2 also lists hit locations (r,θ) for each of the accidents and the phase and type of operation for the aircraft involved.

Tables 3-3 and 3-4 provide the number of takeoffs and landings for scheduled and nonscheduled operations for the period 1956 to 1982 (References 3, 7, 8, and 9). The takeoff and landing crash frequencies, plotted by year in Figure 3-1, show a downtrend in the accident frequencies.

Figure 3-2 is a plot of the radial distribution of crashes based on the data in Table 3-2. The angular distribution for takeoffs and landings are presented in the form of scatter diagrams in Figures 3-3 and 3-4, respectively.

3.3 PLANT TARGET AREA

The following estimates for the TMI station target area for landing and takeoff hits, which are presented in Reference 10, were used in this analysis.

A_L = Landing Target Area = 0.0224 square mile

A_T = Takeoff Target Area = 0.0066 square mile

These areas were calculated by considering "shadow effect" to account for the dependence of the potential target area on the glide angle of the crashing aircraft and the "skid effect" to account for airplanes that might crash in front of the plant and slide into it. The calculated landing and takeoff target areas are based on glide angles of 10 and 45 degrees, respectively.

The area used in the previous study (Reference 3) was calculated for one unit of the TMI station. In this study, the effective target area of both units is used to account for the fact that most of the critical structures of the two units are closely connected, so the crash of a large aircraft into the structures of one unit might have some impact on the structures of the other unit. This, of course, is a conservative assumption.

TABLE 3-3. U.S. AIR CARRIER ACCIDENT RATE FOR SCHEDULED AND NONSCHEDULED LANDINGS IN THE CONTIGUOUS U.S.*

Year	Scheduled			Nonscheduled			Total Landings		
	Operations (10^3)	Accidents	Accident Rate** (10^{-6})	Operations (10^3)	Accidents	Accident Rate** (10^{-6})	Operations (10^3)	Accidents	Accident Rate** (10^{-6})
1956	3,188	2	.627	90	0	0	3,278	2	.610
1957	3,444	2	.581	90	0	0	3,534	2	.566
1958	3,302	2	.606	90	0	0	3,392	2	.590
1959	3,551	5	1.406	90	2	22.2	3,641	7	1.92
1960	3,501	1	.286	125	0	0	3,626	1	.276
1961	3,400	1	.294	140	1	7.14	3,540	2	.565
1962	3,303	3	.908	175	0	0	3,478	3	.863
1963	3,414	2	.586	155	2	12.9	3,569	4	1.12
1964	3,554	2	.563	95	1	10.5	3,649	3	.822
1965	3,772	2	.530	95	1	10.5	3,867	3	.776
1966	3,926	2	.509	85	1	11.8	4,011	3	.748
1967	4,478	1	.223	90	1	11.8	4,568	2	.438
1968	4,836	3	.620	105	1	9.52	4,941	4	.810
1969	4,934	1	.203	115	0	0	5,049	1	.198
1970	4,669	0	0	125	2	16.0	4,794	2	.417
1971	4,558	1	.219	155	0	0	4,713	1	.212
1972	4,601	3	.652	135	0	0	4,736	3	.633
1973	4,651	4	.860	130	0	0	4,781	4	.837
1974	4,275	2	.468	105	0	0	4,380	2	.457
1975	4,269	1	.234	110	0	0	4,379	1	.228
1976	4,411	1	.227	115	0	0	4,526	1	.221
1977	4,560	0	0	125	0	0	4,685	0	0
1978	4,608	1	.217	116	0	0	4,724	1	.212
1979	4,852	0	0	122	2	16.4	4,974	2	.402
1980	4,892	0	0	123	1	8.13	5,015	1	.199
1981	4,664	1	.214	110	0	0	4,774	1	.209
1982	4,455	1	.224	114	1	8.77	4,569	2	.438

*Destruct accidents on or off runway but within 5 miles.

**Accidents per landing.

0177G042985

TABLE 3-4. U.S. AIR CARRIER ACCIDENT RATE FOR SCHEDULED AND NONSCHEDULED TAKEOFFS IN THE CONTIGUOUS U.S.*

Year	Scheduled			Nonscheduled			Total Landings		
	Operations (10^3)	Accidents	Accident Rate** (10^{-6})	Operations (10^3)	Accidents	Accident Rate** (10^{-6})	Operations (10^3)	Accidents	Accident Rate** (10^{-6})
1956	3,188	2	.627	90	0	0	3,278	2	.610
1957	3,444	1	.290	90	0	0	3,534	1	.283
1958	3,302	3	.909	90	0	0	3,392	3	.884
1959	3,551	1	.281	90	0	0	3,641	1	.275
1960	3,501	1	.286	125	1	8.00	3,626	2	.552
1961	3,400	1	.294	140	0	0	3,540	1	.282
1962	3,303	2	.606	175	0	0	3,478	2	.575
1963	3,414	1	.293	155	0	0	3,569	1	.280
1964	3,554	1	.281	95	1	10.5	3,649	2	.548
1965	3,772	1	.265	95	0	0	3,867	1	.259
1966	3,926	0	.0	85	1	11.8	4,011	1	.249
1967	4,478	1	.223	90	1	11.1	4,568	2	.438
1968	4,836	2	.414	105	0	0	4,941	2	.405
1969	4,934	1	.203	115	0	0	5,049	1	.198
1970	4,669	0	0	125	1	8.0	4,794	1	.209
1971	4,558	0	0	155	0	0	4,713	0	0
1972	4,601	1	.217	135	0	0	4,736	1	.211
1973	4,651	0	0	130	0	0	4,781	0	0
1974	4,275	0	0	105	0	0	4,380	0	0
1975	4,269	0	0	110	1	9.09	4,379	1	.228
1976	4,411	0	0	115	0	0	4,526	0	0
1977	4,560	0	0	125	1	8.00	4,685	1	.213
1978	4,608	1	.217	116	0	0	4,724	1	.212
1979	4,852	1	.206	122	1	8.20	4,974	2	.402
1980	4,892	0	0	123	1	8.13	5,015	1	.199
1981	4,664	0	0	110	0	0	4,774	0	0
1982	4,455	0	0	114	0	0	4,569	0	0

*Destruct accidents on or off runway but within 5 miles.

**Accidents per takeoff.

0177G042985

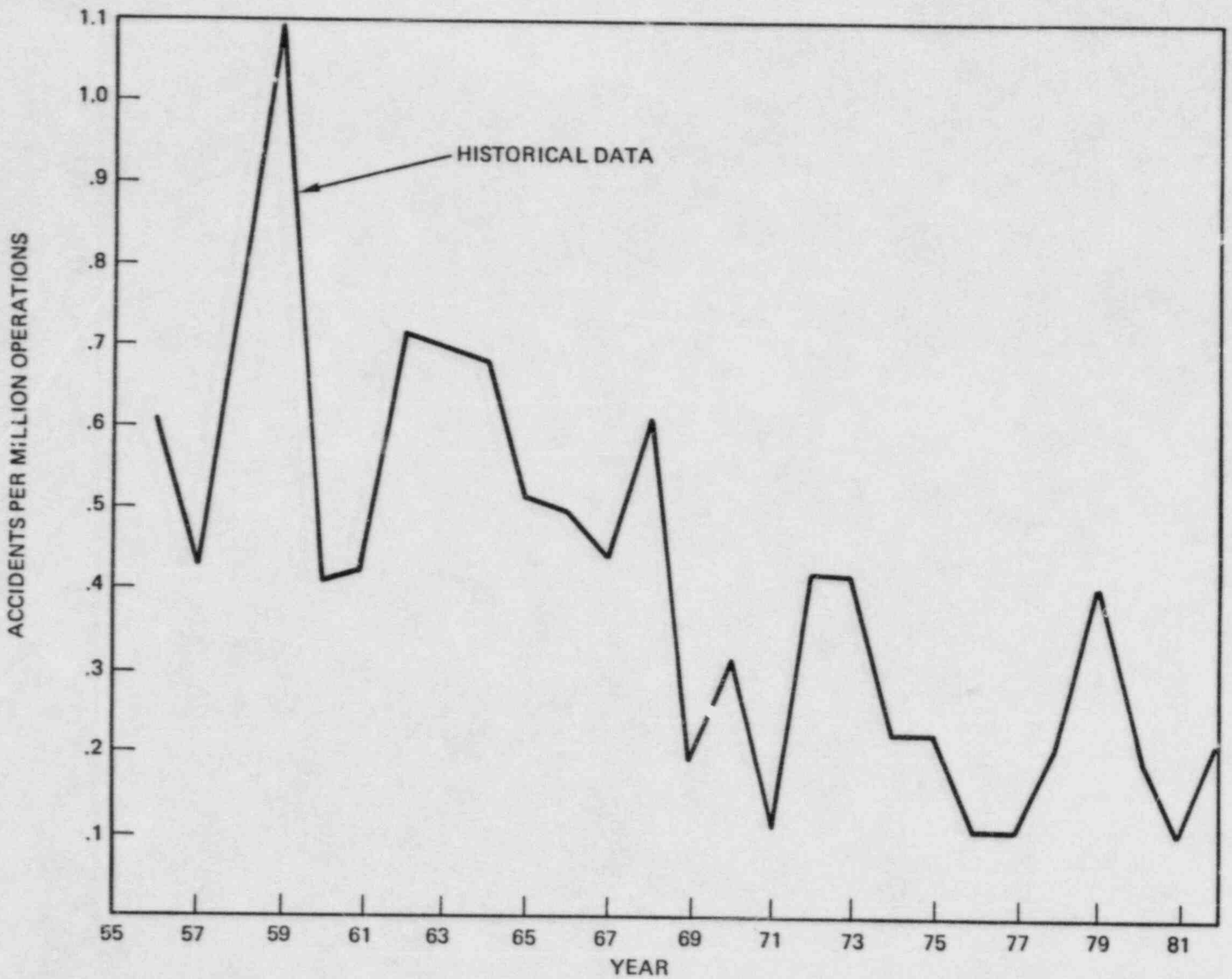

FIGURE 3-1. HISTORICAL ACCIDENT RATE VERSUS TIME - LANDINGS AND TAKEOFFS COMBINED

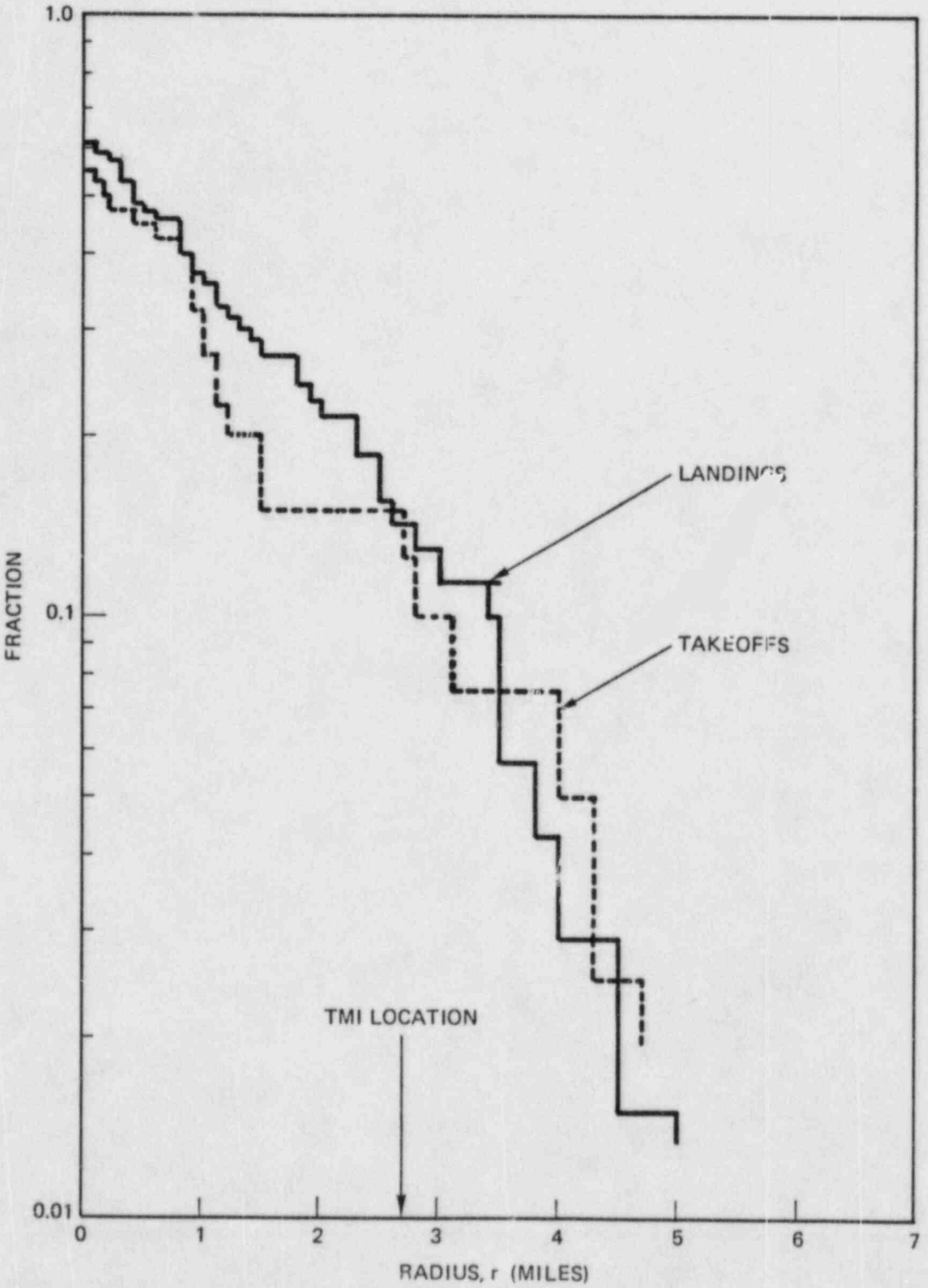

FIGURE 3-2. FRACTION OF CRASHES OCCURRING AT RADIUS r OR GREATER

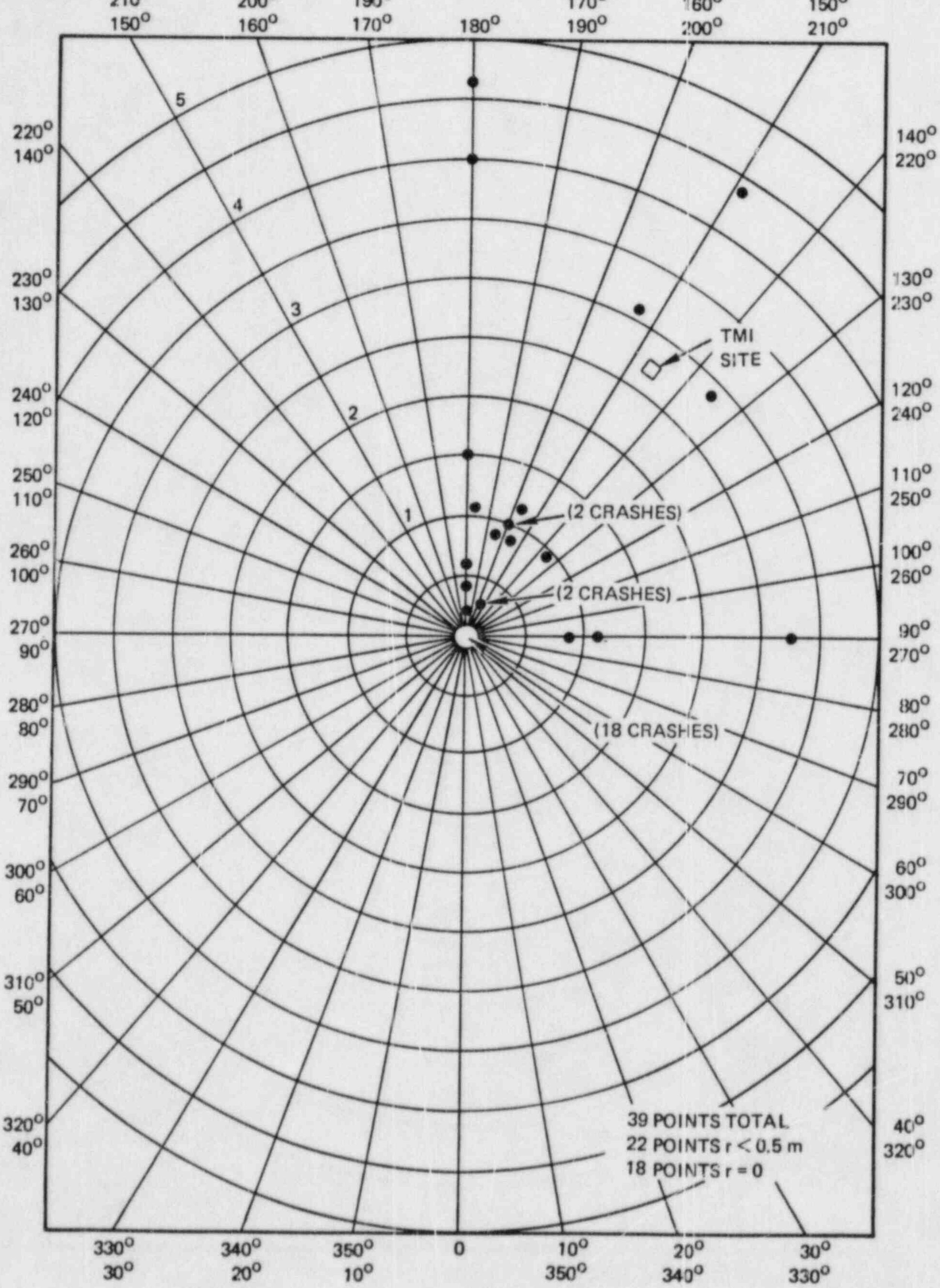

FIGURE 3-3. SCATTER PATTERN FOR TAKEOFF ACCIDENTS (Radius in Miles)

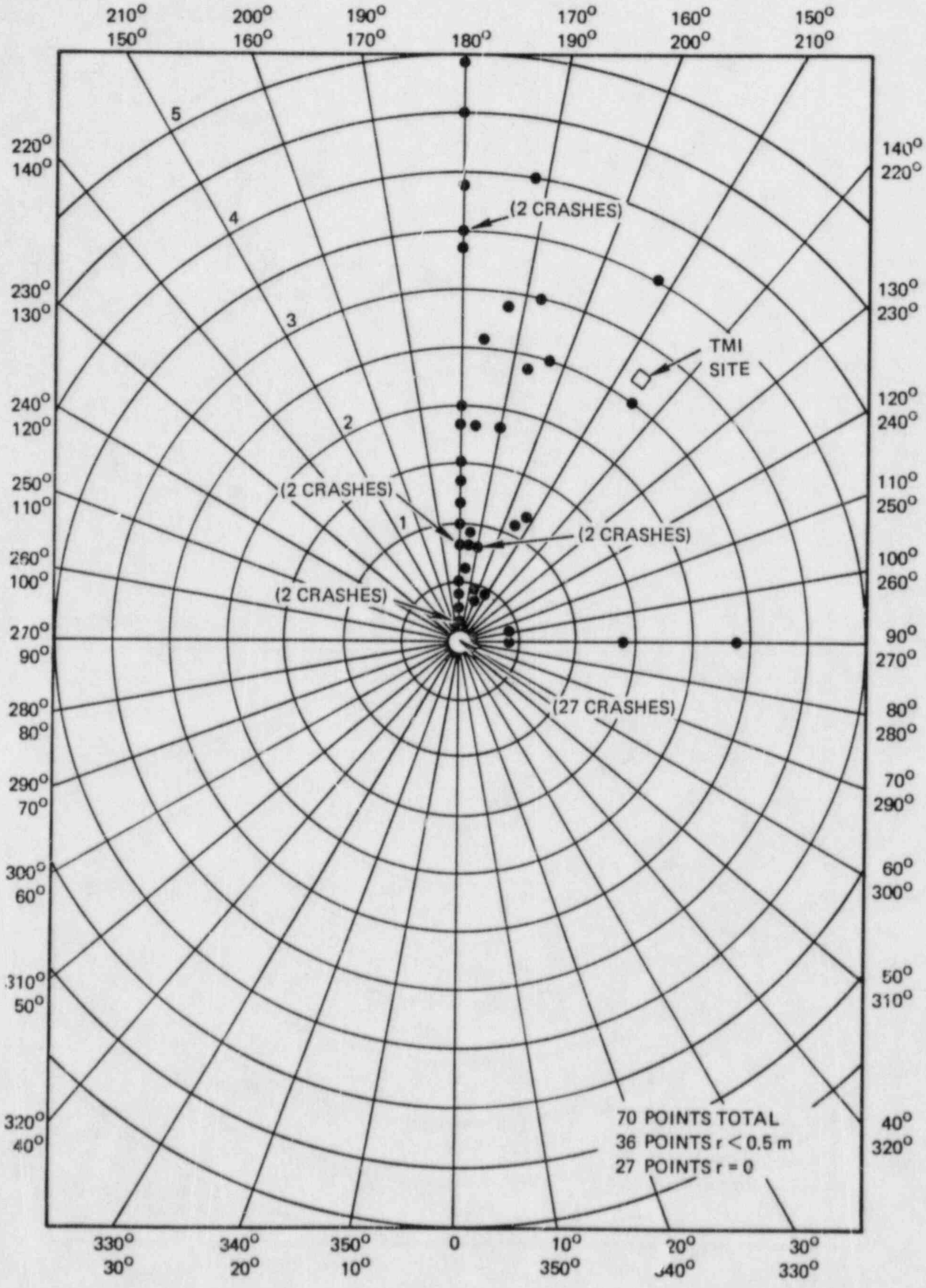

FIGURE 3-4. SCATTER PATTERN FOR LANDING ACCIDENTS (Radius in Miles)

4. ASSESSMENT OF MODEL PARAMETERS

In this section we will use the data presented in the previous section to estimate various components of the aircraft crash frequency model presented in Section 2.

4.1 PREDICTION OF ACCIDENT RATES FROM HISTORICAL DATA

In this section we develop an estimate of the aircraft accident rate, f, applicable to the plant in 1985 and beyond. Since, of course, we do not know the value of f exactly, we express our estimate in the form of a probability curve against f. The location and shape of this curve will then communicate our state of knowledge about the "true" value of f.

The historical data curve in Figure 3-1 shows, beginning in the early 1960s, a clear downward trend in accident rates reflecting, presumably, a steady improvement in aircraft equipment, flight safety technology, and safety consciousness.

A direct linear extrapolation of the curve to the years beyond 1982, however, would yield a crash rate very close to zero. A further extrapolation would go negative. Clearly, then, our extrapolation must reflect a leveling out of the curve. The approach followed in this study for extrapolating the crash frequency is based on Bayesian methods as described in the following:

1. We regard the historical data curve in Figure 3-1 as the result of sampling from an underlying population whose crash frequency is assumed to vary with time according to the functional form:

$$f(t) = a + (b-a)e^{-\lambda(t-t_0)} \quad (4.1)$$

   which reflects a gradual decrease and a leveling out at value a. In other words, we are saying that the "true" frequency in 1965, for example, is f(1965) as calculated from Equation (4.1). In that year, we selected (see Tables 3-3 and 3-4) a sample of 3,867 departures (7,734 operations) out of which we had a total of 4 accidents.

   The parameter b controls the initial or starting value of F(t), λ defines its rate of decrease in time, and, finally, a determines its asymptotic behavior for large t.

2. In this form, Equation (4.1), we shall fix the year t_0, the starting point in time for the fit, and assign a value to b that would be the value of $F(t_0)$. We then determine or "fit" the remaining two parameters, a and λ, using Bayes' theorem. That is, we regard the data in Tables 3-3 and 3-4, the experience of the past, as evidence. On the basis of this evidence, we derive by Bayes' theorem a probability distribution on the space of a, λ pairs.

3. From this probability distribution of a, λ pairs, we shall derive a probability distribution for the crash frequency for any given year in the future. For instance,

$$f(1985) = a + (b-a)^{-\lambda\,(1985\,-\,t_o)} \tag{4.2}$$

is the accident rate in 1985, given a, λ, and b. The probability distribution of f(1985) is found from the distribution of a, λ pairs.

To obtain the quantity in which we are interested; namely, the expected crash frequency over the remaining life of the plant, we calculate

$$\bar{f} = \frac{1}{31} \sum_{t=1985}^{t=2015} f(t) \tag{4.3}$$

The following provides the details of this "Bayesian Extrapolation" process.

Tables 3-3 and 3-4 give us for each year, t, a doublet (n_t, m_t) that tells the numbers of crashes and the number of operations in that year. Denote by B the set of such doublets from the year t_o on:

$$B = \left\{(n_t,\ m_t)\right\}_{t=t_o}^{1982} \tag{4.4}$$

B, then, is the experience of the past. Next we assume that the underlying frequency has the time dependence represented by Equation (4.1) with b and t_o fixed from inspection of the data. We now ask: What can we say about the values of a, λ in light of the experience B?

For this purpose we write Bayes' theorem in the form

$$p(a,\lambda|B) = p(a,\lambda)\left[\frac{p(B|a,\lambda)}{p(B)}\right] \tag{4.5}$$

where

- $p(a,\lambda)$ = the probability we assign to the pair a,λ 'prior' to having information B.
- $p(a,\lambda|B)$ = our probability of a,λ after having information B (the posterior).
- $p(B|a,\lambda)$ = the probability of experiencing B, given the values a,λ.
- $p(B)$ = the prior probability of B.

$$p(B) = \iint_{a\,\lambda} p(a,\lambda)\, p(B|a,\lambda)\, da\, d\lambda. \tag{4.6}$$

To evaluate $p(B|a,\lambda)$ we note that each pair a, λ implies a specific function of time f(t) through Equation (4.1). In any particular year, then, the probability of observing the pair (n_t, m_t) is:

$$p(n_t,m_t|a,\lambda) = \binom{m_t}{n_t} [f(t)]^{n_t} \; [1-f(t)]^{m_t-n_t} \tag{4.7}$$

For the size m_t we are dealing with, the right side of Equation (4.7) may be replaced by

$$p(n_t,m_t|a,\lambda) = \frac{[m_t f(t)]^{n_t}}{n_t!} \; e^{-[m_t f(t)]} \tag{4.8}$$

The probability of experiencing the entire set B is then

$$p(B|a,\lambda) = \prod_{t=t_0}^{1982} \frac{[m_t f(t)]^{n_t}}{n_t!} \; e^{-[m_t f(t)]} \tag{4.9}$$

To carry out the process numerically, we established a discrete grid over the values of a and λ as follows

$$a: \{a_1, a_2 \cdots, a_I\}$$

$$\lambda: \{\lambda_1, \lambda_2, \cdots, \lambda_j\} \; (\text{yrs}^{-1}) \tag{4.10}$$

We then chose a uniform prior over the set of discrete points (a_i,λ_j), saying thus, that as far as our knowledge goes, each such pair is as likely as any other within the grid. With this choice, Equation (4.5) becomes

$$P_{ij} = p(a_i,\lambda_j|B) \quad \frac{p(B|a_i,\lambda_j)}{\sum_{i,j} p(B|a_i,\lambda_j)} \tag{4.11}$$

with the right side computed from Equation (4.9) using the f(t) given by Equation (4.1).

We now calculate the crash frequency for four different categories of aircraft operation: scheduled landings, nonscheduled landings, scheduled takeoffs, and nonscheduled takeoffs and repeat the Bayesian analysis for each category. The historical data in Tables 3-3 and 3-4 are displayed

graphically for each data category in Figures 4-1 through 4-4. The a, λ, and b values used for each category are as follows:

- Scheduled Landings

$$b = 1.0 \times 10^{-6} \quad t_o = 1955$$

$$a = \{0.0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6\} \ (\times 10^{-6})$$

$$\lambda = \left\{\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \cdots \frac{1}{25}\right\} (\text{yrs}^{-1})$$

- Nonscheduled Landings

$$b = 16 \times 10^{-6} \quad t_o = 1955$$

$$a = \{0.0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6\} \ (\times 10^{-6})$$

$$\lambda = \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \cdots \frac{1}{17}, \frac{1}{18} \ \frac{1}{20}\right\} (\text{yrs}^{-1})$$

- Scheduled Takeoffs

$$b = 0.8 \times 10^{-6} \quad t_o = 1955$$

$$a = \{0.0, 0.025, 0.05, 0.075, 0.1, 0.2\} \ (\times 10^{-6})$$

$$\lambda = \left\{\frac{1}{1.0}, \frac{1}{2.0}, \frac{1}{3.0}, \cdots \frac{1}{14}, \frac{1}{15}, \frac{1}{17}, \frac{1}{20}\right\} (\text{yrs}^{-1})$$

- Nonscheduled Takeoffs

$$b = 10 \times 10^{-6} \quad t_o = 1955$$

$$a = \{0.0, 1.0, 2.0, 30.0, 4.0, 5.0, 6.0\} \ (\times 10^{-6})$$

$$\lambda = \left\{\frac{1}{0.1}, \frac{1}{0.5}, \frac{1}{1.0}, \frac{1}{2.0}, \frac{1}{3.0}, \cdots \frac{1}{10}\right\} (\text{yrs}^{-1})$$

The discrete probability distributions on the a, λ grids are given in Tables 4-1 through 4-4. The crash rate versus time is on the left and the distribution of the average crash rate for the years 1985-2015 is on the right. The resulting expected distributions for the predicted average crash frequency between 1985 and 2015 are displayed at the right of Figures 4-1 through 4-4. Each of these distributions was calculated by obtaining a distribution for the value of F(t) for each value of t in the period 1985 through 2015, using the probability distribution on the a, λ grid and then obtaining the value of $\overline{f}$ based on Equation (4.3). The smooth curve on Figures 4-1 through 4-4 is a plot of Equation (4.1), using the mean values of a and λ from the discrete probability distribution. Similar results are provided in Figures 4-5 and 4-6 for

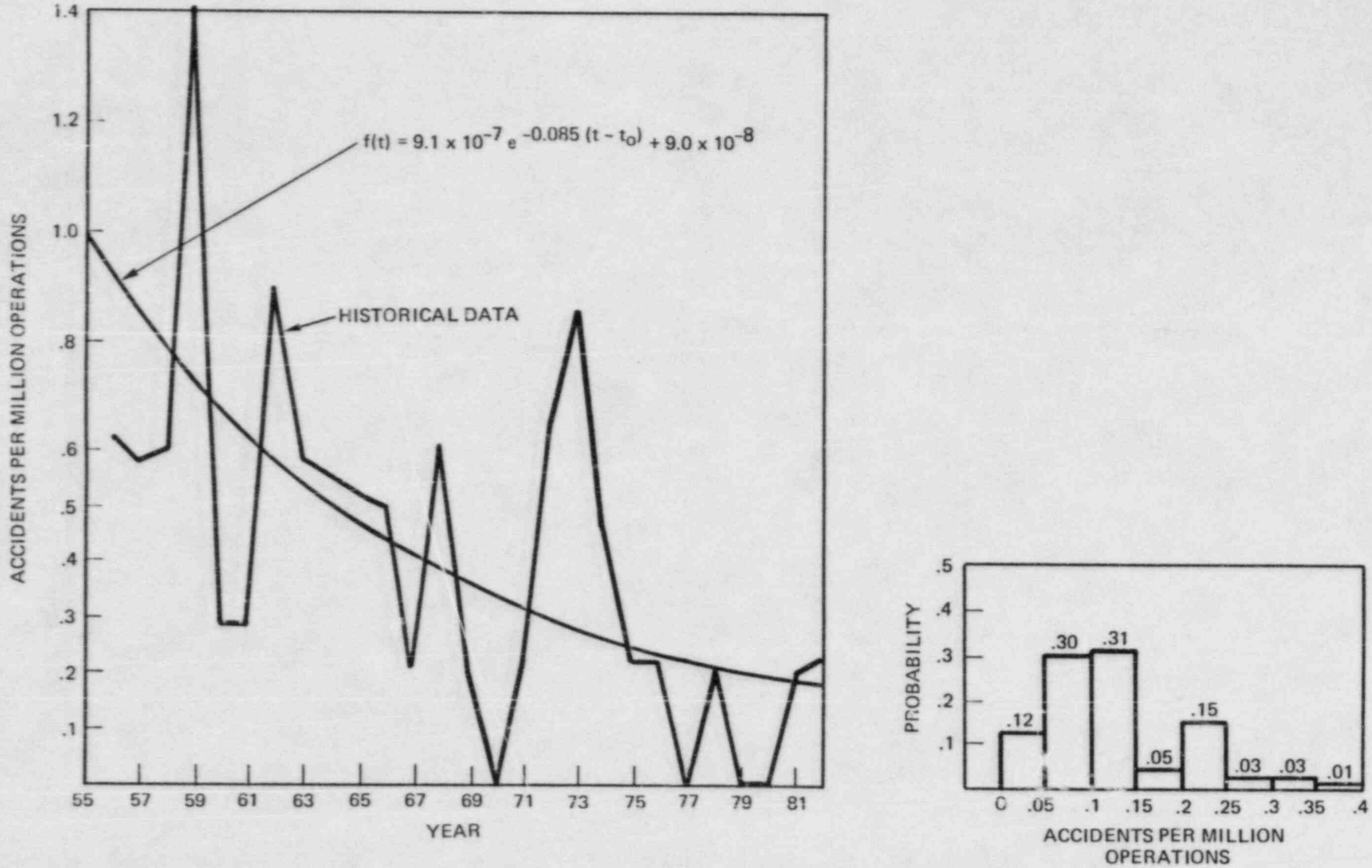

FIGURE 4-1. CRASH RATE VERSUS TIME - SCHEDULED LANDINGS

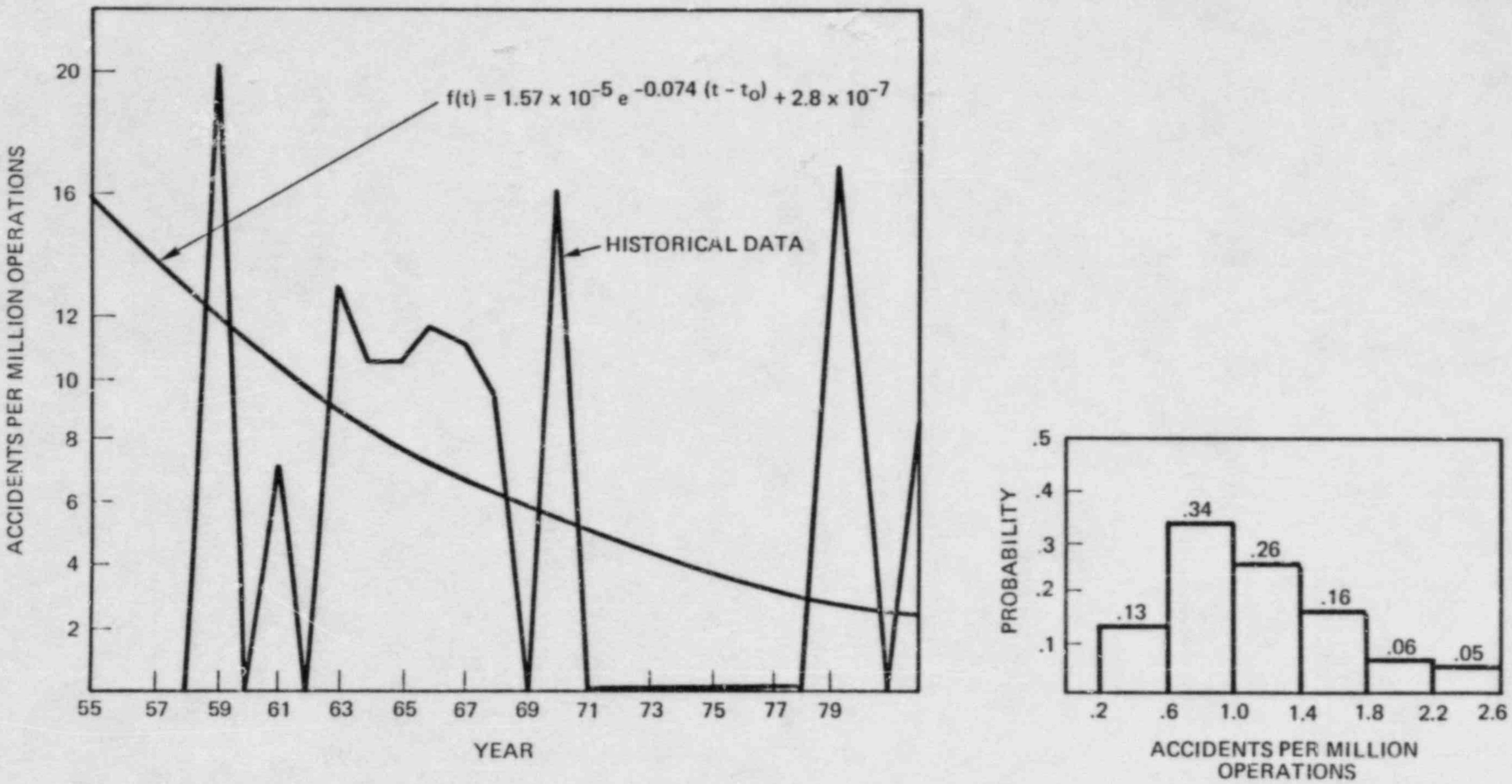

FIGURE 4-2. CRASH RATE VERSUS TIME - NONSCHEDULED LANDINGS

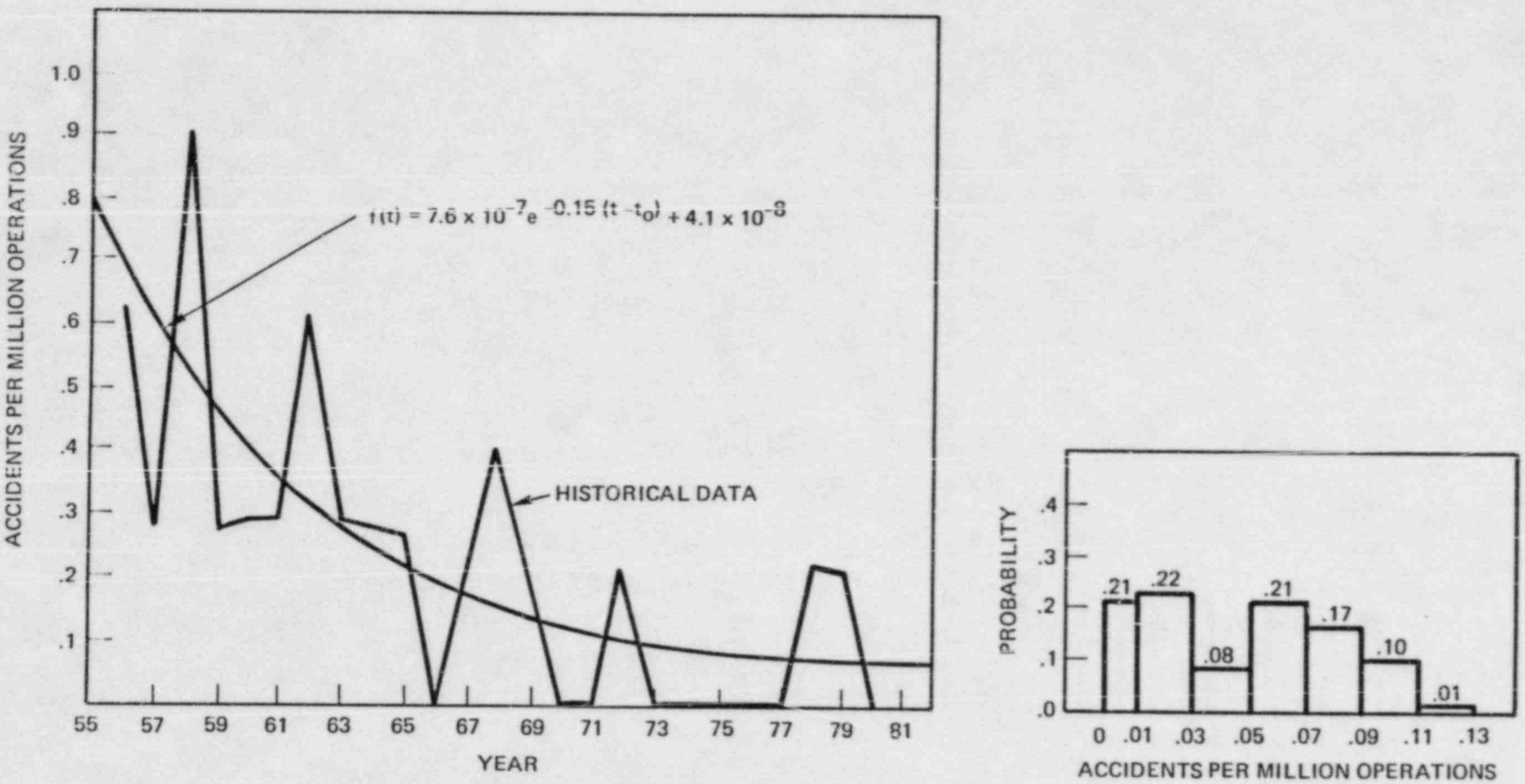

FIGURE 4-3. CRASH RATE VERSUS TIME - SCHEDULED TAKEOFFS

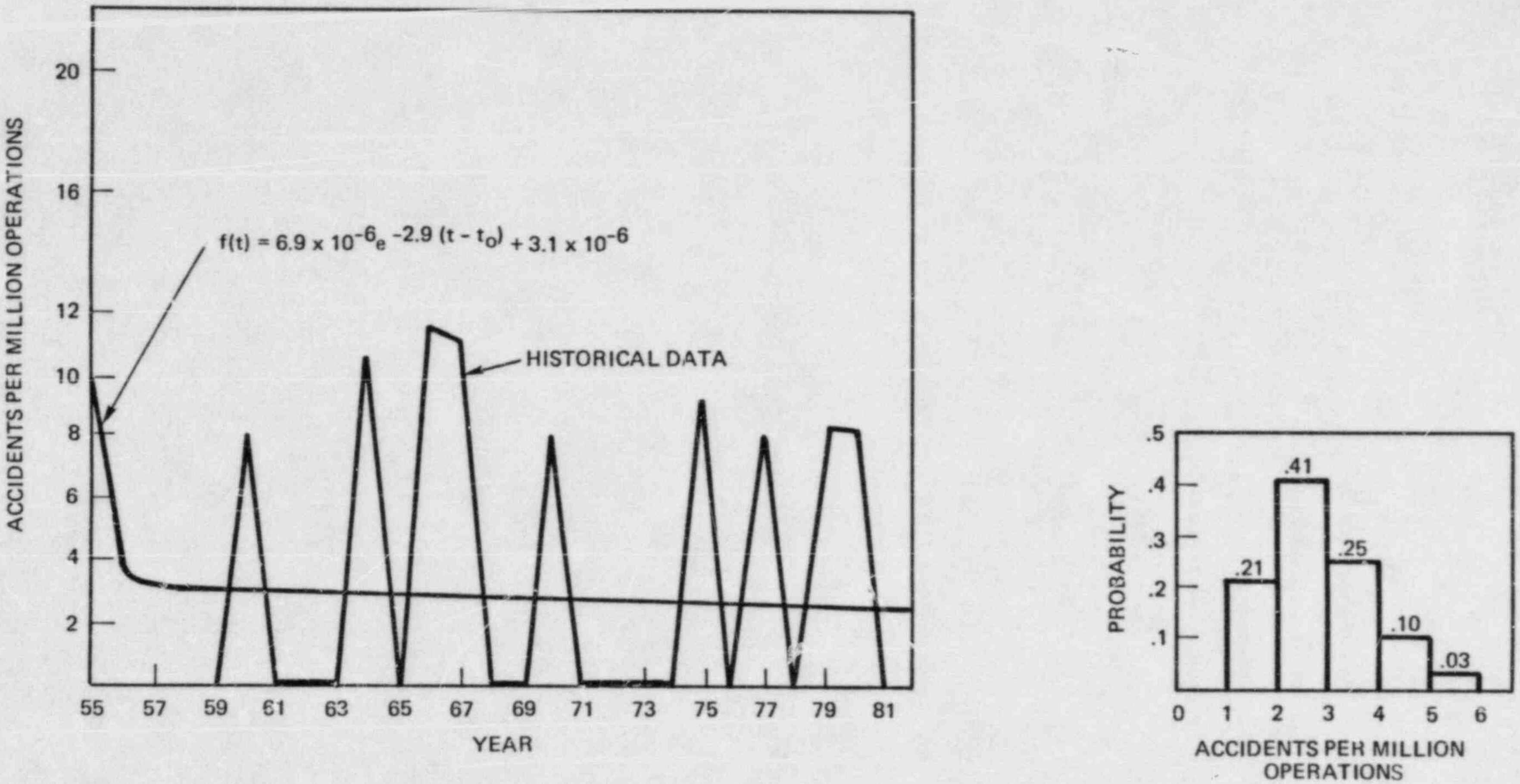

FIGURE 4-4. CRASH RATE VERSUS TIME - NONSCHEDULED TAKEOFFS

TABLE 4-1. BAYESIAN RESULTS FOR ACCIDENT RATE DISTRIBUTION - SCHEDULED LANDINGS

*** VALUES OF PARAMETER A *** B=1.00E-6

1/LAMBDA	*	0.00E-01	5.00E-08	1.00E-07	2.00E-07	3.00E-07	4.00E-07	5.00E-07	6.00E-07	TOTAL
5	*	2.10E-17	9.47E-09	1.35E-05	4.12E-03	9.71E-03	2.10E-03	1.06E-04	1.99E-06	1.60E-02
6	*	7.50E-12	1.30E-06	2.19E-04	1.08E-02	1.15E-02	1.69E-03	7.16E-05	1.28E-06	2.43E-02
7	*	2.98E-08	5.39E-05	1.76E-03	1.96E-02	1.09E-02	1.17E-03	4.37E-05	7.70E-07	3.36E-02
8	*	6.61E-06	7.63E-04	7.49E-03	2.61E-02	8.60E-03	7.27E-04	2.49E-05	4.44E-07	4.37E-02
9	*	2.29E-04	4.64E-03	1.89E-02	2.70E-02	5.91E-03	4.17E-04	1.36E-05	2.51E-07	5.71E-02
10	*	2.23E-03	1.46E-02	3.17E-02	2.31E-02	3.72E-03	2.31E-04	7.35E-06	1.42E-07	7.55E-02
11	*	9.72E-03	2.90E-02	3.93E-02	1.71E-02	2.17E-03	1.23E-04	3.92E-06	8.06E-08	9.74E-02
12	*	2.37E-02	4.06E-02	3.88E-02	1.14E-02	1.21E-03	6.49E-05	2.10E-06	4.61E-08	1.16E-01
13	*	3.82E-02	4.39E-02	3.24E-02	7.05E-03	6.59E-04	3.42E-05	1.14E-06	2.69E-08	1.22E-01
14	*	4.59E-02	3.94E-02	2.39E-02	4.15E-03	3.54E-04	1.81E-05	6.27E-07	1.60E-08	1.14E-01
15	*	4.48E-02	3.10E-02	1.62E-02	2.39E-03	1.91E-04	9.77E-06	3.54E-07	9.74E-09	9.46E-02
16	*	3.75E-02	2.20E-02	1.03E-02	1.33E-03	1.02E-04	5.29E-06	2.02E-07	6.01E-09	7.12E-02
17	*	2.81E-02	1.46E-02	6.23E-03	7.35E-04	5.51E-05	2.92E-06	1.18E-07	3.78E-09	4.97E-02
18	*	1.97E-02	9.29E-03	3.73E-03	4.12E-04	3.06E-05	1.67E-06	7.10E-08	2.46E-09	3.32E-02
19	*	1.28E-02	5.61E-03	2.15E-03	2.26E-04	1.68E-05	9.48E-07	4.28E-08	1.60E-09	2.08E-02
20	*	8.16E-03	3.38E-03	1.25E-03	1.28E-04	9.57E-06	5.61E-07	2.68E-08	1.08E-09	1.29E-02
21	*	5.02E-03	2.00E-03	7.18E-04	7.21E-05	5.49E-06	3.35E-07	1.70E-08	7.35E-10	7.81E-03
22	*	3.10E-03	1.20E-03	4.23E-04	4.22E-05	3.28E-06	2.08E-07	1.11E-08	5.17E-10	4.77E-03
23	*	1.86E-03	7.03E-04	2.45E-04	2.45E-05	1.95E-06	1.29E-07	7.33E-09	3.64E-10	2.84E-03
24	*	1.12E-03	4.18E-04	1.45E-04	1.45E-05	1.19E-06	8.24E-08	4.94E-09	2.62E-10	1.70E-03
25	*	6.71E-04	2.47E-04	8.53E-05	8.66E-06	7.31E-07	5.29E-08	3.36E-09	1.90E-10	1.01E-03
	*									
TOTALS	*	2.83E-01	2.63E-01	2.36E-01	1.56E-01	5.52E-02	6.60E-03	2.76E-04	5.07E-06	1.00E+00

MEAN VALUE FOR A =8.7241E-08

MEAN VALUE FOR LAMBDA =.086 = 1/11.6

TABLE 4-2. BAYESIAN RESULTS FOR ACCIDENT RATE DISTRIBUTION - NONSCHEDULED LANDINGS

*** VALUES OF PARAMETER A ***   B=1.60E-4

1/LAMBDA	0.00E-01	5.00E-08	1.00E-07	2.00E-07	3.00E-07	4.00E-07	5.00E-07	6.00E-07	TOTAL
3	9.74E-20	1.47E-14	5.80E-13	4.37E-11	7.46E-10	6.33E-09	3.53E-08	1.48E-07	1.90E-07
4	9.76E-13	7.15E-11	6.98E-10	1.26E-08	9.07E-08	4.13E-07	1.42E-06	4.01E-06	5.95E-06
5	7.51E-09	4.09E-08	1.42E-07	8.74E-07	3.35E-06	9.76E-06	2.37E-05	5.06E-05	8.85E-05
6	1.64E-06	3.63E-06	7.09E-06	2.12E-05	5.09E-05	1.06E-04	1.97E-04	3.38E-04	7.24E-04
7	4.93E-05	7.54E-05	1.11E-04	2.15E-04	3.78E-04	6.15E-04	9.42E-04	1.37E-03	3.76E-03
8	4.46E-04	5.73E-04	7.22E-04	1.09E-03	1.58E-03	2.18E-03	2.90E-03	3.75E-03	1.32E-02
9	1.86E-03	2.17E-03	2.51E-03	3.28E-03	4.16E-03	5.15E-03	6.24E-03	7.42E-03	3.28E-02
10	4.67E-03	5.15E-03	5.65E-03	6.71E-03	7.84E-03	9.01E-03	1.02E-02	1.14E-02	6.07E-02
11	8.29E-03	8.83E-03	9.36E-03	1.04E-02	1.15E-02	1.26E-02	1.36E-02	1.46E-02	8.93E-02
12	1.16E-02	1.20E-02	1.25E-02	1.34E-02	1.42E-02	1.49E-02	1.56E-02	1.62E-02	1.10E-01
13	1.37E-02	1.40E-02	1.43E-02	1.48E-02	1.53E-02	1.57E-02	1.60E-02	1.63E-02	1.20E-01
14	1.43E-02	1.45E-02	1.46E-02	1.48E-02	1.50E-02	1.51E-02	1.51E-02	1.51E-02	1.18E-01
15	1.38E-02	1.38E-02	1.38E-02	1.38E-02	1.37E-02	1.36E-02	1.35E-02	1.33E-02	1.09E-01
16	1.25E-02	1.24E-02	1.23E-02	1.22E-02	1.20E-02	1.17E-02	1.15E-02	1.12E-02	9.57E-02
17	1.08E-02	1.07E-02	1.06E-02	1.03E-02	1.01E-02	9.80E-03	9.53E-03	9.24E-03	8.10E-02
18	9.09E-03	8.96E-03	8.84E-03	8.58E-03	8.32E-03	8.05E-03	7.78E-03	7.50E-03	6.71E-02
19	7.44E-03	7.32E-03	7.19E-03	6.94E-03	6.70E-03	6.45E-03	6.20E-03	5.96E-03	5.42E-02
20	6.05E-03	5.93E-03	5.82E-03	5.60E-03	5.38E-03	5.16E-03	4.94E-03	4.73E-03	4.36E-02
TOTALS	1.14E-01	1.16E-01	1.18E-01	1.22E-01	1.26E-01	1.30E-01	1.34E-01	1.39E-01	1.00E+00

MEAN VALUE FOR A =2.8214E-07

MEAN VALUE FOR LAMBDA =0.074 = 1/13.5

TABLE 4-3. BAYESIAN RESULTS FOR ACCIDENT RATE DISTRIBUTION - SCHEDULED TAKEOFFS

*** VALUES OF PARAMETER A *** B=8. 0E-7

1/LAMBDA	0. 00E-01	2. 50E-08	5. 00E-08	7. 50E-08	1. 00E-07	2. 00E-07	TOTAL
1	1. 64E-63	1. 64E-11	7. 86E-08	4. 93E-06	4. 82E-05	2. 30E-04	2. 83E-04
2	1. 82E-24	6. 08E-08	1. 64E-05	2. 35E-04	8. 93E-04	6. 73E-04	1. 82E-03
3	2. 86E-12	2. 24E-05	7. 06E-04	3. 29E-03	6. 10E-03	1. 13E-03	1. 13E-02
4	7. 90E-07	1. 19E-03	8. 27E-03	1. 69E-02	1. 82E-02	1. 19E-03	4. 58E-02
5	4. 25E-04	1. 30E-02	3. 29E-02	3. 75E-02	2. 74E-02	8. 40E-04	1. 12E-01
6	1. 03E-02	4. 52E-02	5. 77E-02	4. 38E-02	2. 44E-02	4. 44E-04	1. 82E-01
7	4. 47E-02	6. 99E-02	5. 64E-02	3. 24E-02	1. 49E-02	1. 92E-04	2. 18E-01
8	7. 08E-02	6. 17E-02	3. 68E-02	1. 74E-02	7. 06E-03	7. 31E-05	1. 94E-01
9	6. 02E-02	3. 72E-02	1. 81E-02	7. 56E-03	2. 81E-03	2. 57E-05	1. 26E-01
10	3. 53E-02	1. 77E-02	7. 61E-03	2. 92E-03	1. 03E-03	8. 89E-06	6. 46E-02
11	1. 63E-02	7. 19E-03	2. 85E-03	1. 04E-03	3. 54E-04	3. 03E-06	2. 77E-02
12	6. 52E-03	2. 66E-03	1. 00E-03	3. 54E-04	1. 19E-04	1. 04E-06	1. 07E-02
13	2. 43E-03	9. 42E-04	3. 45E-04	1. 20E-04	4. 02E-05	3. 70E-07	3. 87E-03
14	8. 66E-04	3. 27E-04	1. 18E-04	4. 09E-05	1. 38E-05	1. 35E-07	1. 37E-03
15	3. 10E-04	1. 16E-04	4. 15E-05	1. 45E-05	4. 90E-06	5. 21E-08	4. 87E-04
17	3. 93E-05	1. 47E-05	5. 32E-06	1. 89E-06	6. 61E-07	8. 37E-09	6. 19E-05
20	2. 22E-06	8. 53E-07	3. 23E-07	1. 21E-07	4. 45E-08	7. 41E-10	3. 56E-06
TOTALS	2. 48E-01	2. 57E-01	2. 23E-01	1. 64E-01	1. 04E-01	4. 81E-03	1. 00E+00

MEAN VALUE FOR A =4. 1154E-08

MEAN VALUE FOR LAMBDA =. 15 = 1/6. 67

TABLE 4-4. BAYESIAN RESULTS FOR ACCIDENT RATE DISTRIBUTION - NONSCHEDULED TAKEOFFS

*** VALUES OF PARAMETER A ***   B=1.0E-5

1/LAMBDA	*	0.00E-01	1.00E-06	2.00E-06	3.00E-06	4.00E-06	5.00E-06	6.00E-06	TOTAL
0.1	*	4.96-614	2.39E-03	5.39E-02	9.10E-02	5.32E-02	1.74E-02	3.95E-03	2.22E-01
0.5	*	2.93-117	2.11E-03	4.81E-02	8.25E-02	4.89E-02	1.62E-02	3.74E-03	2.02E-01
1.0	*	2.53E-55	1.58E-03	3.63E-02	6.39E-02	3.92E-02	1.35E-02	3.21E-03	1.58E-01
2.0	*	1.10E-24	1.29E-03	2.35E-02	4.01E-02	2.53E-02	9.16E-03	2.33E-03	1.02E-01
3.0	*	8.48E-15	1.46E-03	1.81E-02	2.77E-02	1.72E-02	6.40E-03	1.71E-03	7.25E-02
4.0	*	4.13E-10	1.94E-03	1.53E-02	2.03E-02	1.22E-02	4.59E-03	1.28E-03	5.55E-02
5.0	*	1.71E-07	2.65E-03	1.34E-02	1.54E-02	8.83E-03	3.36E-03	9.72E-04	4.47E-02
6.0	*	6.60E-06	3.53E-03	1.19E-02	1.19E-02	6.54E-03	2.50E-03	7.48E-04	3.72E-02
7.0	*	6.79E-05	4.45E-03	1.06E-02	9.32E-03	4.92E-03	1.89E-03	5.82E-04	3.18E-02
8.0	*	3.14E-04	5.27E-03	9.29E-03	7.33E-03	3.74E-03	1.44E-03	4.59E-04	2.78E-02
9.0	*	8.64E-04	5.83E-03	8.06E-03	5.80E-03	2.88E-03	1.12E-03	3.67E-04	2.49E-02
10.0	*	1.69E-03	6.09E-03	6.91E-03	4.61E-03	2.24E-03	8.82E-04	2.97E-04	2.27E-02
	*								
TOTALS	*	2.94E-03	3.86E-02	2.55E-01	3.80E-01	2.25E-01	7.85E-02	1.97E-02	1.00E+00

MEAN VALUE FOR A =3.0997E-06

MEAN VALUE FOR LAMBDA =2.9 =1/.34

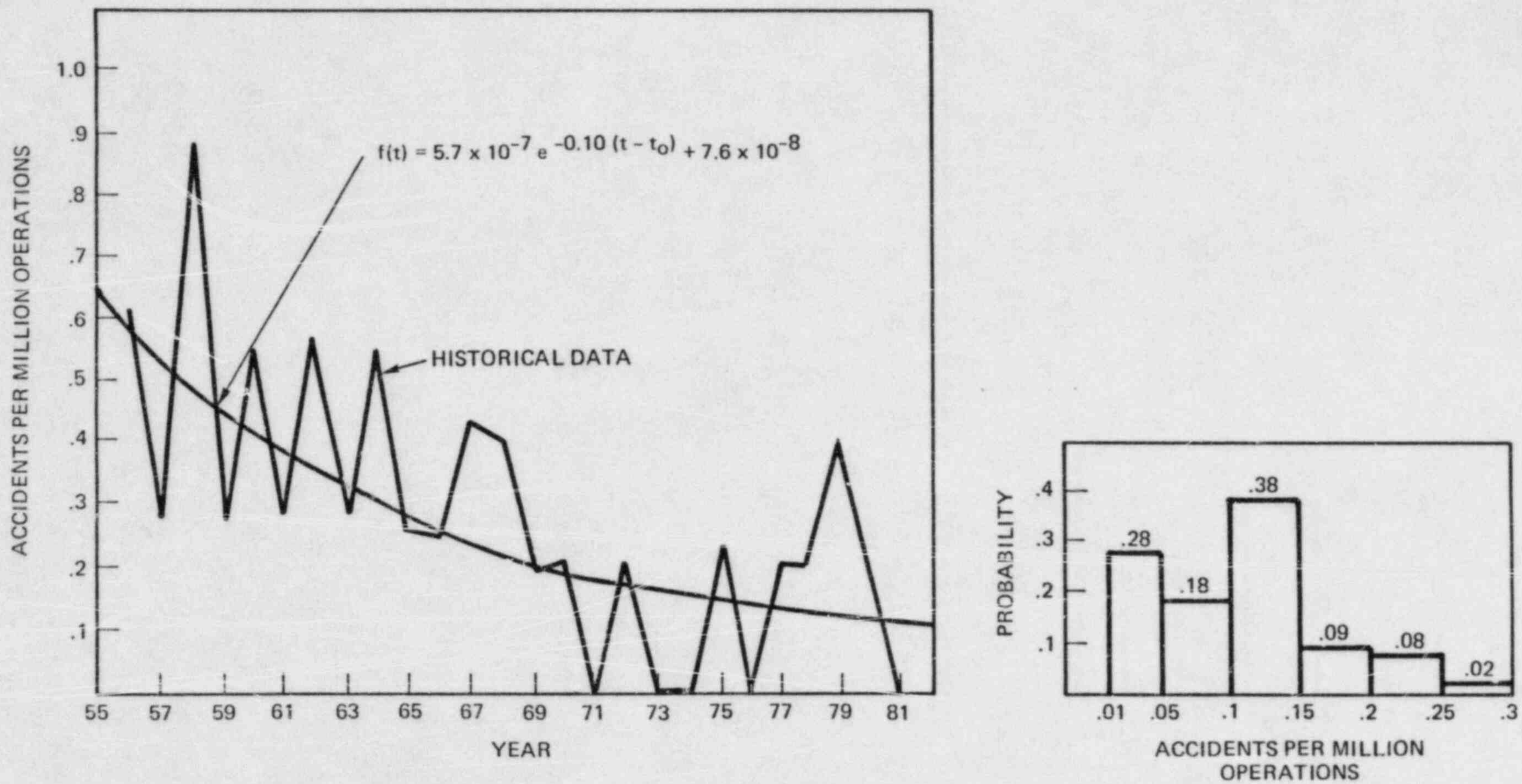

FIGURE 4-5. CRASH RATE VERSUS TIME - TAKEOFFS

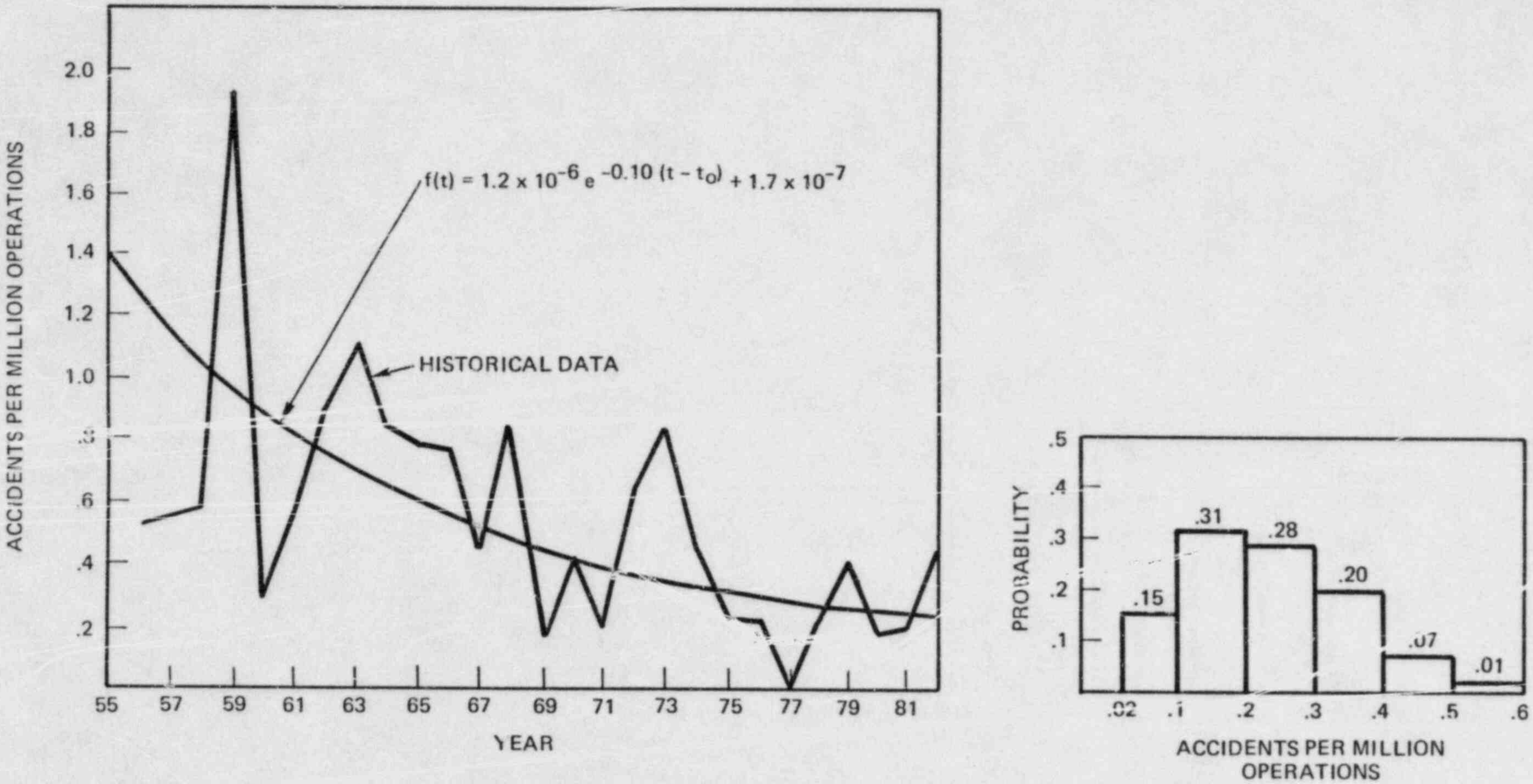

FIGURE 4-6. CRASH RATE VERSUS TIME - LANDINGS

all takeoffs and landings, respectively. Tables 4-5 and 4-6 present the discrete probability distributions on the a, λ grid for these two cases.

The mean annual crash rates for various cases are summarized as follows:

Scheduled Landings = 1.27×10^{-7} Crashes per Year
Nonscheduled Landings = 1.13×10^{-6} Crashes per Year
Scheduled Takeoffs = 4.57×10^{-8} Crashes per Year
Nonscheduled Takeoffs = 3.11×10^{-6} Crashes per Year
All Landings Combined = 2.11×10^{-7} Crashes per Year
All Takeoffs Combined = 9.89×10^{-8} Crashes per Year

4.2 DETERMINATION OF THE SPATIAL DENSITY FUNCTIONS

In this section, we apply the Bayesian approach to find probability distributions for the bracketed quantities in Equation (2.6). We begin with the radial distribution of landing crashes.

4.2.1 THE RADIAL DENSITY $\left[\frac{d}{dr} R(r)\right]_{r = r_o}$

4.2.1.1 General Approach

The data shown in Figure 3-2 suggests that R(r) may be well fit by a step at r = 0, followed by a decaying exponential, i.e.,

$$R(r) = \begin{cases} 1.0 , & r = 0 \\ ae^{-\lambda r}, & r > 0 \end{cases} \quad (4.12)$$

This being so, the derivative of R(r) contains a delta function at r = 0

$$\left[\frac{-d}{dr} R(r)\right] = (1-a)\,\delta(r) + \lambda a e^{-\lambda r} \quad (4.13)$$

We seek to estimate the value of this derivative at the radius of the plant. Thus, we seek

$$D(r) = \left[\frac{-d}{dr} R(r)\right]_{r_o} = \lambda a e^{-\lambda r_o} \quad (4.14)$$

where r_o = 2.7 miles. We will obtain this estimate by first obtaining a discretized probability distribution (DPD) on the space of doublets, (a,λ), and then converting this to a DPD against the desired derivative through Equation (4.14). To begin this process, we discretize the sets of possible a's and λ's

$$a: \{a_1, a_2, a_3 \ldots, a_I\} \quad (4.15)$$

$$\lambda: \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_J\} \quad (4.16)$$

TABLE 4-5. ACCIDENT RATE FOR TAKEOFFS - SCHEDULED AND NONSCHEDULED

*** VALUES OF PARAMETER A *** B=6.5E-7

1/LAMBDA	0.00E-01	1.00E-07	2.00E-07	3.00E-07	4.00E-07	5.00E-07	6.00E-07	TOTAL
1	9.22-122	5.14E-07	1.08E-03	1.13E-03	4.80E-05	3.46E-07	7.98E-10	2.26E-03
2	1.77E-51	1.13E-05	3.42E-03	1.69E-03	5.23E-05	3.35E-07	7.73E-10	5.17E-03
3	1.07E-28	1.52E-04	8.34E-03	2.18E-03	5.18E-05	3.08E-07	7.39E-10	1.07E-02
4	6.41E-18	1.17E-03	1.53E-02	2.36E-03	4.64E-05	2.69E-07	6.94E-10	1.39E-02
5	7.12E-12	5.40E-03	2.15E-02	2.19E-03	3.78E-05	2.24E-07	6.43E-10	2.91E-02
6	3.04E-08	1.55E-02	2.41E-02	1.78E-03	2.87E-05	1.80E-07	5.90E-10	4.14E-02
7	6.78E-06	3.12E-02	2.26E-02	1.32E-03	2.06E-05	1.40E-07	5.37E-10	5.52E-02
8	2.26E-04	4.68E-02	1.87E-02	9.17E-04	1.44E-05	1.08E-07	4.87E-10	6.67E-02
9	2.24E-03	5.60E-02	1.40E-02	6.10E-04	9.87E-06	8.34E-08	4.42E-10	7.29E-02
10	9.65E-03	5.64E-02	9.95E-03	4.00E-04	6.79E-06	6.45E-08	4.02E-10	7.64E-02
11	2.45E-02	5.01E-02	6.74E-03	2.59E-04	4.66E-06	5.01E-08	3.67E-10	8.16E-02
12	4.27E-02	4.05E-02	4.44E-03	1.67E-04	3.23E-06	3.91E-08	3.36E-10	8.78E-02
13	5.68E-02	3.06E-02	2.89E-03	1.09E-04	2.26E-06	3.09E-08	3.09E-10	9.04E-02
14	6.23E-02	2.21E-02	1.87E-03	7.12E-05	1.60E-06	2.47E-08	2.85E-10	8.64E-02
15	5.98E-02	1.55E-02	1.22E-03	4.75E-05	1.16E-06	1.99E-08	2.64E-10	7.66E-02
16	5.20E-02	1.06E-02	7.90E-04	3.18E-05	8.40E-07	1.62E-08	2.46E-10	6.34E-02
17	4.20E-02	7.17E-03	5.16E-04	2.16E-05	6.20E-07	1.33E-08	2.29E-10	4.97E-02
18	3.26E-02	4.87E-03	3.45E-04	1.51E-05	4.67E-07	1.11E-08	2.15E-10	3.78E-02
19	2.41E-02	3.24E-03	2.29E-04	1.05E-05	3.52E-07	9.29E-09	2.02E-10	2.76E-02
20	1.76E-02	2.20E-03	1.56E-04	7.52E-06	2.72E-07	7.87E-09	1.91E-10	1.99E-02
TOTALS	4.26E-01	4.00E-01	1.58E-01	1.53E-02	3.32E-04	2.28E-06	8.75E-09	1.00E+00

MEAN VALUE FOR A =7.63E-8

MEAN VALUE FOR LAMBDA =0.10 = 1/10.0

TABLE 4-6. ACCIDENT RATE FOR LANDINGS - SCHEDULED AND NONSCHEDULED

*** VALUES OF PARAMETER A ***   B=1.40E-6

1/LAMBDA	0.00E-01	1.00E-07	2.00E-07	3.00E-07	4.00E-07	5.00E-07	6.00E-07	7.00E-07	8.00E-07	9.00E-07	1.00E-06	TOTAL
1	3.37-263	3.22E-23	1.54E-11	1.52E-06	2.37E-04	1.03E-03	4.55E-04	4.12E-05	1.16E-06	1.33E-08	7.52E-11	1.77E-03
2	3.48-110	5.25E-19	1.69E-09	1.84E-05	3.53E-04	1.79E-03	5.04E-04	3.47E-05	8.37E-07	9.00E-09	5.07E-11	3.20E-03
3	1.44E-60	6.00E-15	1.76E-07	2.28E-04	3.19E-03	3.22E-03	5.73E-04	2.97E-05	6.10E-07	6.10E-09	3.42E-11	7.24E-03
4	4.41E-37	1.96E-11	9.28E-06	1.80E-03	3.58E-03	4.49E-03	5.34E-04	2.18E-05	3.94E-07	3.76E-09	2.14E-11	1.54E-02
5	6.73E-24	1.45E-08	2.15E-04	6.10E-03	1.53E-02	4.59E-03	3.90E-04	1.31E-05	2.18E-07	2.06E-09	1.23E-11	2.86E-02
6	5.72E-16	2.27E-06	2.12E-03	2.09E-02	1.85E-02	3.53E-03	2.30E-04	6.78E-06	1.07E-07	1.04E-09	6.64E-12	4.53E-02
7	7.79E-11	1.01E-04	1.05E-02	3.42E-02	1.61E-02	2.13E-03	1.14E-04	3.05E-06	4.77E-08	4.82E-10	3.39E-12	6.31E-02
8	1.71E-07	1.47E-03	2.78E-02	3.79E-02	1.08E-02	1.08E-03	4.96E-05	1.26E-06	2.00E-08	2.16E-10	1.69E-12	7.91E-02
9	2.65E-05	8.67E-03	4.51E-02	3.09E-02	5.99E-03	4.80E-04	1.99E-05	4.96E-07	8.17E-09	9.57E-11	8.41E-13	9.13E-02
10	6.69E-04	2.51E-02	4.99E-02	2.03E-02	2.92E-03	2.00E-04	7.80E-06	1.94E-07	3.37E-09	4.31E-11	4.27E-13	9.91E-02
11	5.34E-03	4.36E-02	4.16E-02	1.12E-02	1.29E-03	7.89E-05	2.96E-06	7.53E-08	1.39E-09	1.96E-11	2.19E-13	1.03E-01
12	1.87E-02	5.13E-02	2.80E-02	5.52E-03	5.34E-04	3.02E-05	1.12E-06	2.95E-08	5.83E-10	9.06E-12	1.14E-13	1.04E-01
13	3.61E-02	4.52E-02	1.61E-02	2.51E-03	2.15E-04	1.16E-05	4.30E-07	1.18E-08	2.52E-10	4.33E-12	6.16E-14	1.00E-01
14	4.61E-02	3.22E-02	8.32E-03	1.08E-03	8.46E-05	4.43E-06	1.68E-07	4.86E-09	1.12E-10	2.13E-12	3.40E-14	8.78E-02
15	4.37E-02	1.98E-02	4.02E-03	4.59E-04	3.37E-05	1.75E-06	6.81E-08	2.08E-09	5.20E-11	1.09E-12	1.95E-14	6.80E-02
16	3.33E-02	1.03E-02	1.82E-03	1.88E-04	1.33E-05	6.92E-07	2.79E-08	9.08E-10	2.46E-11	5.66E-13	1.13E-14	4.62E-02
17	2.17E-02	5.49E-03	8.01E-04	7.71E-05	5.33E-06	2.81E-07	1.18E-08	4.09E-10	1.20E-11	3.04E-13	6.77E-15	2.81E-02
18	1.28E-02	2.69E-03	3.55E-04	3.25E-05	2.23E-06	1.20E-07	5.28E-09	1.95E-10	6.17E-12	1.71E-13	4.21E-15	1.59E-02
19	6.84E-03	1.24E-03	1.50E-04	1.33E-05	9.17E-07	5.08E-08	2.35E-09	9.25E-11	3.17E-12	9.63E-14	2.62E-15	8.24E-03
20	3.54E-03	5.74E-04	6.60E-05	5.75E-06	4.00E-07	2.29E-08	1.11E-09	4.66E-11	1.72E-12	5.69E-14	1.70E-15	4.18E-03
TOTALS	2.29E-01	2.48E-01	2.37E-01	1.76E-01	8.47E-02	2.27E-02	2.88E-03	1.52E-04	3.41E-06	3.61E-08	2.07E-1[illegible]	1.00E+00

MEAN VALUE FOR A =1.72E-7

MEAN VALUE FOR LAMBDA =0.10 = 1/10.0

We then consider the space of a, λ doublets

$$\{(a_i, \lambda_j)\} \quad (4.17)$$

On this space, we will establish a discrete probability distribution by assigning a probability, p_{ij}, to each such doublet, i.e.,

$$\{\langle p_{ij}, (a_i,\lambda_j)\rangle\} \quad (4.18)$$

To explain the next step, let us introduce the notation

$$g(a,\lambda) = \lambda a e^{-\lambda r_o} \quad (4.19)$$

and

$$g_{ij} = g(a_i,\lambda_j) = \lambda_j a_i e^{-\lambda_j r_o} \quad (4.20)$$

Then, the DPD Equation (4.18) converts through Equation (4.20) to a DPD for g:

$$\{\langle p_{ij}, g_{ij}\rangle\} \quad (4.21)$$

This is then the DPD for our desired derivative in Equation (4.14).

We obtain the DPD on (a,λ) space by applying Bayes' theorem in the form

$$p(a_i,\lambda_j|B) = p(a_i,\lambda_j) \frac{p(B|a_i,\lambda_j)}{\sum_{i,j} p(a_i,\lambda_j)\, p(B|a_i,\lambda_j)} \quad (4.22)$$

where

- B = the information we get from our historical data.
- $p(a_i,\lambda_j|B)$ = the probability we assign to the doublet (a_i,λ_j) after we have the information B.
- $p(a_i,\lambda_j)$ = the probability we assign to the doublet (a_i,λ_j) prior to having the information B.
- $p(B|a_i,\lambda_j)$ = the likelihood of event B happening, given that a_i,λ_j are true.

In our case, B is the set of radii at which crashes occurred.

We note that B contains a total of 110 points; 45 points have r = 0, and the remainder we will write as

$$\{r_n\} = \{r_1, r_2, \ldots, r_{65}\} \quad (4.23)$$

From Equation (4.13), then, the probability of these 110 crashes occurring the way they did is

$$p(B|a_i,\lambda_j) = (1-a_i)^{45} (a_i \lambda_j)^{65} \exp\left\{-\lambda_j \sum_{n=1}^{65} r_n\right\} \qquad (4.24)$$

Again, for our case we have

$$\sum_{n=1}^{65} r_n = 108.7 \text{ miles} \qquad (4.25)$$

so that

$$p(B|a_i,\lambda_j) = (1-a_i)^{45} (a_i\lambda_j)^{65} e^{(-\lambda_j 108.7)} \qquad (4.26)$$

All that remains before carrying out the calculations using Equation (4.22) is to specify the a_i and λ_j numerically and, then, to set the prior. We choose a_i,λ_j as follows:

$$\{a_i\} \equiv \{0.4, .45, .5, .55, .6, .65, .7\} \qquad (4.27)$$

$$\{\lambda_j\} \equiv \frac{1}{.75}, \frac{1}{1.0}, \frac{1}{1.25}, \ldots, \frac{1}{3.25} \qquad (4.28)$$

To reflect an initial state of knowledge, we shall choose the prior to be uniform over the (a_i,λ_j); Equation (4.22) then reduces, using Equation (4.26), to

$$p(a_i,\lambda_j|B) = \frac{(1-a_i)^{45} (a_i\lambda_j)^{65} e^{-\lambda_j 108.7}}{\sum_{i,j} (1-a_i)^{45} (a_i\lambda_j)^{65} e^{-\lambda_j 108.7}} \qquad (4.29)$$

The results of this calculation are shown in Table 4-7. From these results, we obtain the curve in Figure 4-7 for the function R(r) using the average a and λ. Figure 4-8a shows the DPD-obtained derivative of R(r) at r = 2.7, which is obtained by applying the results in Table 4-7 as in Equations (4.20) and (4.21).

TABLE 4-7. BAYESIAN RESULTS FOR RADIAL DISTRIBUTION - ALL CRASHES COMBINED

*** VALUES OF PARAMETER A ***

1/LAMBDA	4.00E-01	4.50E-01	5.00E-01	5.50E-01	6.00E-01	6.50E-01	7.00E-01	TOTAL
0.75	5.15E-17	2.17E-15	2.80E-14	1.20E-13	1.71E-13	7.65E-14	9.18E-15	4.07E-13
1.00	2.07E-09	8.73E-08	1.13E-06	4.83E-06	6.89E-06	3.08E-06	3.69E-07	1.64E-05
1.25	2.87E-06	1.21E-04	1.56E-03	6.69E-03	9.55E-03	4.26E-03	5.12E-04	2.27E-02
1.50	4.01E-05	1.69E-03	2.18E-02	9.35E-02	1.33E-01	5.96E-02	7.15E-03	3.17E-01
1.75	5.60E-05	2.36E-03	3.05E-02	1.30E-01	1.86E-01	8.31E-02	9.98E-03	4.43E-01
2.00	2.25E-05	9.46E-04	1.22E-02	5.23E-02	7.47E-02	3.33E-02	4.00E-03	1.78E-01
2.25	4.38E-06	1.84E-04	2.38E-03	1.02E-02	1.46E-02	6.50E-03	7.81E-04	3.46E-02
2.50	5.92E-07	2.49E-05	3.22E-04	1.38E-03	1.97E-03	8.79E-04	1.06E-04	4.68E-03
2.75	6.45E-08	2.72E-06	3.51E-05	1.50E-04	2.14E-04	9.58E-05	1.15E-05	5.10E-04
3.00	5.75E-09	2.42E-07	3.13E-06	1.34E-05	1.91E-05	8.55E-06	1.03E-06	4.55E-05
3.25	5.46E-10	2.30E-08	2.97E-07	1.27E-06	1.82E-06	8.11E-07	9.73E-08	4.32E-06
TOTALS	1.26E-04	5.33E-03	6.89E-02	2.95E-01	4.21E-01	1.88E-01	2.25E-02	1.00E+00

MEAN VALUE FOR A =0.589

MEAN VALUE FOR LAMBDA =0.589 = 1/1.70

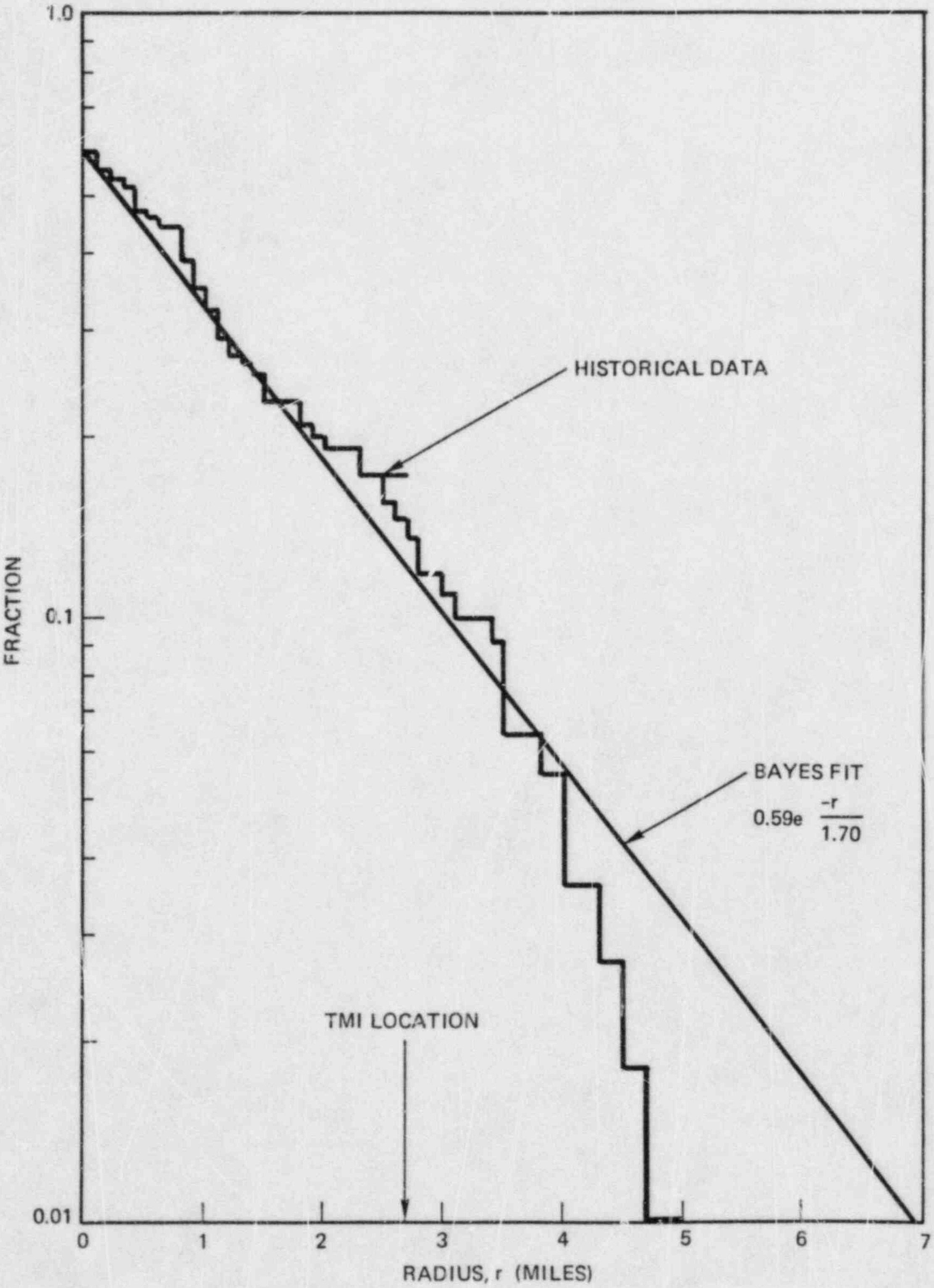

FIGURE 4-7. FRACTION OF CRASHES OCCURRING AT RADIUS r OR GREATER - TAKEOFFS AND LANDINGS COMBINED

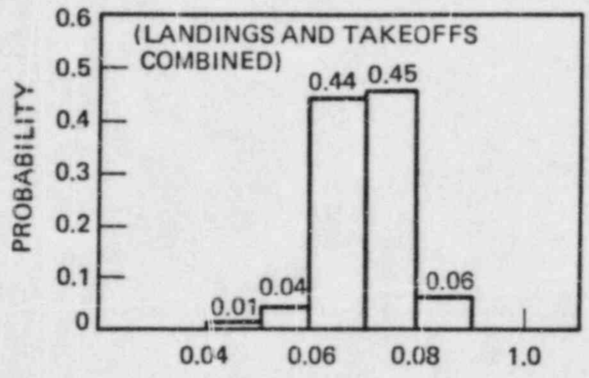

FIGURE 4-8a. THE QUANTITY $\left[\frac{-d}{dr} R(r)\right]_{r = 2.7}$

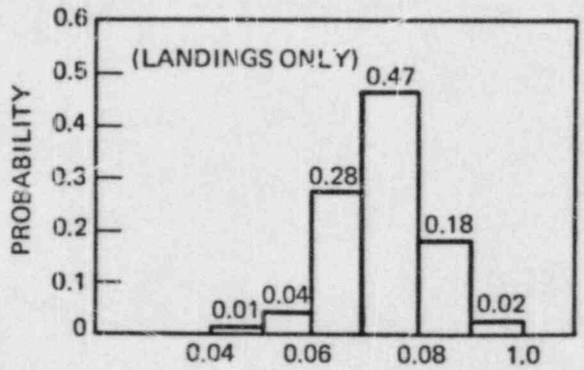

FIGURE 4-8b. THE QUANTITY $\left[\frac{-d}{dr} R_L(r)\right]_{r = 2.7}$

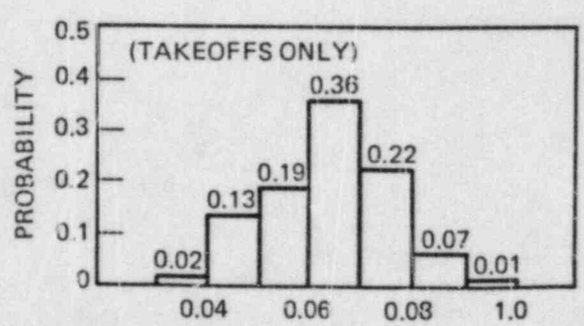

FIGURE 4-8c. THE QUANTITY $\left[\frac{-d}{dr} R_T(r)\right]_{r = 2.7}$

4.2.1.2 The Radial Densities for Landings, $\left[\frac{-d}{dr} R_L(r)\right]_{r=r_o}$, and Takeoffs, $\left[\frac{-d}{dr} R_T(r)\right]_{r=r_o}$

In this section we repeat the analysis of the previous section to determine the radial dependence separately for landing and takeoff accidents.

In the case of landings, B is the set of radii at which landing crashes occurred. Thus, from Table 3-2 we have:

$B = \{0, 0, 3.5, 0.8, 0.4, \ldots, \text{etc.}\}$

We note that B contains a total of 70 points; 27 points have r = 0, and the remainder have sum

$$\sum_{n=1}^{43} r_n = 73.8 \text{ miles} \tag{4.30}$$

Then, as in Equation (4.26), the probability of these 70 crashes occurring as they did is:

$$p(B|a_i,\lambda_j) = (1-a_i)^{27} \ (a_i\lambda_j)^{43} \ e^{\{-\lambda_j \ 73.8\}} \tag{4.31}$$

For this calculation the same a and λ sets were chosen as in the previous section.

The results are shown in Table 4-8 and Figure 4-8b. The Bayes' fit using the mean a, λ is shown as the straight line in Figure 4-9. The staircase function is the historical data.

In the case of takeoff crashes, B, from Table 3-2, is the set:

$B = \{0, 4.7, 0.9, 4.0, 3.1, 0.6, \ldots, \text{etc.}\}$

B for takeoff contains a total of 40 points, 18 having r = 0.

The remainder have the sum

$$\sum_{n=1}^{22} r_n = 34.9 \text{ miles} \tag{4.32}$$

TABLE 4-8. BAYESIAN RESULTS FOR RADIAL DISTRIBUTION - LANDING CRASHES ONLY

*** VALUES OF PARAMETER A ***

1/LAMBDA	4.00E-01	4.50E-01	5.00E-01	5.50E-01	6.00E-01	6.50E-01	7.00E-01	TOTAL
0.75	4.93E-13	7.45E-12	5.27E-11	1.85E-10	3.24E-10	2.75E-10	1.04E-10	9.47E-10
1.00	9.95E-08	1.50E-06	1.06E-05	3.73E-05	6.54E-05	5.55E-05	2.09E-05	1.91E-04
1.25	1.74E-05	2.63E-04	1.86E-03	6.53E-03	1.14E-02	9.71E-03	3.66E-03	3.35E-02
1.50	1.28E-04	1.94E-03	1.37E-02	4.81E-02	8.43E-02	7.16E-02	2.70E-02	2.47E-01
1.75	1.92E-04	2.90E-03	2.05E-02	7.19E-02	1.26E-01	1.07E-01	4.04E-02	3.69E-01
2.00	1.20E-04	1.81E-03	1.28E-02	4.49E-02	7.88E-02	6.69E-02	2.52E-02	2.31E-01
2.25	4 53E-05	6.84E-04	4.84E-03	1.70E-02	2.97E-02	2.52E-02	9.52E-03	8.70E-02
2.50	1.31E-05	1.98E-04	1.40E-03	4.91E-03	8.60E-03	7.30E-03	2.75E-03	2.52E-02
2.75	3.23E-06	4.89E-05	3.46E-04	1.21E-03	2.12E-03	1.80E-03	6.80E-04	6.22E-03
3.00	6.93E-07	1.05E-05	7.42E-05	2.60E-04	4.55E-04	3.87E-04	1.46E-04	1.33E-03
3.25	1.53E-07	2.31E-06	1.64E-05	5.73E-05	1.01E-04	8.54E-05	3.22E-05	2.94E-04
TOTALS	5.20E-04	7.86E-03	5.56E-02	1.95E-01	3.42E-01	2.90E-01	1.09E-01	1.00E+00

MEAN VALUE FOR A =0.609

MEAN VALUE FOR LAMBDA =0.569 = 1/1.76

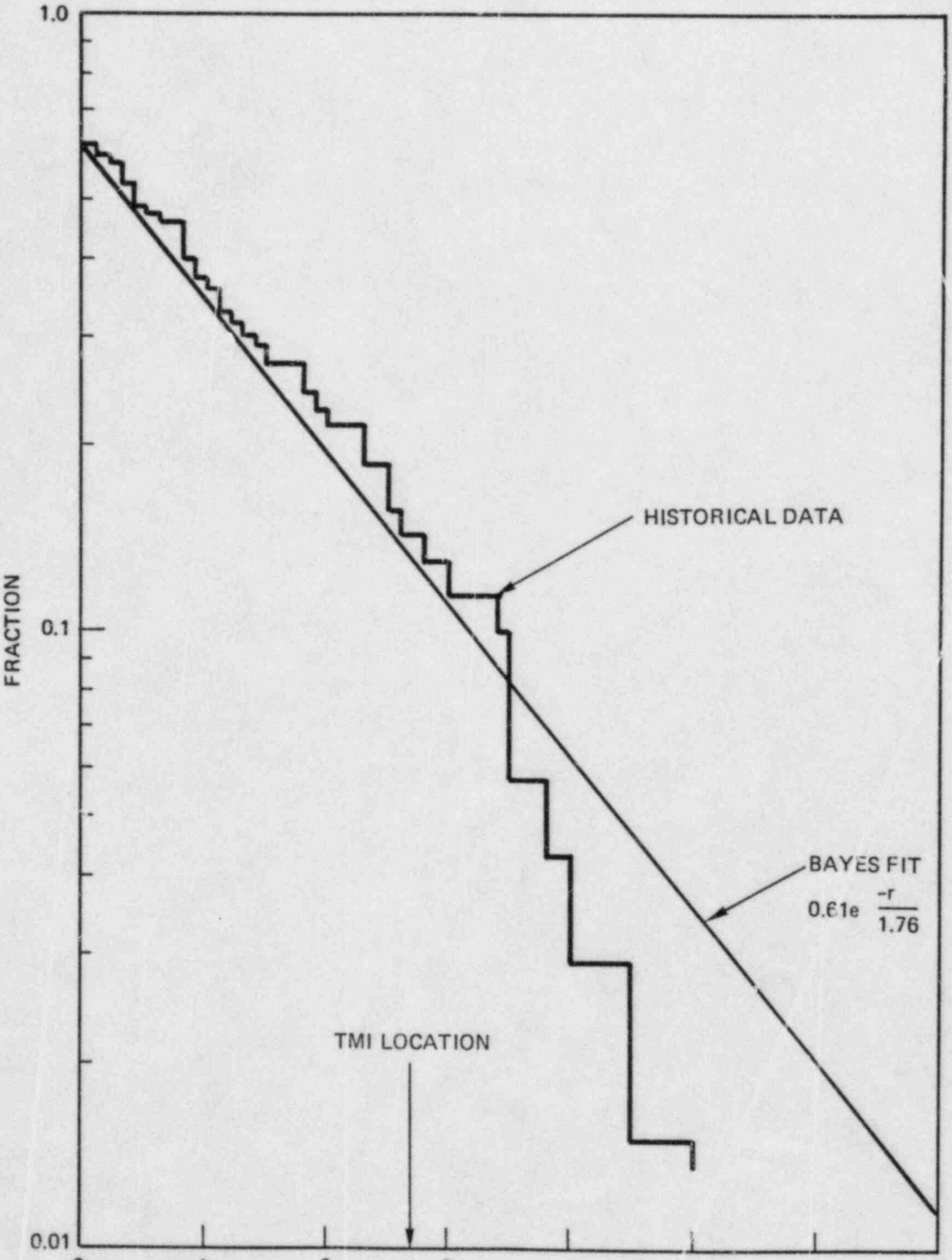

FIGURE 4-9. FRACTION OF LANDING CRASHES OCCURRING AT RADIUS r OR GREATER

The probability of these 40 takeoff crashes occurring as they did is

$$p(B|a_i,\lambda_j) = (1-a_i)^{18} \quad (a_i\lambda_j)^{22} \quad e^{\{-\lambda_j \quad 34.9\}} \tag{4.33}$$

The results for the takeoff calculation are shown in Table 4-9 and Figure 4-8c. Figure 4-10 compares the mean Bayes' fit with the historical takeoff crash data.

The mean value of the distribution of the radial density for various cases are summarized as follows:

Radial Density, Landings = 7.39×10^{-2}
Radial Density, Takeoffs = 6.40×10^{-2}
Radial Density, Combined = 7.04×10^{-2}

4.2.2 THE ANGULAR DENSITY $\left[\frac{d}{d\theta}\Theta(\theta)\right]_{\theta = \theta_0}$

The same kind of reasoning can now be applied to determine the θ dependence, using as data only those crashes occurring at a radius of $\geq$ .5 mile. However, it is evident from Figure 4-11 that a simple exponential is not going to give a good fit to the angular data. Therefore, we need to modify the procedure used for the radial dependence. In doing this, we need to recognize that the important point is that the fit be good in the neighborhood of 34°, the location of the plant. At the same time, we wish to include the experience at the extremes (0° and 90°) of the θ range. Finally, if we can, we prefer to retain a fitting fraction with two parameters, rather than the complication of a three or four-parameter form.

4.2.2.1 General Approach

The following approach appears to satisfy these requirements. We define $\Theta(\theta)$ as the fraction of crashes occurring at angle θ or greater.

We then choose the form

$$\Theta(\theta) = e^{-\lambda\theta} + b, \quad 0° \leq \theta \leq 70° \tag{4.34}$$

and use them to fit the data within the 0° to 70° range. Within this range, we may expect, from Figure 4-11, that this form has the flexibility to give a good fit. Outside the range, of course, it cannot fit since it levels off, whereas the actual data goes to zero. To blend in appropriately at θ = 70°, and to account for the data of 90°, we choose the following b value:

$$b = 0.098 \tag{4.35}$$

We then use a Bayesian procedure to establish probability distributions on a, λ in the following way.

TABLE 4-9. BAYESIAN RESULTS FOR RADIAL DISTRIBUTION - TAKEOFF CRASHES ONLY

*** VALUES OF PARAMETER A ***

1/LAMBDA	4.00E-01	4.50E-01	5.00E-01	5.50E-01	6.00E-01	6.50E-01	7.00E-01	TOTAL
0.75	3.83E-06	1.07E-05	1.95E-05	2.38E-05	1.94E-05	1.02E-05	3.25E-06	9.08E-05
1.00	7.64E-04	2.13E-03	3.89E-03	4.75E-03	3.87E-03	2.03E-03	6.48E-04	1.81E-02
1.25	6.05E-03	1.69E-02	3.08E-02	3.76E-02	3.06E-02	1.61E-02	5.13E-03	1.43E-01
1.50	1.15E-02	3.20E-02	5.84E-02	7.14E-02	5.81E-02	3.06E-02	9.73E-03	2.72E-01
1.75	1.07E-02	2.98E-02	5.45E-02	6.66E-02	5.42E-02	2.85E-02	9.08E-03	2.53E-01
2.00	6.87E-03	1.91E-02	3.50E-02	4.27E-02	3.48E-02	1.83E-02	5.82E-03	1.63E-01
2.25	3.55E-03	9.90E-03	1.81E-02	2.21E-02	1.80E-02	9.46E-03	3.01E-03	8.41E-02
2.50	1.66E-03	4.63E-03	8.45E-03	1.03E-02	8.40E-03	4.42E-03	1.41E-03	3.93E-02
2.75	7.32E-04	2.04E-03	3.73E-03	4.55E-03	3.71E-03	1.95E-03	6.21E-04	1.73E-02
3.00	3.05E-04	8.49E-04	1.55E-03	1.89E-03	1.54E-03	8.11E-04	2.58E-04	7.21E-03
3.25	1.31E-04	3.65E-04	6.66E-04	8.14E-04	6.63E-04	3.48E-04	1.11E-04	3.10E-03
TOTALS	4.22E-02	1.18E-01	2.15E-01	2.63E-01	2.14E-01	1.12E-01	3.58E-02	1.00E+00

MEAN VALUE FOR A =0.548

MEAN VALUE FOR LAMBDA =0.603 = 1/1.66

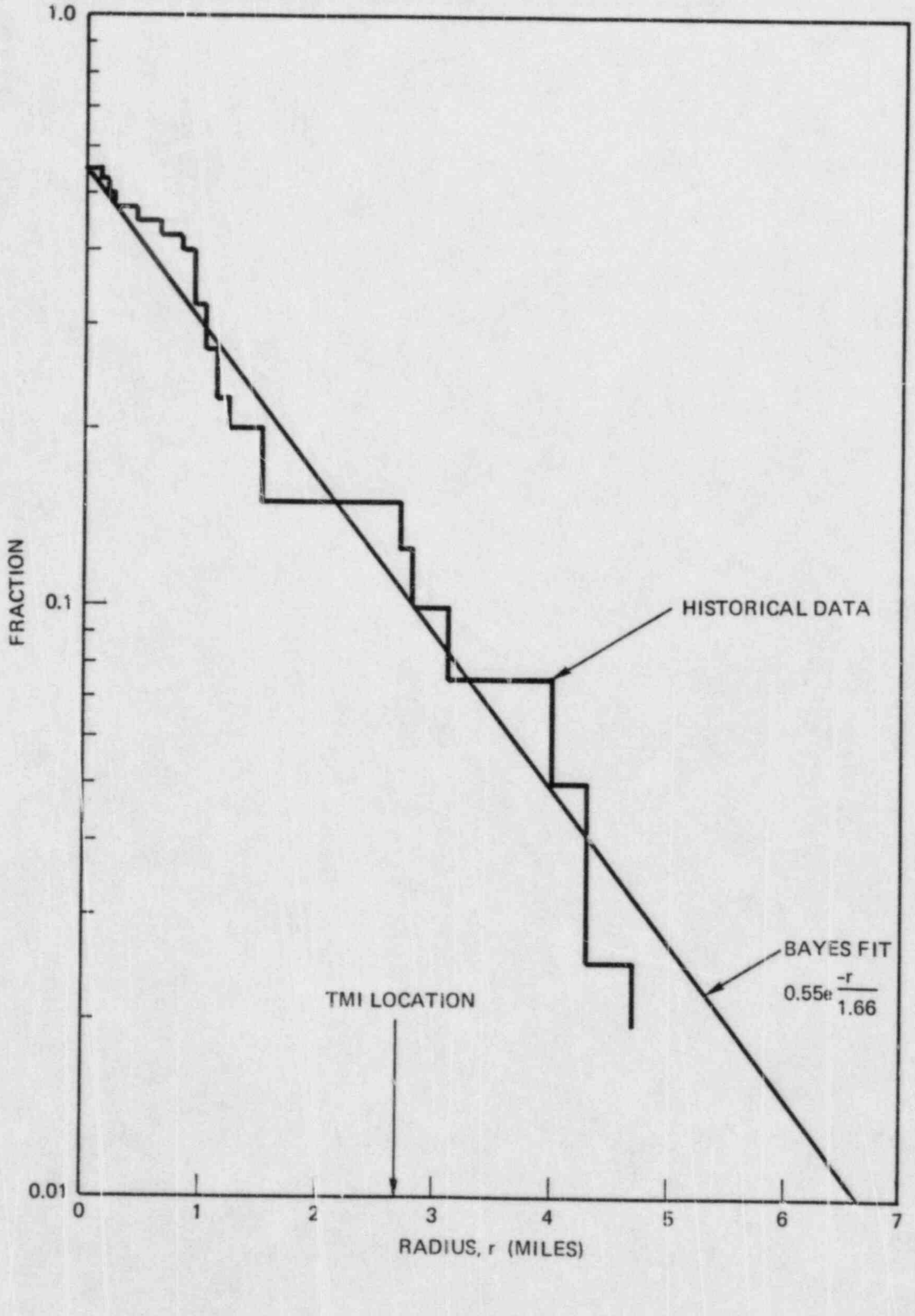

FIGURE 4-10. FRACTION OF TAKEOFF CRASHES OCCURRING AT RADIUS r OR GREATER

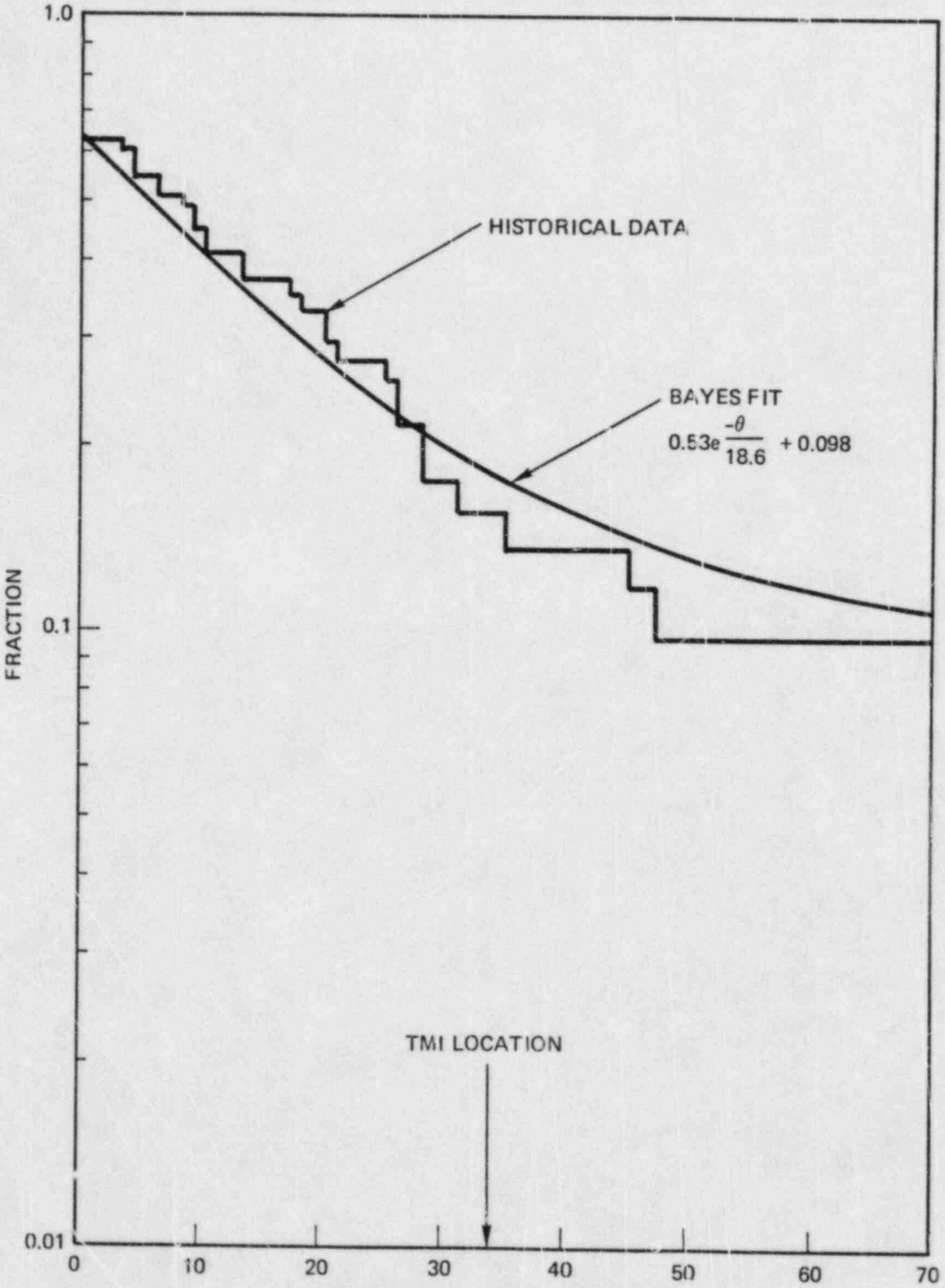

FIGURE 4-11. ANGULAR DISTRIBUTION OF CRASHES - LANDINGS AND TAKEOFFS COMBINED

From Equation (4.34) we have the frequency density

$$\left[\frac{-d}{d\theta}\,\Theta\,(\theta)\right] = (1-a-b)\ \delta(\Theta) + a\lambda e^{-\lambda\theta} \tag{4.36}$$

We now take B to be the set of crash points in the 0° to 70° range (and having $r \geq .5$ mile). Thus, from Table 3-2:

B = {0, 01, 47, 61, 0, 26, 0, ...}

a total of 46 crashes with 19 at $\theta = 0$. Thus,

$$p(B|a,\lambda) = (1-a-b)^{19}(a\lambda)^{27}\ e^{-\lambda}\quad \sum_{1}^{27}\ \Theta_i \tag{4.37}$$

where

$$\sum_{1}^{27} \theta_i = 486 \tag{4.38}$$

The resulting DPDs are shown in Table 4-10. The corresponding distribution for the desired derivative quantities is shown in Figure 4-12a. As a matter of interest, Figure 4-11 shows the goodness of fit using the mean a,λ, to the experimental data.

4.2.2.2 The Quantities $-\left[\frac{d}{d\theta}\,\Theta_L(\theta)\right]_{\theta_o}$ and $\left[\frac{d}{d\theta}\,\Theta_T(\theta)\right]_{\theta_o}$

We now apply the analysis of the previous section to the landing and takeoff data separately. For landings, there is a total of 34 crashes; 15 at $\theta = 0$ and 2 at 90°. We, therefore, set

$$b = \frac{2}{34} = .059 \tag{4.39}$$

and summing over the points less than 90°, we have

$$\sum_{i=1}^{17}\ \theta_i = 230 \tag{4.40}$$

The resulting distribution over the (a,λ) space is shown in Table 4-11. The histogram for the desired derivative is plotted as Figure 4-12b. The Bayes' fit with average a,λ is plotted with the historical data in Figure 4-13.

TABLE 4-10. BAYESIAN RESULTS FOR ANGULAR DISTRIBUTION - LANDINGS AND TAKEOFFS COMBINED

*** VALUES OF PARAMETER A ***   B=0.098

1/LAMBDA	*	2.00E-01	3.00E-01	4.00E-01	5.00E-01	6.00E-01	7.00E-01	TOTAL
	*							
5	*	7.52E-23	2.31E-19	1.73E-17	1.05E-16	6.29E-17	1.94E-18	
10	*	7.17E-10	2.20E-06	1.64E-04	9.99E-04	5.99E-04	1.85E-05	1.87E-16
15	*	1.38E-07	4.22E-04	3.16E-02	1.92E-01	1.15E-01	3.55E-03	1.78E-03
20	*	1.91E-07	5.85E-04	4.38E-02	2.66E-01	1.60E-01	4.92E-03	3.42E-01
25	*	5.96E-08	1.83E-04	1.37E-02	8.30E-02	4.98E-02	1.54E-03	4.75E-01
30	*	1.10E-08	3.36E-05	2.51E-03	1.53E-02	9.15E-03	2.82E-04	1.48E-01
35	*	1.77E-09	5.42E-06	4.06E-04	2.46E-03	1.48E-03	4.56E-05	2.73E-02
40	*	2.69E-10	8.24E-07	6.17E-05	3.75E-04	2.25E-04	6.93E-06	4.40E-03
45	*	4.25E-11	1.30E-07	9.74E-06	5.92E-05	3.55E-05	1.09E-06	6.69E-04
50	*	7.39E-12	2.26E-08	1.70E-06	1.03E-05	6.17E-06	1.90E-07	1.06E-04
	*							1.84E-05
TOTALS	*	4.02E-07	1.23E-03	9.22E-02	5.60E-01	3.36E-01	1.04E-02	1.00E+00

MEAN VALUE FOR A =0.526

MEAN VALUE FOR LAMBDA =0.0537 = 1/18.6

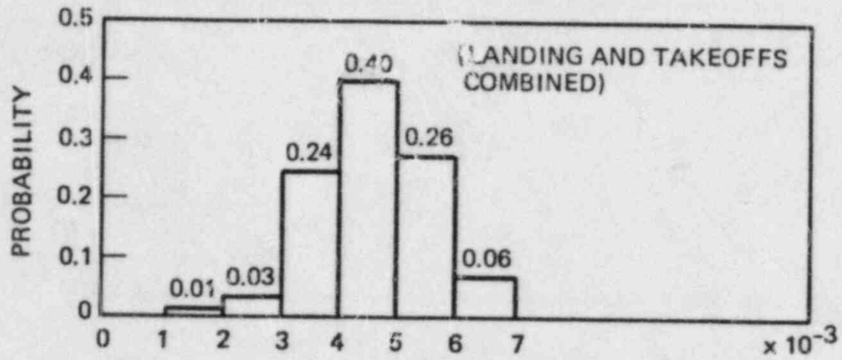

FIGURE 4-12a. THE QUANTITY $\left[\frac{-d}{d\theta}\Theta(\theta)\right]_{\theta = 34°}$

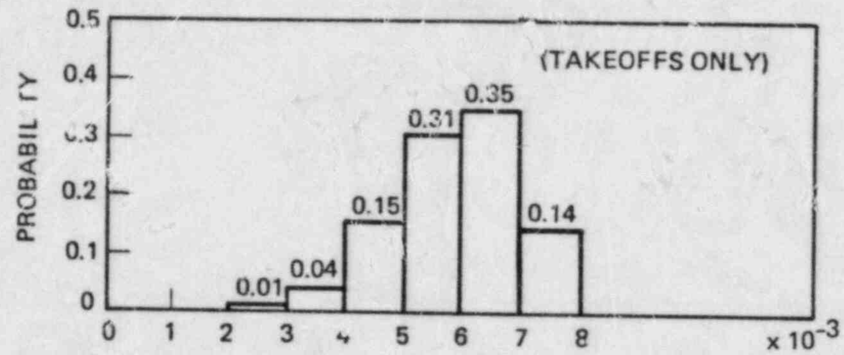

FIGURE 4-12b. THE QUANTITY $\left[\frac{-d}{d\theta}\Theta_T(\theta)\right]_{\theta = 34°}$

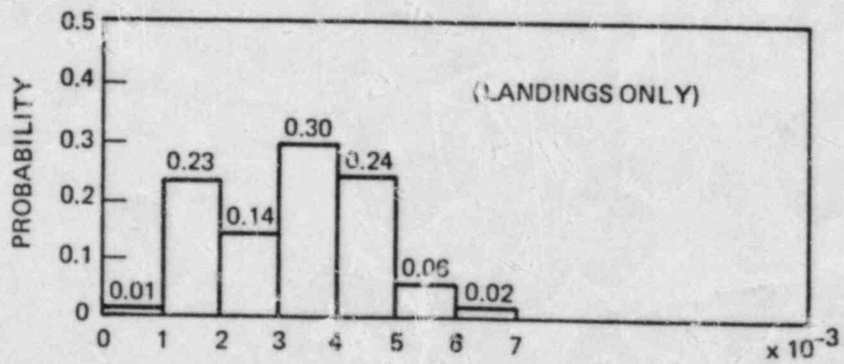

FIGURE 4-12c. THE QUANTITY $\left[\frac{-d}{d\theta}\Theta_L(\theta)\right]_{\theta = 34°}$

TABLE 4-11. BAYESIAN RESULTS FOR ANGULAR DISTRIBUTION - LANDINGS ONLY

*** VALUES OF PARAMETER A ***   B=5. 90E-02

1/LAMBDA	*	2. 00E-01	3. 00E-01	4. 00E-01	5. 00E-01	6. 00E-01	7. 00E-01	TOTAL
	*							
5	*	6. 61E-10	7. 40E-08	7. 73E-07	1. 60E-06	7. 50E-07	5. 65E-08	3. 26E-06
10	*	4. 91E-05	5. 50E-03	5. 75E-02	1. 19E-01	5. 58E-02	4. 20E-03	2. 42E-01
15	*	1. 06E-04	1. 19E-02	1. 24E-01	2. 58E-01	1. 21E-01	9. 09E-03	5. 24E-01
20	*	3. 70E-05	4. 14E-03	4. 33E-02	8. 96E-02	4. 20E-02	3. 16E-03	1. 82E-01
25	*	8. 31E-06	9. 31E-04	9. 72E-03	2. 01E-02	9. 43E-03	7. 11E-04	4. 09E-02
30	*	1. 72E-06	1. 93E-04	2. 01E-03	4. 17E-03	1. 95E-03	1. 47E-04	8. 47E-03
35	*	3. 82E-07	4. 27E-05	4. 46E-04	9. 24E-04	4. 33E-04	3. 26E-05	1. 88E-03
40	*	8. 87E-08	9. 93E-06	1. 04E-04	2. 15E-04	1. 01E-04	7. 59E-06	4. 37E-04
45	*	2. 24E-08	2. 51E-06	2. 62E-05	5. 43E-05	2. 55E-05	1. 92E-06	1. 10E-04
50	*	6. 31E-09	7. 06E-07	7. 38E-06	1. 53E-05	7. 16E-06	5. 40E-07	3. 11E-05
75	*	2. 86E-11	3. 21E-09	3. 35E-08	6. 94E-08	3. 25E-08	2. 45E-09	1. 41E-07
100	*	4. 80E-13	5. 37E-11	5. 62E-10	1. 16E-09	5. 45E-10	4. 11E-11	2. 36E-09
125	*	1. 71E-14	1. 92E-12	2. 00E-11	4. 15E-11	1. 94E-11	1. 46E-12	8. 43E-11
	*							
TOTALS	*	2. 03E-04	2. 27E-02	2. 37E-01	4. 92E-01	2. 30E-01	1. 74E-02	1. 00E+00

MEAN VALUE FOR A =4. 9816E-01

MEAN VALUE FOR LAMBDA =7. 0191E-02 =1/14. 2

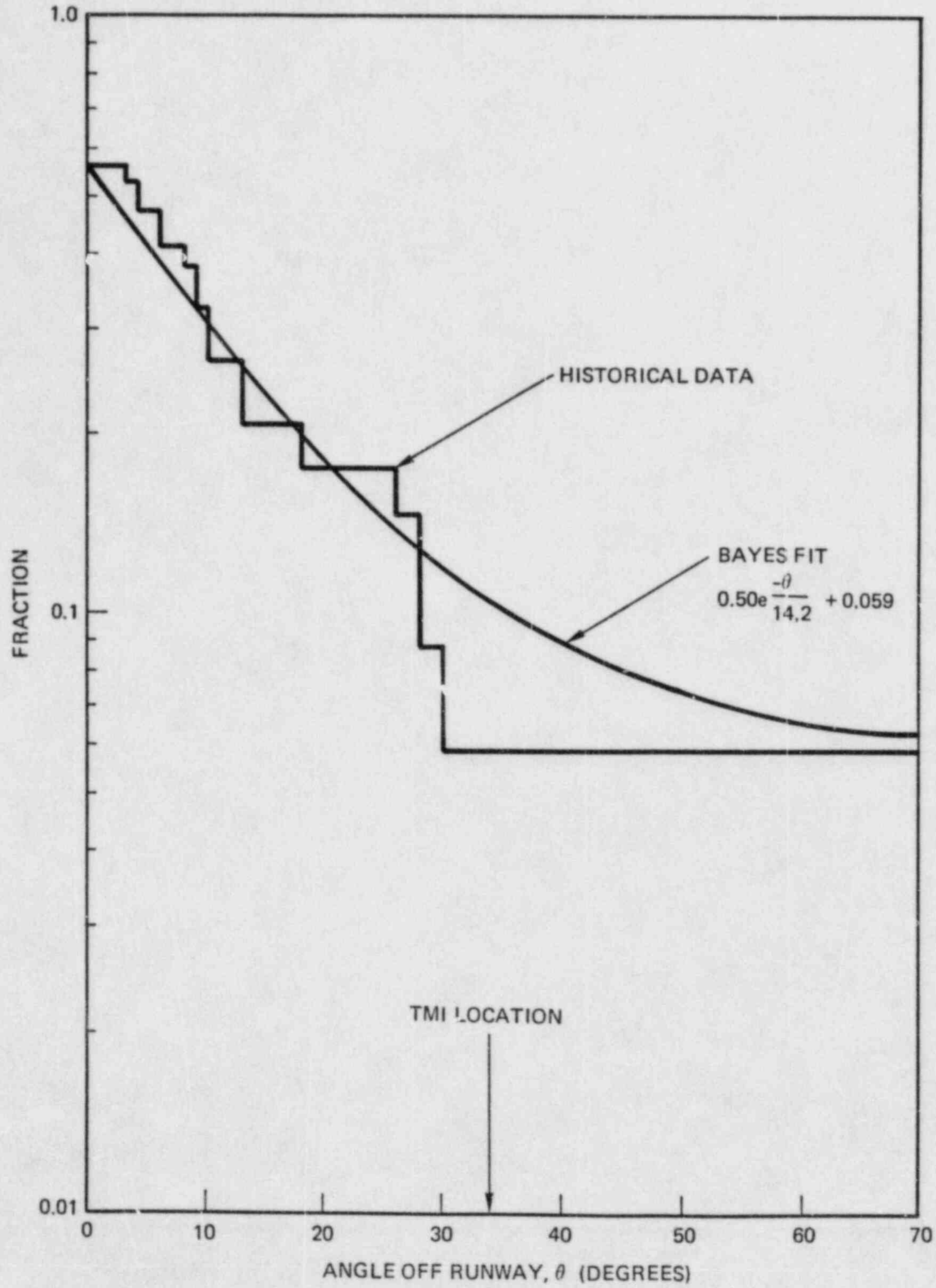

FIGURE 4-13. ANGULAR DISTRIBUTION OF LANDING CRASHES

For takeoffs, there is a total of 17 crashes; 4 at $\theta = 0$ and 3 at 90°. In this case

$$b = \frac{3}{17} = .176 \qquad (4.41)$$

The sum of the angles in this case is

$$\sum_{i=1}^{10} \theta_i = 256 \qquad (4.42)$$

The results are given in Table 4-12 and Figures 4-12c and 4-14. The mean value of the distribution of the angular density for various cases are summarized as follows:

Angular Density, Landings = 3.31×10^{-3}
Angular Density, Takeoff = 5.75×10^{-3}
Angular Density, Combined = 4.52×10^{-3}

TABLE 4-12. BAYESIAN RESULTS FOR ANGULAR DISTRIBUTION - TAKEOFFS ONLY

*** VALUES OF PARAMETER A *** B=0.176

1/LAMBDA	*	2.00E-01	3.00E-01	4.00E-01	5.00E-01	6.00E-01	7.00E-01	TOTAL
5	*	1.57E-15	4.50E-14	3.42E-13	1.09E-12	1.54E-12	6.75E-13	3.69E-12
10	*	2.01E-07	5.76E-06	4.39E-05	1.39E-04	1.97E-04	8.64E-05	4.73E-04
15	*	1.78E-05	5.11E-04	3.89E-03	1.24E-02	1.75E-02	7.67E-03	4.19E-02
20	*	7.11E-05	2.04E-03	1.55E-02	4.93E-02	6.97E-02	3.06E-02	1.67E-01
25	*	9.87E-05	2.63E-03	2.16E-02	6.84E-02	9.68E-02	4.25E-02	2.32E-01
30	*	8.77E-05	2.52E-03	1.92E-02	6.08E-02	8.60E-02	3.77E-02	2.06E-01
35	*	6.38E-05	1.83E-03	1.39E-02	4.42E-02	6.26E-02	2.75E-02	1.50E-01
40	*	4.18E-05	1.20E-03	9.12E-03	2.90E-02	4.10E-02	1.80E-02	9.83E-02
45	*	2.61E-05	7.48E-04	5.69E-03	1.81E-02	2.56E-02	1.12E-02	6.14E-02
50	*	1.61E-05	4.63E-04	3.52E-03	1.12E-02	1.58E-02	6.94E-03	3.79E-02
75	*	1.52E-06	4.35E-05	3.31E-04	1.05E-03	1.49E-03	6.52E-04	3.57E-03
100	*	2.04E-07	5.84E-06	4.45E-05	1.41E-04	2.00E-04	8.77E-05	4.79E-04
125	*	3.65E-08	1.05E-06	7.97E-06	2.53E-05	3.58E-05	1.57E-05	8.59E-05
TOTALS	*	4.25E-04	1.22E-02	9.28E-02	2.95E-01	4.17E-01	1.83E-01	1.00E+00

MEAN VALUE FOR A =0.566

MEAN VALUE FOR LAMBDA =0.0363 = 1/27.5

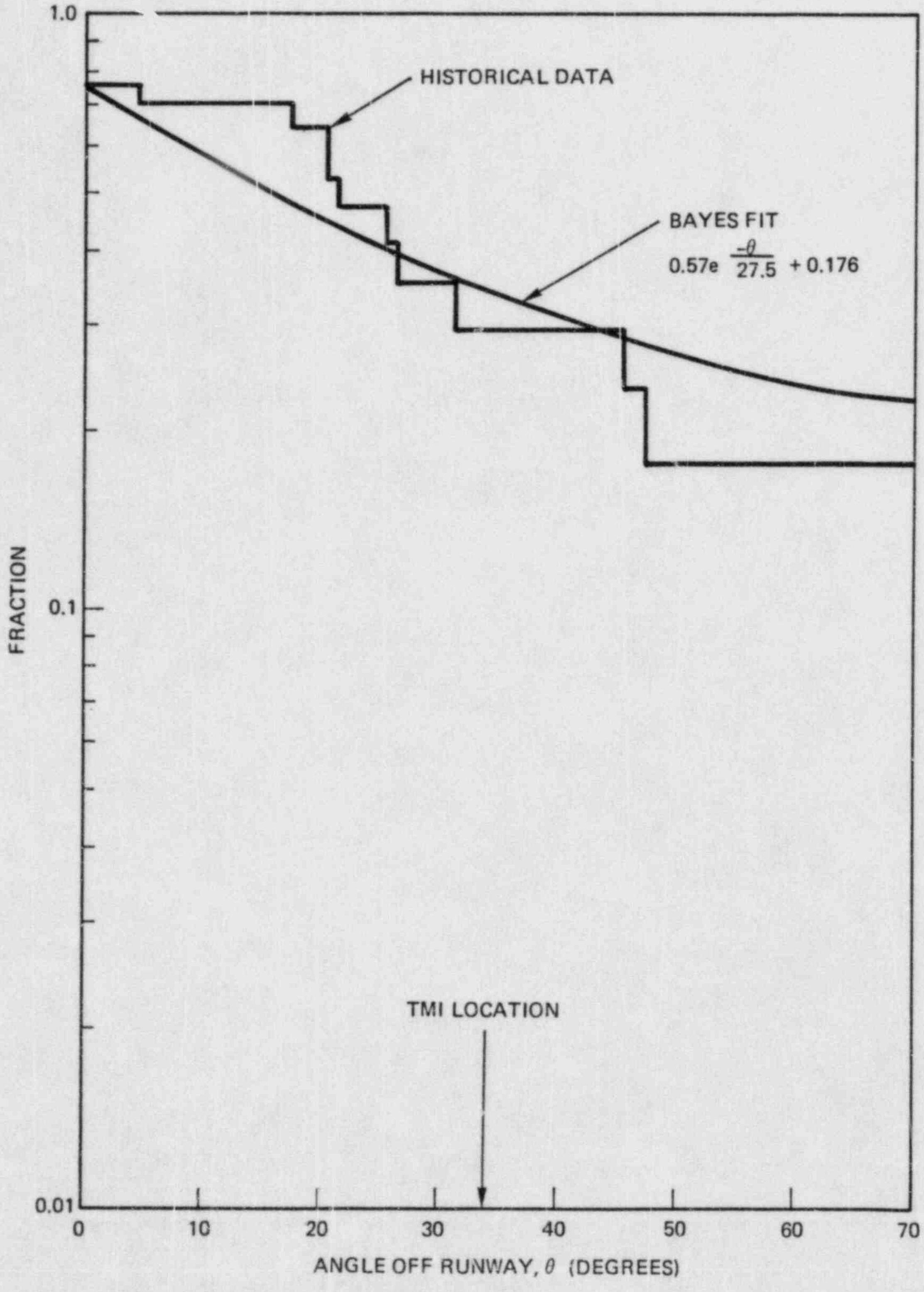

FIGURE 4-14. ANGULAR DISTRIBUTION OF TAKEOFF CRASHES

5. REFERENCES

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