

GENERAL ELECTRIC CO.
Nuclear Energy Business Operations
ENGINEERING CALCULATION SHEET

NUMBER _____ DATE October 1992
SUBJECT ABWR Certification Program BY Dk Henrie SHEET 1 OF 12

ATTACHMENT A

THEORETICAL BASIS

INDEPENDENT SUPPORT MOTION (ISM) TIME HISTORY ANALYSIS

BY

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1.0 NOMENCLATURE

- $\underline{\alpha}$ - Column vector denoted by alpha character underscored by a single bar.
- $\underline{\alpha}^T$ - Row vector denoted by the transpose of a column vector.
- $\underline{\underline{\alpha}}$ - Rectangular matrix denoted by an alpha character underscored by two bars.
- $\underline{\underline{\alpha}}^{-1}$ - Inverse.
- $\underline{\alpha}^T$ - Transpose.
- $\underline{\underline{I}}$ - Diagonal matrix.
- $\underline{\underline{m}}$ - Dynamic model mass matrix, $(n \times n)$.
- $\underline{\underline{K}}$ - Dynamic model stiffness matrix, $(n \times n)$.
- $\underline{\underline{C}}$ - Dynamic model damping matrix, $(n \times n)$.
- $\underline{\underline{x}}$ - Dynamic model nodal displacement vector the components of which include all support and non-support degrees-of-freedom (DOFs), $(n \times 1)$.
- $\underline{\dot{x}}$ - Dynamic model nodal velocity vector, $(n \times 1)$.
- $\underline{\ddot{x}}$ - Dynamic model nodal acceleration vector, $(n \times 1)$.
- $\underline{\underline{x}}^1$ - Dynamic model nodal displacement vector the components of which correspond to the non-support DOFs, $((n-k) \times 1)$.
- $\underline{\underline{x}}^2$ - Dynamic model nodal displacement vector the components of which correspond to the support DOFs, $(k \times 1)$.

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$\underline{F}(t)$ - Dynamic model applied load vector, $(n \times 1)$

\underline{x}_d^1 - Dynamic part of model non-support dof's nodal displacement vector, $((n-k) \times 1)$.

\underline{x}_s^1 - Pseudo static part of model non-support dof's nodal displacement vector, $((n-k) \times 1)$.

Note: All displacement, velocity and acceleration nodal displacement vectors above are absolute; i.e., they are all relative to an inertial reference frame.

$\underline{0}$ - Null column vector all components of which are zero.

$\underline{0}$ - Null matrix all components of which are zero.

$\underline{q}(t)$ - Generalized coordinate vector the components of which are the Generalized Coordinates $q_1(t), q_2(t), \dots, q_n(t)$.

$\underline{\Phi}$ - Matrix of eigen vectors the column vectors of which are the eigenvectors of the dynamic model obtained from the eigenanalysis.

$\underline{\Phi}^n$ - The n^{th} mode eigenvector.

$\underline{\omega}^2$ - Diagonal matrix of dynamic model eigenvalues

$\underline{\omega}$ - Diagonal matrix of dynamic model natural frequencies, rad/sec.

\underline{M} - Diagonal Generalized Mass Matrix.

\underline{K} - Diagonal Generalized Stiffness Matrix.

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- $\underline{\underline{D}}$ - Diagonal Generalized Damping Matrix.
- $\underline{\underline{\lambda}}$ - Diagonal Modal Damping Matrix, each component of which is the modal damping ratio with respect to critical.
- $\underline{\underline{I}}_1^i$ - Column vector ($n \times 1$) in which the i^{th} component, corresponding to the dof of the applied load is unity and all other components are zero.
- $\underline{\underline{I}}_2^k$ - Column vector ($n \times 1$) in which the component corresponding to the k^{th} component of the ISM is unity and all other components are zero.
- $F_i(t)$ - Scalar force time history applied to the dynamic model i^{th} model degree-of-freedom.
- $\ddot{x}_k^2(t)$ - Scalar acceleration time history corresponding to the k^{th} component of the ISM excitation.
- $\underline{\underline{P}}_r^i$ - The r^{th} mode Force Participation Factor corresponding to the i^{th} dof component of the dynamic model applied load vector ($i = 1, 2, \dots, n$).
- $\underline{\underline{P}}_r^k$ - The r^{th} mode Modal Participation Factor corresponding to the k^{th} component of ISM.

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2.0 ISM EQUATIONS OF MOTION

The governing equations of motion for the ISM method of analysis are given by

$$\underline{m} \ddot{\underline{x}} + \underline{c} \dot{\underline{x}} + \underline{k} \underline{x} = \underline{F}(t) \quad \dots (1)$$

Partition (1) to separate the non-support and the support degrees-of-freedom.

$$\begin{bmatrix} \underline{m}_{11} & \underline{m}_{12} \\ \underline{m}_{21} & \underline{m}_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{x}}^1 \\ \ddot{\underline{x}}^2 \end{Bmatrix} + \begin{bmatrix} \underline{c}_{11} & \underline{c}_{12} \\ \underline{c}_{21} & \underline{c}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\underline{x}}^1 \\ \dot{\underline{x}}^2 \end{Bmatrix} + \begin{bmatrix} \underline{k}_{11} & \underline{k}_{12} \\ \underline{k}_{21} & \underline{k}_{22} \end{bmatrix} \begin{Bmatrix} \underline{x}^1 \\ \underline{x}^2 \end{Bmatrix} = \begin{Bmatrix} \underline{F}_1(t) \\ \underline{F}_2(t) \end{Bmatrix} \quad \dots (2)$$

Decompose the nodal displacement vector corresponding to the non-support dof's into dynamic and pseudo-static components as follows

$$\underline{x}^1 = \underline{x}_d^1 + \underline{x}_s^1 \quad \dots (3)$$

Substitute (3) into (2) to yield

$$\begin{bmatrix} \underline{m}_{11} & \underline{m}_{12} \\ \underline{m}_{21} & \underline{m}_{22} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{x}}_d^1 + \ddot{\underline{x}}_s^1 \\ \ddot{\underline{x}}^2 \end{Bmatrix} + \begin{bmatrix} \underline{c}_{11} & \underline{c}_{12} \\ \underline{c}_{21} & \underline{c}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\underline{x}}_d^1 + \dot{\underline{x}}_s^1 \\ \dot{\underline{x}}^2 \end{Bmatrix} + \begin{bmatrix} \underline{k}_{11} & \underline{k}_{12} \\ \underline{k}_{21} & \underline{k}_{22} \end{bmatrix} \begin{Bmatrix} \underline{x}_d^1 + \underline{x}_s^1 \\ \underline{x}^2 \end{Bmatrix} = \begin{Bmatrix} \underline{F}_1 \\ \underline{F}_2 \end{Bmatrix} \quad \dots (4)$$

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The first equation implicit to (4) becomes

$$\underline{m}_{11} \ddot{\underline{x}}_1^1 + \underline{m}_{11} \ddot{\underline{x}}_2^1 + \underline{m}_{12} \ddot{\underline{x}}^2 + \underline{c}_{11} \dot{\underline{x}}_1^1 + \underline{c}_{11} \dot{\underline{x}}_2^1 + \underline{c}_{12} \dot{\underline{x}}^2 + \underline{k}_{11} \underline{x}_1^1 + \underline{k}_{11} \underline{x}_2^1 + \underline{k}_{12} \underline{x}^2 = \underline{F}_1 \quad \dots (5)$$

Assuming no inertial coupling between the non-support and the support d.o.f.s means $\underline{m}_{12} = \emptyset$. Therefore

$$\underline{m}_{12} \ddot{\underline{x}}^2 = \emptyset \quad \dots (6)$$

Next it is assumed that

$$\underline{k}_{11} \underline{x}_s^1 + \underline{k}_{12} \underline{x}^2 = \emptyset \quad \dots (7)$$

which yields

$$\underline{x}_s^1 = - \underline{k}_{11}^{-1} \underline{k}_{12} \underline{x}^2 \quad \dots (8)$$

The pseudo static nodal response vector $\underline{x}_s^1(t)$ of all non-support degrees-of-freedom is then calculated by (8) in terms of the motion of the support degrees-of-freedom $\underline{x}^2(t)$.

Finally, it is assumed that

$$\underline{c}_{11} \dot{\underline{x}}_s^1 + \underline{c}_{12} \dot{\underline{x}}^2 = \emptyset \quad \dots (9)$$

Equation (9) is a higher order approximation in which (1) the higher order damping forces, $\underline{c}_{11} \dot{\underline{x}}_s^1$, associated with the pseudo-static part of the response are neglected, and (2) the cross-coupling damping forces, $\underline{c}_{12} \dot{\underline{x}}^2$, between the support and non-support degrees-of-freedom are also neglected. These assumptions (i.e., approximations) are very good

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for lightly damped systems, such as piping, for which the system damping is less than say 10% of critical.

Substituting (6), (7) and (9) into (5) yields

$$\underline{m}_{s1} \ddot{\underline{x}}_d^1 + \underline{C}_{s1} \dot{\underline{x}}_d^1 + \underline{K}_{s1} \underline{x}_d^1 = \underline{F}_1 + \underline{m}_{s1} \underline{K}_{s1}^{-1} \underline{K}_{s2} \ddot{\underline{x}}^e(t) \quad \dots (10)$$

Note: The second matrix equation implicit to (4) is given by

$$\underline{m}_{s2} (\ddot{\underline{x}}_d^1 + \ddot{\underline{x}}_s^1) + \underline{m}_{e2} \ddot{\underline{x}}^2 + \underline{C}_{s2} (\dot{\underline{x}}_d^1 + \dot{\underline{x}}_s^1) + \underline{C}_{e2} \dot{\underline{x}}^2 + \underline{K}_{s2} (\underline{x}_d^1 + \underline{x}_s^1) + \underline{K}_{e2} \underline{x}^2 = \underline{F}_2 \quad \dots (11)$$

The ISM support motion time history vector $\underline{x}^2(t)$ is given or known. Also $\underline{x}_s^1(t)$ is given in terms of $\underline{x}^2(t)$ by (8) and $\underline{x}_d^1(t)$ is obtained as the solution of (10). Equation (11) contains no unknown time history motions and is satisfied for the $\underline{x}^2(t)$, $\underline{x}_s^1(t)$ and $\underline{x}_d^1(t)$ displacement time history vectors. Also $\underline{x}^2(t)$ is implicitly dependent on \underline{m}_{e2} , \underline{C}_{e2} , \underline{K}_{e2} , \underline{m}_{s2} , \underline{C}_{s2} and \underline{K}_{s2} which are all generally unknown, except for \underline{K}_{s2} , for the ISM analysis.

Equation (10) is now solved by the Method of Normal Modes; i.e., by modal superposition.

3.0 EIGEN TRANSFORMATION

The equation of motions (10) for the dynamic response of the non-support

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degrees-of-freedom are transformed to the generalized coordinate system via the matrix of the eigenvectors $\underline{\Phi}$. The dynamic model \underline{m}_{11} , \underline{c}_{11} and \underline{k}_{11} matrices are diagonal in the generalized coordinate system. Consequently, the equations of motion are decoupled in the generalized coordinate system. The Eigen Transformation is defined by

$$\underline{x}_d^1 = \underline{\Phi} \underline{q} \quad \dots (12)$$

4.0 ORTHOGONALITY CONDITIONS

$$\underline{\Phi}^T \underline{m}_{11} \underline{\Phi} = \underline{M}_1 = \underline{I}_1 \quad \dots (13)$$

$$\underline{\Phi}^T \underline{k}_{11} \underline{\Phi} = \underline{K}_1 = \underline{\omega}_1^2 \underline{M}_1 \quad \dots (14)$$

$$\underline{\Phi}^T \underline{c}_{11} \underline{\Phi} = \underline{C}_1 = 2 \underline{\lambda}_1 \underline{\omega}_1 \underline{M}_1 \quad \dots (15)$$

5.0 SOLUTION OF EQUATIONS OF MOTION (10)

Substitute the Eigen Transformation (12) into (10) and premultiply through the resulting equations by $\underline{\Phi}^T$ and, in turn, apply the orthogonality conditions (13), (14) and (15) to yield

$$\underline{M}_1 \ddot{\underline{q}} + 2 \underline{\lambda}_1 \underline{\omega}_1 \underline{M}_1 \dot{\underline{q}} + \underline{\omega}_1^2 \underline{M}_1 \underline{q} = \underline{\Phi}^T \underline{F}_1 + \underline{\Phi}^T \underline{m}_{11} \underline{K}_{11}^{-1} \underline{K}_{12} \ddot{\underline{x}}^2(t) \quad \dots (16)$$

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Pre multiply through (16) by \underline{M}^{-1} to yield

$$\ddot{\underline{q}} + 2\underline{\lambda} \underline{\omega} \dot{\underline{q}} + \underline{\omega}^2 \underline{q} = \underline{M}^{-1} \underline{\Phi}^T \underline{F}_1 + \underline{M}^{-1} \underline{\Phi}^T \underline{m}_{11} \underline{K}_{11}^{-1} \underline{K}_{12} \ddot{\underline{x}}^2(t) \quad \dots (17)$$

Take

$$\begin{aligned} \underline{F}_1 &= \underline{I}_1^1 \underline{F}_1(t) + \underline{I}_1^2 \underline{F}_2(t) + \dots + \underline{I}_1^n \underline{F}_n(t) \\ &= \underline{I}_1^i \underline{F}_i(t) \quad , \quad (i=1,2,\dots,n) \end{aligned} \quad \dots (18)$$

and

$$\ddot{\underline{x}}^2 = \underline{I}_2^k \ddot{x}_k^2(t) \quad , \quad (k=1,2,\dots,k) \quad \dots (19)$$

The Repeated Indices tensor summation convention is utilized in Equations (18) and (19). Substituting (18) and (19) into (17) yields

$$\ddot{\underline{q}} + 2\underline{\lambda} \underline{\omega} \dot{\underline{q}} + \underline{\omega}^2 \underline{q} = \underline{M}^{-1} \underline{\Phi}^T \underline{I}_1^i \underline{F}_i(t) + \underline{M}^{-1} \underline{\Phi}^T \underline{m}_{11} \underline{K}_{11}^{-1} \underline{K}_{12} \underline{I}_2^k \ddot{x}_k^2(t) \quad \dots (20)$$

The n^{th} decoupled equation from (20) becomes

$$\ddot{q}_n + 2\lambda_n \omega_n \dot{q}_n + \omega_n^2 q_n = \frac{\underline{\Phi}_n^T \underline{I}_1^i}{\underline{\Phi}_n^T \underline{m}_{11} \underline{\Phi}_n} \underline{F}_i(t) + \frac{\underline{\Phi}_n^T \underline{m}_{11} \underline{K}_{11}^{-1} \underline{K}_{12} \underline{I}_2^k}{\underline{\Phi}_n^T \underline{m}_{11} \underline{\Phi}_n} \ddot{x}_k^2(t) \quad \dots (21)$$

Note that the denominator of the two terms on the right side of (21) is the n^{th} mode generalized mass. Also, the bar which underscores the n subscript is used to nullify the repeated indice tensor summation convention.

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Define

$$\underline{P}_r^i \equiv \underline{\phi}_r^T \underline{I}_1^i \quad \dots (22)$$

= The r^{th} mode Force Participation Factor corresponding to the i^{th} dof component of the dynamic model applied load vector ($i = 1, 2, \dots, n$).

and

$$\underline{\Gamma}_r^k = \underline{\phi}_r^T \underline{m}_{11}^{-1} \underline{k}_{11} \underline{k}_{12} \underline{I}_2^k \quad \dots (23)$$

= The r^{th} mode Modal Participation Factor corresponding to the k^{th} component of ISM.

Substitute (22) and (23) into (21) to yield the final form of the r^{th} decoupled equations of motion; i.e.,

$$\ddot{z}_r + 2\lambda_r \omega_r \dot{z}_r + \omega_r^2 z_r = \frac{\underline{P}_r^i}{M_r} F_i(t) + \frac{\underline{\Gamma}_r^k}{M_r} \ddot{x}_k^2(t) \quad \dots (24)$$

The Convolution Solution of (24) then becomes

$$\begin{aligned} z_r(t) &= \frac{\underline{P}_r^i}{M_r \omega_r} \int_0^t F_i(\tau) e^{-\lambda_r \omega_r(t-\tau)} \sin \omega_r(t-\tau) d\tau \\ &+ \frac{\underline{\Gamma}_r^k}{M_r \omega_r} \int_0^t \ddot{x}_k^2(\tau) e^{-\lambda_r \omega_r(t-\tau)} \sin \omega_r(t-\tau) d\tau \quad \dots (25) \end{aligned}$$

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where z is summed from 1 to n and k from 1 to k .

The final solution for the absolute nodal displacement vector, $\underline{x}^1(t)$, of dynamic model non-support degree-of-freedom is obtained by substituting $\underline{x}_s^1(t)$ from (8) and $\underline{x}_d^1(t)$ from the Eigen Transformation (12) into (3) to yield

$$\underline{x}^1(t) = \underline{x}_d^1(t) + \underline{x}_s^1(t) \quad \dots (26)$$

which becomes

$$\underline{x}^1(t) = \underline{\phi} \underline{q}(t) - \underline{K}_{11}^{-1} \underline{K}_{12} \underline{x}^2(t) \quad \dots (27)$$

6.0 NODAL ABSOLUTE ACCELERATION TIME HISTORY VECTOR

Taking the second derivative of (27) with respect to time and substitute $\underline{\ddot{q}}(t)$ from (17) into the resulting equation to yield

$$\begin{aligned} \underline{\ddot{x}}^1(t) &= \underline{\phi} \left[-2\underline{\lambda}_s \underline{\omega}_s \dot{\underline{q}}(t) - \underline{\omega}_s^2 \underline{q}(t) + \underline{M}_s^{-1} \underline{\phi}^T \underline{F}_1(t) \right. \\ &\quad \left. + \underline{M}_s^{-1} \underline{\phi}^T \underline{m}_{11} \underline{K}_{11}^{-1} \underline{K}_{12} \underline{\ddot{x}}^2(t) \right] - \underline{K}_{11}^{-1} \underline{K}_{12} \underline{\ddot{x}}^2(t) \quad \dots (28) \end{aligned}$$

$$\begin{aligned} &= -2\underline{\phi} \underline{\lambda}_s \underline{\omega}_s \dot{\underline{q}}(t) - \underline{\phi} \underline{\omega}_s^2 \underline{q}(t) + \underline{\phi} \underline{M}_s^{-1} \underline{\phi}^T \underline{F}_1(t) \\ &\quad + \underline{\phi} \underline{M}_s^{-1} \underline{\phi}^T \underline{m}_{11} \underline{K}_{11}^{-1} \underline{K}_{12} \underline{\ddot{x}}^2(t) - \underline{K}_{11}^{-1} \underline{K}_{12} \underline{\ddot{x}}^2(t) \quad \dots (29) \end{aligned}$$

Note

$$\underline{\phi} \underline{M}_s^{-1} \underline{\phi}^T = \underline{\phi} \left[\underline{\phi}^T \underline{m}_{11} \underline{\phi} \right]^{-1} \underline{\phi}^T = \underline{\phi} \underline{\phi}^{-1} \underline{m}_{11}^{-1} \underline{\phi} \underline{\phi}^T = \underline{m}_{11}^{-1} = \underline{\phi} \underline{\phi}^T \quad (30)$$

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Substitute (30) into (29) to obtain

$$\ddot{\underline{x}}^1(t) = -2 \underline{\phi} \underline{\lambda} \underline{\omega} \dot{\underline{q}}(t) - \underline{\phi} \underline{\omega}^2 \underline{q}(t) + \underline{\phi} \underline{\phi}^T \underline{F}_1(t) \dots (31)$$

7.0 SUMMARY

For ISM support motion only $\underline{F}_1(t) = \underline{0}$. The pseudo static displacement vector from (8), the inertia dynamic displacement vector from the Eigen Transformation (12), the absolute displacement vector $\underline{x}^1(t)$ from (27), and the absolute acceleration time history vector from (31) are summarized as follows:

$$\underline{x}_s^1(t) = - \underline{K}_{s1}^{-1} \underline{K}_{s2} \ddot{\underline{x}}^2(t) \dots (8)$$

$$\underline{x}_d^1(t) = \underline{\phi} \underline{q}(t) \dots (12)$$

$$\underline{x}^1(t) = \underline{\phi} \underline{q}(t) - \underline{K}_{u1} \underline{K}_{s2} \underline{x}^2(t) \dots (27)$$

$$\ddot{\underline{x}}^1(t) = -2 \underline{\phi} \underline{\lambda} \underline{\omega} \dot{\underline{q}}(t) - \underline{\phi} \underline{\omega}^2 \underline{q}(t) \dots (31)$$

The n^{th} Generalized Coordinate time history $q_n(t)$ is given by (25).

(END)

FIBWR CERTIFICATION PROGRAM PIPING ANALYSIS

ITEM NO. : A-25

Acceleration Level for Calculation of Missing Mass Contributions
Due to Truncated Higher Frequency Modes

GE RESPONSE

by
DK HENRIE
October 1992

GE NUCLEAR ENERGY
SAN JOSE, CALIFORNIA

1.0 DESCRIPTION OF CONCERN

ITEM NO. A-25. "Why does piping analysis use ZPA for high frequency effects, rather than the ^{spectral} acceleration at the highest frequency at which the modal analysis ends?"

2.0 GE NUCLEAR ENERGY RESPONSE

The acceleration levels used in the GE calculations for the missing mass response contributions due to truncated higher frequency modes in the modal analysis are taken as the peak spectral accelerations, in the input motion spectra, above the analysis cut-off frequency.

If the analysis cut-off frequency is equal to the input motion ZPA frequency, the accelerations used in the missing mass calculations are identical to the ZPA's.

GE's missing mass calculations follow the same procedure as recommended in Appendix A to SRP Section 3.7.2, Rev. 9, August 1987.