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Downers Grove, IL 60515-5701

ComEd

March 21, 1996

Office of Nuclear Reactor Regulation
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

Attn: Document Control Desk

Subject: Byron Station Units 1 and 2
Braidwood Station Units 1 and 2
Supplemental Response to the NRC Request for Additional Information Regarding Ampacity Derating Analyses
NRC Docket Numbers 50-454/455; 50-456/457

Reference:
1) February 15, 1995, M. J. Vonk letter to USNRC
2) March 28, 1995, K. L. Kaup letter to USNRC
3) March 29, 1995, K. L. Graesser letter to USNRC
4) November 2, 1995, R. R. Assa letter to D. L. Farrar
5) December 4, 1995, G. F. Dick letter to D. L. Farrar
6) December 15, 1995, D. Saccomando letter to USNRC

Reference (1) provided the Commonwealth Edison Company(ComEd) White Paper that compared the NRC test results provided in Nuclear Regulatory Commission(NRC) Information Notice 94-22 for determining the ampacity derating factors for cable trays wrapped with three-hour rated Thermo-Lag 330-1 fire barriers with the ComEd analytical techniques and results used to derate the ampacities of cables installed in wrapped trays. Reference (1) also provided the calculation that determined the ampacity derating factors for the potential Darmatt KM-1 Fire Barrier System to be installed at Braidwood Station.

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Reference (2) provided the response to the NRC Request For Additional Information, pursuant to 10CFR50.54(f), dated December 29,1994, for Braidwood Station. Reference (3) provided the response to the NRC Request For Additional Information(RAI), pursuant to 10CFR50.54(f), dated December 29,1994, for Byron Station. Reference (4) is the NRC Request For Additional Information regarding the ampacity derating analyses performed for Braidwood Station. Reference (5) is the NRC Request For Additional Information regarding the ampacity derating analyses performed for Byron Station.

Reference 6 transmitted ComEd's request to respond to the Braidwood and Byron RAIs (references 4 and 5) concurrently because many of the issues discussed in the Braidwood RAI apply to Byron Station. The information being transmitted in this letter is ComEd's response to the RAIs.

The following provides some general clarification regarding the RAIs transmitted in references 4 and 5 :

General: Though reference (1) was provided in response to phone conversations between ComEd and the NRC staff with respect to the planned Darmatt KM-1 installations at Braidwood, reference (2) subsequently notified the NRC staff that the re-routing of safe-shutdown cables to eliminate the need for installing fire-rated barriers had been chosen as the preferred option to achieve resolution of the Thermo-Lag 330-1 issues for Braidwood Station. Similarly, reference (3) notified the NRC staff that the re-routing of safe-shutdown cables to eliminate the need for installing additional fire-rated barriers had been chosen as the preferred option to achieve resolution of the Thermo-Lag 330-1 for Byron Station, for those applications that had not been previously protected with the Darmatt KM-1 fire barrier. At the Byron and Braidwood stations, the installed Thermo-Lag 330-1 fire barriers are to be abandoned in place and no credit is being taken at either station for it as a fire barrier.

With the selection and pursuit of the re-routing option, no Darmatt KM-1 fire barrier is planned to be installed at Braidwood station and the cited Calculation G-63, Revision 2 no longer applies to Braidwood. However, as noted above, some Darmatt KM-1 is installed at Byron Station and the NRC questions apply to these Byron installations. Therefore, the NRC questions have been answered based on the Byron installation configurations. [Please note that the current revision of calculation G-63 is Revision 2. Revision 3 of this calculation does not exist, as cited in reference (4).]

ComEd has been reevaluating the Byron and Braidwood Thermo-Lag 330-1 ampacity analyses and has concluded that additional actions are necessary. This reevaluation has determined that the existing analyses may not completely envelope all installed configurations. Specifically, the as-installed Thermo-Lag 330-1 board thickness, in some cases, may be greater than the nominal value supplied by TSI. This thicker Thermo-Lag 330-1 board value had not been used in prior ampacity derating calculations. Also, some installations may have an air gap between the Thermo-Lag panel and the cable tray that had not been modeled in the prior ampacity derating calculations. These identified conditions are being addressed at both stations and the appropriate calculations will be revised as necessary. During this reevaluation, ComEd will compare the analytical methodology to valid industry test data. ComEd intends to utilize existing industry data that is readily available and has no plans to perform additional ampacity derating tests.

The following are the ComEd responses to the specific NRC requests and observations as presented in references (4) and (5).

A) From Reference (4):

1) **Request(Ref.: Item 1, Pages 1 & 2):** *"Given that the referenced 1982 ampacity experiments were performed using solid bottom cable trays and those experimental results are bases for determining the internal resistance between cables and the surface of a covered tray, the subject analysis must be considered to be limited to that application. In fact, the 1982 American Power Conference paper, 'Tests At Braidwood Station on the Effects of Fire Stops on Ampacity Rating of Power Cables', makes note of the fact that the industry ampacity tables were found to be nonconservative for some of the tested configurations.*

Based on the above discussion, the licensee is requested to confirm that all of the cable trays under consideration for Braidwood Station are solid bottom trays of the type used in the original tests performed for Braidwood Station as reported in the aforementioned 1982 paper. If other types of cable trays are applicable for Braidwood Station, then a specific and detailed justification for the applicability of the licensee's methodology should be submitted by the licensee."

Answer:

For Darmatt KM-1 Installations:

All of the cable trays for Byron Station that are protected by the Darmatt KM-1 fire barrier are solid bottom trays of the type used in the original tests performed at Braidwood Station, and are governed by the methodology provided to the NRC staff in reference (1). As stated above, Braidwood Station currently has no plans to install any Darmatt KM-1 fire barriers. Therefore, the question of cable tray wrapped with Darmatt KM-1 at Braidwood is not applicable.

For Thermo-Lag 330-1 Installations:

The existing Thermo-Lag 330-1 installations at the Byron and Braidwood Stations will be abandoned in place. Essentially all fire wrapped cables trays are solid bottom of the type used in the original tests performed at Braidwood Station. However, there is one 30 inch long section of ladderback tray, that provides a transition between two solid bottom cable tray sections, installed in Unit 2 at both stations. Because this section of ladderback cable tray, with respect to the total cable tray run, is a relatively small length, ComEd believes its effect with respect to the ampacity derating values is insignificant . However, as stated above, ComEd is reevaluating the ampacity calculations for the as-installed condition of the Thermo-Lag 330-1 installed at the Byron and Braidwood Stations, which will include an evaluation of this transition section.

2) **Observation/Recommendation(Ref.: Item 2, Page 3):** *"Although the licensee's methodology contains many conservative features, the staff questions whether the licensee's White Paper provides an adequate basis for validation of the cable tray analysis method. Although the staff would not require a validation of the cable tray analysis assuming that the 1982 experiments performed for Braidwood station bound Thermo-Lag cable tray types, it is recommended that these calculations be revisited with valid industry test data. There are clearly more appropriate tests for which a more representative comparison and validation can be made (e.g., Comanche Peak Steam Electric Station, Unit 2. ampacity derating tests). It would clearly be desirable to see the licensee's analysis methodology validated against experimental data."*

Answer: ComEd concurs that the methodology developed to calculate ampacity derating factors is conservative, however, concerns regarding the actual installed configurations have been identified. These identified concerns are being addressed at both stations and the appropriate calculations will be revised as necessary. As previously stated, a comparison of the analytical methodology against valid industry test data will be made. ComEd intends to utilize existing industry data that is readily available and has no plans to perform additional ampacity derating tests.

3) **Observation/Request(Ref.: Item 3, Pages 3, 4 & 5):** *"SNL noted an apparent error was made in the treatment of the air gap between the conduit and the fire barrier system. The licensee's analysis utilizes a 'trick' which is commonly applied to steady state rectangular problems. The 'trick' involves a mathematical manipulation where the air gap is converted to an equivalent thickness of Darmatt based on the ratio of their thermal conductivities.". . . "Unfortunately, this approach can not be applied directly to annular regions.". . "Hence, the conversion of an annular gap of air into an annular gap of Darmatt must be consistent with the above logarithmic form.". . . "SNL recalculated the values for the 1 hour and 3 hour barrier systems for a 3/4" conduit using the licensee provided data...." "The licensee is requested to address the above apparent discrepancy and to revise its analysis accordingly."*

Answer: ComEd acknowledges that the SNL derived equation does provide a more accurate determination of the thermal resistance values of the conduit covered with the Darmatt KM-1 and will incorporate it into calculation G-63.

In calculation G-63, Revision 2, the postulated 1/16" air gap was modeled as an assumed condition that is not expected to exist in the actual installation, since the Darmatt KM-1 is installed in direct contact with the conduit surface. This assumed value provides additional margin into the calculation and the simplified approach of converting the air gap to an equivalent thickness of Darmatt KM-1 was considered to be an appropriate model for calculating the total thermal resistance.

The difference in the resultant thermal resistance between the model used and the calculation and the exact formula used by the NRC staff is 0.73 hr-ft-°F/BTU for a 3 hour Darmatt KM-1 barrier on a 3/4" conduit. However, it appears that the value of thermal conductivity used in the NRC staff calculation only accounted for heat transfer across the air gap

by conduction. Heat will also be transferred across the air gap by radiation, which, based on the information provided, was not taken into account in the calculation. When the heat transfer by radiation is taken into account, the effective thermal resistance of the air gap will be decreased. This reduces the difference between the thermal resistance determined in Calculation G-63, Revision 2 and the thermal resistance calculated by the NRC staff. Therefore, since the SNL derived equation does provide a more accurate determination of the thermal resistance values of the conduit covered with the Darmatt KM-1, ComEd will incorporate this equation into calculation G-63.

4) **Observation/Question(Ref.: Item 4, Page 5):** *"Given the information provided, the nature of the cable insulation and jacket resistance calculations are not clear. Specifically, the calculations presented as the top six lines of page 130 of Calculation G-63 require clarification. Although the calculations attempt to account for the cable insulation and jacket regions as annular regions, why are the multipliers of 2 and 3 applied to various parts of the resistance? How does the licensee justify simply adding the various components without consideration of parallel path heat transfer and the fact that heat is not flowing from the center of each conductor radially through each individual conductor, but rather non-uniformly through the multi-conductor cable as a whole?"*

Answer: The first expression calculates the temperature drop between the conductor and the insulation and individual jacket surrounding each conductor:

$$0.00522 \times 2\left[\rho_i \ln\left(\frac{d_i}{d}\right) + \rho_j \ln\left(\frac{d_j}{d_i}\right)\right]$$

For a single conductor cable, the insulation and jacket would be concentric annular layers, and the thermal resistance in consistent units would be:

$$\frac{1}{2\pi}\left[\rho_i \ln\left(\frac{d_i}{d}\right) + \rho_j \ln\left(\frac{d_j}{d_i}\right)\right]$$

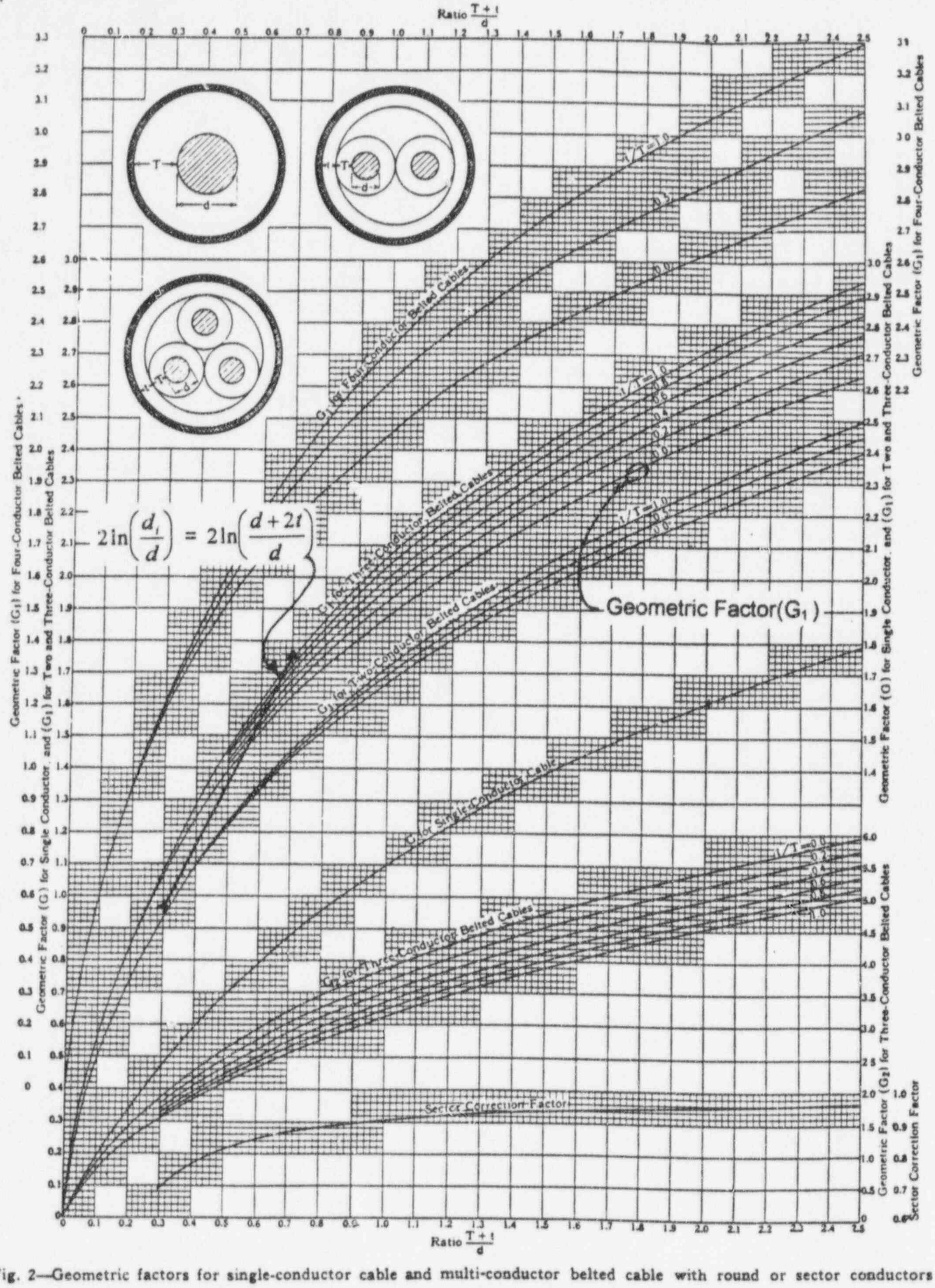

Fig. 2—Geometric factors for single-conductor cable and multi-conductor belted cable with round or sector conductors

Geometric factors can be obtained by calculating the ratios $(T + t)/d$ and t/T (d being defined for sector cables as the diameter of a round conductor of the same area as the sector), and then reading the required value of geometric factor from a curve above. The value thus obtained will be the correct geometric factor for a round-conductor cable. For sector conductors the values so obtained should be multiplied by the sector correction factor. In cables of the non-type H form without belts, such as multi-conductor rubber cables, the ratio becomes T/d, and $t/T = 0$.

For the mixed units often used for cable calculations, the inclusion of the required unit conversion factors gives rise to the coefficient 0.00522. For a three conductor cable, the heat flow will not be uniform due to the proximity of the other heated conductors, and the effective thermal resistance will be increased. The multiplier of 2 accounts for this increase. *This factor is based on formulations referenced in a 1957 paper by Neher and McGrath*[1]. *The Neher-McGrath paper uses* equations for this temperature drop of the form:

$$\frac{1}{2\pi}\rho \cdot G$$

where, G is a geometric factor that takes the non-uniform heat flow into account. The expression $2\ln\left(\frac{d_i}{d}\right)$ closely approximates the values of G, as can be seen in the plot of G taken from *Figure 2, page 13 of the D. M. Simmons paper*[2].

$$2\ln\left(\frac{d_i}{d}\right) = 2\ln\left(\frac{d+2t}{d}\right)$$

The second term accounts for the temperature drop of the three phase conductors passing through the overall jacket:

$$0.00522 \times 3\rho_j \ln\left(\frac{d_{3jo}}{d_{3ji}}\right)$$

This resistance is applied to the heat generated by a single conductor. However, the overall jacket surrounds three heat generating conductors. Therefore, the factor of 3 is required to calculate the proper amount of heat flowing through the jacket when the equation is multiplied by the heat generated by one conductor to obtain the temperature drop.

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[1] Neher, J. H. and McGrath, M. H. 1957. "The Calculation of the Temperature Rise and Load Capability of Cable Systems". *AIEE Transactions on Power Apparatus and Systems* 76 (October):752-72.

[2] Donald M. Simmons, General Cable Corporation, "Calculation of the Electrical Problems of Underground Cables", reprinted from The Electric Journal, issues of May to November, 1932, inclusive

5) **Observation(Ref.: Item 4, Page 5):** *"For this geometry, a cable resting on the bottom inside of a conduit, treatment of the problem as one of purely annular regions, which apparently are cascaded one upon another, is not correct and appears to ignore the inherent 2-dimensionality of the problem."*

Answer: The expression used to calculate this temperature drop is equation 41A of the Neher-McGrath paper:

$$R_{ac} = \frac{A'}{D_s' + B'}$$

where D_s' is the circumscribed diameter of the cables in the conduit and A' and B' are constants that depend on the conduit material. This equation has been in general use and is accepted industry practice. The derivation of the formula is given in Appendix I of the Neher-McGrath paper. As can be seen in this appendix, while the formula is partly derived from theoretical considerations, it is calibrated using experimental data. Additional information that was used in the derivation of this formula is given by two other papers[3,4].

6) **Request(Ref.: Item 4, Page 5):** *"Based on the above discussion, the licensee is requested to submit a copy of Sargent & Lundy(S&L) Standard ESA -105. Further, the licensee should explain in greater detail, the full nature of the cable-to-conduit thermal resistance calculation process. This description should include a detailed explanation of both the basis and intent of calculations(e.g., the first six lines on page 130 of the*

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[3] Buller, F. H. And Neher, J. H. 1950. The Thermal Resistance Between Cables and a Surrounding Pipe or Duct Wall. *AIEE Transactions, Volume I.* 69:342-349

[4] Greebler, P. And Barnett, G. F. 1950. Heat Transfer Study on Power Cable Ducts and Duct Assemblies. *AIEE Transactions, Volume I.* 69:357-367.

ComEd Calculation G-63) and an explanation and justification for merging the two separate calculations into a single expression."

Answer: ComEd believes that the above answers and discussions address the cable-to-conduit thermal resistance calculation process. The relevant portions of S&L Standard ESA-105 have been addressed or summarized, along with justification for the formulae involved. However in response to the NRC Staff request, S&L Standard ESA-105 is provided in Attachment B. ComEd is notifying the NRC Staff that S&L Standard ESA-105 is a proprietary document to S&L and requests that the staff control this document, accordingly.

7) **Observation/Request(Ref.: Item 5, Pages 5 & 6):** *"Another concern is the value assumed for the emissivity of the outer surface of the conduit. In both the cable tray and the conduit analyses, a lower bound value of 0.23 is used. In the case of the cable tray analysis this was concluded to be a conservative approach. However, in the case of the conduit analysis, this approach is actually nonconservative.". . . . . ."For conservatism, the conduit baseline case analyses should consider the maximum possible conduit surface emissivity rather than the minimum value." "Based on the above discussion, the licensee is requested to assess the impact on the calculated ampacity derating factors by using an upper bound emissivity value(i.e., 0.8 - 0.9) in its baseline conduit calculations."*

Answer: ComEd acknowledges that utilizing an upper bound emissivity value (i.e., 0.8 - 0.9) would add conservatism to the conduit and cable tray analysis. However, postulating an emissivity value other than 0.23 is believed to add unnecessary conservatism to the calculation that does not properly represent the galvanized steel material from which the conduit and cable tray is fabricated. The cable tray and conduit emissivity value of 0.23 utilized in Calculation G-63, Revision 2 was taken from the, "Standard Handbook for Mechanical Engineers", Baumeister and Marks, Seventh Edition. This is listed as reference #7, on page 9 of 215 of the calculation. An emissivity value of 0.23 is applicable to the steel conduit modeled in the calculation and installed in the plant and is considered to be appropriate as utilized in the calculation.

8) **Observation/Request(Ref.: Item 6, Page 6):** *"The licensee did not provide any experimental validation of the analytical methodology for conduits based on actual test data. The licensee is requested to evaluate the validity of their analytical methodology using available industry test data."*

Answer: The Neher-McGrath methodology used in Calculation G-63, Revision 2 and discussed above, has been used by the electrical industry for calculating the ampacity of cables since the time it was derived. As mentioned in the papers referenced above, key parts of this methodology have been calibrated using test data. This methodology has been accepted in the *National Electrical Code* [NFPA 70-1996, Clause B-310-15(b)(2)]. However, as previously stated, an attempt to validate the analytical methodology against valid industry test data will be made. ComEd intends to utilize existing industry data that is readily available and has no plans to perform additional ampacity derating tests.

<u>B) From Reference (5):</u>

1) **Request(Ref.: Item 1, Page 2):** *"Please provide a copy of the typical calculation(s) depicting the use of the subject analytical methodology which were used to determine the ampacity derating parameters for the Thermo-Lag fire barriers that are installed at Byron Station."*

Answer: Though the methodology described above formed the basis for determining the ampacity derating parameters for the Thermo-Lag 330-1 fire barriers installed at the Byron and Braidwood Stations, a specific Byron and Braidwood calculation utilizing that methodology was not prepared. Rather the 33% derating factor for the Thermo-Lag 330-1 cited in the White Paper was determined by performing an independent evaluation of the raw data from an ampacity test, I.T.L. Report No. 82-5-355F, dated July, 1982. This derating factor was compared to the calculated value of 32%, determined in the calculation(19-AI-8), referenced in the White Paper. Using the derating factor of 33%, detailed calculations for the cable routing points were then performed to confirm that the previously determined ampacities were still acceptable.

As previously stated, ComEd is reevaluating the as-installed condition of the Thermo-Lag 330-1 installed at the Byron and Braidwood Stations, and concludes that additional actions are necessary. This reevaluation has determined that the existing analyses may not completely envelope all installed configurations. The identified conditions are being addressed at both stations and the appropriate calculations will be revised as necessary. These will be made available for NRC Staff review.

2) **Request(Ref.: Item 2, Page 2):** *"In its submittal of December 16, 1994, the licensee referred to a site specific comparison regarding the acceptability of plant ampacity derating parameters when compared to the test results cited in IN 94-22. The staff recognizes that most licensees may have excess ampacity margin using valid test data. However, those licensees who utilize industry test data must evaluate whether installed configurations are representative of the tested configurations. The subject evaluations should also analyze any deviations of the installed configuration with respect to the test configuration. It should be noted that the methodology used in the ampacity test differs significantly from the methodology utilized by the draft industry test procedure IEEE P848.*

In the event that the licensee wishes to use the test results cited in IN 94-22, the licensee must indicate whether the subject test configuration is representative of Thermo-Lag enclosed configurations which are installed at Byron Station."

Answer: As previously stated, ComEd is reevaluating the ampacity calculations for the as-installed condition of the Thermo-Lag 330-1 at the Byron and Braidwood Stations. During this reevaluation, a comparison of the analytical methodology against valid industry test data will be made. ComEd intends to utilize existing industry data that is readily available and has no plans to perform additional tests. During this re-evaluation, any industry test data utilized for comparison, including the IN 94-22 data, will be evaluated for applicability to our analytical model.

Please note *Sargent & Lundy(S&L) Standard ESA -105* contains information proprietary to Sargent & Lundy, it is supported by an affidavit signed by Sargent & Lundy, the owner of the information (see Attachment B). The affidavit sets forth the basis on which the information may be withheld from public disclosure by the Commission and addresses with specificity the considerations listed in Paragraph (b)(4) of Section 2.790 of the Commission's regulations. Accordingly, it is respectfully requested that the information which is proprietary to Sargent & Lundy be withheld from public disclosure in accordance with 10 CFR 2.790 of the Commission's regulations.

Correspondence with respect to the proprietary aspects of the items should be addressed to K. Kostal, Executive Vice President, Sargent and Lundy, 55 East Monroe Street, Chicago, IL. 60603-5780.

To the best of my knowledge and belief, the statements contained in this document are true and correct. In some respects these statements are not based on my personal knowledge, but on information furnished by other ComEd employees, contractor employees, and/or consultants. Such information has been reviewed in accordance with company practice, and I believe it to be reliable.

If there are any further questions concerning this matter, please contact this office.

Sincerely,

Denise Saccomando

Denise Saccomando
Senior Nuclear Licensing Administrator

OFFICIAL SEAL
MARY JO YACK
NOTARY PUBLIC, STATE OF ILLINOIS
MY COMMISSION EXPIRES 11/29/97

Mary Jo Yack 3-22-96

Attachments:

cc: H. Miller, Regional Administrator - RIII
G. Dick, Byron Project Manager-NRR
R. Assa, Braidwood Project Manager-NRR
H. Peterson, Senior Resident Inspector-Byron
C. Phillips, Senior Resident Inspector-Braidwood
Office of Nuclear Facility Safety - IDNS

Attachment "A"

Documents cited In the responses to Items (10) and (11):

1) Neher, J. H. and McGrath, M. H. 1957. The calculation of the Temperature Rise and Load Capability of Cable Systems. *AIEE Transactions on Power Apparatus and Systems* 76 (October):752-72.

2) Donald M. Simmons, General Cable Corporation, "Calculation of the Electrical Problems of Underground Cables", reprinted from The Electric Journal, issues of May to November, 1932, inclusive

3) Buller, F. H. And Neher, J. H. 1950. The Thermal Resistance Between Cables and a Surrounding Pipe or Duct Wall. *AIEE Transactions, Volume I.* 69:342-349.

4) Greebler, P. And Barnett, G. F. 1950. Heat Transfer Study on Power Cable Ducts and Duct Assemblies. *AIEE Transactions, Volume I.* 69:357-367.

The Calculation of the Temperature Rise and Load Capability of Cable Systems

J. H. NEHER
MEMBER AIEE

M. H. McGRATH
MEMBER AIEE

IN 1932 D. M. Simmons[1] published a series of articles entitled, "Calculation of the Electrical Problems of Underground Cables." Over the intervening 25 years this work has achieved the status of a handbook on the subject. During this period, however, there have been numerous developments in the cable art, and much theoretical and experimental work has been done with a view to obtaining more accurate methods of evaluating the parameters involved. The advent of the pipe-type cable system has emphasized the desirability of a more rational method of calculating the performance of cables in duct in order that a realistic comparison may be made between the two systems.

In this paper the authors have endeavored to extend the work of Simmons by presenting under one cover the basic principles involved, together with more recently developed procedures for handling such problems as the effect of the loading cycle and the temperature rise of cables in various types of duct structures. Included as well are expressions required in the evaluation of the basic parameters for certain specialized allied procedures. It is thought that a work of this type will be useful not only as a guide to engineers entering the field and as a reference to the more experienced, but particularly as a basis for setting up computation methods for the preparation of industry load capability and a-c/d-c ratio compilations.

The calculation of the temperature rise of cable systems under essentially steady-state conditions, which includes the effect of operation under a repetitive load cycle, as opposed to transient temperature rises due to the sudden application of large amounts of load, is a relatively simple procedure and involves only the application of the thermal equivalents of Ohm's and Kirchoff's Laws to a relatively simple thermal circuit. Because this circuit usually has a number of parallel paths with heat flows entering at several points, however, care must be exercised in the method used of expressing the heat flows and thermal resistances involved, and differing methods are used by various engineers. The method employed in this paper has been selected after careful consideration as being the most consistent and most readily handled over the full scope of the problem.

All losses will be developed on the basis of watts per conductor foot. The heat flows and temperature rises due to dielectric loss and to current-produced losses will be treated separately, and, in the latter case, all heat flows will be expressed in terms of the current produced loss originating in one foot of conductor by means of multiplying factors which take into account the added losses in the sheath and conduit.

In general, all thermal resistances will be developed on the basis of the per conductor heat flow through them. In the case of underground cable systems, it is convenient to utilize an effective thermal resistance for the earth portion of the thermal circuit which includes the effect of the loading cycle and the mutual heating effect of the other cable of the system. All cables in the system will be considered to carry the same load currents and to be operating under the same load cycle.

The system of nomenclature employed is in accordance with that adopted by the Insulated Conductor Committee as standard, and differs appreciably from that used in many of the references. This system represents an attempt to utilize in so far as possible the various symbols appearing in the American Standards Association Standards for Electrical Quantities, Mechanics, Heat and Thermo-Dynamics, and Hydraulics, when these symbols can be used without ambiguity. Certain symbols which have long been used by cable engineers have been retained, even though they are in direct conflict with the above-mentioned standards.

Nomenclature

(AF) = attainment factor, per unit (pu)
A_s = cross-section area of a shielding tape or skid wire, square inches
α = thermal diffusivity, square inches per hour
CI = conductor area, circular inches
d = distance, inches
d_{12} etc. = from center of cable no. 1 to center of cable no. 2 etc.
d_{12}' etc. = from center of cable no. 1 to image of cable no. 2 etc.
d_{1i} etc. = from center of cable no. 1 to a point of interference
d_{1i}' etc. = from image of cable no. 1 to a point of interference
D = diameter, inches
D_o = inside of annular conductor
D_c = outside of conductor
D_i = outside of insulation
D_s = outside of sheath
D_{sm} = mean diameter of sheath
D_j = outside of jacket
D_s' = effective (circumscribing circle) of several cables in contact
D_p = inside of duct wall, pipe or conduit
D_e = diameter at start of the earth portion of the thermal circuit
D_x = fictitious diameter at which the effect of loss factor commences
E = line to neutral voltage, kilovolts (kv)
ϵ = coefficient of surface emissivity
ϵ_r = specific inductive capacitance of insulation
f = frequency, cycles per second
F, F_{int} = products of ratios of distances
$F(x)$ = derived Bessel function of x (Table III and Fig. 1)
G = geometric factor
G_1 = applying to insulation resistance (Fig. 2 of reference 1)
G_2 = applying to dielectric loss (Fig. 2 of reference 1)
G_b = applying to a duct bank (Fig. 2)
I = conductor current, kiloamperes
k_s = skin effect correction factor for annular and segmental conductors
k_p = relative transverse conductivity factor for calculating conductor proximity effect
l = lay of a shielding tape or skid wire, inches
L = depth of reference cable below earth's surface, inches
L_b = depth to center of a duct bank (or backfill), inches
(lf) = load factor, per unit
(LF) = loss factor, per unit
n = number of conductors per cable
n' = number of conductors within a stated diameter
N = number of cables or cable groups in a system
P = perimeter of a duct bank or backfill, inches
$\cos\phi$ = power factor of the insulation
q_s = ratio of the sum of the losses in the conductors and sheaths to the losses in the conductors
q_e = ratio of the sum of the losses in the conductors, sheath and conduit to the losses in the conductors
R = electrical resistance, ohms
R_{dc} = d-c resistance of conductor
R_{ac} = total a-c resistance per conductor
R_s = d-c resistance of sheath or of the parallel paths in a shield-skid wire assembly
$\bar{R}$ = thermal resistance (per conductor losses) thermal ohm-feet
$\bar{R}_i$ = of insulation
$\bar{R}_j$ = of jacket
$\bar{R}_{sd}$ = between cable surface and surrounding enclosure

Paper 57-660, recommended by the AIEE Insulated Conductors Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Summer General Meeting, Montreal, Que., Canada, June 24–28, 1957. Manuscript submitted March 20, 1957; made available for printing April 18, 1957.

J. H. NEHER is with the Philadelphia Electric Company, Philadelphia, Pa., and M. H. McGRATH is with the General Cable Corporation, Perth Amboy, N. J.

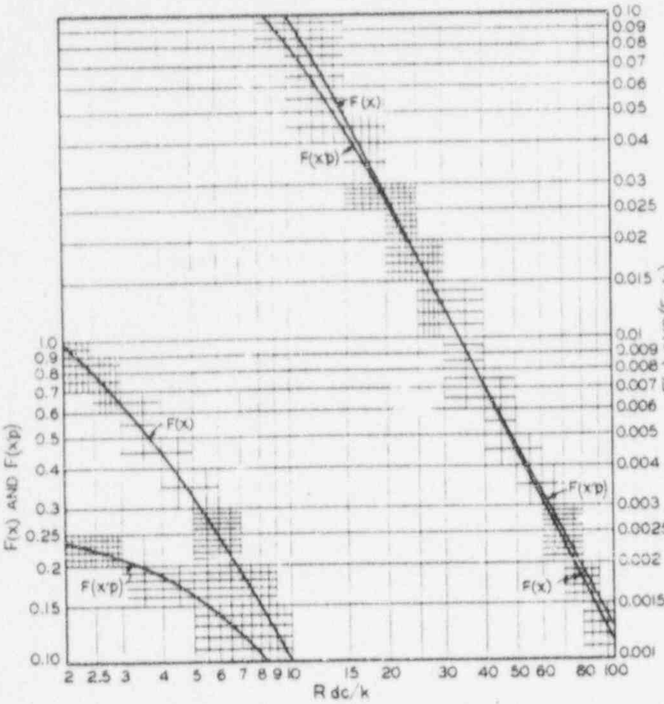

Fig. 1 (above). F(x) and F(x_p') as functions of R_{dc}/k

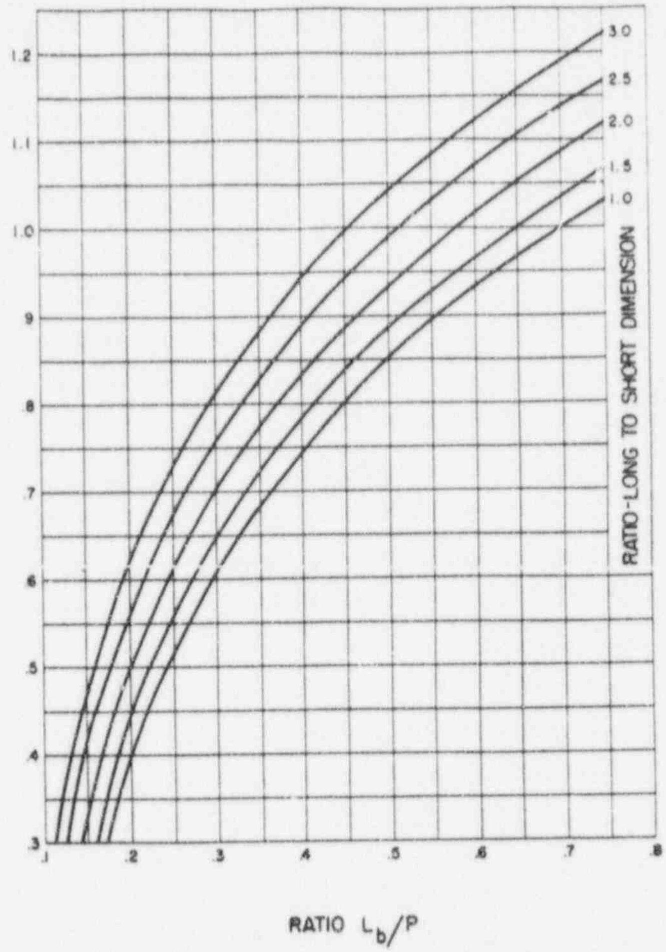

Fig. 2 (right). G_b for a duct bank

$\bar{R}_d$ = of duct wall or asphalt mastic covering
$\bar{R}_{sd}$ = total between sheath and diameter D_e including $\bar{R}_j$, $\bar{R}_{sd}$ and $\bar{R}_d$
$\bar{R}_e$ = between conduit and ambient
$\bar{R}_e'$ = effective between diameter D_e and ambient earth including the effects of loss factor and mutual heating by other cables
$\bar{R}_{ca}'$ = effective between conductor and ambient for conductor loss
R_{ct}' = effective transient thermal resistance of cable system
$\bar{R}_{da}'$ = effective between conductor and ambient for dielectric loss
$\bar{R}_{int}$ = of the interference effect
$\bar{R}_{ya}$ = between a steam pipe and ambient earth
ρ = electrical resistivity, circular mil ohms per foot
$\bar{\rho}$ = thermal resistivity, degrees centigrade centimeters per watt
s = distance in a 3-conductor cable between the effective current center of the conductor and the axis of the cable, inches
S = axial spacing between adjacent cables, inches
t, T = thickness (as indicated), inches
T = temperature, degrees centigrade
T_a = of ambient air or earth
T_c = of conductor
T_m = mean temperature of medium
ΔT = temperature rise, degrees centigrade
ΔT_c = of conductor due to current produced losses
ΔT_d = of conductor due to dielectric loss
ΔT_{int} = of a cable due to extraneous heat source
τ = inferred temperature of zero resistance, degrees centigrade (C) (used in correcting R_{dc} and R_s to temperatures other than 20 C)
V_w = wind velocity, miles per hour
W = losses developed in a cable, watts per conductor foot
W_c = portion developed in the conductor
W_s = portion developed in the sheath or shield
W_p = portion developed in the pipe or conduit
W_d = portion developed in the dielectric
X_m = mutual reactance, conductor to sheath or shield, microhms per foot
Y = the increment of a-c/d-c ratio, pu
Y_c = due to losses originating in the conductor, having components Y_{cs} due to skin effect and Y_{cp} due to proximity effect
Y_s = due to losses originating in the sheath or shield, having components Y_{sc} due to circulating current effect and Y_{se} due to eddy current effect
Y_p = due to losses originating in the pipe or conduit
Y_a = due to losses originating in the armor

General Considerations of the Thermal Circuit

The Calculation of Temperature Rise

The temperature rise of the conductor of a cable above ambient temperature may be considered as being composed of a temperature rise due to its own losses, which may be divided into a rise due to current produced (I^2R) losses (hereinafter referred to merely as losses) in the conductor, sheath and conduit ΔT_c and the rise produced by its dielectric loss ΔT_d. Thus

$$T_c - T_a = \Delta T_c + \Delta T_d \text{ degrees centigrade} \tag{1}$$

Each of these component temperature rises may be considered as the result of a rate of heat flow expressed in watts per foot through a thermal resistance expressed in thermal ohm feet (degrees centigrade feet per watt); in other words, the radial rise in degrees centigrade for a heat flow of one watt uniformly distributed over a conductor length of one foot.

Since the losses occur at several positions in the cable system, the heat flow in the thermal circuit will increase in steps. It is convenient to express all heat flows in terms of the loss per foot of conductor, and thus

$$\Delta T_c = W_c(\bar{R}_i + q_s\bar{R}_{sd} + q_e\bar{R}_e) \text{ degrees centigrade} \tag{2}$$

in which W_c represents the losses in one conductor and $\bar{R}_i$ is the thermal resistance of the insulation, q_s is the ratio of the sum of the losses in the conductors and sheath to the losses in the conductors, $\bar{R}_{sd}$ is the total thermal resistance between sheath and conduit, q_e is the ratio of the sum of the losses in conductors, sheath and conduit, to the conductor losses, and $\bar{R}_e$

is the thermal resistance between the conduit and ambient.

In practice, the load carried by a cable is rarely constant and varies according to a daily load cycle having a load factor (lf). Hence, the losses in the cable will vary according to the corresponding daily loss cycle having a loss factor (LF). From an examination of a large number of load cycles and their corresponding load and loss factors, the following general relationship between load factor and loss factor has been found to exist.[3]

$$(LF)=0.3\ (lf)+0.7\ (lf)^2 \text{ per unit} \qquad (3)$$

In order to determine the maximum temperature rise attained by a buried cable system under a repeated daily load cycle, the losses and resultant heat flows are calculated on the basis of the maximum load (usually taken as the average current for that hour of the daily load cycle during which the average current is the highest, i.e. the daily maximum one-hour average load) on which the loss factor is based and the heat flow in the last part of the earth portion of the thermal circuit is reduced by the factor (LF). If this reduction is considered to start at a point in the earth corresponding to the diameter D_x,[3] equation 2 becomes

$$\Delta T_c=W_c[\bar{R}_i+q_s\bar{R}_{sd}+q_e(\bar{R}_{ex}+(LF)\bar{R}_{xa})] \text{ degrees centigrade} \qquad (4)$$

In effect this means that the temperature rise from conductor to D_x is made to depend on the heat loss corresponding to the maximum load whereas the temperature rise from diameter D_x to ambient is made to depend on the average loss over a 24-hour period. Studies indicate that the procedure of assuming a fictitious critical diameter D_x at which an abrupt change occurs in loss factor from 100% to actual will give results which very closely approximate those obtained by rigorous transient analysis. For cables or duct in air where the thermal storage capacity of the system is relatively small, the maximum temperature rise is based upon the heat flow corresponding to maximum load without reduction of any part of the thermal circuit.

When a number of cables are installed close together in the earth or in a duct bank, each cable will have a heating effect upon all of the others. In calculating the temperature rise of any one cable, it is convenient to handle the heating effects of the other cables of the system by suitably modifying the last term of equation 4. This is permissible since it is assumed that all the cables are carrying equal currents, and are operating on the same load cycle. Thus for an N-cable system

$$\Delta T_c=W_c(\bar{R}_i+q_s\bar{R}_{se}+q_e[\bar{R}_{ex}+(LF)\times(\bar{R}_{xa})+(N-1)\bar{R}_{pa}]) \qquad (5)$$

$$=W_c(\bar{R}_i+q_e\bar{R}_{se}+q_e\bar{R}_e') \text{ degrees centigrade} \qquad (5A)$$

where the term in parentheses is indicated by the effective thermal resistance $\bar{R}_e'$.

The temperature rise due to dielectric loss is a relatively small part of the total temperature rise of cable systems operating at the lower voltages, but at higher voltages it constitutes an appreciable part and must be considered. Although the dielectric losses are distributed throughout the insulation, it may be shown that for single conductor cable and multiconductor shielded cable with round conductors the correct temperature rise is obtained by considering for transient and steady state that all of the dielectric loss W_d occurs at the middle of the thermal resistance between conductor and sheath or alternately for steady-state conditions alone that the temperature rise between conductor and sheath for a given loss in the dielectric is half as much as if that loss were in the conductor. In the case of multiconductor belted cables, however, the conductors are taken as the source of the dielectric loss.[1]

The resulting temperature rise due to dielectric loss ΔT_d may be expressed

$$\Delta T_d=W_d\bar{R}_{da}' \text{ degrees centigrade} \qquad (6)$$

in which the effective thermal resistance $\bar{R}_{da}'$ is based upon $\bar{R}_i$, $\bar{R}_{se}$, and $\bar{R}_e'$(at unity loss factor) according to the particular case. The temperature rise at points in the cable system other than at the conductor may be determined readily from the foregoing relationships.

The Calculation of Load Capability

In many cases the permissible maximum temperature of the conductor is fixed and the magnitude of the conductor current (load capability) required to produce this temperature is desired. Equation 5(A) may be written in the form

$$\Delta T_c=I^2R_{dc}(1+Y_c)\bar{R}_{ca}' \text{ degrees centigrade} \qquad (7)$$

in which the quantity $R_{dc}\ (1+Y_c)$ which will be evaluated later represents the effective electrical resistance of the conductor in microhms per foot, and which when multiplied by I^2 (I in kiloamperes) will equal the loss W_c in watts per conductor foot actually generated in the conductor; and $\bar{R}_{ca}'$ is the effective thermal resistance of the thermal circuit.

$$\bar{R}_{ca}'=\bar{R}_i+q_s\bar{R}_{se}+q_e\bar{R}_e' \text{ thermal ohm-feet} \qquad (8)$$

From equation 1 it follows that

$$I=\sqrt{\frac{T_c-(T_a+\Delta T_d)}{R_{dc}(1+Y_c)\bar{R}_{ca}'}} \text{ kiloamperes} \qquad (9)$$

Table I. Electrical Resistivity of Various Materials

Material	ρ Circular Mil Ohms per Foot at 20 C	τ, C
Copper (100% IACS*)	10.371	234.5
Aluminum (61% IACS)	17.002	228.1
Commercial Bronze (43.6% IACS) (90 Cu–10 Zn)	23.8	564
Brass (27.3% IACS) (70 Cu–30 Zn)	38.0	912
Lead (7.84% IACS)	132.3	236

* International Annealed Copper Standard.

Calculation of Losses and Associated Parameters

Calculation of D-C Resistances

The resistance of the conductor may be determined from the following expressions which include a lay factor of 2%; see Table I.

$$R_{dc}=\frac{1.02\rho_c}{CI} \text{ microhms per foot at 20 C} \qquad (10)$$

$$=\frac{12.9}{CI} \text{ for 100\% IACS copper conductor at 75 C} \qquad (10A)$$

$$=\frac{21.2}{CI} \text{ for 61\% IACS aluminum at 75 C} \qquad (10B)$$

where CI represents the conductor size in circular inches and where ρ_c represents the electrical resistivity in circular mil ohms per foot. To determine the value of resistance at temperature T multiply the resistance at 20 C by $(\tau+T)/(\tau+20)$ where τ is the inferred temperature of zero resistance.

The resistance of the sheath is given by the expressions

$$R_s=\frac{\rho_s}{4D_{sm}t} \text{ microhms per foot at 20 C} \qquad (11)$$

$$R_s=\frac{37.9}{D_{sm}t} \text{ for lead at 50 C} \qquad (11A)$$

$$=\frac{4.75}{D_{sm}t} \text{ for 61\% aluminum at 50 C} \qquad (11B)$$

where D_{sm} is the mean diameter of the sheath and t is its thickness, both in inches

$$D_{sm}=D_s-t \text{ inches} \qquad (12)$$

The resistance of intercalated shields or skid wires may be determined from the expression

$$R_s\ (\text{per path})=\frac{\pi\rho_s}{4A_s}\sqrt{1+\left(\frac{\pi D_{sm}}{l}\right)^2} \text{ microhms per foot at 20 C} \qquad (13)$$

where A_s is the cross-section area of the

tape or skid wire and l is its lay. The over-all resistance of the shield and skid wire assembly, particularly for nonintercalated shields, should be determined by electrical measurement when possible.

Calculation of Losses

It is convenient to develop expressions for the losses in the conductor, sheath and pipe or conduit in terms of the components of the a-c/d-c ratio of the cable system which may be expressed as follows[4]

$$R_{ac}/R_{dc} = 1 + Y_c + Y_s + Y_p \tag{14}$$

The a-c/d-c ratio at conductor is $1 + Y_c$

and at sheath or shield is $1 + Y_c + Y_s$

and at pipe or conduit is $1 + Y_c + Y_s + Y_p$

The corresponding losses physically generated in the conductor, sheath, and pipe are

$$W_c = I^2 R_{dc}(1 + Y_c) \text{ watts per conductor foot} \tag{15}$$

$$W_s = I^2 R_{dc} Y_s \text{ watts per conductor foot} \tag{16}$$

$$W_p = I^2 R_{dc} Y_p \text{ watts per conductor foot} \tag{17}$$

This permits a ready determination of the losses if the segregated a-c/d-c ratios are known, and conversely, the a-c/d-c ratio is readily obtained after the values of Y_c, Y_s and Y_p have been calculated.

It follows from the definitions of q_s and q_e that

$$q_s = \frac{W_c + W_s}{W_c} = 1 + \frac{Y_s}{1 + Y_c} \tag{18}$$

$$q_e = \frac{W_c + W_s + W_p}{W_c} = 1 + \frac{Y_s + Y_p}{1 + Y_c} \tag{19}$$

The factor Y_c is the sum of two components, Y_{cs} due to skin effect and Y_{cp} due proximity effect.

$$W_c = I^2 R_{dc}(1 + Y_{cs} + Y_{cp}) \text{ watts per conductor foot} \tag{20}$$

The skin effect may be determined from the skin effect function $F(x)$

$$Y_{cs} = F(x_s) \tag{21}$$

$$x_s = 0.875\sqrt{\frac{f k_s}{R_{dc}}} = \frac{6.80}{\sqrt{R_{dc}/k_s}} \text{ at 60 cycles} \tag{22}$$

in which the factor k_s depends upon the conductor construction. For solid or conventional conductors appropriate values of k_s will be found in Table II. The function $F(x)$ may be obtained from Table III or from the curves of Fig. 1 in terms of the ratio R_{dc}/k at 60 cycles.

For annual conductors

$$k_s = \frac{D_c - D_o}{D_c + D_o}\left(\frac{D_c + 2D_o}{D_c + D_o}\right)^2 \tag{23}$$

in which D_c and D_o represent the outer and inner diameters of the annular conductor. In comparison with the rigorous Bessel function solution for the skin effect in an isolated tubular conductor, it has been found that the 60-cycle skin effect of annular conductor when computed by equation 23 will not be in error by more than 0.01 in absolute magnitude for copper or aluminum IPCEA (Insulated Power Cable Engineers Association) filled

Table II. Recommended Values of k_s and k_p

Conductor Construction	Coating on Strands	Treatment	k_s	k_p
Concentric round	None	None	1.0	1.0
Concentric round	Tin or alloy	None	1.0	1.0
Concentric round	None	Yes	1.0	0.80
Compact round	None	Yes	1.0	0.6
Compact segmental	None	None	0.435	0.6
Compact segmental	Tin or alloy	None	0.5	0.7
Compact segmental	None	Yes	0.435	0.37
Compact sector	None	Yes	1.0	(see note)

Notes:

1. The term "treated" denotes a completed conductor which has been subjected to a drying and impregnating process similar to that employed on oaper power cable.

2. Proximity effect on compact sector conductors may be taken as one-half of that for compact round having the same cross-sectional area and insulation thickness.

3. Proximity effect on annular conductors may be approximated by using the value for a concentric round conductor of the same cross-sectional area and spacing. The increased diameter of the annular type and the removal of metal from the center decreases the skin effect but, for a given axial spacing, tends to result in an increase in proximity.

4. The values listed above for compact segmental refer to four segment constructions. The "uncoated-treated" values may also be taken as applicable to four segment compact segmental with hollow core (approximately 0.75 inch clear). For "uncoated-treated" six segment hollow core compact segmental limited test data indicates k_s and k_p values of 0.39 and 0.33 respectively.

Table III. Skin Effect in % in Solid Round Conductor and in Conventional Round Concentric Strand Conductors

100 F(x), Skin Effect %

x	0	1	2	3	4	5	6	7	8	9
0.3	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.4	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03
0.5	0.03	0.04	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.06
0.6	0.07	0.07	0.08	0.08	0.09	0.10	0.10	0.11	0.11	0.12
0.7	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.19	0.20
0.8	0.21	0.22	0.24	0.25	0.26	0.28	0.29	0.30	0.31	0.33
0.9	0.34	0.36	0.38	0.39	0.41	0.43	0.45	0.47	0.48	0.50
1.0	0.52	0.54	0.56	0.58	0.61	0.63	0.65	0.68	0.70	0.73
1.1	0.76	0.79	0.81	0.84	0.87	0.90	0.94	0.97	1.00	1.03
1.2	1.07	1.11	1.14	1.18	1.22	1.26	1.30	1.34	1.38	1.42
1.3	1.47	1.52	1.56	1.61	1.66	1.71	1.76	1.81	1.86	1.92
1.4	1.97	2.02	2.08	2.14	2.20	2.26	2.32	2.39	2.45	2.52
1.5	2.58	2.65	2.72	2.79	2.86	2.93	3.01	3.08	3.16	3.24
1.6	3.32	3.40	3.49	3.57	3.66	3.75	3.83	3.92	4.02	4.11
1.7	4.21	4.30	4.40	4.50	4.60	4.70	4.81	4.91	5.02	5.13
1.8	5.24	5.35	5.47	5.58	5.70	5.82	5.94	6.06	6.19	6.31
1.9	6.44	6.57	6.70	6.83	6.97	7.11	7.24	7.38	7.53	7.67
2.0	7.82	7.96	8.11	8.26	8.42	8.57	8.73	8.89	9.05	9.21
2.1	9.38	9.54	9.71	9.88	10.05	10.22	10.40	10.58	10.76	10.94
2.2	11.13	11.31	11.50	11.69	11.88	12.07	12.27	12.47	12.67	12.87
2.3	13.07	13.27	13.48	13.68	13.90	14.11	14.33	14.54	14.76	14.98
2.4	15.21	15.43	15.66	15.89	16.12	16.35	16.58	16.82	17.16	17.30
2.5	17.54	17.78	18.03	18.27	18.52	18.78	19.03	19.28	19.54	19.80
2.6	20.06	20.32	20.58	20.85	21.12	21.38	21.65	21.93	22.20	22.48
2.7	22.75	23.03	23.31	23.60	23.88	24.17	24.45	24.74	25.03	25.33
2.8	25.62	25.92	26.21	26.51	26.81	27.11	27.42	27.72	28.03	28.34
2.9	28.65	28.96	29.27	29.58	29.90	30.21	30.53	30.85	31.17	31.49
3.0	31.81	32.13	32.45	32.78	33.11	33.44	33.77	34.10	34.43	34.77
3.1	35.10	35.44	35.78	36.11	36.45	36.79	37.13	37.47	37.82	38.16
3.2	38.50	38.85	39.20	39.55	39.89	40.24	40.59	40.94	41.29	41.65
3.3	42.00	42.35	42.71	43.06	43.42	43.78	44.14	44.49	44.85	45.21
3.4	45.57	45.93	46.29	46.66	47.02	47.38	47.74	48.11	48.47	48.84
3.5	49.20	49.57	49.94	50.30	50.67	51.04	51.40	51.77	52.14	52.51
3.6	52.88	53.25	53.62	53.99	54.36	54.73	55.10	55.48	55.85	56.22
3.7	56.59	56.96	57.33	57.71	58.08	58.45	58.82	59.20	59.57	59.94
3.8	60.31	60.69	61.06	61.44	61.81	62.18	62.56	62.93	63.30	63.68
3.9	64.05	64.42	64.80	65.17	65.55	65.92	66.29	66.67	67.04	67.41
4.0	67.79	68.16	68.53	68.91	69.28	69.65	70.03	70.40	70.77	71.14
4.1	71.52	71.89	72.26	72.63	73.00	73.38	73.75	74.12	74.49	74.86
4.2	75.23	75.60	75.97	76.34	76.71	77.08	77.45	77.82	78.19	78.56
4.3	78.93	79.30	79.67	80.04	80.41	80.78	81.14	81.51	81.88	82.25
4.4	82.61	82.98	83.35	83.61	84.08	84.45	84.81	85.18	85.55	85.91
4.5	86.28	86.64	87.01	87.37	87.73	88.10	88.46	88.82	89.19	89.55
4.6	89.91	90.28	90.64	91.00	91.37	91.73	92.09	92.45	92.81	93.17
4.7	93.53	93.89	94.25	94.61	94.97	95.33	95.69	96.05	96.41	96.77
4.8	97.13	97.49	97.85	98.21	98.57	98.92	99.28	99.64	100.00	100.35
4.9	100.71	101.07	101.42	101.78	102.14	102.49	102.85	103.21	103.56	103.92

Table IV. Mutual Reactance at 60 Cycles, Conductor to Sheath (or Shield)

$D_{sm}/2S$	0	1	2	3	4	5	6	7	8	9
0.4	21.1	20.5	19.9	19.4	18.9	18.3	17.8	17.4	16.9	16.4
0.3	27.7	26.9	26.2	25.5	24.8	24.1	23.5	22.9	22.2	21.6
0.2	37.0	35.9	34.8	33.8	32.8	31.9	31.0	30.1	29.3	28.4
0.1	52.9	50.7	48.7	46.9	45.2	43.6	42.1	40.7	39.4	38.2

core conductors up through 5.0 *CI* and for hollow core concentrically stranded copper or aluminum oil-filled cable conductors up through 4.0 *CI*.

For values of x_p below 3.5, a range which appear to cover most cases of practical interest at power frequencies, the conductor proximity effect for cables in equilateral triangular formation in the same or in separate ducts may be calculated from the following equation based on an approximate expression given by Arnold[6] (equation 7) for a system of three homogeneous, straight, parallel, solid conductors of circular cross section arranged in equilateral formation and carrying balanced 3-phase current remote from all other conductors or conducting material. The empirical transverse conductance factor k_p is introduced to make the expression applicable to stranded conductors. Experimental results suggest the values of k_p shown in Table II.

$$Y_{cp} = F(x_p)\left(\frac{D_c}{S}\right)^2 \times \left[\frac{1.18}{F(x_p)+0.27}+0.312\left(\frac{D_c}{S}\right)^2\right] \quad (24)$$

$$x_p = \frac{6.80}{\sqrt{R_{dc}/k_p}} \text{ at 60 cycles} \quad (25)$$

When the second term in the brackets is small with respect to the first term as it usually is, equation 24 may be written

$$Y_{cp} = 4F(x_p)\left[\frac{0.295(D_c/S)^2}{F(x_p)+0.27}\right] = 4\left(\frac{D_c}{S}\right)^2 F(x_p') \quad (24\text{A})$$

where the function $F(x_p')$ is shown in Fig. 1.

The average proximity effect for conductors in cradle configuration in the same duct or in separate ducts in a formation approximating a regular polygon may also be estimated from equation 24 and 24(A). In such cases, S should be taken as the axial spacing between adjacent conductors.

The factor Y_s is the sum of two factors, Y_{sc} due to circulating current effect and Y_s due to eddy current effects.

$$W_s = I^2 R_{dc}(Y_{sc}+Y_{se}) \text{ watts per conductor foot} \quad (26)$$

Because of the large sheath losses which result from short-circuited sheath operation with appreciable separation between metallic sheathed single conductor cables, this mode of operation is usually restricted to triplex cable or three single-conductor cables contained in the same duct. The circulating current effect in three metallic sheathed single-conductor cables arranged in equilateral configuration is given by

$$Y_{sc} = \frac{R_s/R_{dc}}{1+(R_s/X_m)^2} \quad (27)$$

When $(R_s/X_m)^2$ is large with respect to unity as usually is the case of shielded non-leaded cables, equation 27 reduces to

$$Y_{sc} = \frac{X_m}{R_s R_{dc}} \text{ approximately} \quad (27\text{A})$$

$$X_m = 0.882 f \log 2S/D_{sm} \text{ microhms per foot} \quad (28)$$

$$= 52.9 \log 2S/D_{sm} \text{ microhms per foot at 60 cycles} \quad (28\text{A})$$

where S is the axial spacing of adjacent cables. For a cradled configuration X_m may be approximated from

$$X_m = 52.9 \log \frac{2.52S}{D_{sm}} \sqrt[4]{1-\left(\frac{S}{D_p-S}\right)^2} \text{ microhms per foot at 60 cycles} \quad (29)$$

$$= 52.9 \log 2.3\ S/D_{sm} \text{ approximately} \quad (29\text{A})$$

Table IV provides a convenient means for determining X_m for cables in equilateral configuration.

The eddy-current effect for single-conductor cables in equilateral configuration with open-circuited sheaths is

$$Y_{se} = \frac{3R_s/R_{dc}}{\left(\frac{5.2R_s}{f}\right)^2+\frac{1}{5}\left(\frac{2S}{D_{sm}}\right)} \times \left(\frac{D_{sm}}{2S}\right)^2\left[1+\frac{5}{12}\left(\frac{D_{sm}}{2S}\right)^2\right] \quad (30)$$

when $(5.2\ R_s/f)^2$ is large in respect to 1/5 $(2\ S/D_{sm})$ as in the case of lead sheaths.

$$Y_{se} = \frac{396}{R_s R_{ds}}\left(\frac{D_{sm}}{2S}\right)^2\left[1+\frac{5}{12}\left(\frac{D_{sm}}{2S}\right)^2\right] \text{ approximately at 60 cycles} \quad (30\text{A})$$

When the sheaths are short-circuited, the sheath eddy loss will be reduced and may be approximated by multiplying equations 30 or 30(A) by the ratio

$R_s^2/(R_s^2+X_m^2)$

In computing average eddy current for cradled configuration, S should be taken equal to the axial spacing and not to the geometric-mean spacing. Equations 30 and 30(A) may be used to compute the eddy-current effect for single-conductor cables installed in separate ducts. Strictly speaking, these equations apply only to three cables in equilateral configuration but can be used to estimate losses in large cable groups when latter are so oriented as to approximate a regular polygon.

The eddy-current effect for a 3-conductor cable is given by Arnold.[6]

$$Y_{se} = \frac{3R_s}{R_{dc}}\left[\frac{(2s/D_{sm})^2}{\left(\frac{5.2R_s}{f}\right)^2+1}+\frac{(2s/D_{sm})^4}{4\left(\frac{5.2R_s}{f}\right)^2+1}+\frac{(2s/D_{sm})^6}{16\left(\frac{5.2R_s}{f}\right)^2+1}\cdots\right] \quad (31)$$

When $(5.2R_s/f)^2$ is large with respect to unity,

$$Y_{se} = \frac{396}{R_s R_{dc}}\left(\frac{2s}{D_{sm}}\right)^2 \text{ approximately at 60 cycles} \quad (31\text{A})$$

$s = 1.155T + 0.60 \times$ the V gauge depth for compact sectors

$= 1.155T + 0.58\ D_c$ for round conductors (32)

and T is the insulation thickness, including thickness of shielding tapes, if any. While equation 31(A) will suffice for lead sheath cables, equation 31 should be used for aluminum sheaths.

On 3-conductor shielded paper lead cable it is customary to employ a 3- or 5-mil copper tape or bronze tape intercalated with a paper tape for shielding and binder purposes. The lineal d-c resistance of a copper tape 5 mils by 0.75 inch is about 2,200 microhms per foot of tape at 20 C. The d-c resistance per foot of cable will be equal to the lineal resistance of the tape multiplied by the lay correction factor as given by the expression under the square-root sign in equation 13. In practice the lay correction factor may vary from 4 to 12 or more resulting in shielding and binder assembly resist-

Table V. Specific Inductive Capacitance of Insulations

Material	ϵ_r
Polyethylene	2.3
Paper insulation (solid type)	3.7 (IPCEA value)
Paper insulation (other types)	3.3–4.2
Rubber and rubber-like compounds	5 (IPCEA value)
Varnished cambric	5 (IPCEA value)

ances of approximately 10,000 or more microhms per foot of cable. Even on the assumption that the assembly resistance is halved because of contact with adjacent conductors and the lead sheath computations made using equations 27 and 30 show that the resulting circulating and eddy current losses are a fraction of 1% on sizes of practical interest. For this reason it is customary to assume that the losses in the shielding and binder tapes of 3-conductor shielded paper lead cable are negligible. In cases of nonleaded rubber power cables where lapped metallic tapes are frequently employed, tube effects may be present and may materially lower the resistance of the shielding assembly and hence increase the losses to a point where they are of practical significance.

An exact determination of the pipe loss effect Y_p in the case of single-conductor cables installed in nonmagnetic conduit or pipe is a rather involved procedure as indicated in reference 7. Equation 31 may be used to obtain a rough estimate of Y_p for cables in cradled formation on the bottom of a nonmagnetic pipe, however by taking the average of the results obtained for wide triangular spacing with $s=(D_p-D_s)/2$ and for close triangle spacing at the center of the pipe with $s=0.578\,D_s$. The mean diameter of the pipe and its resistance per foot should be substituted for D_{sm} and R_s respectively.

For magnetic pipes or conduit the following empirical relationships[8] may be employed

$$Y_p=\frac{1.54s-0.115D_p}{R_{dc}}\quad \text{(3-conductor cable)} \qquad (33)$$

$$Y_p=\frac{0.89S-0.115D_p}{R_{dc}}\quad \text{(single-conductor, close triangular)} \qquad (34)$$

$$Y_p=\frac{0.34S+0.175D_p}{R_{dc}}\quad \text{(single-conductor, cradled)} \qquad (35)$$

These expressions apply to steel pipe[8] and should be multiplied by 0.8 for iron conduit.[9]

The expressions given for Y_c and Y_s above should be multiplied by 1.7 to find the corresponding in-pipe effects for magnetic pipe or conduit for both triangular and cradled configurations.

Calculation of Dielectric Loss

The dielectric loss W_d for 3-conductor shielded and single-conductor cable is given by the expression

$$W_d=\frac{0.00276E^2\epsilon_r\cos\phi}{\log(2T+D_c)/D_c}\ \text{watts per conductor foot at 60 cycles} \qquad (36)$$

and for 3-conductor belted cable by[1]

$$W_d=\frac{0.019E^2\epsilon_r\cos\phi}{G_1}\ \text{watts per conductor foot at 60 cycles} \qquad (37)$$

where E is the phase to neutral voltage in kilovolts, ϵ_r is the specific inductive capacitance of the insulation (Table V) T is its thickness and $\cos\phi$ is its power factor. The geometric factor G_1 may be found from Fig. 2 of reference 1.

For compact sector conductors the dielectric loss may be taken equal to that for a concentric round conductor having the same cross-sectional area and insulation thickness.

Calculation of Thermal Resistance

Thermal Resistance of the Insulation

For a single conductor cable,

$$\bar{R}_i=0.012\bar{\rho}_i\log D_i/D_c\quad \text{thermal ohm-feet} \qquad (38)$$

where $\bar{\rho}_i$ is the thermal resistivity of the insulation (Table VI) and D_i is its diameter. In multiconductor cables there is a multipath heat flow between the conductor and sheath. The following expression[1] represents an equivalent value which, when multiplied by the heat flow from one conductor, will produce the actual temperature elevation of the conductor above the sheath.

$$\bar{R}_i=0.00522\bar{\rho}_iG_1\quad \text{thermal ohm-feet} \qquad (39)$$

Values of the geometric factor G_1 for 3-conductor belted and shielded cables are given in Fig. 2 and Table VIII respectively of reference 1. On large size sector conductors with relatively thin insulation walls (i.e. ratios of insulation thickness to conductor diameter of the order of 0.2 or less); values of G_1 for 3-conductor shielded cable as determined by back calculation, on the basis of an assumed insulation resistivity, from laboratory heat-run temperature-rise data, have not always confirmed theoretical values, and, in some cases, have yielded G_1 values which approach those for a nonshielded, nonbelted construction.

Table VI. Thermal Resistivity of Various Materials

Material	$\bar{\rho}$, C Cm/Watt
Paper insulation (solid type)	700 (IPCEA value)
Varnished cambric	600 (IPCEA value)
Paper insulation (other types)	500–550
Rubber and rubber-like	500 (IPCEA value)
Jute and textile protective covering	500
Fiber duct	480
Polyethylene	450
Transite duct	200
Somastic	100
Concrete	85

Thermal Resistance of Jackets, Duct Walls, and Somastic Coatings

The equivalent thermal resistance of relatively thin cylindrical sections such as jackets and fiber duct walls may be determined from the expression

$$\bar{R}=0.0104\bar{\rho}n'\left(\frac{t}{D-t}\right)\quad \text{thermal ohm-feet} \qquad (40)$$

with appropriate subscripts applied to $\bar{R}$, $\bar{\rho}$, and D in which D represents the outside diameter of the section and t its thickness. n' is the number of conductors contained with the section contributing to the heat flow through it.

Thermal Resistance between Cable Surface and Surrounding Pipe, Conduit, or Duct Wall

Theoretical expressions for the thermal resistance between a cable surface and a surrounding enclosure are given in reference 10. As indicated in Appendix I, these have been simplified to the general form

$$\bar{R}_{sd}=\frac{n'A}{1+(B+CT_m)D_s'}\quad \text{thermal ohm-feet} \qquad (41)$$

in which A, B, and C are constants, D_s' represents the equivalent diameter of the cable or group of cables and n' the number of conductors contained within D_s'. T_m is the mean temperature of the intervening medium. The constants A, B, and C

Table VII Constants for Use in Equations 41 and 41(A)

Condition	A	B	C	A′	B′
In metallic conduit	17	3.6	0.029	3.2	0.19
In fiber duct in air	17	2.1	0.016	5.6	0.33
In fiber duct in concrete	17	2.3	0.024	4.6	0.27
In transite duct in air	17	3.0	0.014	4.4	0.26
In transite duct in concrete	17	2.9	0.029	3.7	0.22
Gas-filled pipe cable at 200 psi	3.1	1.16	0.0053	2.1	0.68
Oil-filled pipe cable	0.84	0	0.0065	2.1	2.45

D_s' = 1.00 × diameter of cable for one cable
1.65 × diameter of cable for two cables
2.15 × diameter of cable for three cables
2.50 × diameter of cable for four cables

given in Table VII have been determined from the experimental data given in references 10 and 11.

If representative values of $T_m = 60$ C are assumed, equation 41 reduces to

$$\bar{R}_{sd} = \frac{n'A'}{D_s' + B'} \text{ thermal ohm-feet} \qquad (41A)$$

It should be noted that in the case of ducts, $\bar{R}_{sd}$ is calculated to the inside of the duct wall and the thermal resistance of the duct wall should be added to obtain $\bar{R}_{se}$.

Thermal Resistance from Cables, Conduits, or Ducts Suspended in Air

The thermal resistance $\bar{R}_a$ between cables, conduits, or ducts suspended in still air may be determined from the following expression which is developed in Appendix I.

$$\bar{R}_a = \frac{15.6n'}{D_s'[(\Delta T/D_s')^{1/4} + 1.6\epsilon(1 + 0.0167T_m)]} \text{ thermal ohm-feet} \qquad (42)$$

In this equation ΔT represents the difference between the cable surface temperature T_s and ambient air temperature T_a in degrees centigrade, T_m the average of these temperatures and ϵ the coefficient of emissivity of the cable surface. Assuming representative values of $T_s = 60$ and $T_a = 30$ C, and a range in D_s' of from 2 to 10 inches, equation 42 may be simplified to

$$\bar{R}_a = \frac{9.5n'}{1 + 1.7D_s'(\epsilon + 0.41)} \text{ thermal ohm-feet} \qquad (42A)$$

The value of ϵ may be taken as equal to 0.95 for pipes, conduits or ducts, and painted or braided surfaces, and from 0.2 to 0.5 for lead and aluminum sheaths, depending upon whether the surface is bright or corroded. It is interesting to note that equation 42(A) checks the IPCEA method of determining R_a very closely with $\epsilon = 0.41$ for diameters up to 3.5 inches. In the IPCEA method $\bar{R}_a = 0.00411\ n'B/D_s'$ where $B = 650 + 314\ D_s'$ for

$D_s' = 0 - 1.75$ inches and $B = 1{,}200$ for larger values of D_s'

Effective Thermal Resistance Between Cables, Ducts, or Pipes, and Ambient Earth

As previously indicated, an effective thermal resistance $\bar{R}_e'$ may be employed to represent the earth portion of the thermal circuit in the case of buried cable systems. This effective thermal resistance includes the effect of loss factor and, in the case of a multicable installation, also the mutual heating effects of the other cables of the system. In the case of cables in a concrete duct bank, it is desirable to further recognize a difference between the thermal resistivity of the concrete $\bar{\rho}_c$ and the thermal resistivity of the surrounding earth $\bar{\rho}_e$.

The thermal resistance between any point in the earth surrounding a buried cable and ambient earth is given by the expression[11]

$$\bar{R}_{pa} = 0.012\bar{\rho}_e \log d'/d \qquad \text{thermal ohm-feet} \qquad (43)$$

in which $\bar{\rho}_e$ is the thermal resistivity of the earth, d' is the distance from the image of the cable to the point P, and d is the distance from the cable center to P. From this equation and the principles discussed in references 3, 12, and 13, the following expressions may be developed, applicable to directly buried cables and to pipe-type cables.

$$\bar{R}_e' = 0.012\bar{\rho}_e n' \times \left[\log\frac{D_x}{D_e} + (LF)\log\left[\left(\frac{4L}{D_x}\right)F\right]\right] \text{ thermal ohm-feet} \qquad (44)$$

in which D_e is the diameter at which the earth portion of the thermal circuit commences and n' is the number of conductors contained within D_e. The fictitious diameter D_x at which the effect of loss factor commences is a function of the diffusivity of the medium α and the length of the loss cycle.[3]

$$D_x = 1.02\sqrt{\alpha(\text{length of cycle in hours})} \text{ inches} \qquad (45)$$

The empirical development of this equation is discussed in Appendix III. For a daily loss cycle and a representative value of $\alpha = 2.75$ square inches per hour for earth, D_x is equal to 8.3 inches. It should be noted that the value of D_x obtained from equation 45 is applicable for pipe diameters exceeding D_x, in which case the first term of equation 44 is negative.

The factor F accounts for the mutual heating effect of the other cables of the cable system, and consists of the product of the ratios of the distance from the reference cable to the image of each of the other cables to the distance to that cable. Thus,

$$F = \left(\frac{d_{12}'}{d_{12}}\right)\left(\frac{d_{13}'}{d_{13}}\right) \cdots \left(\frac{d_{1N}'}{d_{1N}}\right)(N-1 \text{ terms}) \qquad (46)$$

It will be noted that the value of F will vary depending upon which cable is selected as the reference, and the maximum conductor temperature will occur in the cable for which $4LF/D_x$ is a maximum. N refers to the number of cables or pipes, and F is equal to unity when $N = 1$.

When the cable system is contained within a concrete envelope such as a duct bank, the effect of the differing thermal resistivity of the concrete envelope is conveniently handled by first assuming that the thermal resistivity of the medium is that of concrete $\bar{\rho}_c$ throughout and then correcting that portion lying beyond the concrete envelope to the thermal resistivity of the earth $\bar{\rho}_e$. Thus

$$\bar{R}_e' = 0.012\bar{\rho}_c n' \times \left[\log\frac{D_x}{D_e} + (LF)\log\left[\left(\frac{4L}{D_x}\right)F\right]\right] + 0.012(\bar{\rho}_e - \bar{\rho}_c)n'N(LF)G_b \text{ thermal ohm-feet} \qquad (44A)$$

The geometric factor G_b, as developed in Appendix II is a function of the depth to the center of the concrete enclosure L_b and its perimeter P, and may be found conveniently from Fig. 2 in terms of the ratio L_b/P and the ratio of the longest to short dimension of the enclosure.

For buried cable systems T_a should be taken as the ambient temperature at the depth of the hottest cable. As indicated in reference 12, the expressions used throughout this paper for the thermal resistance and temperature rise of buried cable systems are based on the hypothesis suggested by Kennelly applied in accordance with the principle of superposition. According to this hypothesis, the isothermal-heat flow field and temperature rise at any point in the soil surrounding a buried cable can be represented by the steady-state solution for the heat flow between two parallel cylinders (constituting a heat source and sink) located in a vertical plane in an infinite medium of uniform temperature and thermal resistivity with an axial separation between cylinders of twice the actual depth of burial and with source and sink respectively generating and absorbing heat at identical rates, thereby resulting in the temperature of the horizontal midplane between cylinders (i.e., corresponding to the surface of the earth) remaining, by symmetry, undisturbed.

The principle of superposition, as applied to the case at hand, can be stated in thermal terms as follows: If the thermal network has more than one source of temperature rise, the heat that flows at any point, or the temperature drop between any two points, is the sum of the heat flows and temperature drops at these points which would exist if each source of temperature rise were considered separately. In the case at hand, the sources of heat flow and temperature rise to be superimposed are, namely, the heat

from the cable, the outward flow of heat from the core of the earth, and the inward heat flow solar radiation, and, when present, the heat flow from interfering sources. By employing as the ambient temperature in the calculations the temperature at the depth of burial of the hottest cable, the combined heat flow from earth core and solar radiation sources is superimposed upon that produced at the surface of the hottest cable by the heat flow from that cable and interfering sources which are calculated separately with all other heat flows absent. The combined heat flow from earth core and solar sources results in an earth temperature which decreases with depth in summer; increases with depth in winter; remains about constant at any given depth on the average over a year; approximates constancy at all depths at midseason, and in turn results in flow of heat from cable sources to earth's surface, directly to surface in midseason and winter and indirectly to surface in summer.

Factors which tend to invalidate the combined Kennelly-superposition principle method are departure of the temperature of the surface of earth from a true isothermal (as evidenced by melting of snow in winter directly over a buried steam main) and nonuniformity of thermal resistivity (due to such phenomena as radial and vertical migration of moisture). The extent to which the Kennelly-superposition principle method is invalidated, however, is not of practical importance provided that an over-all or effective thermal resistivity is employed in the Kennelly equation.

Special Conditions

Although the majority of cable temperature calculations may be made by the foregoing procedure, conditions frequently arise which require somewhat specialized treatment. Some of these are covered herein.

Emergency Ratings

Under emergency conditions it is frequently necessary to exceed the stated normal temperature limit of the conductor T_c and to set an emergency temperature limit T_c'. If the duration of the emergency is long enough for steady-state conditions to obtain, then the emergency rating I' may be found by equation 9 substituting T_c' for T_c and correcting R_{dc} for the increased conductor temperature.

If the duration of the emergency is less than that required for steady-state conditions to obtain, the emergency rating of the line may be determined from

$$I' = \sqrt{\frac{T_c' - I^2 R_{dc}(1+Y_c)(\bar{R}_{ca}' - \bar{R}_{ct}') - (T_a + \Delta T_d)}{R_{dc}'(1+Y_c)\bar{R}_{ct}'}} \quad \text{kiloamperes} \quad (47)$$

in which $\bar{R}_{ct}'$ is the effective transient thermal resistance of the cable system for the stated period of time. Procedures for calculating $\bar{R}_{ct}'$ for times up to several hours are given in reference 14, and for longer times in references 15-17.

The Effect of Extraneous Heat Sources

In the case of multicable installations the assumption has been made that all cables are of the same size and are similarly loaded. When this is not the case the temperature rise or load capability of one particular equal cable group may be determined by treating the heating effect of other cable groups separately, introducing an interference temperature rise ΔT_{int} in equations 1 and 9. Thus

$$T_c - T_a = \Delta T_c + \Delta T_d + \Delta T_{int} \quad \text{degrees centigrade} \quad (1A)$$

$$I = \sqrt{\frac{T_c - (T_a + \Delta T_d + \Delta T_{int})}{R_{dc}(1+Y_c)\bar{R}_{ca}'}} \quad \text{kiloamperes} \quad (9A)$$

in which ΔT_{int} represents the sum of a number of interference effects, for each of which

$$\Delta T_{int} = [W_c q_s(LF) + W_d]\bar{R}_{int} \quad \text{degrees centigrade} \quad (48)$$

$$\bar{R}_{int} = 0.012\bar{\rho}_e n' \log F_{int} \quad \text{thermal ohm-feet} \quad (49)$$

$$F_{int} = \frac{(d_{11}')(d_{21}')(d_{31}')\ldots d_{N1}'}{(d_{11})(d_{21})(d_{31})\ldots d_{N1}} \quad (N \text{ terms}) \quad (50)$$

where the parameters apply to each system which may be considered as a unit. For cables in duct

$$\bar{R}_{int} = 0.012n'[\bar{\rho}_c \log F_{int} + N(\bar{\rho}_e - \bar{\rho}_c)G_b] \quad \text{thermal ohm-feet} \quad (49A)$$

Because of the mutual heating between cable groups, the temperature rise of the interfering groups should be rechecked. If all the cable groups are to be given mutually compatible ratings, it is necessary to evaluate W_c for each group by successive approximations, or by setting up a system of simultaneous equations, substituting for W_c its value by equation 15 and solving for I.

In case ΔT_{int} or a component of it is produced by an adjacent steam main, the temperature of the steam T_v rather than the heat flow from it is usually given. Thus

$$\Delta T_{int} = \left[\frac{T_v - T_a}{\bar{R}_{va}}\right]\bar{R}_{int} \quad \text{degrees centigrade} \quad (51)$$

where $\bar{R}_{va}$ is the thermal resistance between the steam pipe and ambient earth.

Aerial Cables

In the case of aerial cables it may be desirable to consider both the effects of solar radiation which increases the temperature rise and the effect of the wind which decreases it.[24] Under maximum sunlight conditions, a lead-sheathed cable will absorb about 4.3 watts per foot per inch of profile[18] which must be returned to the atmosphere through the thermal resistance $\bar{R}_e/n'$. This effect is conveniently treated as an interference temperature rise according to the relationship

$$\Delta T_{int} = 4.3 D_s' \bar{R}_e / n' \quad \text{degrees centigrade} \quad (47A)$$

For black surfaces this value should be increased about 75%.

As indicated in Appendix II, the following expression for $\bar{R}_e$ may be used where V_w is the velocity of the wind in miles per hour

$$\bar{R}_e = \frac{3.5n'}{D_s'(\sqrt{V_w/D_s'} + 0.62\epsilon)} \quad \text{thermal ohm-feet} \quad (42B)$$

Use of Low-Resistivity Backfill

In cases where the thermal resistivity of the earth is excessively high, the value of $\bar{R}_e'$ may be reduced by backfilling the trench with soil or sand having a lower value of thermal resistivity. Equation 44(A) may be used for this case if $\bar{\rho}_f$, the thermal resistivity of the backfill is substituted for $\bar{\rho}_c$, and G_b applies to the zone having the backfill in place of the zone occupied by the concrete.

Single-Conductor Cables in Duct with Solidly Bonded Sheaths

The relatively large and unequal sheath losses in the three phases which may result from this type of operation may be determined from Table VI of reference 1. It will be noted that

$$Y_{sc1} = \left(\frac{R_s}{R_{dc}}\right)\left(\frac{I_{s1}^2}{I^2}\right); \quad Y_{sc2} = \left(\frac{R_s}{R_{dc}}\right)\left(\frac{I_{s2}^2}{I^2}\right); \quad Y_{sc3} = \left(\frac{R_s}{R_{dc}}\right)\left(\frac{I_{s3}^2}{I^2}\right) \quad (52)$$

where expressions for I_{s1}^2/I^2 etc., appear in the table. The resulting unequal values of Y_c in the three phases will yield unequal values of q_s, and equation 5 becomes for phase no. 1, the instance given as equation 5(A) on the following page.

$$\Delta T_{c1} = W_c[\bar{R}_1 + q_{s1}\{\bar{R}_{sd} + \bar{R}_{es} + (LF)\bar{R}_{sp}\} + Nq_{sa}(LF)\bar{R}_{pa}] \quad \text{thermal ohm-feet} \quad (5A)$$

where q_{sa} is the average of q_{s1}, q_{s2}, and q_{s3}.

Armored Cables

In multiconductor armored cables a loss occurs in the armor which may be considered as an alternate to the conduit or pipe loss. If the armor is nonmagnetic, the component of armor loss Y_a to be used instead of Y_p in equations 14 and 19 may be calculated by the equations for sheath loss substituting the resistance and mean diameter of the armor for those of the sheath. In calculating the armor resistance, account should be taken of the spiralling effect for which equation 13 suitably modified may be used. If the armor is magnetic, one would expect an increase in the factors Y_c and Y_s in equation 14 since this occurs in the case of magnetic conduit. Unfortunately, no simple procedure is available for calculating these effects. A rough estimate of the inductive effects may be made by using the procedure given above for magnetic conduit.

A simple method of approximating the losses in single conductor cables with steel-wire armor at spacings ordinarily employed in submarine installations is to assume that the combined sheath and armor current is equal to the conductor current.[1] The effective a-c resistance of the armor may be taken as 30 to 60% greater than its d-c resistance corrected for lay as indicated above. If more accurate calculations are desired references 19 and 20 will be found useful.

Effect of Forced Cooling

The temperature rise of cables in pipes or tunnels may be reduced by forcing air axially along the system. Similarly, in the case of oil-filled pipe cable, oil may be circulated through the pipe. Under these conditions, the temperature rise is not uniform along the cable and increases in the direction of flow of the cooling medium. The solution of this problem is discussed in reference 21.

Appendix I

Development of Equations 41, 42, and Table VII

Theoretical and semiempirical expressions for the thermal resistance between cables and an enclosing pipe or duct wall are given in reference 10. Further data on the thermal resistance between cables and fiber and transite ducts are given in reference 11. For purposes of cable rating, it is desirable to develop standardized expressions for these thermal resistances based upon all of the data available and including the effect of the temperature of the intervening medium.

Table VIII. Constants for Use in Equation 53

Condition	a	b	c	Average ΔT
Cable in metallic conduit	0.07	0.121	0.0017	20
Cable in fiber duct in air	0.07	0.036	0.0009	20
Cable in fiber duct in concrete	0.07	0.043	0.0014	20
Cable in transite duct in air	0.07	0.086	0.0008	20
Cable in transite duct in concrete	0.07	0.079	0.0016	20
Gas-filled pipe-type cable at 200 psi	0.07	0.121	0.0017	10

The theoretical expression for the case where the intervening medium is air or gas as presented in reference 10 may be generalized in the following form:

$$\bar{R}_{sd} = \frac{n'}{D_s'\left[a\left(\dfrac{\Delta T P^2}{D_s'}\right)^{1/4} + b + cT_m\right]} \quad (53)$$

where

$\bar{R}_{sd}$ = the effective thermal resistance between cable and enclosure in thermal ohm-feet

D_s' = the cable diameter or equivalent diameter of three cables in inches

ΔT = the temperature differential in degrees centigrade

P = the pressure in atmospheres

T_m = mean temperature of the medium in degrees centigrade

n' = number of conductors involved

The constants a, b, and c in this equation have been established empirically as follows: Considering $b+cT_m$ as a constant for the moment, the analysis given in reference 10 results in a value of $a=0.07$. With a thus established, the data given in reference 10 for cable in pipe, and in reference 11 for cable in fiber and transite ducts were analyzed in similar manner to give the values of b and c which are shown in Table VIII.

In order to avoid a reiterative calculation procedure, it is desirable to assume a value for ΔT since its actual value will depend upon $\bar{R}_{sd}$ and the heat flow. Fortunately, as ΔT occurs to the 1/4 power in equation 53, the use of an average value as indicated in Table VIII will not introduce a serious error.

By further restricting the range of D_s' to 1–4 inches for cable in duct or conduit and to 3–5 inches for pipe-type cables, equation 53 is reduced to equation 41.

$$\bar{R}_{sd} = \frac{n'A}{1+(B+CT_m)D_s'} \quad \text{thermal ohm-feet} \quad (41)$$

in which the values of the constants A, B, and C appear in Table VII.

In the case of oil-filled pipe cable, the analysis given in reference 10 gives the following expression

$$\bar{R}_{sd} = \frac{n'}{0.60+0.025(D_s'^3 T_m^3 \Delta T)^{1/4}} \quad \text{thermal ohm-feet} \quad (54)$$

Assuming an average value of $\Delta T = 7$ C and a range of 150–350 for $D_s'T_m$, equation 54 reduces to equation 41 with the values of A, B, and C given in Table VII.

In the case of cables or pipes suspended in still air, the heat loss by radiation may be determined by the Stefan-Bolzmann formula

$$n'W(\text{radiation}) = 0.138 D_s'\epsilon[(T_s+273)^4-(T_a+273)^4]10^{-8} \quad \text{watts per foot} \quad (55)$$

where ϵ is the coefficient of emissivity of the cable or pipe surface. Over the limited temperature range in which we are interested, equation 55 may be simplified to[10]

$$n'W(\text{radiation}) = 0.102 D_s'\Delta T\epsilon \times (1+0.0167T_m) \quad \text{watts per foot} \quad (55A)$$

Over the same temperature range the heat loss by convection from horizontal cables or pipes is given with sufficient accuracy by the expression

$$n'W(\text{convection}) = 0.064\, D_s'\Delta T(\Delta T/D_s')^{1/4} \quad \text{watts per foot} \quad (56)$$

in which the numerical constant 0.064 has been selected for the best fit with the carefully determined test results reported by Heilman[22] on 1.3, 3.5 and 10.8-inch diameter black pipes ($\epsilon=0.95$). Incidentally, this value also represents the best fit with the test data on 1.9–4.5 inch diameter black pipes reported by Rosch.[23] For vertical cables or pipes the value of this numerical constant may be increased by 22%.[22]

Combining equations 55(A) and 56 we obtain the relationship

$$\bar{R}_e = \frac{\Delta T}{n'W(\text{total})} = \frac{15.6n'}{D_s'[(\Delta T/D_s')^{1/4}+1.6\epsilon(1+0.0167T_m)]} \quad \text{thermal ohm-feet} \quad (42)$$

If the cable is subjected to wind having a velocity of V_w miles per hour, the following expression derived from the work of Schurig and Frick[24] should be substituted for the convection component.

$$n'W(\text{convection}) = 0.286 D_s'\Delta T\sqrt{V_w/D_s'} \quad \text{watts per foot} \quad (56A)$$

Combining equations 55(A) and 56(A) with $T_m = 45$ C

$$\bar{R}_e = \frac{\Delta T}{n'W(\text{total})} = \frac{3.5n'}{D_s'(\sqrt{V_w/D_s'}+0.62\epsilon)} \quad \text{thermal ohm-feet} \quad (42B)$$

Appendix II

Determination of the Geometric Factor G_b for Duct Bank

Considering the surface of the duct bank to act as an isothermal circle of radius r_b, the thermal resistance between the duct bank and the earth's surface will be a logarithmic function of r_b and L_b the distance of the center of the bank below the surface. Using the long form of the Kennelly Formula[13] we may define the geometric factor G_b as

$$G_b = \log \frac{L_b{}^2+\sqrt{L_b{}^2-r_b{}^2}}{r_b}$$

$$= \log\left[L_b/r_b+\sqrt{(L_b/r_b)^2-1}\right] \quad (57)$$

In order to evaluate r_b in terms of the dimensions of a rectangular duct bank, let the smaller dimension of the bank be x and the larger dimension by y. The radius of a circle inscribed within the duct bank touching the sides is

$$r_1 = x/2 \quad (58)$$

and the radius of a larger circle embracing the four corners is

$$r_2 = \frac{\sqrt{x^2+y^2}}{2} \quad (59)$$

Let us assume that the circle of radius r_b lies between these circles and the magnitude of r_b is such that it divides the thermal resistance between r_1 and r_2 in direct relation to the portions of the heat field between r_1 and r_2 occupied and unoccupied by the duct bank. Thus

$$\log \frac{r_b}{r_1} = \frac{xy-\pi r_1{}^2}{\pi(r_2{}^2-r_1{}^2)}\left(\log \frac{r_2}{r_1}\right) \quad \text{or}$$

$$\log \frac{r_2}{r_b} = \frac{\pi r_2{}^2-xy}{\pi(r_2{}^2-r_1{}^2)}\left(\log \frac{r_2}{r_1}\right)$$

from which

$$\log\ r_b = \frac{1}{2}\,\frac{x}{y}\left(\frac{4}{\pi}-\frac{x}{y}\right)\log\left(1+\frac{y^2}{x^2}\right)+\log \frac{x}{2} \quad (60)$$

It is desirable to derive r_b in terms of the perimeter P of the duct bank. Thus

$$P = 2(x+y) = 4\frac{x}{2}(1+y/x)$$

and therefore

$$\log \frac{x}{2} = \log \frac{P}{4(1+y/x)} \quad (61)$$

The curves of Fig. 2 have been developed from equations 57, 60, and 61 for several values of the ratio y/x. It should be noted in passing that the value of $r_b = 0.112P$ used in reference 13 applies to a y/x ratio of about 2/1 only.

Appendix III

Empirical Evaluation of D_x

In order to evaluate the effect of a cyclic load upon the maximum temperature rise of a cable system simply, it is customary to assume that the heat flow in the final portion of the thermal circuit is reduced by a factor equal to the loss factor of the cyclic load. The point at which this reduction commences may be conveniently expressed in terms of a fictitious diameter D_x. Thus

$$\bar{R}_{ca}' = \bar{R}_{cx}+(LF)\bar{R}_{xa} \text{ thermal ohm-feet} \quad (62)$$

For greater accuracy, it is desirable to establish the value of D_x empirically rather than to assume that D_x is equal to the diameter D_e at which the earth portion of the thermal circuit commences.

Equation 62 may be written in the form

$$\bar{R}_{ca}' = \bar{R}_{ce}+\bar{R}_{ex}+(LF)(\bar{R}_{ea}-\bar{R}_{ex}) \text{ thermal ohm-feet} \quad (62A)$$

In terms of the attainment factor (AF), one may write

$$\bar{R}_{ca}' = (AF)\bar{R}_{ca} = (AF)(\bar{R}_{ce}+\bar{R}_{ea}) \text{ thermal ohm-feet} \quad (63)$$

Equating equations 62(A) and 63 obtains the relationship

$$\bar{R}_{ex} = (1-x)\bar{R}_{ea}-x\bar{R}_{ce} \text{ thermal ohm-feet} \quad (64)$$

where

$$x = \frac{1-(AF)}{1-(LF)} \quad (65)$$

Since

$$\bar{R}_{ex} = 0.012n'\bar{\rho}\log D_x/D_e \text{ thermal ohm-feet} \quad (66)$$

$$\log D_x/D_e = \frac{83}{n'\bar{\rho}}[(1-x)\bar{R}_{ea}-x\bar{R}_{ce}] \quad (67)$$

The first paper of reference 3 presents the results of a study in which a number of typical daily loss cycles and also sinusoidal loss cycles of the same loss factor were applied to a number of typical buried cable systems. The results indicated that in all cases the sinusoidal loss cycle of the same loss factor adequately expressed the maximum temperature rise which was obtained with any of the actual loss cycles considered.

An analysis by equations 65 and 67 of the calculated values of attainment factors for sinusoidal loss cycles given in Table II and the corresponding cable system parameters given in Table I of the first paper of reference 3 yields a most probable value of $D_x = 8.3$ inches. As indicated in the third paper of reference 3, however, theoretically D_x should vary as the square root of the product of the diffusivity and the time length of the loading cycle. Hence as the diffusivity was taken as 2.75 square inches per hour in the above,

$$D_x = 1.02 \times \sqrt{\alpha_e \times \text{length of cycle in hours}} \text{ inches} \quad (45)$$

Table IX. Comparison of Values of % (AF) for Sinusoidal Loss Cycles at 30% Loss Factor

System	Description, Inches	% (AF) Neher	Shanklin	Wiseman
I	4.5 pipe	63/63	61/62	63/65
II	6.6 pipe	56/56	60/57	53/60
III	8.6 pipe	56/56	59/58	54/63
IV	10.6 pipe	58/58	61/59	55/53
V	0.5 cable		80/80	
VI	1.5 cable	77/76	77/76	77/77
VII	1.9 cable	71/71		
VIII	2.0 cable		63/62	
IX	2.0 cable		75/74	
X	3.4 cable		77/78	
Xa*	3.4 cable	83/80	83/81	
XI	3.7 cable	76/74	74/73	
XII	4.2 cable	70/66	70/67	
XIII	4.5 cable	69/64	65/64	61/63

* Diffusivity = 4.7 square inches per hour.

Table IX presents a comparison of the values of per cent attainment factor for sinusoidal loss cycles at 30% loss factor as calculated by equations 45, 66, 62(A), and 63 and as they appear in Table II of the first paper of reference 3.

Appendix IV. Calculations for Representative Cable Systems

15-Kv 350-MCM—3-Conductor Shielded Compact Sector Paper and Lead Cable Suspended in Air

$D_c = 0.616$ (equivalent round); V = gauge depth = 0.539 inch

$D_s = 2.129$; $T = 0.175$ inch; $t = 0.120$ inch

$$T_c = 81\text{ C};\ R_{dc} = \frac{12.9}{0.350}\left(\frac{234.5+81}{234.5+75}\right) = 37.6 \text{ microhms per foot} \quad (\text{Eq. 10A})$$

$$D_{sm} = 2.129-0.120 = 2.009 \text{ inches} \quad (\text{Eq. 12})$$

$$R_s = \frac{37.9}{2.009(0.120)} = 157 \text{ microhms per foot at 50 C} \quad (\text{Eq. 11A})$$

$k_s = 1.0$; $k_p = 0.6$ (equivalent round) (Table II)

$R_{dc}/k_s = 37.6$; $Y_{cs} = 0.008$ (Eq. 21 and Fig. 1)

$$S = 0.616+2(0.175+0.008) = 0.982 \text{ inches}$$

$R_{dc}/k_p = 62.6$; $F(x_p') = 0.003$ (Fig. 1)

$$Y_{cp} = \frac{1}{2}\left[4\left(\frac{0.616}{0.982}\right)^2\right]0.003 = 0.002$$

(Eq. 24A, and note to Table II)

$$1+Y_c = 1+0.008+0.002 = 1.010$$

$$s = 1.155(0.175+0.008)+0.60(0.539) = 0.534 \text{ inch} \quad (\text{Eq. 32})$$

$$Y_s = Y_{se} = \frac{396}{157(37.6)}\left\{\frac{2(0.534)}{2.009}\right\}^2 = 0.019 \quad (\text{Eq. 31A})$$

$$R_{ac}/R_{dc} = 1.010+0.019 = 1.029 \quad (\text{Eq. 14})$$

$$q_s = q_e = 1+\frac{0.019}{1.010} = 1.019 \quad (\text{Eqs. 18–19})$$

$\epsilon_r = 3.7$ (Table V); $E = 15/\sqrt{3} = 8.7$; $\cos\phi = 0.022$

$$W_d = \frac{0.00276\,(8.7)^2[3.7(0.022)]}{\log \dfrac{2(0.175)+0.681}{0.681}} = 0.094 \text{ watt per conductor foot}$$

(Eq. 36 and text)

(Note: In computing dielectric loss on

sector conductors, the equivalent diameter of the conductor is taken equal to that of a concentric round conductor, i.e., 0.681 inch for 350 MCM.)

$\bar{\rho}_i = 700$ (Table VI); $G_1 = 0.45$
(Table VIII of reference 1)

$\bar{R}_i = 0.00522\{700(0.45)\} = 1.64$
thermal ohm-feet (Eq. 39)

$n' = 3$; $\epsilon = 0.41$ (assumed)

$$\bar{R}_e = \frac{9.5(3)}{1+1.7[2.129(0.41+0.41)]}$$

$= 7.18$ thermal ohm-feet (Eq. 42A)

$\bar{R}_{ca} = 1.64 + 1.019(7.18) = 8.96$
thermal ohm-feet (Eq. 8)

$\Delta T_d = 0.094(0.82+7.18) = 0.75$ C (Eq. 6)

$T_a = 40$ C (assumed)

$$I = \sqrt{\frac{81-(40+0.8)}{37.6[1.010(8.96)]}}$$

$= 0.344$ kiloampere (Eq. 9)

If the cable is outdoors in sunlight and subjected to an 0.84 mile per hour wind

$$\bar{R}_e = \frac{3.5(3)}{2.129[\sqrt{0.84/2.129}+0.62(0.41)]}$$

$= 5.59$ thermal ohm-feet (Eq. 42B)

$\bar{R}_{ca}' = 1.64 + 1.019(5.59) = 7.34$
thermal ohm-feet (Eq. 8)

$$\Delta T_{int} = (4.3)(2.129)\left(\frac{5.59}{3}\right) = 17.1 \text{ C}$$

(Eq. 47A)

$T_a = 30$ C (assumed)

$$I = \sqrt{\frac{81-(30+0.6+17.1)}{(37.6)(1.010)(7.34)}}$$

$= 0.346$ kiloampere (Eq. 9)

In this particular case the net effect of solar radiation and an 0.84 mile per hour wind is to effectively raise the ambient temperature by 10 degrees, which is a rough estimating value commonly used. It should be noted, however, that this will not always be true, and the procedure outlined above is preferable.

69-Kv 1,500-MCM—Single-Conductor Oil-Filled Cable in Duct

Two identical cable circuits will be considered in a 2 by 3 fiber and concrete duct structure having the dimensions shown in Fig. 3.

$D_o = 0.600$; $D_c = 1.543$; $D_i = 2.113$;
$T = 0.285$; $D_s = 2.373$; $t = 0.130$ inches

$T_c = 75$ C; $R_{dc} = \frac{12.9}{1.50} = 8.60$
microhms per foot (Eq. 10A)

$D_{sm} = 2.373 - 0.130 = 2.243$ inches (Eq. 12)

$$R_s = \frac{37.9}{(2.243)(0.130)} = 130 \text{ microhms}$$

per foot at 50 C (Eq. 11A)

$$k_s = \frac{1.543-0.600}{1.543+0.600}\left(\frac{1.543+1.200}{1.543+0.600}\right)^2$$

$= 0.72$; $k_p = 0.8$ (Eq. 23 and Table II)

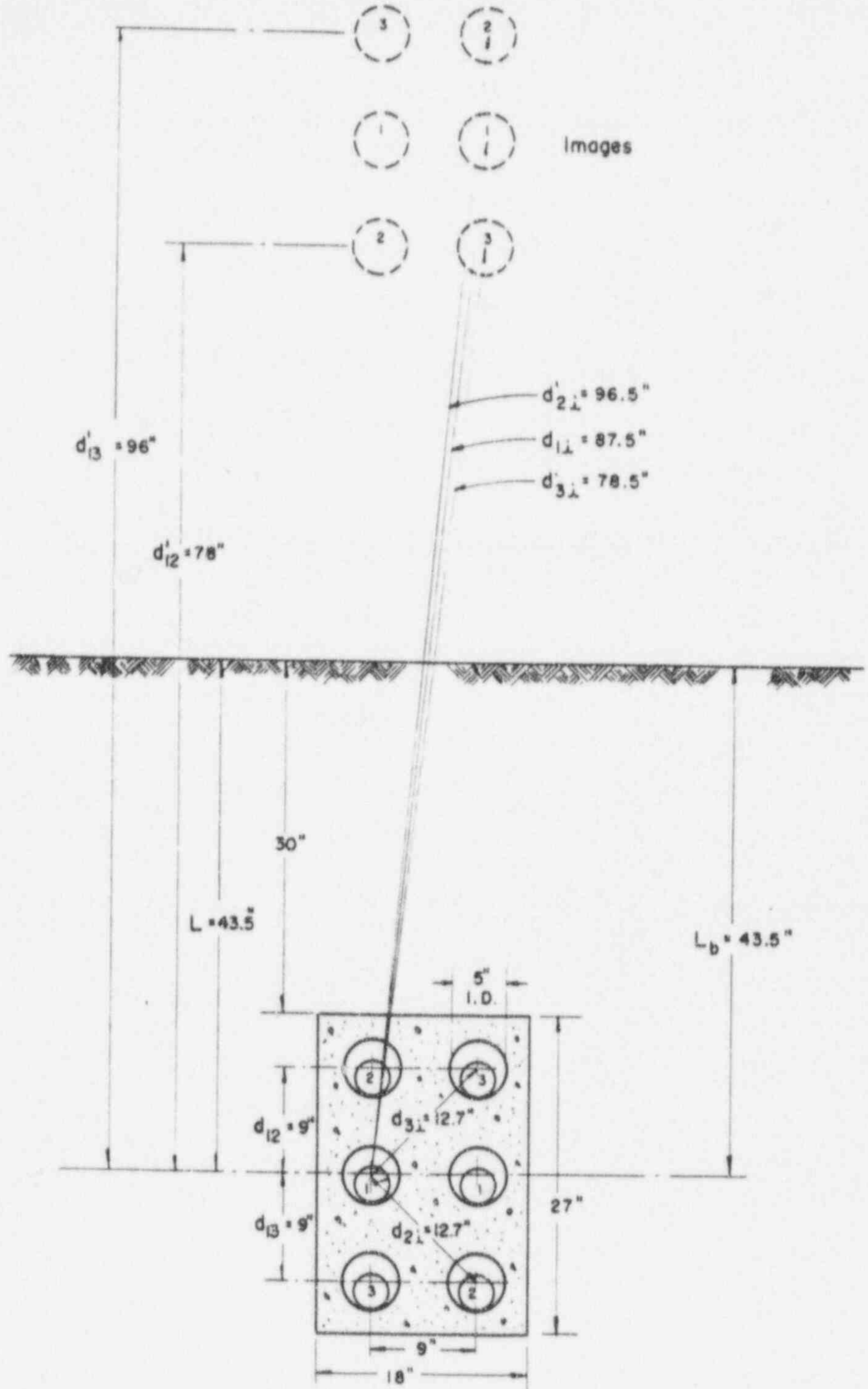

Fig. 3. Assumed duct bank configuration for typical calculations on 69-kv 1,500-MCM oil-filled cable (Appendix IV)

$R_{dc}/k_s = 11.9$; $Y_{cs} = 0.075$
(Eq. 21 and Fig. 1)

$S = 9.0$ (Fig. 3); $R_{dc}/k_p = 10.75$;
$F(x_p') = 0.075$ (Fig. 1)

$$Y_{cp} = 4\left(\frac{1.412}{9.0}\right)^2 0.075 = 0.007 \quad \text{(Eq. 24A)}$$

$1 + Y_c = 1 + 0.075 + 0.007 = 1.082$

Assuming the sheaths to be open-circuited, $Y_{sc} = 0$

$$Y_s = Y_{se} = \frac{396}{130(8.60)}\left(\frac{2.243}{2(9.0)}\right)^2 \times$$

$$\left[1+\frac{5}{12}\left(\frac{2.243}{2(9.0)}\right)^2\right] = 0.006 \quad \text{(Eq. 30A)}$$

$R_{ac}/R_{dc} = 1.082 + 0.006 = 1.088$ (Eq. 14)

$$q_s = q_e = 1 + \frac{0.006}{1.082} = 1.006 \quad \text{(Eqs. 18–19)}$$

$\epsilon_r =$ (Table V); $E = 69/\sqrt{3} = 40$;
$\cos\phi = 0.005$

$$W_d = \frac{0.00276(40)^2(3.5)(0.005)}{\log\frac{2.113}{1.543}}$$

$= 0.57$ watt per conductor foot (Eq. 36)

$\bar{\rho}_1 = 550$ (Table VI)

$$\bar{R}_1 = 0.012\left(550 \log \frac{2.113}{1.543}\right) = 0.90 \text{ thermal ohm-foot} \quad \text{(Eq. 38)}$$

$$n' = 1;\ \bar{R}_{sd} = \frac{1(4.6)}{2.37+0.27} = 1.74 \text{ thermal ohm-feet} \quad \text{(Eq. 41A)}$$

$\bar{\rho}_d = 480$ (Table VI); $t = 0.25$; $D_e = 5.0+0.5 = 5.50$ for fiber duct

$$\bar{R}_d = \frac{0.0104(480)(0.25)}{5.50-0.25} = 0.24 \text{ thermal ohm-foot} \quad \text{(Eq. 40)}$$

$\bar{\rho}_e = 120$ (asumed); $\bar{\rho}_c = 85$ (Table VI); $L = L_b = 43.5$ inches (Fig. 3)

$N = 6$; $(LF) = 0.80$ (assumed);

$$F = \left(\frac{96}{9}\right)\left(\frac{78}{9}\right)\left(\frac{96.5}{12.7}\right)\left(\frac{87.5}{9}\right)\left(\frac{78.5}{12.7}\right) = 42{,}200 \quad \text{(Fig. 3 and Eq. 46)}$$

$$L_b/P = \frac{43.5}{2(18+27)} = 0.483;\ \frac{27}{18} = 1.5;\ G_b = 0.87 \quad \text{(Fig. 2)}$$

$$R_e' \text{ (at 80\% loss factor)} = (0.012)(85)(1)\times\left(\log\frac{8.3}{5.5} + 0.80 \log\left[\frac{4(43.5)}{8.3}(42{,}200)\right]\right) + 0.012(120-85)(1)(6)(0.80)(0.87) = 6.79 \text{ thermal ohm-feet} \quad \text{(Eq. 44A)}$$

$\bar{R}_e'$ (at unity loss factor) $= 8.44$ thermal ohm-feet (Eq. 44A)

$$\bar{R}_{ca}' = 0.90 + 1.006\,(1.74+0.24+6.79) = 9.72 \text{ thermal ohm-feet} \quad \text{(Eq. 8)}$$

$$\Delta T_d = 0.57\left(\frac{0.90}{2} + 1.74 + 0.24 + 8.44\right) = 6.2 \text{ C} \quad \text{(Eq. 6)}$$

$T_a = 25$ C (assumed);

$$I = \sqrt{\frac{75-(25+6.2)}{8.60(1.082)(9.72)}} = 0.696 \text{ kiloampere} \quad \text{(Eq. 9)}$$

To illustrate the case where the cable circuits are not identical, consider the second circuit to have 750-MCM conductors. For the first circuit,

$N = 3$; $(LF) = 0.80$ (assumed);

$$F = \left(\frac{96}{9}\right)\left(\frac{78}{9}\right) = 92.4 \quad \text{(Eq. 46)}$$

$$\bar{R}_e' = 0.012(85)(1)\times\left[\log\frac{8.3}{5.5} + 0.80 \log\left(\frac{4(43.5)}{8.3}\ 92.4\right)\right] + 0.012(120-85)(1)(3)(0.80)(0.87) = 3.74 \text{ thermal ohm-feet} \quad \text{(Eq. 44A)}$$

$$F_{int} = \left(\frac{96.4}{12.7}\right)\left(\frac{87.5}{9}\right)\left(\frac{78.5}{12.7}\right) = 456 \quad \text{(Eq. 50)}$$

$$\bar{R}_{int} = 0.012(1)\times[85 \log 456 + 3(120-85)(0.87)] = 3.81 \text{ thermal ohm-feet} \quad \text{(Eq. 49)}$$

$$\bar{R}_{ca}' = 0.90 + 1.006(1.74+0.24+3.74) = 6.65 \text{ thermal ohm-feet} \quad \text{(Eq. 8)}$$

$$\Delta T_d = 0.57\,(0.45+1.75+0.24+4.63) = 4.0 \text{ C} \quad \text{(Eq. 6)}$$

$$W_{c1} = (I_1^2)(8.60)(1.082) = 9.31\, I_1^2 \text{ watts per conductor foot} \quad \text{(Eq. 15)}$$

$$\Delta T_{int} = (9.31 I_1^2[(1.006)(0.80)+0.57])3.81 = 2.17 + 28.5 I_1^2 \text{ degrees centigrade in circuit no. 2} \quad \text{(Eq. 48)}$$

Similar calculations for the second circuit yield the following values.

$\bar{R}_{ca}' = 7.18$; $\Delta T_d = 3.4$; $W_{c2} = 17.44 I_2^2$; $\Delta T_{int} = 1.71 + 53.2 I_2^2$ in circuit no. 1

$$I_1^2 = \frac{75-(25+4.0+1.71+53.2I_2^2)}{(9.31)(6.65)}\times = 0.715 - 0.859 I_2^2 \quad \text{(Eq. 9A)}$$

$$I_2^2 = \frac{75-(25+3.4+2.17+28.5I_1^2)}{(17.44)(7.18)}\times = 0.355 - 0.228 I_1^2 \quad \text{(Eq. 9A)}$$

Solving simultaneously $I_1 = 0.714$; $I_2 = 0.487$ kiloampere.

138-Kv 2,000-MCM High-Pressure Oil-Filled Pipe-Type Cable 8.625-Inch-Outside-Diameter Pipe

The cable shielding will consist of an intercalated 7/8(0.003)-inch bronze tape—1-inch lay, and a single 0.1(0.2)-inch D-shaped brass skid wire—1.5-inch lay. The cables will lie in cradled configuration.

$D_c = 1.632$; $D_i = 2.642$; $T = 0.505$; $D_s = 2.661$; $D_p = 8.125$

$$T_c = 70 \text{ C};\ R_{dc} = \left(\frac{12.9}{2.00}\right)\left(\frac{234.5+70}{234.5+75}\right) = 6.35 \text{ microhms per foot} \quad \text{(Eq. 10A)}$$

For shielding tape $A_s = 7/8(0.003) = 0.00263$; $l = 1.0$; $\rho = 23.8$; $\tau = 564$ (Table I)

$$R_s = \frac{23.8\pi}{4(0.00263)}\sqrt{1+\left(\frac{2.66\pi}{1}\right)^2}\times\left(\frac{564+50}{564+20}\right) = 62{,}900 \text{ microhms per foot at 50 C} \quad \text{(Eq. 13)}$$

For skid wire $A_s = \frac{1}{2}\pi(0.1)^2 = 0.0157$; $l = 1.5$; $\rho = 38$; $\tau = 912$ (Table I)

$$R_s = \frac{38\pi}{4(0.0157)}\sqrt{1+\left(\frac{2.66\pi}{1.5}\right)^2}\times\left(\frac{912+50}{912+20}\right) = 11{,}100 \text{ microhms per foot at 50 C} \quad \text{(Eq. 13)}$$

$$R_s \text{ (net)} = \left[\frac{(62.9)(11.1)}{(62.9)(11.1)}\right]1{,}000 = 9{,}435 \text{ microhms per foot at 50 C}$$

$k_s = 0.435$; $k_p = 0.37$ (Table II)

$R_{dc}/k_s = 14.6$; $Y_{cs} = 0.052(1.7) = 0.088$ (Eq. 21, Fig. 1, and text)

$S = 2.66 + 0.10 = 2.76$; $R_{dc}/k_p = 17.2$; $F(x_p') = 0.035$ (Fig. 1)

$$Y_{cp} = 4\left(\frac{1.632}{2.76}\right)^2(0.035)(1.7) = 0.083 \quad \text{(Eq. 24A and text)}$$

$1 + Y_c = 1 + 0.088 + 0.083 = 1.171$

$$X_m = 52.9 \log\frac{(2.3)(2.76)}{2.66} = 20.0 \text{ microhms per foot} \quad \text{(Eq. 29A)}$$

$$Y_s = Y_{sc} = \frac{(20.0)^2(1.7)}{(9.435)(6.35)} = 0.011 \quad \text{(Eq. 27A and text)}$$

$$Y_p = \frac{(0.34)(2.76)+(0.175)(8.13)}{6.35} = 0.372 \quad \text{(Eq. 35)}$$

$$R_{ac}/R_{dc} = 1.171 + 0.011 + 0.372 = 1.554 \quad \text{(Eq. 14)}$$

$$q_s = 1 + \frac{0.011}{1.171} = 1.009;\ q = 1 + \frac{0.011+0.372}{1.171} = 1.327 \quad \text{(Eqs. 18–19)}$$

$\epsilon_r = 3.5$ (Table V); $E = 138/\sqrt{3} = 80$; $\cos\phi = 0.005$

$$W_d = \frac{0.00276(80)^2(3.5)(0.005)}{\log\frac{2.642}{1.632}} = 1.48 \text{ watts per conductor foot} \quad \text{(Eq. 36)}$$

$$\bar{\rho}_i = 550 \text{ (Table VI)};\ \bar{R}_i = 0.012\times\left(550 \log\frac{2.642}{1.632}\right) = 1.38 \text{ thermal ohm-feet} \quad \text{(Eq. 38)}$$

$n' = 3$; $D_s' = 2.15(2.66) = 5.72$;

$$R_{sd} = \frac{3(2.1)}{5.72+2.45} = 0.77 \text{ thermal ohm-foot} \quad \text{(Eq. 41A)}$$

$\rho_d = 100$ (Table VI); $t = 0.50$; $D_e = 8.63 + 1.0 = 9.63$ for 1/2-inch wall of asphalt mastic

$$\bar{R}_d = \frac{0.0104(100)(3)(0.50)}{9.63-0.50} = 0.17 \text{ thermal ohm-foot} \quad \text{(Eq. 40)}$$

Assume $\bar{\rho}_e = 80$, $L = 36$ inches, $(LF) = 0.85$; $N = 1$, $F = 1$

$$\bar{R}_e' \text{ (at 85\% loss factor)} = 0.012(80)(3)\times\left[\log\frac{8.3}{9.63} + 0.85 \log\left(\frac{4(36)}{8.3}(1)\right)\right] = 2.85 \text{ thermal ohm-feet} \quad \text{(Eq. 44)}$$

$\bar{R}_e'$ (at unity loss factor) $= 3.38$ thermal ohm-feet (Eq. 44)

$$\bar{R}_{ca}' = 1.38 + 1.009(0.77) + 1.327(0.17+2.85) = 6.17 \text{ thermal ohm-feet} \quad \text{(Eq. 8)}$$

$$\Delta T_d = 1.48(0.69+0.77+0.17+3.38) = 7.4 \text{ C} \quad \text{(Eq. 6)}$$

$T_a = 25$ C (assumed);

$$I = \sqrt{\frac{70-(25+7.4)}{(6.35)(1.171)(6.17)}} = 0.905 \text{ kiloampere} \quad \text{(Eq. 9)}$$

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Discussion

C. C. Barnes (Central Electricity Authority, London, England): This paper is an excellent and up-to-date study of a most important subject. For 25 years D. M. Simmons' articles have been used for fundamental study on current rating problems, but the numerous cable developments and changes in installation techniques introduced in recent years have made a modern assessment of this subject very necessary. The essential duty of a power cable is that it should transmit the maximum current (or power) for specified installation conditions. There are three main factors which determine the safe continuous current that a cable will carry.

1. The maximum permissible temperature at which its components may be operated with a reasonable factor of safety.

2. The heat-dissipating properties of the cable.

3. The installation conditions and ambient conditions obtaining.

In Great Britain the basic reference document is ERA (The British Electrical and Allied Industries Research Association) report *F/T131*[1] published in 1939, and in 1955 revised current rating tables for solid-type cables up to and including 33 kv were published in ERA report *F/T183*. A more detailed report summarizing the method of computing current ratings for solid-type, oil-filled, and gas-pressure cables is now being finalized and will be published as ERA report *F/T187* some time in 1958.

Until recent years current ratings in Great Britain have usually been considered on a continuous basis, but the importance of taking into consideration cyclic ratings has now been carefully studied, since continued high metal prices have forced cable users to review carefully the effects of cyclic loadings. A report has recently been issued in which a simple method is presented for the rapid calculation of cyclic ratings.[3]

Table V gives specific inductive capacitance values for paper as: paper insulation (solid type), 3.7 (IPCEA value); paper insulation (other type), 3.3-4.2. Is it possible to list the other types and their appropriate specific inductive capacitance values or alternatively simply use an average specific inductive capacitance value of 3.7, for example, for all types of paper insulation?

Reference is made to the adoption of the hypothesis suggested by Kennelly as the basis of the paper—this is a logical approach but it appears to differ from the basis of computing ratings hitherto adopted in the United States. An amplification of the authors' viewpoint on this important issue will be welcomed.

With reference to the use of low-resistivity backfill, recent studies in Great Britain have shown that the method of backfilling cable trenches deserves careful consideration as attention to this point can result in increases up to 20% in load currents.

Equation 43 gives the thermal resistance between any point in the earth surrounding a buried cable and ambient earth. It is

Table X. Temperature Limits* for Belted-, Screened- and HSL†-Type Cables

System Voltage and Type of Cable	Laid Direct or in Air: Lead Sheathed, Armored	Laid Direct or in Air: Lead Sheathed, Unarmored	Laid Direct or in Air: Aluminum Sheathed, Armoured or Unarmored	In Ducts: Lead Sheathed, Armored	In Ducts: Lead Sheathed, Unarmored	In Ducts: Aluminum Sheathed, Armored or Unarmored
1.1 kv						
Single-core		80	80		60	80
Twin and multicore belted	80	80	80	80	60	80
3.3 kv and 6.6 kv						
Single-core		80	80		60	80
Three-core belted-type	80	80	80	80	60	80
11 kv						
Single-core		70	70		50	70
Three-core belted-type	65	65	65	65	50	65
Three-core screened-type	70	70	70	70	50	70
22 kv						
Single-core		65	65		50	65
Three-core belted-type	55	55		55	50	
Three-core screened-type	65	65	65	65	50	65
Three-core (SL‡ or SA§)	65		65	65		65
33 kv (screened)						
Single-core		65			50	
Three-core	65	65		65	50	
Three-core HSL	65			65		

* Measured in degrees centigrade.
† Hochstater separate lead.
‡ Separate lead sheathed.
§ Separate aluminum sheathed.

not clear, however, what value of soil thermal resistivity is used in this expression and information on this important point is desirable.

In Great Britain a value of soil thermal resistivity (g) of 120 C cm/watt is generally used but further test data are being slowly acquired, and where tests have indicated that a lower value, e.g., 90 C cm/watt, is justified, this value is used. Current loading tables in ERA report *F/T183* provide data for soil thermal resistivity values of 90 and 120 C cm/watt, and correction factors for other values of soil thermal resistivity are also provided.

In the United States buried cables are usually pulled into duct banks, but there must be many cases where direct burial, as normally used in Great Britain, will result in lower installation costs. Formulas dealing with this installation technique are a desirable addition. Permissible temperature limits for the various types of cables and installation conditions used in the United States will be a helpful appendix, and it is suggested that this information should be added to the paper. For comparison purposes, the limits recommended in Great Britain are summarized in Table X and in the following:

Plastic-insulated power cables............ 70 C maximum conductor temperature

Gas-pressure and oil-filled cable systems (all types)............................ 85 C maximum conductor temperature

Finally, it will be helpful to know if adoption of the formulas in the paper will necessitate revision or amplification of existing rating tables and, if so, when the revised tables will be published.

REFERENCES

1. CURRENT RATING OF CABLES FOR TRANSMISSION AND DISTRIBUTION, S. Whitehead, E. E. Hutchings. *Report, Reference F/T131*, The British Electrical and Allied Industries Research Association, Leatherhead, England, 1939; also *Journal*, Institution of Electrical Engineers, London, England, vol. 83, 1938, p. 517.

2. THE CALCULATION OF CYCLIC RATING FACTORS FOR CABLES LAID DIRECT OR IN DUCTS, H. Goldenberg. *Proceedings*, Institution of Electrical Engineers, London, England, vol. 104, pt. C, 1957, p. 154.

H. Goldenberg (Electrical Research Association, Leatherhead, England): The calculation of cable ratings is a subject of prime importance to cable engineers. Nevertheless, it seems that until recently the American standard work on this subject has been that of Simmons,[1] while the corresponding British standard work has been recorded by Whitehead and Hutchings.[2] These papers have been supplemented by scattered published papers, including developments dealing with cyclic loading.

The paper by Mr. Neher and Mr. McGrath records up to date American cable-rating practice in a manner that will prove invaluable to engineers for many years to come. It is a pleasing feature that the authors are especially competent to deal with this subject in view of their valuable contributions to the cable-rating field over a number of years. Modern British cable rating practice has recently been recorded in an ERA report[3] dealing with continuous current ratings, and in two IEE (Institution of Electrical Engineers) papers[4,5] (based on ERA reports) dealing with cyclic loading, but the majority of this work is in process of printing and publication.

An obvious difference in British and American technique is the method of cyclic rating factor calculation. Mr. Neher and Mr. McGrath's method is based on an equivalence between typical daily loss cycles and sinusoidal loss cycles of the same loss factor, while a method recently introduced in Britain[4,5] takes full account of the form of a daily load cycle. Both methods are considerably shorter than any that have been available hitherto. Nevertheless without further study I would not feel certain that for British-type cables, subject to their typical daily cycles, the form of the cyclic load can be adequately taken into account by use of the loss factor independently of the cyclic load wave form giving rise to it. In fact the conclusion reached in my second IEE paper,[5] is that a knowledge of the cyclic load wave form for the 6 hours prior to peak conductor temperature, together with the loss factor, are adequate for cyclic rating factor calculation. However, it would be unfair to assess any of the relative merits of the two methods prior to the publication of one of them.

The difference between British and American cable rating technique is not so marked for continuous current rating calculation as might appear to be the case at first sight. In fact, such differences as exist are principally due to the different types of cables employed on each side of the Atlantic, and to the different standard a-c frequencies in use. Nevertheless a comparison of the present paper with the ERA report dealing with continuous current ratings[3] gives rise to certain observations.

The present paper is principally directed to the calculation of a single current rating, but one use to which it might well be put is the large-scale preparation of current rating tables, with rating factors for nonstandard conditions. For such an application it is often preferable to introduce explicit formulas for the rating factors, as these formulas might be independent of some of the thermal resistances or loss factors involved, with a consequent saving in calculation time.

The method employed for external thermal resistance calculation for grouped cables laid direct in the ground differs somewhat from that recommended in a recent paper of mine.[6] For the preparation of group rating factors for the more commonly occurring groups of cables dealt with in an ERA report,[7] the combination of certain simplified external thermal resistance formulas and my recommended method has led to a substantial saving in calculation time. I do not favor the introduction of a geometric-mean distance, or its equivalent, as it is inconvenient for unequally loaded cables.

A brief résumé of other points is that the thermal resistivity values given in Table VI for thermal resistance calculation are generally somewhat lower than the corresponding British values, that the proximity effect on cylindrical hollow conductors appears to me to be best obtained from Arnold's paper,[8] that where sheath and nonferrous reinforcement losses occur a parallel combination of sheath and reinforcement resistance permits the calculation of a single loss factor, that a simple formula has been derived for the external thermal resistance of one of three cables in trefoil touching formation laid direct in the ground,[6] and that sector correction factors are often used in British practice for 3-core cable rating calculations.

REFERENCES

1. See reference 1 of the paper.

2. See reference 1 of Mr. Barnes' discussion.

3. THE CALCULATION OF CONTINUOUS RATINGS AND RATING FACTORS FOR TRANSMISSION AND DISTRIBUTION CABLES, H. Goldenberg. *Report, Reference F/T187* ERA, London, England, (to be published).

4. See reference 2 of Mr. Barnes' discussion.

5. THE CALCULATION OF CYCLIC RATING FACTORS AND EMERGENCY LOADING FOR ONE OR MORE CABLES LAID DIRECT OR IN DUCTS, H. Goldenberg. *Monograph no. 251*, Institution of Electrical Engineers, July 1951.

6. THE EXTERNAL THERMAL RESISTANCE OF BURIED CABLES, H. Goldenberg. *Beama Journal*, London, England, vol. 64, no. 1, Feb. 1957, p. 36.

7. CURRENT RATINGS FOR PAPER-INSULATED CABLES TO B.S.480, 1954; VARNISHED-CAMBRIC-INSULATED CABLES TO B.S. 608, 1955. *Report, Reference F/T183*, The British Electrical and Allied Industries Research Association, Leatherhead, England.

8. See reference 5 of the paper.

Elwood A. Church (Boston Edison Company, Boston, Mass.): The authors present a large amount of useful data and formulas for the calculation of cable thermal constants and suggest a new approach to the problem of calculation of temperature rise for various loss factors including steady-load or 100% loss factor. Cable engineers usually agree on the factors to be taken into account and the methods of calculation for steady loads. However, there appears still to be disagreement on the problem of cyclic loading.

At the AIEE General Meeting in January 1953, a group of papers[1] was presented suggesting various approaches to the problems of cyclic loading on buried cables and on pipe-type cable. Of the methods suggested in these papers, the one which appealed to the author the most was Mr. Neher's method using sinusoidal loss cycles. In his paper it was shown that this method yields reasonably accurate results for the higher loss factors. For a low loss factor sharply peaked cycle, the results are not as accurate.

A modification of this method would be to represent the load cycle more accurately by splitting it into harmonics and computing the temperature rise for each harmonic separately. This entails more work, but with modern methods of machine calculation it is economical to use the most accurate method available and let the machine perform the laborious calculations. In fact, it takes very little more time on the machine when the more rigorous methods are used instead of any of the approximate methods which have been suggested.

The author has investigated the various methods of calculation of the cyclic component of temperature rise of 1,250-MCM

Table XI. Thermal Impedance Functions

1,250-MCM 115-Kv Cable Enclosed in $6^3/_4$-Inch-Outside-Diameter Pipe

Harmonic	T_1/Q_c	T_2/Q_c	T_3/Q_c	T_4/Q_c
0*	$8.03\underline{\vert 0^\circ}$	$6.55\underline{\vert 0^\circ}$	$5.97\underline{\vert 0^\circ}$	$5.50\underline{\vert 0^\circ}$
0†	$10.56\underline{\vert 0^\circ}$	$9.08\underline{\vert 0^\circ}$	$8.50\underline{\vert 0^\circ}$	$8.03\underline{\vert 0^\circ}$
1	$2.88\underline{\vert -30^\circ}$	$1.67\underline{\vert -43^\circ}$	$1.24\underline{\vert -54^\circ}$	$0.93\underline{\vert -61^\circ}$
2	$2.29\underline{\vert -38^\circ}$	$1.19\underline{\vert -54^\circ}$	$0.82\underline{\vert -68^\circ}$	$0.57\underline{\vert -77^\circ}$
3	$1.94\underline{\vert -43^\circ}$	$0.94\underline{\vert -61^\circ}$	$0.61\underline{\vert -79^\circ}$	$0.39\underline{\vert -87^\circ}$
4	$1.68\underline{\vert -50^\circ}$	$0.76\underline{\vert -67^\circ}$	$0.48\underline{\vert -87^\circ}$	$0.29\underline{\vert -95^\circ}$

* Steady-state component for single pipe.

† Steady-state component for two pipes, 18 inches apart.

Q_c = watts copper loss per conductor per foot
T_1 = temperature rise of conductor
T_2 = temperature rise of shielding tape
T_3 = temperature rise of oil in pipe
T_4 = temperature rise of pipe

Table XIII. Maximum Temperature Rise for Cyclic Loading

Method of Calculation	Conductor Temperature, C		Pipe Temperature, C	
	1 Pipe	2 Pipes	1 Pipe	2 Pipes
For Loss Cycle 1				
1	39.1	49.2	24.1	34.3
2	39.8	49.9	24.6	34.8
3	39.9	50.1	23.2*	33.4*
For Loss Cycle 2				
1	30.9	37.5	17.1	23.8
2	32.6	39.2	18.2	24.9
3	32.8	39.5	16.1*	22.8*

These figures do not include the temperature rise due to dielectric loss, which would be added to the steady-state component.

* These are average temperatures. It is not possible to compute the maximum temperature of the pipe by this method.

115-kv cables enclosed in $6^3/_4$-inch-outside-diameter pipe buried in the earth. The results of three such methods for two representative load cycles are presented in this discussion for comparison. The three methods compared are: (1) the Harmonic method using Bessel functions to compute the heat-flow constants of the cable for each harmonic of the temperature cycle, (2) the sinusoidal method suggested by Mr. Neher in his 1953 paper, and (3) the latest method suggested by Mr. Neher and Mr. McGrath in their current paper. Space in this discussion does not permit a complete derivation of the heat-flow equations for the harmonic components of the heat-flow cycle, but only the results, as calculated by an IBM (International Business Machines) *650*, are tabulated in Table XI. It may be noted that the machine time to solve the eight simultaneous equations necessary for the solution of the temperatures and heat flows for each harmonic was approximately 5 minutes per matrix, with a separate solution necessary for each harmonic. The whole cost of the job in rental time on the machine and punching the data on the cards for insertion in the machine was $150 for three

Table XII. Harmonic Components of Loss Cycles

Harmonic	Loss Cycle 1		Loss Cycle 2	
	Loss, Watts	Phase Angle, Degrees	Loss, Watts	Phase Angle, Degrees
0	4.03		2.64	
1	2.50	0	2.31	−20
2	1.10	+30	0.43	+185
3	0.20	−90	0.60	+65
4	0.53	+40	0.53	−35

Example: The equation of loss cycle 1 using the foregoing data is as follows: (Maximum $Q_c = 6.6$ watts per foot per conductor)

$$Q_c = 4.03 + 2.50 \sin \omega t + 1.10 \sin (2\omega t + 30^\circ) + 0.20 \sin (3\omega t - 90^\circ) + 0.53 \sin (4\omega t + 40^\circ) \text{ watts}$$

Corresponding temperature cycle for conductor temperature is as follows for a single pipe: (Maximum $T_1 = 39.1^\circ$)

$$T_1 = 32.4 + 7.24 \sin (\omega t - 30^\circ) + 2.57 \sin (2\omega t - 8^\circ) + 0.39 \sin (3\omega t - 133^\circ) + 0.89 \sin (4\omega t - 7^\circ) \text{ degrees centigrade}$$

Zero time = 9.00 a.m. in the foregoing expressions.

different sizes of cable (a total of 12 matrixes). The cost of programming was small since the general program for solution of complex simultaneous equations was already available in the IBM library, and only a small amount of work was necessary to set up this particular problem.

The components of the loss cycles with which the data in Table XI was multiplied to obtain the temperature cycles are given in Table XII. These loss cycles are illustrated in Figs. 4 and 5, with the corresponding temperature cycles of the conductor and pipe.

In all future calculations of this sort, it is planned to carry the programming still further and have the machine calculate the temperature cycle for each size of cable and determine its maximum value. This has been estimated to cost approximately $500 for programming and $15 extra per size of cable to compute.

Usually only the temperature of the conductor and the pipe are significant in calculation of the current-carrying capability but the electronic calculator automatically computes the other values listed in Table XI, and they are recorded for whatever use may be made of them.

A tabulation of maximum temperatures for the foregoing two load cycles and the three different methods of calculation listed previously are tabulated in Table XIII in the same order. Examination of this table will reveal that the sinusoidal method yields results which are nearer to the more accurate harmonic method than the latest method proposed in the paper. The agreement between the various methods is seen to be better at the higher loss factors.

It may be argued that the agreement is close enough between the three methods for all practical purposes and that the accuracy of the original thermal constants from which the computations were made does not warrant the extra work necessary to use the harmonic method. However, the danger in using an approximate method is that someone unfamiliar with its derivation and its limitations will use it where it does not apply. The author does not consider the agreement close enough for 40% loss factor.

The computation of the pipe temperature is just as important as the conductor temperatures, especially in summer when high earth temperatures prevail and where higher daily loss factors are more likely to be encountered. If the earth next to the pipe exceeds an average of 50 C, there is danger of drying out the soil causing thermal instability. Calculations of current-carrying capability should take this limit into account.

REFERENCE

1. See reference 3 of the paper.

R. J. Wiseman (The Okonite Company, Passaic, N. J.): The authors are to be commended for this very fine technical paper. The need for an up-to-date compilation of engineering formulas and constants for the calculation of current-carrying capacities of cables has been of increasing importance every year. When Dr. Simmons wrote his series of papers about 25 years ago we might say the electrical cable industry was young in engineering knowledge, the types of cable furnished were not too great in number, and the characteristics of the cables were not too well known. Today our knowledge of cable design, materials, and operating conditions along with new types of cables is far in advance of 25 years ago. We have been using the formulas as they became known and it was desirable to bring them together in one place and, in addition, all of us who have occasion to make these calculations will be using the same formulas and electrical and thermal constants. Also, this paper will be of great help to younger men coming into the cable industry. Although it summarizes the formulas, anyone wishing to get a clearer appreciation of the text can refer to the bibliography and study the original papers.

To make any text of this kind generally useful, it is desirable that the procedure be easy to follow and the formulas readily applied. Theoretical formulas involving higher mathematics can be used, but they take time, and very often it is not possible to take the time to work up a case. Again conditions of installation are variable daily, so if we attempt to make a field check of calculations we can find differences; therefore, exactness to a high degree is

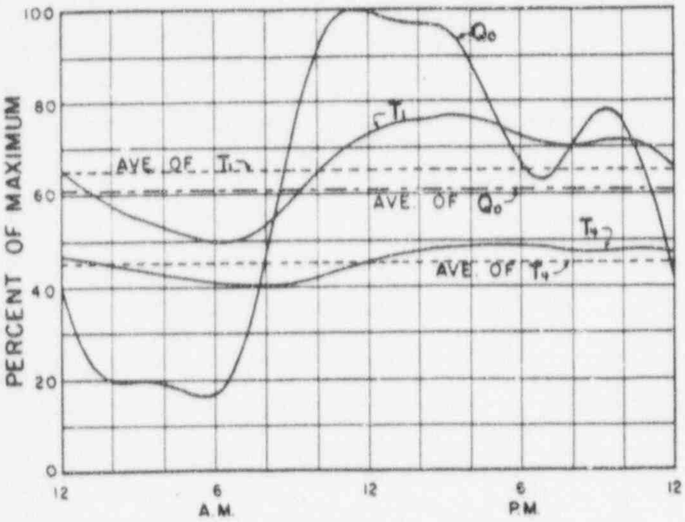

Fig. 4. Loss and temperature cycles for 75% load factor, summer load cycle

Q_0 = copper loss cycle
T_1 = temperature of conductor
T_4 = temperature of pipe

Temperatures are in per cent of copper temperature corresponding to steady load equal to the maximum.

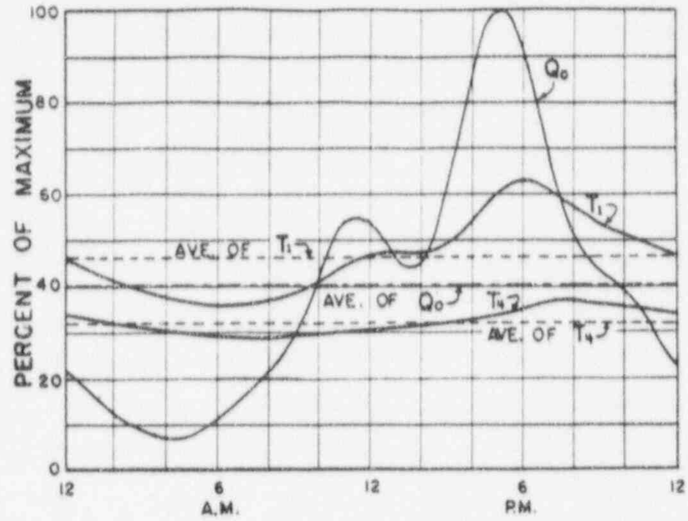

Fig. 5. Loss and temperature cycles for 60% load factor winter load cycle

Values same as in Fig. 4

not necessary. It has been suggested that it is now possible to use computers on these problems. This is true for those who have a computer, but here also time is taken for setting up the problem for the computer. Also we must show how to calculate the currents and in a form that will be used.

You will note that many of the formulas are new to most of you. These formulas were developed to make the calculations easily and quickly and yet do not cause a large error in the final answer from the highly theoretical formula. It is natural that the formulas may be a compromise and some may feel that a particular formula that they use may be superior to that recommended. Likewise the thermal constants may be a compromise. This is true as far as I am concerned, yet we are willing to accept the recommendations given in the paper. The calculation of the various losses existing in a cable system and the location of these losses is well done and should be carefully studied by all new engineers.

The section dealing with the calculation of some of the thermal resistances need careful study in order to appreciate them as they depart from the usual manner in which a thermal resistances are calculated. For example: the thermal resistance between a cable and a surrounding wall, such as a duct wall or a pipe; see equations 41 and 41(A). Heretofore, we used $\bar{R}_{sd} = 0.00411\ B/D$, and referred to as the IPCEA method. This has been revised to take into consideration the condition existing and the materials. Equation 41(A) is a general one, and by inserting the correct values of A' and B' as given in Table I, we can get R_s. This is an example of how we can accept a compromise in order to get agreement. We at Okonite made tests years ago to determine the thermal constants for the oil or gas medium surrounding cables in a pipe. We tried to use the cylindrical log formula and found the apparent thermal resistivity varied due to the convection effects of the oil. If we took the simple formula $R_{sd} = 1.60/D$ where D is the diameter over the shielding tape we found we got good agreement with test. We neglected temperature effects as the actual value of R_{sd} as compared to the thermal resistance of the insulation is very low, many times in the order of one-tenth; therefore, temperature effects are small. For a gas medium using 200 pounds per square inch we use the equation $\bar{R}_{sd} = 2.58/D$. How do these formulae compare with equation 41(A) proposed by the authors?

Consider two cases, one having a diameter over the shielding tape of 1 inch and another having a diameter of 2.5 inches. The following table compares the two types of equations.

Medium		Diameter = 1 Inch, Thermal Ohm-Foot	Diameter = 2.5 Inches, Thermal Ohm-Foot
Oil	Okonite	1.60 t.o.f.	0.64 t.o.f.
	Neher and McGrath	1.37	0.80
Gas	Okonite	2.58 t.o.f.	1.03 t.o.f.
	Neher and McGrath	2.22	1.04

The differences are not great and when considered in relation to the total thermal resistance, they are negligible. We can accept the authors' equations.

I am glad to see the authors place the duct system in proper relationship to a buried cable system and that the same soil thermal resistivity will be used when making comparisons. This was the weakness in the duct heating constants originally set up by NELA and later known as IPCEA constants. Also a better understanding of the effect of multiple cables in a duct bank is obtainable, and the determination of the cable having the highest thermal resistance is possible.

Appendix III discusses the derivation of D_x, a fictitious diameter in the soil up to which it is assumed that a steady heat load exists and outside which the loss factor of the load is taken into consideration. I have not been able to accept this assumption. It is an endeavor to obtain a thermal resistance for the soil that will check with a study that Messrs. Neher, Buller, Shanklin and myself made and is referred to in reference 3 in the bibliography of this paper. A study of the previous papers will show that the attainment factor is not exactly the same for all types of cables studied and all shapes of load curves.

The authors tabulate in Table IX a comparison of the attainment factor for three methods of calculation for a loss factor of 30% for several cable designs. Rather than give results for one loss factor only, it would have been better if they had covered the range of loss factors which were studied in 1953. If these attainment factors were plotted against loss factor as I did in my paper, it would have been noted that a straight line could be drawn giving a good representation of how (AF) varies with loss factor, namely, $(AF) = 0.43 + 0.57\ (lf)$ for my method. This equation follows the plot of (AF) and loss factor very well down to about 35% loss factor, and in some cases, it gave a higher value and other cases a lower value than actually calculated. The (AF) values I reported are based on careful calculations from the exact load curve and no assumption that a single sine-wave curve can be taken as representing any load curve. As it is a rarity that cables are designed for loss factors as low as 30% (50% load factor), my formula gives results as accurate as when using D_x and easier to use. However, for the sake of uniformity in methods of calculation, we will accept the authors' method.

In this connection, I would like to raise a question which I hope will be taken up by others interested in this subject. The use of the equation involving D_x is an

attempt to increase the thermal resistance for the soil for cables or small pipe sizes; in other words, the computed value of thermal resistance is too low. Is it not likely that we are leaving out of our equation a term involving a surface contact between the surface of the cable or pipe and the soil. This term would be of the same form as we now use for the case of cables in air, namely, $\bar{R}=0.00411\ B/D$. If we add this term to the log formula for soil thermal resistance, we will get a higher total resistance and the influence of the diameter of the cable or pipe will be greater, the lower the diameter. It will be necessary to determine the value of B. The idea of such a term is shown in the paper[1] by Mr. Mather and his coauthors. In Table I they give some thermal data obtained from tests made by them on a pipe-type cable. They give a value of B for surface of Somastic to water of 218 thermal ohms per cm[2]. I like this. Is it not likely that we have a surface resistivity between the cable and the soil in immediate contact?

REFERENCE

1. BONNEVILLE POWER ADMINISTRATION HIGH-VOLTAGE CABLE STUDIES, R. J. Mather, F. J. McCanna, E. Demirjian. *AIEE Transactions*, vol. 76, pt. III, 1957 (*Paper no. 57-739*).

E. R. Thomas (Consolidated Edison Company of New York, Inc., New York, N. Y.): The authors are to be congratulated in setting up mathematical equations to evaluate load carrying capabilities of cable systems. I regret that no mention was made of the pioneer work by Wallace B. Kirke in the middle 1920's on the rating of cables installed in duct banks. This work, I believe, furnished the basis of cable rating of the NELA and present IPCEA published ratings of cable. The work of Kirke was presented before the AIEE and published in the Journal.[1]

The work on ratings of cable by Kirke was based on thousands of field measurements in the New York City area and later field measurements furnished by utilities throughout the country furnished data which lead to the NELA-IPCEA rating value. This was done before the general use of pipe-type cable. It should be obvious that the answer obtained by mathematical solution is never any better than the assumptions on which the equations are developed and the constants used with the equations.

I believe the actual heat flow in underground cable systems is considerably more complex than has been assumed in this paper and, therefore, actual ratings which are obtained may be different from those obtained by this calculation.

REFERENCE

1. THE CALCULATION OF CABLE TEMPERATURES IN SUBWAY DUCTS, Wallace B. Kirke. *AIEE Journal*, vol. 49, Oct. 1930, pp. 855–59.

H. D. Short (Canada Wire and Cable Company, Toronto, Ont., Canada): Several of the engineers who work with me at Canada Wire have been studying the Neher-McGrath paper over the past few months, and have arrived at certain conclusions, some of which are discussed in the following paragraph.

The determination of the losses in the conductor, shield, sheath or pipe, and the dielectric have been well established by the authors and bear no further comment. The calculation of the thermal resistances of direct buried cable and pipe-cable installations appear to have been well founded; although the method of arriving at the effect of cyclic loading seems to be in question amongst the various investigators (reference 3 of the paper). However, as far as duct bank installations are concerned, the difference between the NELA or IPCEA current rating method and that proposed by the authors is so great that one cannot help but wonder at the dearth of practical data in the paper.

In reading references 10, 12, 13, 16, and 17 of the paper, there seems to be very little data on cable temperature measurements taken in the field, such as was done by the various utilities when the NELA values were established. The work reported in these references is almost all theoretical, and laboratory measurements and analogue methods used are all approximations.

I am given to understand that there is a movement afoot to have this Neher-McGrath method accepted and to revise the IPCEA current rating tables accordingly. I am not sure that this is the case—but if it is, then perhaps the authors can tell us upon what factual data their method is based.

We have used the method given in the paper to compute the current rating of quite a number of high-voltage cable circuits in a duct bank and find complete disagreement with the NELA or IPCEA method. In every case the Neher-McGrath method results in a larger conductor size for a given current rating, in some cases as much as 30% more conductor metal is required by the Neher-McGrath method.

Here is where our dilemma begins. One of two things prevails: either Mr. Neher and Mr. McGrath have cornered the nonferrous metal market or they are attempting to make a pipe-type cable carry the same load as a duct-bank installation. Yet on the face of it, it is incomprehensible how anyone can conceive of a 3-conductor high-voltage cable (and a pipe-type cable is in fact a direct-buried 3-conductor cable) competing on a current rating basis with single-conductor high-voltage cables separately spaced in a duct bank where a-c losses are a minimum and heat dissipation a maximum. In either event we cannot understand why so much time should be spent on developing a new method of current rating calculation for duct-bank systems without first having at least obtained some actual in-service field measurements to substantiate their formulas.

On the other hand, we must sincerely commend the authors for attempting to arrive at a realistic comparison between duct-bank and direct-buried systems. It is unfortunate, however, that in doing so they have not based their formula development on extensive field survey data as was done at the time the NELA duct constants were established.

The only way in which we have as yet been able to make the Neher-McGrath method track with the old and well proved NELA method is to reduce the soil thermal resistivity to the order of 40 C to 75 C cm/watt. The actual value which one would use to arrive at the same conductor size as determined by the NELA method appears to depend upon the number of cables in the duct bank and the value of the daily load factor chosen. In contradistinction, Mr. Neher in reference 13 of the paper states that his method agrees within 10% of the NELA method if a $\rho_e=75$ C cm watt is used.

We have made some calculations of the thermal resistance of cables in a duct bank from the sheath to ground (or sink) using the Neher-McGrath method and the average conditions on which the NELA duct constants were obtained. The average conditions were:

1. Most of the measurements were taken under paved streets with the depth of pavement between 10 and 12 inches.
2. Majority of ducts were made of fibre.
3. Average duct inner diameter = 3.75 inches.
4. Concrete spacer between ducts 2 inches, with duct wall = 1/4-inch, 3-inch outer concrete shell. Spacing between duct centres = $6^1/_4$ inches.
5. Average depth of burial to top of duct bank = 30 inches.
6. Most measurements with 3-conductor lead sheathed cables from 2 inches to 3 inches outside diameter. Average diameter 2.5 inches.
7. All loaded cables in outside ducts, all equally loaded.
8. Soil thermal resistivity (*in situ*) = 120 C cm/watt.

Two cases were studied and the results are summarized in the following:

Case I—Three cables in 2 by 2 duct bank (one of lower ducts empty).

NELA Value (i.e. $4.93/D_s'+L_fNH$)

Loss factor	100%	62.5%	33%
Rth_{S-A} thermal/ohms-feet	5.09	3.92	3.00

Neher-McGrath Value

Loss factor	100%	62.5%	33%
Upper cables Rth_{S-A} thermal /ohms-feet	6.68	5.02	3.71
Lower cable Rth_{S-A}	6.63	4.99	3.70
Average values	6.66	5.01	3.71

In order for Neher-McGrath values of thermal resistances to be equal to NELA values, soil resistivity would have to be:

At 100% loss factor $\rho_e=65$ C cm/watt
At 62.5% loss factor $\rho_e=60$ C cm/watt
At 33.0% loss factor $\rho_e=45$ C cm/watt

Case II—Six cables in 2 wide by 3 deep duct bank.

NELA Value

Loss factor	100%	62.5%	33.0%
Rth_{S-A} thermal /ohms-feet	6.89	5.05%	3.60

Neher-McGrath Value

Loss factor	100%	62.5%	33.0%

Upper layer $Rth_{S\text{-}A}$ thermal/ohms-feet	10.23	7.24	4.88
Middle layer $Rth_{S\text{-}A}$ thermal/ohms-feet	10.95	7.69	5.12
Lower layer $Rth_{S\text{-}A}$ thermal/ohms-feet	10.63	7.49	5.02
Average values	10.60	7.47	5.01

In order for Neher-McGrath values of thermal resistances to be equal to NELA values, soil resistivity would have to be:

At 100% loss factor $\rho_e = 53$ C cm/watt
At 62.5% loss factor $\rho_e = 50$ C cm/watt
At 33% loss factor $\rho_e = 43$ C cm/watt

Other calculations on single-conductor high-voltage cables varying in conductor size from 300 to 1,150 MCM installed in outside ducts in a normal duct-bank systems it was necessary to assume a $\rho_e = 75$ C cm/watt in order to make the Neher-McGrath formulas agree with the current ratings calculated by the NELA method.

The NELA method is of course strictly empirical and the duct constants determined from an average of a large number of field surveys. It has been in use for well over 25 years; and there must of a consequence be many thousands of miles of cables operating at current ratings calculated by the use of these duct constants. So far as our experience in Canada is concerned we know of no hot-spot failures with high-voltage cables in duct-bank installations. On the contrary one is led to read with great interest the recent paper by Brookes and Starrs.[1]

Do the authors expect utility engineers operating duct-bank installations to adopt the method put forward in the paper and forthwith reduce their loads accordingly? This is a question of great importance, and we should have a categorical statement from the authors in this specific regard.

In Appendix IV the authors give a specimen calculation for a typical duct-bank installation and also a similar calculation for a pipe-type installation. In the one they use a ρ_e of 120 and in the other a ρ_e of 80. Would the authors enlighten me on the significance of these two different values for ρ_e. On this point Dr. Wiseman stated in his discussion of the paper that he was glad to learn that we can now base the duct-bank calculations on the same basis of ρ_e as pipe-type cable, but the authors have not done this in their Appendix IV.

The use of the Kennelly formula in the practical case of cables buried in the earth is at best an approximation. For the theoretical case of a heat source in a medium that is homogeneous, of uniform resistivity and temperature, the formula would apply. However, for the practical case of cables in the earth, there is considerable deviation from the ideal case such as nonuniform medium, seasonal variation of temperature gradient in the earth, nonuniform distribution of moisture in the earth, moisture migration, and other factors, which render the Kennelly formula more or less inaccurate. Thus in its use one must bear in mind these limitations.

In Europe the Kennelly formula has been used extensively, but the apparent thermal resistivity inserted in the calculations are based on that value obtained *in situ*, as measured in accordance with recommended methods. To get a very accurate value of the apparent thermal resistivity, it seems that the method to be used should exactly duplicate the cable and its operating conditions; i.e., the same diameter as the cable, the same watts loss dissipated, the same depth of burial, and at the time when the thermal conditions are most onerous. Thus in the calculation of thermal resistance from cable to ambient, it appears that the Kennelly formula can be used to a high degree of accuracy if an apparent thermal resistivity of the soil in situ is used. This measurement should automatically take into account all the factors that otherwise limit the Kennelly formula to a theoretical exercise.

There has been a great deal of investigation into the influence of moisture on soil resistivity. However, as yet there seems to be no general agreement on another basic problem, and that is the direction of the heat flow. The authors and others maintain that the heat flow is to the surface of the earth whereas other investigators claim some heat flow is downwards to a deep isothermal, about 30 to 50 feet below the earth's surface. In reference 12 Mr. Neher obtains the heat field pattern by superimposing the field based on the Kennelly formula on the temperature gradient. It is obvious from the field patterns that in the summer the heat flow is predominantly down, whereas in the winter the heat flow is to the surface. The authors give no quantitative method of evaluating the effect of the temperature gradient on the apparent soil resistivity. This could be one of the reasons for the difference between the resistivity as measured in the laboratory and in the field. An indication of the effect of change of apparent thermal resistivity is shown in a paper by de Haas, Sandiford, and Cameron,[2] wherein the effect of introducing a deep isothermal (ground water) in combination with the earth's surface as the sink has a thermal resistance of approximately 25% less than if the earth's surface was the only sink. This would indicate that the thermal resistivity of the medium is changed whereas the change in temperature distribution due to the temperature gradient should be investigated.

It should be emphasized that the Kennelly formula is applicable to steady-state conditions only. The authors realize this, of course, and attempt to compensate for this short-coming by applying a cyclical loading factor to the external thermal path. The factor they use is based upon measured values obtained on direct buried and/or pipe-type cables. Since the thermal circuit of a duct bank is quite different from that of direct buried cables, we do not agree that this same cyclical loading factor (as measured on direct buried cables) can be applied to a duct-bank installation.

Finally it is pertinent to point out that the Kennelly formula is premised upon all the heat energy flowing to the earth's surface. One must then ask the authors what they mean by ambient soil temperature. Theoretically at least the temperature of the earth at the cable depth of burial is not the ambient to be used in the Kennelly formula if the sink is the earth's surface. Why is the earth's surface temperature not the true ambient to use when applying the Kennelly formula? Is the British use of a 2/3 factor in reality a correction for the virtual sink temperature, or sink temperatures if the deep isothermal theory is valid.

REFERENCE

1. THERMAL AND MECHANICAL PROBLEM ON 138-KV PIPE CABLE IN NEW JERSEY, A. S. Brookes, T. E. Starrs. *AIEE Transactions*, vol. 76, pt. III, Oct. 1957, pp. 773–84.

2. AN ANOLOGUE SOLUTION OF CABLE HEAT FLOW PROBLEMS, E. de Haas, P. J. Sandiford, A. W. W. Cameron. *Ibid.*, vol. 74, pt. III, June 1955, pp. 315–22.

F. O. Wollaston (British Columbia Engineering Company, Ltd., Vancouver, B. C., Canada): This discussion is confined to the parts of the paper dealing with cables in ducts. The paper is in many respects most admirable, notably the coverage of skin effect in conductors of special types, proximity and eddy current effects, mutual heating effect of multicable installations, and the effect of extraneous heat sources. For the first time these are all adequately treated in one paper. The methods of calculation must, however, be critically examined before being accepted. I am disturbed to find that the methods given for rating cables in ducts lead to substantially larger conductor sizes than does the IPCEA-NELA method. By the IPCEA-NELA method I mean the method given in an Anaconda publication.[1] I believe this method is identical to that used in preparing the existing IPCEA current ratings for cables.

The Neher-McGrath method leads to much higher values for the duct heating constant (the thermal resistance from duct-bank to earth ambient) than does the IPCEA-NELA method, when the thermal resistivity of the earth is taken as 120 C cm/watt in the Neher-McGrath calculation. The value to be used for earth thermal resistivity is of paramount importance and will be discussed in more detail later. A few illustrations of the difference between the two methods will first be given.

The first application of the Neher-McGrath method which we made was to determine the conductor size for a proposed 230-kv cable installation. The calculated conductor size was 1,500 MCM, whereas by the IPCEA-NELA method the calculated size was 1,150 MCM. Some 42 miles of cable were involved in the proposed project, so the Neher-McGrath result would have meant substantial extra cost for the cable compared to the IPCEA-NELA result.

In another instance, the Neher-McGrath method was used to determine the required size of cable leads for a 75-mva transformer. The calculated size was so large as to be considered physically impractical, whereas by the IPCEA-NELA method the calculated size was practical. Rather than risk possible trouble if the IPCEA-NELA result were adopted, it was decided to use aerial bus instead of cable for these leads.

In a third case, the cable leads of a 50-mva 13.8-kv generator were to be changed

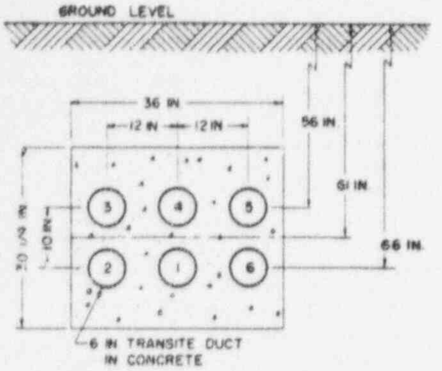

Fig. 6. Cross section of duct bank

because the associated 230-kv step-up transformer was being replaced with a 345-kv unit. The existing leads consist of two 2,500-MCM cables per phase installed in a 6-duct bank. According to the Neher-McGrath method, these cables should be approximately 3,500 MCM each if the AEIC allowable temperature of 76 C is not to be exceeded at full load in summer time. The unit has run at full load for long periods on many occasions since going into service in 1949. If our application of the Neher-McGrath method is correct, one must conclude that the existing cables have been severely overloaded many times during their service period of 8 years. No evidence of such overloading has been seen; the cables have been entirely trouble-free. There are two other units at this plant, identical in all respects to the one described above except that one of them has been in service slightly longer, the other not quite as long. No trouble has occurred on the leads of these units.

It was decided to make a temperature survey to establish the correct facts. The unit was run at full load for 5 days. Test results showed that the duct structure attained equilibrium temperature in 24 hours. The bulb of a recording thermometer was inserted 20 feet in the bottom middle duct. The details of the duct bank and cable are given in Fig. 6 and

Table XIV. Cable and Loss Data

2,500-MCM Segmental Copper Conductor, Paper Insulated-Lead-Sheathed Solid-Type, 13.8 Kv

Cable No.	Current During Test, Amperes	Watts Loss Per Foot of Cable
1	1,035	5.79
2	975	5.13
3	965	5.03
4	905	4.43
5	911	4.49
6	1,029	5.73
	Total	30.60
	Per cable average	5.1

Notes:

Ambient earth temperature during test was 19.5 C. Cables are paired 2–3 for A-phase, 1–4 for B-phase, 5–6 for C-phase.

Diameter over conductor, inches	2.000
Cotton tape thickness, inches	0.017
Insulation thickness, inches	0.210
Diameter over insulation, inches	2.454
Copper tape thickness, inches	0.003
Sheath thickness, inches	0.125
Over-all diameter, inches	2.710

A–C resistance at 65 C = 5.41 (10^{-4}) ohms-feet

Table XIV. It was necessary to measure the air temperature in an occupied duct, since there were no empty ducts. The loading on the machine was recorded and the current division between the six cables was determined. The maximum departure from equal loading of the two cables on each phase was only 2%. After 5 days the duct air temperature was 43 C. The ambient ground temperature was 19.5 C at the same depth as the center of the duct bank. Dividing the temperature rise by 1/6 of the total losses, a thermal resistance of 4.6 ohms is obtained. Table XV shows the thermal resistances pertinent to this case as determined by the Neher-McGrath method and the IPCEA-NELA method. The experimental value (occupied duct air to earth ambient of Table XV) is in good agreement with the IPCEA-NELA value given in "duct wall to earth ambient" of Table XV, while the Neher-McGrath value is much higher. The Neher-McGrath value should be approximately equal to the IPCEA-NELA value if the two methods are to give the same results, as is obvious by inspection of Table XV. The Neher-McGrath value should be lower than our experimental value, since the former represents the thermal resistance from the outside surface of the occupied duct wall to earth ambient, while the latter represents this same resistance plus the thermal resistance from occupied duct air to the outside surface of the occupied duct wall.

One is not entitled to say that the discrepancy between the Neher-McGrath value and the IPCEA-NELA value is real unless the value of the specific thermal resistivity of the earth ρ_e is the same for both. The Neher-McGrath value in the tabulation is obtained when a value of earth thermal resistivity $\rho_e = 120$ C cm/watt and thermal resistivity of concrete $\rho_c = 85$ are used in equation 44(A) of the paper. There has never been any general agreement on what value of earth thermal resistivity is inherent in the IPCEA-NELA duct constants. Several years ago Mr. G. B. Shanklin and his coworkers in the General Electric Company investigated this extensively and concluded that the value is about 180 C cm/watt. If this conclusion is correct the discrepancy between the Neher-McGrath result and the IPCEA-NELA duct heating constant is real and serious. Our test result cited above does not give any information on this point because the earth thermal resistivity was not measured, due to lack of facilities.

If the discrepancy is real, one is led to question the soundness of the Kennelly formula used by the authors. It is based on the premise that all heat generated in the cable escapes to the surface of the earth. Some competent engineers have argued that part of the heat escapes by another path, namely to a sink deep in the earth. Mathematical development of this premise gives a result for the thermal resistance between duct bank and earth that is only about two-thirds as large as the result by the Kennelly formula. According to this, we might expect the Neher-McGrath method to agree with the NELA value if the earth thermal resistivity is taken equal to $2/3 \times 180 = 120$ C cm/watt in equation 44(A). It turns out that agreement occurs when

Table XV. Thermal Resistances Pertaining to Test

Thermal Resistance, C per Watt/Foot	Neher-McGrath	IPCEA-NELA	Experimental
Insulation	0.75	0.75	
Sheath to duct	1.52	1.82	
Duct wall	0.13		
Duct wall to earth ambient	8.75*	4.9	
Occupied duct air to earth ambient			4.61

* Calculated from equation 44(A) using $\bar{\rho}_e = 120$ C cm/watt.

the earth resistivity is taken as 55 C cm/watt in equation 44(A). It does not seem likely that the value of 55 is representative of typical soil around duct banks. Many measurements in several laboratories have consistently shown that the specific thermal resistivity of earth varies from about 100 C cm/watt for a moisture content of 15%, to about 300 or 400 C cm/watt for zero moisture content. A value of 180 C cm/watt seems fairly representative of average conditions. I conclude that the validity of the Neher-McGrath method of calculating the thermal resistance from duct bank to earth ambient should be demonstrated by tests wherein the earth thermal resistivity is definitely known. Have the authors verified their findings by such tests?

REFERENCE

1. CURRENT RATINGS FOR ELECTRICAL CONDUCTORS. *Anaconda Publication C-51*, McGraw-Hill Book Company, Inc., New York, N. Y., first edition, Oct. 1942.

J. H. Neher and **M. H. McGrath**: We are indebted to Mr. Barnes and Mr. Goldenberg for their discussions in which they summarize the present cable rating practices in Great Britain and point out some differences with American practice. From this it would appear that in most respects the practices in the two countries are similar. While the method of handling group cable ratings developed by Mr. Goldenberg may appear to differ from the method of the paper, actually both methods are derived from the same basic principles and should give identical results for the same set of conditions.

To answer their questions with regard to temperature limits and the relationship of this paper to the published rating tables, we may say that IPCEA, in collaboration with the AIEE, has under active consideration a revision of the existing current rating tables based on the methods of calculation set forth in this paper. The temperature limits will be those already adopted by IPCEA, AEIC, etc., in industry specifications.

Mr. Church has outlined a procedure for determining the effect of the loading cycle on cable ratings which will be, we fear, an enigma to most cable engineers despite the fact that it represents a challenge to those mathematically inclined. Mr. Goldenberg also has referred to a different but nevertheless mathematically involved procedure for doing this. For normal cable calculations, the tremendous amount of computations required for each individual

case is simply not warranted even if a digital computer were available to the cable engineer.

If the application of a particular load cycle to a given cable system is to be studied, we suggest that this may be done more simply, more rapidly, and more economically by using an analog computer designed for the purpose. We feel, however, that the accuracy of the method given in the paper as compared to all exact calculations which we have examined, including those of Mr. Church, is sufficient, particularly in view of the fact that any particular load cycle may never repeat itself.

The method given in the paper is an approximation, admittedly, but it has been derived from the same fundamental principles which underlie Mr. Church's method through a series of carefully considered simplifications. It should be understood that there is nothing sacred about the value of 8.3 inches used for the fictitious diameter D_x. This value happens to be the best single value to use based on the studies described in reference 3. For Mr. Church's case values of 7.1 for the 75% load factor cycle, and of 5.1 for the 60% load factor cycle are indicated. The errors in using 8.3, however, amount to only 2 and 5% high, respectively, in the conductor loss component of conductor temperature rise, which would be offset by a 10% error in the value of earth thermal resistivity employed.

Dr. Wiseman's comments in this connection are most interesting since he has often expressed the opinion that, practically, it was sufficient to consider D_x to be equal to D_e, or in other words to apply the loss factor to all of the earth portion of the thermal circuit. We can agree with this in respect to pipe-type cables, but, as he has indicated, we do not consider this further simplification desirable in the case of small directly buried cables. Neither do we consider the formula which he gives for obtaining attainment factor directly from loss factor suitable in this case. This is readily apparent from Fig. 2 of the first paper of reference 3 in our paper. Since the use of D_x has considerable theoretical justification in our opinion, we feel that it should be made a part of the general procedure for calculating the effect of the loading cycle.

The introduction of an additional thermal resistance to care for surface effects between cable and earth is an entirely different matter since this will increase the temperature rise both for steady and for cyclic loads, whereas the use of D_x is intended to give the correct result for cyclic loads on the assumption that the total thermal resistance in the circuit which is unchanged by the value of D_x is correct for steady loading. It is quite possible that such a surface effect term is present and that it may attain an appreciable magnitude in the case of small directly buried cables. We concur in the hope that this matter will be investigated further.

Mr. Thomas has noted the pioneer work of W. B. Kirke in connection with cable in duct and indicates that this work formed the basis of the present NELA-IPCEA method. Employing a duct bank configuration such as shown by Wollaston and utilizing equations 14 and 17 of the Kirke article, we find that Kirke would use a resultant thermal resistance from loaded duct wall to earth ambient of 9.0 for the worst soil in metropolitan New York and 6.00 for the best soil. These values, when compared with NELA constant of 4.9, scarcely confirm Mr. Thomas' statement to the effect that the present IPCEA-NELA method is based on or is even closely related to Kirke's work. While Kirke made some attempt to take into account the configuration of the duct-bank structure, he did not utilize resistivity as such, and as previously indicated we believe that a knowledge of this and other parameters ignored by Kirke is essential to a realistic method of handling this problem, particularly when one considers the problem of comparison between different types of systems.

As Mr. Thomas has suggested, the heat flow in a duct structure is complex, but this complexity results from the superposition of a number of heat flows any one of which, due to a particular cable, is readily determined as indicated in reference 12. We are not interested in these heat flows *per se*, but only in the resulting temperature difference between a reference cable and ambient and the corresponding thermal resistance which is fully expressed by the relatively simple equation given. True, the situation is complicated by the concrete envelope, but here extensive studies, both mathematical and on a field plotter, indicate that the equation 44(A) is sufficiently accurate in view of the inherent errors in fixing the earth resistivity and loss factor in a particular situation.

Mr. Short, at the start of his discussion, states in effect that he considers the method for determining the load capability of direct earth-buried or pipe-type cable to be "well founded" for a 100% load factor but, because of questions raised by various investigators in reference 3 of our paper, does not seem to be too sure that this is the case for other load and loss factors. All four investigators who undertook to study the problem for the Insulated Conductor Committee, however, are on record as recommending or agreeing to the method given in the present paper. In accepting the given method for buried and pipe-type cable, Mr. Short does not seem to realize that this method is based on the Kennelly formula because in the latter portion of his discussion he questions the applicability of this premise to current rating determinations for any type of underground installation, and proceeds to attempt to resurrect a number of the ghosts which plagued the Insulated Conductor Committee some 10 years ago when the latter started work on a critical review of the basic parameters involved in load capability calculations. These ghosts were subsequently laid to rest, at least to the satisfaction of the vast majority of engineers in this country. Even at that time the Kennelly formula had been in existence for over 50 years. Despite the fact that this formula is based on scientific principles found in most text books on physics and electrical engineering, some cable engineers had misgivings as to its applicability mainly because calculations by it did not appear to check with measurements in the field. This situation is discussed in reference 12 of our paper wherein it is shown that the disagreement was not due to the formula but to the fact that the field measurements had not been carried to a steady state, and that laboratory determinations of the earth resistivity were not representative of the soil *in situ*. Also, the apparent discrepancy (which appears because the direction of heat flow implied in the formula is toward the surface whereas in summer the total heat flow in the earth is obviously in the reverse direction) is explained by the application of the principle of superposition to the separate heat fields involved. As a result, cable engineers, with very few exceptions, have accepted the formula for calculations involving pipe-type and directly buried cable systems. The method of handling cables in duct, given in the paper, is a logical extension of the principles underlying the Kennelly formula in order to include in the calculations two very important variables which are not a part of the NELA-IPCEA method, namely the duct configuration and the thermal resistivity of the surrounding soil. This method is also not new. It was first described by N. P. Bailey in a paper in 1929[1] and subsequently in reference 13 of our paper.

Mr. Short also mentions the two-thirds factor, another resurrected ghost of the past. Long ago the British established that the two-thirds factor represents a difference between laboratory and *in situ* measurements of soil resistivity and that it does not stem from any lack of applicability of the Kennelly formula to the problem. Numerous British publications point out that the two-thirds factor is not to be used where the resistivity is measured *in situ* by buried sphere or by long or short cylinder. In addition, in recent years the British have developed a new laboratory sampling procedure[2] which checks not only with the buried sphere, the buried cylinder, the transient needle, but in addition also checks with results obtained on loaded cable installations.

Another ghost mentioned by Mr. Short is the deep isothermal approach (a proposal which was first suggested by Levy in 1930)[3] citing the de Haas, Sandiford, and Cameron[4] paper to give new life to this old suggestion. However, in so doing Mr. Short fails to point out that the deep isothermal in this case consists of a conducting paint electrode of an analogue model connected electrically to another electrode representing the earth's surface and hence simulating a *flowing* (not stationary) ground water sink, a somewhat unusual condition that is scarcely pertinent to the problem at hand. Incidentally, Table I of this paper gives results of an excellent analog check of the given method as applied to a duct bank.

We wish to assure Mr. Short that we have not cornered the nonferrous metal market, nor are we saying that three single-conductor cables of a given size installed in a buried pipe must have the same rating as three conductors of the same size installed in separate ducts. We should point out, however, that this has been a rule of thumb for the past 10 years or more and there are now many miles of high-voltage pipe cable in successful service which are rated and are being operated at a load capability level which Mr. Short considers incomprehensible.

Mr. Short's dilemma results solely from

the fact that he is attempting to compare the results of calculations made under a set of assumed conditions with the results of a procedure for which those same conditions are not stated and in fact are unknown. This is a situation which existed immediately following the war and is one of the ghosts previously mentioned. Conductor size determinations for cable in duct utilizing the NELA constants require no knowledge nor consideration of soil resistivity as such. On the other hand, such determinations for pipe-type cable systems by any practical method require a specific numerical assumption to be made as to the value of soil resistivity in order to arrive at an answer. By taking the stand that the concealed resistivity in the NELA constants is 120 or more, it is thus possible to obtain an advantage in favor of duct-lay cable.

Furthermore, because of the use of cable spacing factors and earth and concrete thermal resistivities in the proposed method, it will be obvious that calculations by the given method will check with those of the IPCEA method only for certain combinations of the variable parameters in the method. Since these parameters were not fixed and in fact are now unknown as regards the NELA duct heating constants, it is obviously impossible to make a factual comparison of the results obtained by the two methods. Here again, by assuming earth resistivities of 120 or 180 as both Mr. Short and Mr. Wollaston have done, the given method will result in larger conductor sizes than the IPCEA method.

Despite the fact that both Mr. Short and Mr. Thomas refer to the presumably large amount of factual data which underlie the NELA duct constants, we have been unable to ascertain the specific conditions on which these constants were based nor is there any indication that earth resistivity measurements were taken as a part of the data. About all that can be done, therefore, is to assume representative cable and duct configurations and then to calculate the earth resistivity required in the given method to match the value calculated by the IPCEA method. We cannot agree to the values given as "the average conditions on which the NELA duct constants were obtained" as stated by Mr. Short. Rather, we believe that the conditions assumed in reference 13 are much more representative, on the basis of which an average earth resistivity of 75 was obtained at 100% load factor.

We take the position, therefore, that the validity of the proposed method is not to be judged by whether or not the calculations made by it using parameters arbitrarily picked by Mr. Short (or by Mr. Wollaston) agree with calculations made by the IPCEA method. Rather we feel that the applicability of the IPCEA method to a particular case depends upon how well it checks with the method which we have proposed, and which takes into account more properly the essential parameters which are pertinent to the case at hand.

With respect to Mr. Short's specific question, we hope that utility engineers will adopt the proposed method but we do not think that they will find it necessary to reduce loads unless they have very high values of earth resistivity. Regarding the need for reduction in loads on existing circuits, it should be kept in mind that it is only relatively recently that AEIC specifications have made provision for increased permissible temperature limits for emergency periods, and for the greater portion of the period that these emergency limits have been in effect the number of companies who have utilized them is relatively small. As a result, the greater portion of the cables now in service have been selected on the basis that normal permissible copper temperature would not be exceeded under emergency conditions. Moreover, in recent years a number of *in situ* measurements have been made with the transient needle, the sphere, or the buried cylinder. Theoretical studies have shown that measurement of ultimate soil resistivity can be obtained readily with such devices. While in many cases these have been made in connection with pipe-type cable installations, they apply equally well to duct bank installations in so far as the resistivity of the soil itself is concerned. The values in general range from 50 to 100 with some higher values as the exception at certain times of the year. Moreover, over the past decade a number of pipe-type installations have been installed in this country with design resistivities in the 70 to 90 range. Under the circumstances, we do not believe that it will be found necessary in most cases to reduce the loads on existing circuits. However, we do believe that engineers will be well advised to take steps to ascertain the values of thermal resistivity which are applicable for their conditions because with the more liberal use of emergency temperature limits and the tendency for shift in many areas in the load peak from winter to summer, the existing margin may be reduced to a low level in the not too distant future.

The values of soil resistivity of 80 and 120 used in the examples of Appendix IV were chosen merely for purposes of illustration and the value of 120 rather than 80 was used in the duct lay case in order to emphasize the effect of a difference between the resistivity of earth at 120 and concrete at 85.

Unlike Mr. Short, Mr. Wollaston is very careful in his discussion to make it quite clear that his comments relating to a comparison of the results obtained by the given method and the NELA-IPCEA method is premised on his own arbitrary assumption of a concealed soil resistivity of 120 in the NELA constants and on his impression, presumably based largely on an unpublished 1947 memorandum by G. B. Shanklin, that a resistivity of 180 is representative of average conditions; consequently, the value of 55 which was obtained by back calculation from the given method utilizing his test results indicates a discrepancy in the method. We believe that if Mr. Wollaston will consult some[1,4] of the many references which have appeared in the technical literature over the past few years on determinations of soil resistivity in connection with experimental duct bank, buried cable and pipe-type cable installations, either alone or in conjunction with buried cylinders, spheres or transient needles, that he will find that there is no longer any justification for an inferred resistivity of the order of 120 in the NELA constants or for his impression that a resistivity of 180 is representative of average conditions.

In as much as no actual measurement was made of soil resistivity at the site at which Mr. Wollaston obtained an indicated value of 55, there are, of course, several possible explanations that suggest themselves. Assuming the temperature measurements were made accurately, perhaps the soil actually had a resistivity of this order of magnitude. From recent studies on soils and the effects of such matters as composition, density, compaction, particle size, etc., it is evident that it is very difficult to estimate the resistivity of a soil from appearance alone. Alternatively, it could be that the measured value of resistivity is not the ultimate value as a constant load applied for 5 days would not suffice to bring the duct structure to its ultimate temperature rise over ambient, unless, of course, it had been carrying substantially full load for some time prior to the test in question. Mr. Wollaston mentions that the temperature was measured 20 feet from the manhole but does not indicate the length of the duct run on which the test was conducted. This raises a question as to whether in his particular case there could have been any alleviation of temperature rise by longitudinal heat flow or, alternatively, by longitudinal convection effects such as were found in the tests made with ducts open and plugged.[5]

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Heat Transfer Study on Power Cable Ducts and Duct Assemblies

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STUDIES of heat transfer on power cable in duct have indicated the need for more experimental data to establish thermal resistivity values for several components of the duct system, particularly that portion of the heat flow path from the cable sheath to the outer duct wall surface. Theoretical considerations of heat transfer from the cable to the outer duct wall surface have treated the duct wall as an isothermal surface; and, generally, no consideration has been given to the effect of the thermal resistivity of the duct material upon the heat flow from the cable. These approximations are made necessary in theoretical studies of cable systems by the complex geometry of the duct assembly, the cable sheath resting nonuniformly at the bottom of the duct. While such studies, supported by the experimental evidence available, yield reasonably accurate thermal resistivity values, the effects of a nonisothermal duct wall and of the thermal resistivity of the duct material can be taken into account by performing measurements on a duct assembly.

In 1938 a study of heat flow in duct systems was begun at the Johns-Manville Research Center to determine the thermal resistance of the components of the heat flow path in a duct assembly. Transite and fiber duct, the latter possessing the higher thermal resistance, were selected for the tests.* Preliminary experiments were performed with each type of duct encased in concrete and buried in earth, an iron pipe enclosing a heater resting on the bottom of each duct. The temperature of the iron pipe in the Transite was sufficiently different from that in the fiber duct, both measured under conditions of equal heater load, to justify further study of the effects of the thermal resistivity of the duct upon the thermal resistance from sheath to outer duct wall surface in a cable system.

After about two years, during which time tests and procedures were established, the control apparatus was designed and assembled; and work was started on the first stage of the 2-stage program, which was planned to include investigation of (1) ducts in air; (2) single duct encased in concrete. This paper contains a description of the tests covering this program, and the application of the test results is indicated to the solution of heat flow problems in duct banks.

Thermocouple Assembly

Eight-foot lengths of Transite and fiber duct were employed for the tests in air. The main components for this test are the duct, the lead sheath resting on the bottom of the duct and containing a Calrod heater along its axis, and the apparatus for regulating the power supply and measuring the thermocouple electromotive forces.

Since the geometry of the duct assembly is not radially symmetric, it was realized early in the study of this problem that, in order to obtain an accurate average temperature around a duct surface, it would be necessary to use a large number of thermocouples spaced circumferentially around the surface. Furthermore, it was recognized that the temperature distribution along the length of the duct would be nonuniform due to the irregular contact between the inner duct wall surface and the lead sheath; and so it was necessary to space the thermocouples along the duct length as well. A 2-foot length of duct, placed in the center of the 8-foot assembly, was employed for the measurements. Thus 3-foot sections of duct on each side of this test length served as guards, reducing end effects and thereby allowing the test area to more closely represent a small section of a long duct.

A preliminary survey of the temperature distribution around the duct surfaces showed that 48 thermocouples along the 2-foot length placed in a 3-turn helix of 16 couples per turn would provide enough temperature readings to yield a true

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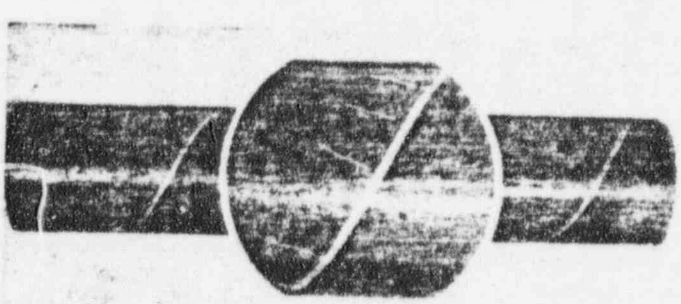


Figure 1. Thermocouple assembly on outside duct surface

* The respective ducts referred to herein are type I fiber conduit covered in Federal Specification W-C-581, August 1, 1939; and asbestos cement conduit type I (J-M Korduct) covered in Federal Specification W-C-571, April 3, 1942. Both ducts are of 0.31-inch wall thickness.

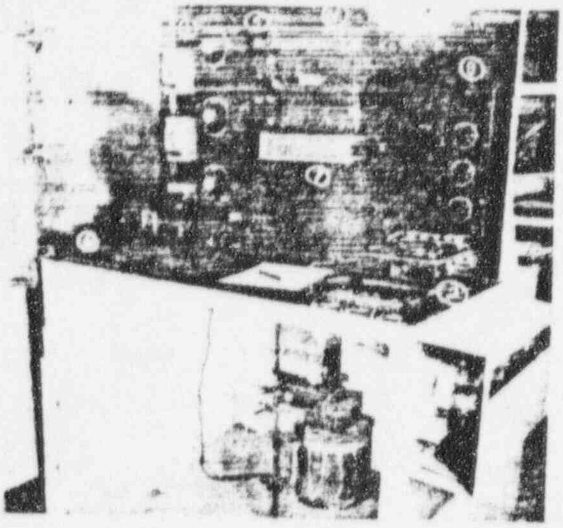


Figure 2 (left). Front view of control board. Power and thermocouple circuit components are numbered as follows: (1) fuse boxes and transtats, (2) ammeter, (3) integrating wattmeters, (4) celectray temperature controller (5) pilot lights (6) type *K-2* Leeds and Northrup potentiometer (7) galvanometer spot reference glass (8) type *R* Leeds and Northrup galvanometer and optical system, and (9) thermocouple selector switches

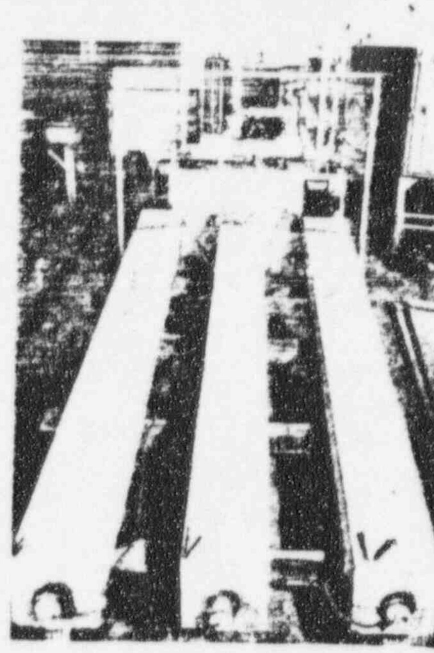


Figure 4. Ducts encased in concrete. The lead sheaths are seen protruding from the duct assemblies. The ducts are not clearly visible since their ends have been packed with rock wool. An experimental conduit is shown in test position in addition to the Transite and fiber duct assemblies

average surface temperature. Figure 1 shows the assembly of the temperature measuring thermocouples on a test length of fiber duct. The constantan wires are soldered to a narrow strip of thin copper foil, which is cemented to the outside surface of the duct. An identical thermocouple assembly is employed on the inside surface. As can be seen in Figure 1, the constantan wires on the outside surface helix are run about two inches along the surface and then inserted into holes taking them inside the duct.

All of the constantan leads were carried inside the duct to reduce the possibility of abrasion when set in concrete. Also, the leads were run along the duct surface for at least two inches to avoid change in the junction temperature by conduction along the wire. In order to permit the measurement of the average surface temperature of a duct by connecting the couples on that surface in parallel, the constantan leads were all cut to lengths of equal resistance. Consideration was given to the effect the mass of wires inside the duct might have on the surface temperature of the lead sheath and of the inner duct surface. Since the total cross-sectional area of the 96, number 30 Brown and Sharpe gauge constantan wires is equal to that of a single 10-gauge wire, it was concluded that this quantity of wire has negligible effect upon the heat flow.

The Transite and fiber ducts employed in the tests each had a 4-inch inside diameter and 0.31-inch wall thickness.

The Heating Unit

The lead sheath used in the duct assembly was cut from a lead pipe of $3^1/_2$-inch outside diameter and 1/8-inch wall thickness. The pipe was stored in the laboratory for over a year before the tests were begun, and so the lead surface had sufficient time to attain a high emissivity oxide coating. A 7.75-foot length Calrod heater, consisting of a coaxial nichrome wire and metallic shield separated by a refractory material, was placed along the axis of the lead sheath and held in place by thin Transite disk spacers. The shield was used as the return current lead to eliminate the possibility of inducing potentials in the thermocouple leads. The resistance of the Calrod heater was measured as approximately 3.4 ohms per foot with ten watts per foot heat flow in the assembled duct.

After some investigation, sand was found to be a suitable medium with which to surround the Calrod heater inside the lead sheath. Five copper-con-

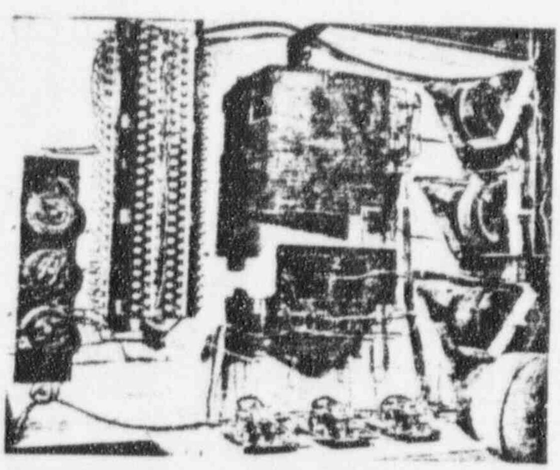


Figure 3 (left). Rear view of control board. Numbered elements are (1) the thermocouple terminal board (2) the survey couple selector switch assembly, and (3) the power circuit relays

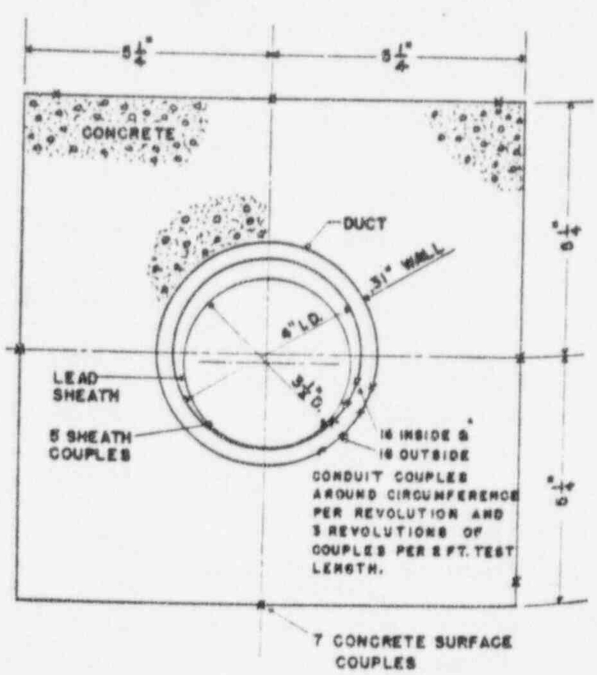


Figure 5 (right). Cross-section detail of duct encased in concrete for "Test Series 1." A 12-inch concrete casing was employed in "Test Series 2." The lead sheath encloses a centered Calrod heater

stantan thermocouples made from number 24 Brown and Sharpe gauge wire were placed in a helical arrangement along a 2-foot length in the middle of the lead pipe. The duct assembly was then completed by placing the sheath inside the duct, the lead pipe resting on the bottom and forming an air crescent with the inner wall surface.

Apparatus

Figures 2 and 3 show the front and rear respectively of the control board. The power circuit controls and the thermocouple circuit instruments are numbered to simplify their identification.

The lead sheath temperature was maintained constant during a test run by means of the Celectray temperatue controller, which surveyed the control couples of each duct assembly once a minute. The control couples were mounted on the lead sheath, and the controller functioned to reduce the current through the Calrod heater when the lead sheath temperature exceeded the set value and to increase it when the temperature fell below the setting. Pilot lights on the control board showed which duct assembly was being surveyed by the controller at any given time. Although the controller has a sensitivity of 0.1 degree centigrade, the temperature variations of the sheath exceeded this quantity because of the appreciable thermal capacity of the duct system. The deviation from the set value was held, however, within 1 per cent of the sheath temperature setting, varying periodically between high and low values.

Energy dissipation in a duct heater during a test interval was measured with integrating wattmeters, and the average power was computed from the time interval of the test run. Thermocouple electromotive forces were measured with a type *K-2* Leeds and Northrup potentiometer, and thermocouple switches were connected so that the electromotive forces of each couple could be measured separately or an average surface temperature reading be made by connecting in parallel the couples on that surface.

Method of Test

All temperature measurements were made under steady-state conditions of heat flow. Thermal equilibrium was generally attained about 72 hours after the initial setting of the controller and power transtats, minor power adjustments being made during the first 48 hours of this period. This steady-state condition was maintained throughout the test period of three to six hours by controlling the temperature of the lead sheath and that of ambient air.

The ends of the duct were packed with rock wool to cut down convection, contributing to the effectiveness of the guard sections in eliminating end effects. Temperature measurements along the test length of the lead sheath during the test period showed variations from the mean temperature of about one per cent. These temperature variations were random along the test length of the sheath, with no indication of a maximum temperature near the center, indicating that the guard sections were effective in eliminating end effects. Along the top of the duct, the temperature was nearly constant along the 2-foot test length. Appreciable temperature differences were measured along the length on the bottom of the duct, but these differences were random and can be attributed to the nonuniform contact between lead sheath and duct wall.

Three "points" each at four different sheath temperatures were obtained for each duct, Transite and fiber, after thermal equilibrium had been obtained. A point consisted of obtaining a record, by means of the integrating wattmeter, of the power dissipated in each duct throughout the test period, recording the surface temperatures of the sheath and duct wall at intervals during the test period, and noting the ambient air temperature. The tests were run in a constant temperature room (34 degrees centigrade) so that little change was noted in the ambient air temperature.

Test Assembly

The assembled ducts were suspended side by side in air, with a separation of one foot, behind the control board. Temperature measurements in the air space between the Transite and fiber ducts showed that their mutual heating effects were negligible. Thermocouple leads were carried through the inside of each duct and connected to the terminal board.

The second stage of the investigation, namely the study of single duct encased in a concrete envelope, was carried out employing the following concrete mix: 440 pounds cement; 1,295 pounds sand; and 2,100 pounds 3/4-inch stone.

This is approximately a 1:3:5 mix, based upon the following densities: sand 100 pounds per cubic foot, stone 100 pounds per cubic foot, and cement 94 pounds per cubic foot. No allowance was made for moisture content of the sand.

The concrete casings were aged before tests were started until the electrical resistance of wood blocks containing metal electrodes, which were cast in the concrete, reached an equilibrium condition. The aging period, as judged by this method, was approximately four months.

Seven thermocouples were placed around the surface of the concrete envelopes to obtain surface temperatures. With the additional observation of outside concrete surface temperature, the test method was identical with that described for ducts in air.

Figure 4 shows the concrete encased ducts in test position behind the panel board. The concrete envelopes are spaced ten inches apart, and the temperature in the air space midway between the encased ducts exceeded room temperature by less than 1 degree centigrade. A duct bank of experimental conduit is shown in addition to the Transite and fiber duct assemblies. Figure 5 shows a cross-section detail of ducts in concrete.

A second series of tests was conducted on Transite and fiber duct, using two additional samples, to add weight to the original data obtained. The test assembly in air was identical with that described before except that the length of the duct assembly was reduced to six feet and the number of lead sheath thermocouples was increased to seven. An additional change made for the tests on ducts in concrete was to increase the size of the concrete envelope so that the total cross-section of the duct bank was a 12-inch square. It is believed that this larger concrete envelope more closely represents a single-duct bank buried underground. The method of test employed in the second test series was identical with that of the first series except that "points" were taken at a larger number of lead sheath temperatures with only one "point" for each temperature. Also the tests were run in a lower ambient room temperature (about 26 degrees centigrade). Data pertaining to the first and second series of tests will be designated respectively as Series 1 and Series 2.

Results

Table I contains typical observations and calculations from the tests on ducts in air and in concrete. Although this table contains only a small number of the total test points employed in the investigation, it shows the method of calculation used to obtain the values of thermal resistance and resistivity employed in the illustrations.

Values of thermal resistance are plotted

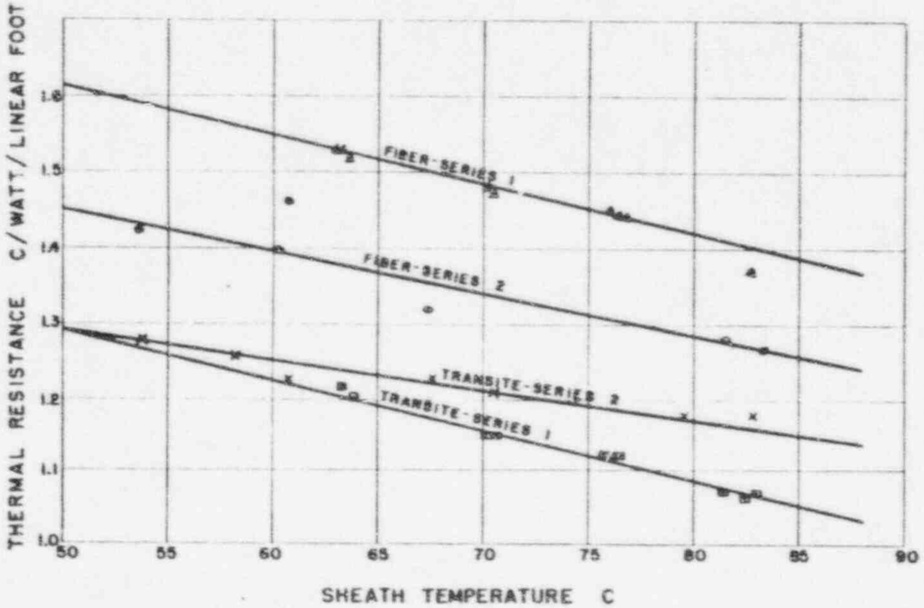


Figure 6. Thermal resistance from lead sheath to inside duct surface versus lead sheath temperature for Transite and fiber ducts in air

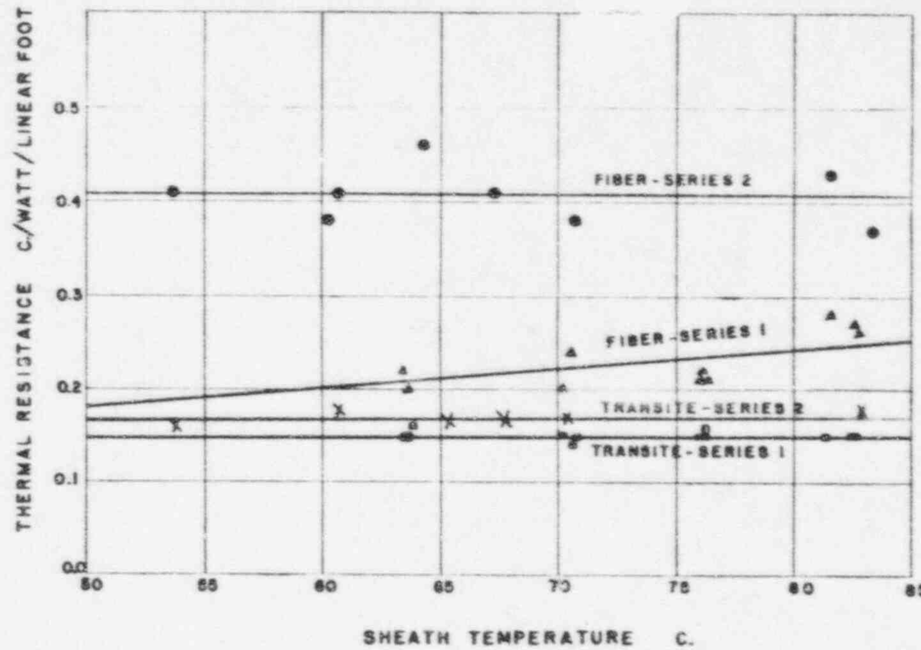


Figure 8. Thermal resistance of the duct wall versus lead sheath temperature for Transite and fiber ducts in air

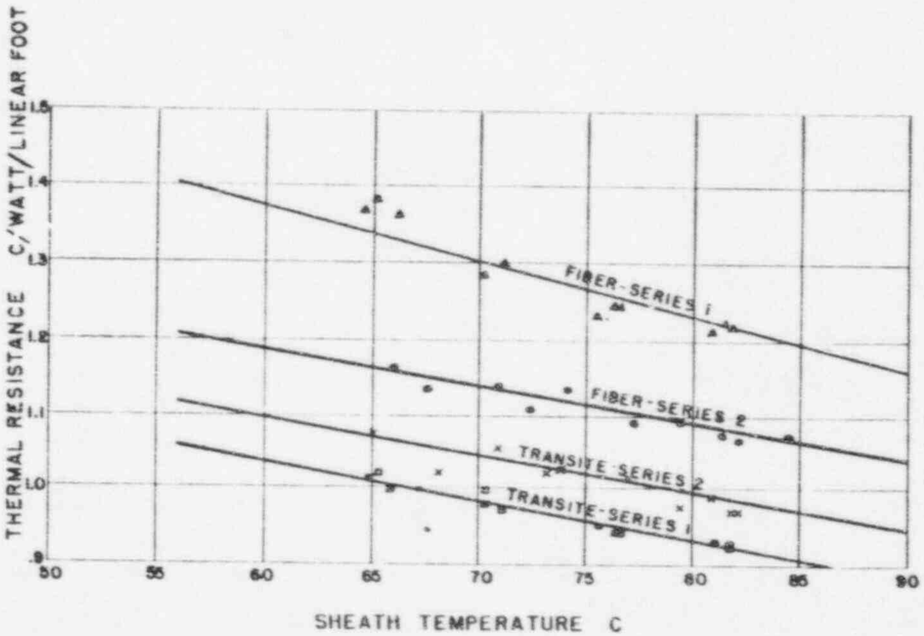


Figure 7. Thermal resistance from lead sheath to inside duct surface versus lead sheath temperature for Transite and fiber ducts encased in concrete

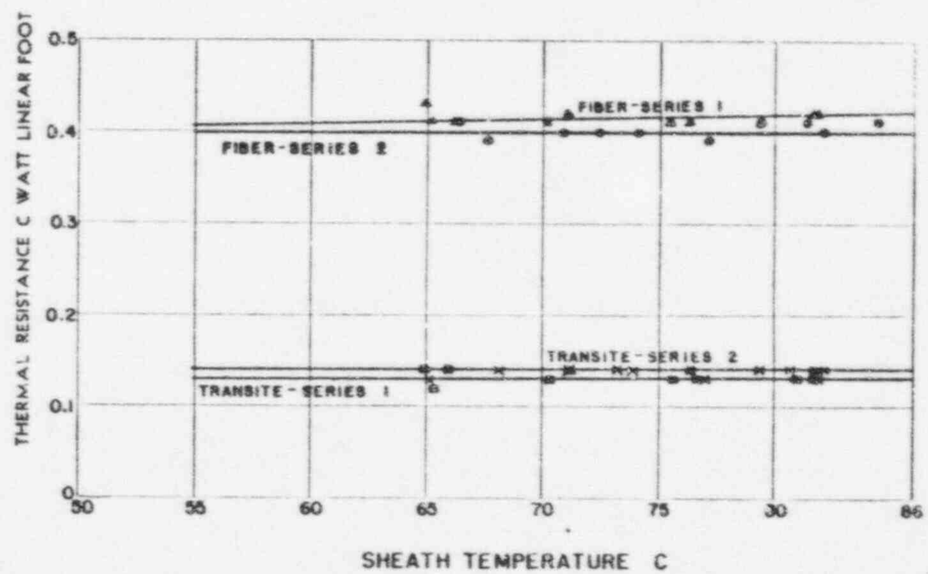


Figure 9. Thermal resistance of the duct wall versus lead sheath temperature for Transite and fiber ducts encased in concrete

Table I. Typical Experimental Data

Duct Material*	Watts Per Ft	Temperature, °C: Sheath	Inside Duct Wall Surface	Outside Duct Wall Surface	Outside Concrete Surface	Ambient	Thermal Resistance, °C Per Watt Per Linear Ft: Sheath to Duct	Duct Wall	Duct to Air	Concrete Envelope	Thermal Resistivity**: Surface Resistivity C cm²/watt: β	β_d	β_e	Volume Resistivity C cm/watt: ρ_d	ρ_e
Ducts in Air															
Fiber (I)	10.5	63.4	47.4	45.1		33.9	1.53	0.22	1.07		1,300	1,200		300	
Fiber (I)	13.2	70.6	51.1	47.9		35.0	1.48	0.24	0.98		1,260	1,100		320	
Fiber (I)	18.3	82.7	57.4	52.6		34.4	1.38	0.26	0.99		1,170	1,110		350	
Fiber (II)	12.5	60.7	42.4	37.2		25.6	1.47	0.41	0.93		1,250	1,050		550	
Fiber (II)	13.5	64.3	43.9	37.7		25.0	1.51	0.46	0.94		1,280	1,060		620	
Fiber (II)	21.9	81.6	53.6	44.2		25.1	1.28	0.43	0.87		1,090	980		580	
Transite (I)	12.6	63.4	48.2	46.3		33.9	1.21	0.15	0.98		1,030	1,100		200	
Transite (I)	16.3	70.1	51.6	49.1		33.9	1.14	0.15	0.93		970	1,050		200	
Transite (I)	19.1	76.2	54.7	51.7		33.9	1.12	0.16	0.93		950	1,050		200	
Transite (II)	14.0	58.2	40.7	38.6		25.9	1.25	0.15	0.90		1,060	1,010		220	
Transite (II)	18.3	67.6	45.3	42.3		27.6	1.22	0.17	0.80		1,040	900		200	
Transite (II)	24.0	79.7	51.7	48.2		28.3	1.17	0.15	0.83		1,000	930		230	
Ducts in Concrete															
Fiber (I)	12.4	64.9	48.3	43.0	37.3	33.5	1.35	0.43		0.46	1,150		1,000	580	83
Fiber (I)	14.9	71.1	51.8	45.6	38.5	33.5	1.30	0.42		0.48	1,110		1,100	560	87
Fiber (I)	19.9	81.9	57.6	49.3	40.3	33.5	1.22	0.42		0.45	1,040		1,110	560	81
Fiber (II)	16.7	66.2	46.8	40.0	30.6	26.1	1.16	0.41		0.56	990		1,000	550	90
Fiber (II)	21.6	77.2	53.6	45.2	33.0	26.1	1.09	0.39		0.56	930		1,180	520	90
Fiber (II)	23.6	82.5	57.3	47.6	34.4	27.2	1.07	0.41		0.56	910		1,130	550	90
Transite (I)	16.7	65.3	48.2	46.1	39.7	33.5	1.02	0.12		0.39	870		1,210	160	71
Transite (I)	19.9	71.1	51.8	49.1	41.2	33.5	0.97	0.14		0.39	830		1,260	190	71
Transite (I)	26.3	81.7	57.3	53.7	43.5	33.5	0.93	0.14		0.39	790		1,240	190	71
Transite (II)	19.5	65.1	44.3	41.7	30.6	26.1	1.07	0.13		0.57	910		860	180	92
Transite (II)	25.2	77.0	51.5	48.1	33.8	26.1	1.01	0.13		0.57	860		1,130	180	92
Transite (II)	27.0	80.7	54.1	50.3	34.7	27.2	0.99	0.14		0.58	840		1,030	190	93

* Number in parentheses represents the test series from which the data was taken.

** β, β_d, and β_e are surface resistivity factors respectively from sheath to duct wall, from duct wall to ambient air, and from concrete to ambient air. ρ_d and ρ_c are volume resistivities of duct and concrete respectively. These thermal resistivity values are computed from the following equations

$$\beta = \frac{R_{ths} D_s}{0.00411},\quad \beta_d = \frac{R_{tha} D_d}{0.00411},\quad \beta_c = \frac{R_{tha}' D_e}{0.00411},\quad \rho_d = \frac{R_{thd}}{0.012 \log_{10} D_d/D_i},\quad \rho_c = \frac{R_{thc}}{0.012 \log_{10} D_e/D_d}$$

where R_{ths}, R_{tha}, R_{tha}', R_{thd}, and R_{thc} are the respective thermal resistance values shown above from sheath to duct wall, duct wall to air, concrete to air, of the duct wall, and the concrete envelope; and D_s, D_i, D_d, and D_e are in inches the respective diameters of sheath, inner duct wall surface, outer duct wall surface, and equivalent concrete diameter.

against lead sheath temperature for the components of the heat flow path in the ducts, Figures 6 through 9. Figures 6 and 7 show the variation with sheath temperature of the thermal resistance from sheath to inside duct wall for ducts in air and ducts in concrete respectively. Since radiation through the air crescent between sheath and inside duct surface is an important factor in the heat transfer between these surfaces, the thermal resistance values decrease with increasing lead sheath temperature. It was not expected that the heat transfer due to convection would increase so rapidly with increasing sheath temperature as that due to radiation, the former varying nearly linearly with the temperature difference of the two surfaces, the latter with the difference in the fourth powers of the surface temperatures.[1]

Two general conclusions can be drawn from these curves regarding the relative thermal resistance values from the lead sheath to the inside duct surface. For a given sheath temperature, the thermal resistance is consistently higher for ducts in air than in concrete. Also the fiber duct consistently gives a higher resistance from sheath to inside duct wall than does the Transite duct. The influence of the concrete casing can be attributed to its effectiveness in producing more uniformity of the heat flow lines in the entire duct assembly, since its thermal resistivity is relatively low. Differences between the fiber and Transite assemblies can be attributed to the heat transfer from sheath to duct by direct conduction. The lead sheath "snakes" throughout the bottom of the duct, making a nonuniform contact with it. The duct material of lower thermal resistivity more readily conducts heat away from the lead sheath.

The difference in the thermal resistance from lead sheath to inside duct wall as between the Transite and fiber ducts is considerably greater in Series 1 tests than in Series 2. The deviation in the two test values for the fiber duct assembly is about 15 per cent of the resistance from sheath to duct wall, and we attribute it to the variable factor of contact area between lead sheath and the duct containing

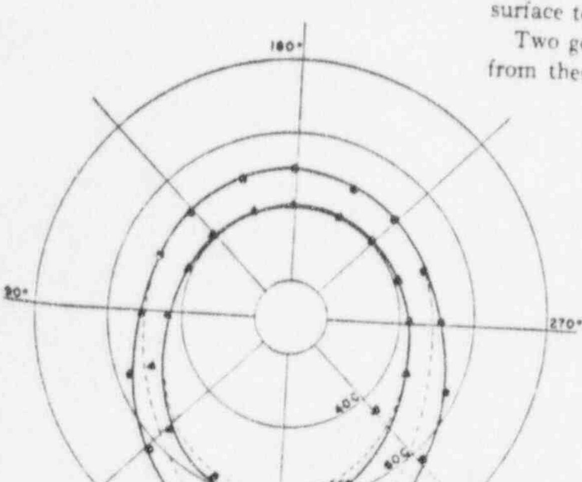


Figure 10 (left). Temperature distribution around the fiber duct surfaces at 82 degrees centigrade lead sheath temperature. The solid lines represent the duct encased in concrete, the dotted lines in air

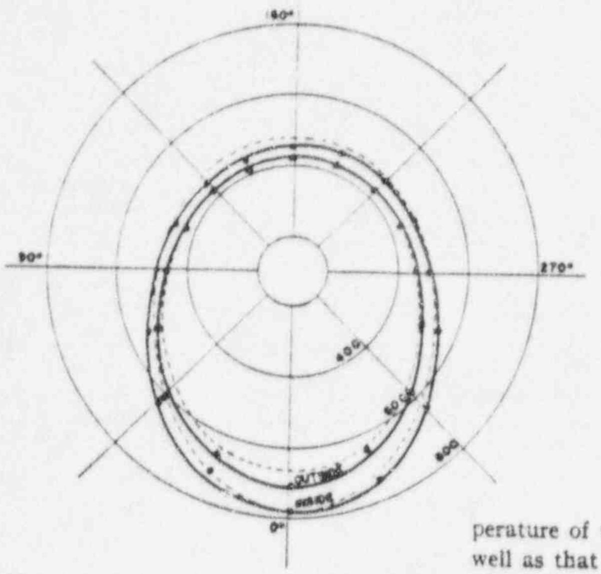


Figure 11 (left). Temperature distribution around the Transite duct surfaces at 82 degrees centigrade lead sheath temperature. The solid lines represent the duct encased in concrete, the dotted lines in air

it. This deviation is not unreasonably high, since considerations of the heat transfer from sheath to duct by Whitehead and Hutchings[3] indicate that variations in spacing between the sheath and the bottom of the duct produce considerable effect upon the thermal resistance of this component of the heat flow path.

Figures 8 and 9 show the thermal resistance of the duct itself as a function of lead sheath temperature for ducts in air and ducts in concrete respectively. This factor is small compared with the total resistance of the heat flow path from the sheath to ambient surroundings, but it serves to measure the effect of the thermal resistance of the duct upon the over-all heat flow through the duct assembly. The Transite duct possesses a lower resistance in concrete than in air, possibly due to the reduction in its outside surface resistance when in contact with the concrete envelope, Transite and concrete both being cement products. An anomalous curve in the set shown in Figures 8 and 9 is that for the Series 1 tests on fiber duct in air. Moisture content, among other factors, was considered as an explanation for this anomaly, but it was discounted since the Transite and fiber ducts were exposed to identical environments before and during the tests. Since the fiber duct samples employed in the two tests were identically assembled, no satisfactory explanation for the anomalous curve has been found.

The thermal circuit for ducts in air is completed with the outside duct wall surface resistance, typical values for which are shown in Table I. These values give the order of magnitude of the outside surface resistance of the duct wall, and in an actual installation, the exact value of this resistance will depend upon the temperature of the ambient surroundings as well as that of the outside duct surface. The resistance of the concrete envelope completes the thermal circuit for ducts in concrete in the laboratory installation, although the thermal resistance of the surrounding earth would also be considered in an underground duct bank. Table I also contains the thermal resistance values for the concrete envelopes, the values for the Series 2 tests being greater since the larger concrete envelope was employed for these tests.

Figures 10 and 11 are polar graphs showing the temperature distribution around the duct walls respectively for fiber and Transite ducts at a sheath temperature of 82 degrees centigrade. The solid lines represent ducts in concrete, dotted lines, ducts in air. The average temperature of the three thermocouples at each 22.5 degree radial position (corresponding to each turn of the three-turn helix of couples) was employed to obtain the points for these curves. The departure from uniform radial heat flow in the duct is shown by the large temperature difference between the top and bottom of the duct wall. Also since the lead sheath does not rest uniformly on the bottom of the duct, the heat flow is not entirely symmetrical about the vertical axis. The temperature distribution around the duct walls for ducts in air at the same sheath temperature is similar to that in concrete, except that a higher percentage of the total heat flow occurs near the bottom.

Figures 12 and 13 show reduced scale drawings of the fiber and Transite ducts with their lead sheaths. The temperatures on the duct walls at each 30 degree radial position were taken from the solid line polar graphs, Figures 10 and 11, representing ducts encased in concrete, and recorded on the drawings. The temperature falls off rapidly on the bottom half of the duct from the region of contact with the lead sheath, and it remains nearly constant on the top half. The direction of the temperature gradient through the duct is indicated by the arrows.

Calculations of thermal resistance for the heat flow components of a duct system are usually based upon accepted values of thermal surface or volume resistivity. Resistivity values are more useful to the cable engineer than are resistance values since the former are independent of the dimensions of the components of the duct system. The thermal resistance values obtained from the tests have been employed to calculate surface resistivity of the lead sheath, volume resistivity of the duct material and of the concrete, and surface resistivity of the outer duct wall in air. These values are shown in Table I, and since there is some interest in the surface resistivity of the outside of the concrete casing in air, these latter values are also given. Lead sheath surface resistivity values in fiber and

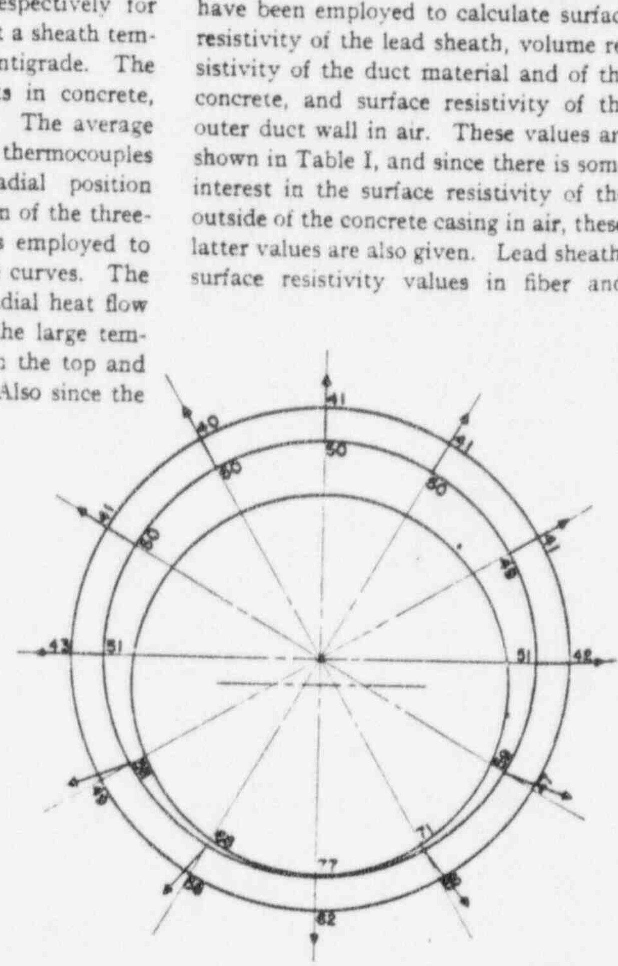


Figure 12 (right). Reduced scale diagram of sheath and duct showing the temperature distribution around the fiber duct encased in concrete at 82 degree centigrade sheath temperature. Arrows show the direction of the temperature gradient

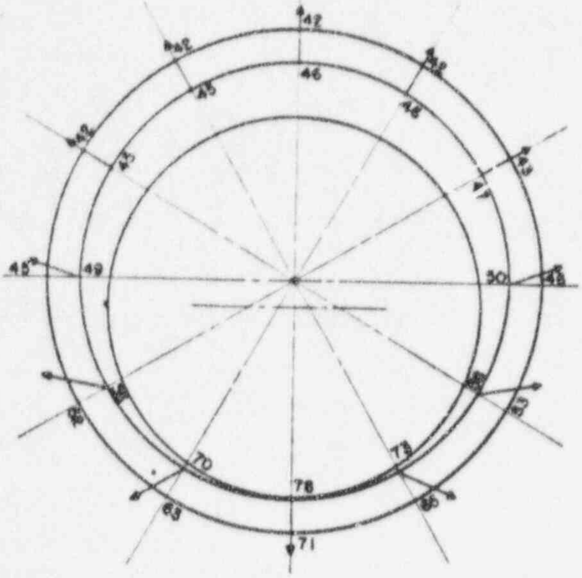


Figure 13 (left). Reduced scale diagram of sheath and duct showing the temperature distribution around the Transite duct encased in concrete at 82 degree centigrade sheath temperature. Arrows show the direction of the temperature gradient

Table II. Thermal Resistivity Factors—10 Watts Per Foot Heat Flow

Thermal Component	Symbol	Resistivity
Lead sheath in fiber duct, surface resistivity	β	1,200 C cm²/watt
Lead sheath in Transite duct, surface resistivity	β	1,020 C cm²/watt
Duct in air, surface resistivity	β_d	1,110 C cm²/watt
Concrete in air, surface resistivity	β_c	1,120 C cm²/watt
Fiber duct, volume resistivity	ρ_d	480 C cm/watt
Transite duct, volume resistivity	ρ_d	200 C cm/watt
Concrete, volume resistivity	ρ_c	84 C cm/watt

Transite ducts are computed, as is customary in cable calculations, by formulating the thermal resistance from sheath to inner duct wall in terms of the sheath surface resistivity. The surface resistivity of the concrete envelope in air, and also its volume resistivity, is calculated by employing a diameter of a circle whose perimeter is equal to that of the square concrete casing. The concrete surface resistivity values are computed from the least precise of all the temperature measurements made on the duct assemblies, and the data should be weighted accordingly.

Figure 14 shows thermal surface resistivity values for various duct assembly components versus heat flow in watts per foot of duct. Curves of lead sheath surface resistivity in fiber and Transite duct are based upon average values from the four sets of tests; that is, Series 1 and 2 for ducts in air and in concrete. The curves of outside surface resistivity of the duct wall and the concrete envelope are each based upon average values from four sets of tests on both Transite and fiber. For a given value of sheath temperature, the outer duct wall surface resistivity is lower for Transite than for fiber since greater heat flow occurs in the assembly of the former, giving its outer duct wall surface a higher temperature. However, since resistivity values in Figure 14 are plotted against heat flow in watts per foot of duct, the outside duct wall surface resistivities for Transite and fiber duct in air are nearly equal and are represented by a single curve. It is interesting to note that the surface resistivity values of the various components are of the same order of magnitude. The curves in Figure 14 are extrapolated to a heat flow of ten watts per foot, and the values thus obtained may be used conveniently as duct constants in cable calculations. These values, along with the volume resistivities of duct wall and concrete obtained from the tests, are contained in Table II.

The temperature rise of a cable in a duct in air is given by the equation

$$T_c = \left(R_{th1} + 0.00411 \frac{\beta}{D_s} + 0.012 \rho_d \times \log_{10} \frac{D_d}{D_1} + 0.00411 \frac{\beta_d}{D_d} \right) Q$$

where the four terms in parentheses represent the thermal resistances respectively from copper to lead, lead sheath to inside duct wall, across the duct wall, and outer duct wall to ambient air; and Q is the heat flow in watts per foot of cable. The symbols employed in the last three terms in the parentheses are defined in Table I, and the resistivity values are contained in Table II. The equation assumes that the sheath to duct resistance is independent of the duct diameter; and since theoretical considerations show that the sheath diameter is the primary variable in the heat transfer from sheath to duct, this assumption probably does not introduce an appreciable error in cable calculations. In temperature measurements on a cable sheath and the inside of an enclosing fiber duct encased in concrete, Barenscher[3] obtained data indicating the correctness of the inverse relationship between sheath diameter and thermal resistance from sheath to duct wall. The temperature rise of a given cable in a buried duct bank above ambient earth is obtained by replacing the fourth term in the equation with the expression derived by Neher[4], the latter taking into consideration the thermal resistance of concrete and surrounding earth and the heating produced by the other loaded cables in the duct bank.

Further tests on various duct materials should be of great value to cable engineers in establishing duct constants for these materials, and should add weight to the data submitted here on Transite and fiber duct. A more complete investigation is needed of the thermal resistance of the heat flow path from the cable sheath to the outside of the duct wall, including experimental data on the effect of variations in lead sheath and duct diameters and in thermal resistivity of the duct wall material. Tests on groups of ducts in a duct bank would also be of considerable interest, although the heat flow path outside the duct wall can be treated, with

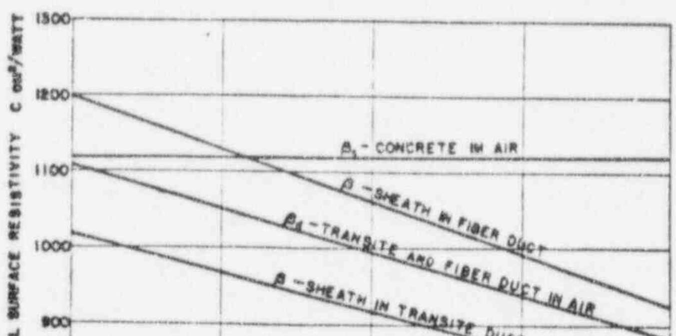


Figure 14. Thermal surface resistivity versus heat flow for lead sheath in Transite duct, lead sheath in fiber duct, outer duct surface in air, and outer concrete surface in air

considerable degree of accuracy, with conventional methods of heat flow calculations.

References

1. HEAT TRANSMISSION (book). W. H. McAdams. McGraw-Hill Book Company, New York, N. Y., second edition, 1942.

2. CURRENT RATINGS OF CABLES FOR TRANSMISSION AND DISTRIBUTION. J. B. Whitehead. *Journal*, Institution of Electrical Engineers (London, England), volume 83, 1938, page 539.

3. A STUDY OF THE TEMPERATURE DISTRIBUTION IN ELECTRIC CABLES IN UNDERGROUND DUCTS. P. J. Barenscher. *Thesis*, University of Wisconsin (Madison, Wis.), 1925.

4. THE TEMPERATURE RISE OF CABLES IN A DUCT BANK. J. H. Neher. *AIEE Transactions*, volume 68, part I, 1949, pages 540–49.

Discussion

V. V. Mason (The Hydro-Electric Power Commission of Ontario, Toronto, Ontario, Canada): This discussion deals with two points which may not bear greatly on the practical value of the results attained by Mr. Greebler and Mr. Barnett but which are of some importance as regards the experimental method and the presentation of the results.

PRESENTATION OF RESULTS

The authors present their results for the sheath-to-duct thermal resistance, R_{ths}, as a graph with the sheath temperature as the independent variable, and express some concern over the lack of agreement of these curves for the Series I and Series II tests. To find a more suitable quantity against which to plot R_{ths} consider that the rate at which heat is transferred from sheath to duct, being partly by conduction, partly by convection, and partly by radiation, may be written thus

$$P = k_1\Delta T + k_2(\Delta T)^{1.25} + k_3\Delta(T^4) \qquad (1)$$

where k_1, k_2, k_3 are constants, and T is the absolute temperature in degrees Kelvin. The exponent of the temperature difference for the convection term actually may vary from 1 to 1.5, but 1.25 is probably fairly near the true value.

The thermal resistance R_{ths}, then, is given by

$$\frac{1}{R_{ths}} = \frac{P}{\Delta T} = k_1 + k_2(\Delta T)^{0.25} + k_3(T_s + T_w)\times (T_s^2 + T_w^2) \qquad (2)$$

where T_s and T_w are the temperatures in degrees Kelvin of the sheath and duct wall respectively, so that $\Delta T = T_s - T_w$ and $\Delta(T^4) = (T_s - T_w)(T_s + T_w)(T_s^2 + T_w^2)$.

We see then that R_{ths} is a function of

$$(\Delta T)^{0.25} + k(T_s + T_w)(T_s^2 + T_w^2)$$

where $k = k_3/k_2$.

In plotting their values for R_{ths} against sheath temperature, the authors have assumed that the duct wall temperature T_w is constant and that the convection term is negligibly small. To see how these quantities affect the appearance of the curves, Figures 1 to 4 of the discussion have been prepared from the complete test records which Mr. Greebler very kindly made available for this purpose. R_{ths} is plotted against $(\Delta T)^{0.25}$ in Figures 1 and 3 and against $(T_s + T_w)(T_s^2 + T_w^2)$ in Figures 2 and 4. Figures 1 and 2 are for fiber while Figures 3 and 4 are for Transite. Curves for duct in air and in concrete are given in each illustration because, for the purpose of this discussion, it is of most interest to compare all the values for a given kind of duct. Figures 1 and 3, R_{ths} versus $(\Delta T)^{0.25}$, would hold if the heat transfer due to radiation were small compared to that by convection, while Figures 2 and 4 would hold for negligible convection. That is, these figures represent extremes; any practical case will be some combination of the two, obtained by assigning a suitable value to k.

Examination of these figures indicates that for fiber ducts, the agreement between Series I and II in both air and concrete is considerably better when plotted against $(\Delta T)^{0.25}$ than when plotted against T_s or $(T_s + T_w)(T_s^2 + T_w^2)$. Transite, on the other hand, exhibits the most reasonable behaviour when $(T_s + T_w)(T_s^2 + T_w^2)$ is used for the abscissa. This would seem to indicate that with the fiber duct a large part of the heat transfer is due to convection, while with Transite, radiation is the more important mechanism. This difference may be ascribed to the difference in the thermal emissivities of the Transite and fiber surfaces.

The agreement between the two series of runs is still far from perfect, but does appear to be considerably improved. Another possible point of difference between the various tests not considered by the authors is the presence of the thermocouple leads with regard to their effect on the convection currents. Small changes in their position could affect materially the magnitude of the convection component of heat transfer.

TEST METHODS

The authors state that the power input was controlled to keep the sheath temperature constant and that both the sheath and ambient temperatures were controlled to maintain steady-state heat-flow conditions. The actual power input was obtained by measuring the energy input over a period of three to six hours. If the rate of heat transfer were accurately proportional to the temperature differences, this would be an entirely justifiable method. However, due to the nonlinear relations involved, the validity of an arithmetic average for the input power is questionable. Moreover, it would have been far simpler to keep the input power and the ambient temperature constant for a given run and to let the sheath temperature fall where it would.

R. W. Burrell (Consolidated Edison Company of New York, Inc., New York, N. Y.): The data presented by the authors are of considerable interest to cable engineers, and the Johns-Manville Company and the authors are to be congratulated for the presentation of this data. The painstaking care used in making these tests is commendable. The results would have been of still greater value if the tests could have been carried out on more than one cable size or at least on a more representative combination than a $3^1/_2$-inch cable in 4-inch duct. The data presented contribute significantly to our understanding of the thermal drop from cable sheath to duct wall. Of particular interest are the polar graphs of Figures 10 to 11 of the paper showing the temperature distribution around loaded duct walls. A need for further exploration of this phenomenon is indicated when these data are compared with the values shown in Figure 1 of Mr. F. V. Smith's discussion of a paper by J. H. Neher[1] presented in 1949. It should be noted that the temperatures given in Mr. Smith's discussion are outside duct wall values, but these show little departure from an isothermal surface in contrast with the values for outside wall shown in Figures 10 and 11 of the paper by Mr. Greebler and Mr. Barnett. Unfortunately, in the case of Mr. Smith's data, the heat loss per foot of cable, 3.0 watts, is relatively low for an average cable of a diameter of 2.52 inches. The comparison is also obscured by the possible variations as between single and multiduct structures.

There are a number of effects indicated in the data presented by the authors which

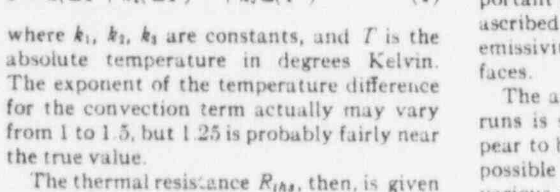
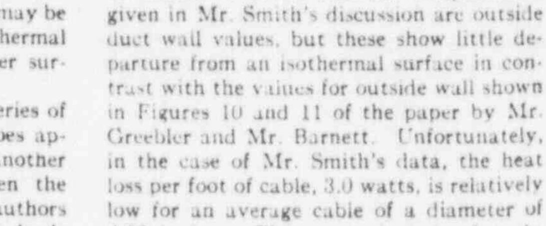


Figure 1 (left). Thermal resistance versus $\Delta T^{0.25}$ for fiber ducts

Figure 2 (right). Thermal resistance versus $(T_s + T_w)(T_s^2 + T_w^2)\times 10^{-8}$ for fiber ducts



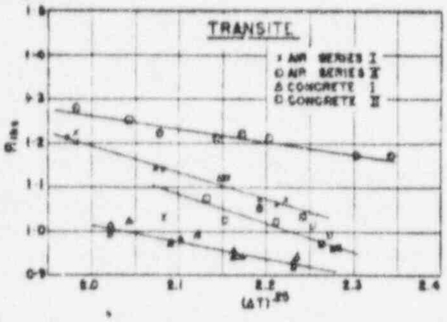


Figure 3. Thermal resistance versus $\Delta T^{0.25}$ for Transite ducts

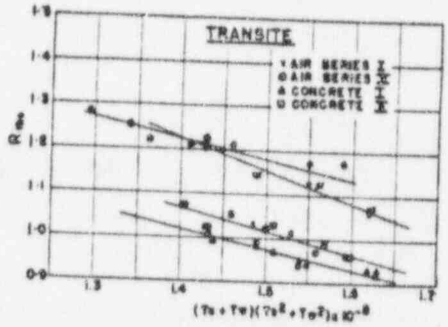


Figure 4. Thermal resistance versus $(T_s+T_w)(T_s^2+T_w^2)\times10^{-8}$ for Transite ducts

possibly may be taken as random deviation rather than true effect. In Figure 6 of the paper, the indicated change in thermal resistance from sheath to duct wall, for the case of fiber ducts in air, as between Series I and Series II, indicates the need for tests on a representative number of samples before firm conclusions can be drawn, although there is no doubt that the thermal resistance is higher in the case of dry fiber ducts as against Transite ducts.

The data on the volume thermal resistivity of Transite and fiber material are of interest. The average value for Transite is less than that formerly used by the Consolidated Edison Company in rating calculations, and we have adopted the lower value since these data were first released by the Johns-Manville Company some few years ago. The value for fiber is higher than our normal standard, and it is to be noted that the values derived from these tests are for the dry condition and may be appreciably lower when wet.

Considering the various values of β presented so far from different sources, apparently for the present we must accept a possible 10- to 20-per cent plus or minus variation in β for a given installation. Additional test data of the detailed type presented by the authors, on single and multiduct structures, in air and especially in earth, with cables of representative size and loading, are required before positive conclusions can be drawn.

Reference

1. Discussion by **Frank V. Smith** of The Temperature Rise of Cables in a Duct Bank, J. H. Neher. *AIEE Transactions*, volume 68, part 1, 1949, pages 547–48.

R. J. Wiseman (The Okonite Company, Passaic, N. J.): This paper is a welcome addition to the investigation which has been going on in recent years as to the thermal resistance of duct systems. It, along with Mr. J. H. Neher's paper,[1] can well be studied for more exact thermal constants for ducts. Today there is a desire to obtain every ampere possible out of a cable. As the physical structure of the duct bank is important in determining the total thermal resistance of the duct bank, better knowledge of the influence of the kind of duct is desirable. The paper by Mr. Greebler and Mr. Barnett furnishes this information to us.

I wish the authors had used a heating load which would have given a sheath temperature of 40 to 45 degrees centigrade, as this is the range of temperature actually obtained in practice, and not the 82 degrees centigrade reported in the paper. We then would have obtained thermal constants more likely to exist.

In Table II of the paper the authors give thermal resistivity values. The value of surface resistivity for the lead sheath should be qualified that it is for a sheath diameter of 3.5 inches. Past tests indicate that it is actually a function of diameter. In the Insulated Power Cable Engineers Association we take cognizance of this by assuming that it increases from $\beta=660$ ohms per square centimeter for $D=0$, to 1,200 ohms per square centimeter for $D=1.75$ inches and constant thereafter. According to Table II, these values may apply to fiber duct.

For Transite duct perhaps we should have a reduction of 15 to 20 per cent. The volume resistivity values for fiber duct, Transite duct, and concrete are very welcome. We have been taking a value of 100 thermal ohms per centimeter for concrete, so were on the conservative side.

Reference

1. See reference 4 of the paper.

F. H. Buller (General Electric Company, Schenectady, N. Y.): There are two points in connection with this paper which are particularly worthy of notice. All tests were taken on ducts in air, or encased in a concrete envelope situated in air, and the ends of the ducts were plugged up to prevent longitudinal convection. These statements also apply to the Barenscher investigation.[1]

Let us examine the significance of these points. Mr. Greebler and Mr. Barnett have presented curves (Series 2 tests) which show the effect of increasing the mass of concrete around the duct with the thought that the heavier mass of material simulated somewhat more closely the effect of underground burial of the duct bank. In this thought they are probably correct, and it will be noted from Figure 2 of the paper that the surface thermal resistance of the cable falls off materially in the case of the fiber duct and increases slightly in the case of the Transite duct, thus bringing these two types of duct closer together in their thermal properties. Presumably, burial in earth would exaggerate this effect further; it is quite possible, however, that such burial might ultimately reduce the values of surface thermal resistance in both cases below those given here for ducts in air. Further research is needed along these lines.

Moreover, it is noted that these tests were made on an isolated duct. There is reason to believe that in a multiple-duct bank, the value of β may be materially affected by the modification of the heat field in the duct structure by the presence of additional loaded ducts in close proximity, so that a test in air on a single duct is not necessarily representative of a multiple-duct bank buried in the ground.

The authors also have shown clearly that the drop through the fiber duct wall is greater than that through the Transite duct wall. However, these tests were taken on dry ducts. Presumably water saturation, which frequently occurs in the field, would tend to reduce the drop through the fiber duct wall while affecting the Transite duct wall comparatively little.

Next, consider the question of plugging up the duct ends. The authors were undoubtedly justified in this procedure, since on such a short length of duct any appreciable longitudinal flow of air probably would have distorted observed results to a point where they became meaningless. However, it is not customary to seal up duct mouths in service, although it is sometimes done. Hence, in service, we may expect some longitudinal movement of air, which would tend to reduce the value of β below what was observed by the authors. Pipe cables, on the other hand, are necessarily sealed in service, since they operate under pressure.

These facts may explain, at least in part, the difference between the β values obtained by the authors and the lower ones observed in field tests on cable-installed underground in multiple-duct banks, notably those reported by Mr. Halperin.[2]

To sum the matter up, while it would appear that these tests were very carefully made, and while the technique used can probably be properly taken as a model for any future tests made on ducts in air, the results are not necessarily representative for the case of duct banks buried in the ground and operated under service conditions. They might possibly approximate more closely to the case of duct banks in air as sometimes used indoors in power stations and elsewhere.

References

1. See reference 3 of the paper.

2. Load Ratings of Cable, **Herman Halperin**. *Electrical Engineering (AIEE Transactions)*, volume 58, October 1939, pages 535–56.

J. H. Neher (Philadelphia Electric Company, Philadelphia, Pa.): This paper describes the procedure employed in obtaining the test results on cables in fiber and Transite ducts which I cited in my recent paper,[1] and which also appears in a paper by F. H. Buller and J. H. Neher.[2] I wish to compliment the authors of this paper on the thoroughness with which they have worked out this procedure, which may well serve as a model for future work along these lines.

Unfortunately, the tests were conducted with a single, and larger than average, cable size so that some extrapolation is necessary to convert the results to normal practice. The authors have attempted to do this in Figure 14 of the paper by expressing β as a function of the heat flow, but completely

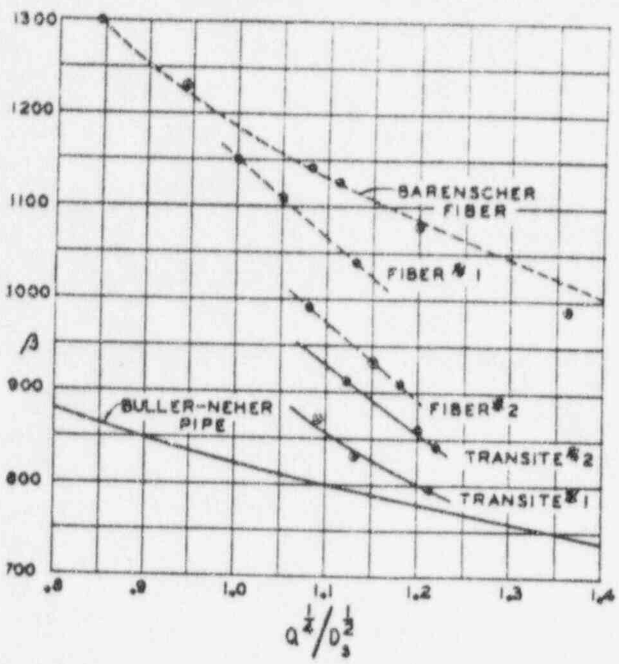


Figure 5. Comparison of test data on fiber and Transite ducts in concrete

disregarding the effect of the cable diameter upon β. Mr. Buller and I have shown in our paper[2] that as a best approximation β is dependent upon the fourth root of the heat flow and the square root of the cable diameter.

Figure 5 of the discussion shows the data of Mr. Greebler and Mr. Barnett plotted as a function of the parameter $Q^{1/4}/D_s^{1/2}$ (Q in watts per foot; D_s in inches) together with the Barenscher[1] data for fiber ducts and the curve for cable in pipe given in the Buller and Neher paper.[2]

It should be noted that the Buller and Neher curve for fiber duct is based on the Barenscher data with which the fiber Series 1 test data of Mr. Greebler and Mr. Barnett is in fairly good agreement. Similarly, the Greebler and Barnett Transite Series 1 test data is so close to the cable in pipe curve that a distinction between the two is scarcely warranted. Only the Series I Greebler and Barnett tests were available to Mr. Buller and myself when our paper was prepared.

The second set of tests by the authors, falling as they do between the first tests on Figure 5 of the discussion, would seem to indicate that in the case of both fiber and Transite duct, β is a not too definite quantity ranging from 900 to 1,100 degrees centigrade square centimeters per watt for a typical cable installation in fiber duct and from 750 to 850 with a Transite duct.

I heartily indorse the authors' concluding paragraph to the effect that further tests along these lines would be valuable.

References

1. See reference 4 of the paper.

2. The Thermal Resistance Between Cables and a Surrounding Pipe or Duct Wall, F. H. Buller, J. H. Neher. *AIEE Transactions*, volume 69, part I, 1950, pages 342–49.

3. See reference 5 of the paper.

Paul Greebler and Guy F. Barnett: Probably the most common criticism of the test procedure arises from the fact that the sheath diameter employed was somewhat large. This cable size was taken from an actual installation in the field, but it is larger than that commonly employed in a 4-inch duct. The earlier experimental work by P. J. Barenscher,[1] as well as the more recent theoretical study by F. H. Buller and J. H. Neher,[2] indicates that the inverse relationship between the sheath diameter and the thermal resistance from the sheath to the duct is valid, as a good approximation, over an appreciable range of cable diameters. Additional tests on Transite and fiber ducts, using several smaller cable diameters, would add greatly to the value of these experimental and theoretical investigations.

Mr. Mason has presented data in his plots of R_{ths} versus variables in the convection and radiation terms that are very helpful toward understanding the mechanisms of heat transfer from the cable to the duct. We have not assumed in our paper, however, that the duct wall temperature is constant nor that the convection term is negligibly small. It was stated in the paper that variations in the convection term were small compared with those in the radiation term. This statement is verified by calculations carried out by Mr. Greebler in his discussion of a paper by Buller and Neher.[3] Values for R_{ths} were plotted against sheath temperature since this temperature is the principal variable in the thermal resistance from the cable to the duct for a given cable in duct geometry. Mr. Mason is entirely correct in pointing out that the arithmetic average of a periodically varying power input does not necessarily give the same results as an equivalent constant-power input, since the relationship between thermal resistance from cable to duct and power input is not linear. In our tests, however, the periodic power fluctuations were very small compared with the average input power, justifying the use of the latter as a highly accurate first-order approximation.

Mr. Burrell has discussed the polar graphs of Figures 10 and 11 of the paper showing the temperature distribution around loaded duct walls, and has compared these with similar data presented by Mr. F. V. Smith in a discussion of a paper by Mr. Neher.[4] The temperature distribution around the duct wall depends upon all of the thermal and geometrical variables that affect the cable-to-duct resistance, and it would be influenced by the mutual heating effects in a multiduct structure. It is expected that considerable departure from isothermal-duct surfaces occurs in any practical duct installation carrying an appreciable load and employing nonmetallic ducts with the cables resting on or near the bottom of their respective duct walls.

Mr. Buller has pointed out several factors in field installations, which we could not duplicate in our laboratory tests, that would modify the heat field and, therefore, influence the value of the cable thermal surface resistivity β. These factors tend, in general, to lower the value of β, and so the values given in our paper are slightly on the conservative side when applied to current rating calculations in buried duct structures.

Both Mr. Burrell and Mr. Buller have discussed the influence of moisture on the thermal resistivity of the duct materials in the field. It is true that the presence of moisture would produce a decrease in the thermal resistivity of the duct wall. However, for design purposes it would appear advisable to base calculations on the limiting case of dry ducts because it is not uncommon to encounter such conditions in practice, particularly for extended periods during the summer months. For continuously submerged ducts, special experimental determinations may be justified and may be expected to show a significant reduction in the magnitude of the thermal resistance component from sheath to duct wall.

The tests described in our paper cover the range of cable sheath temperatures from 50 to 80 degrees centigrade for duct in air and 60 to 80 degrees centigrade for duct encased in concrete. Dr. Wiseman has pointed out that temperatures for cable sheaths obtained in practice are normally below the range of temperatures covered in our tests. It is pertinent to point out that more and more emphasis is being placed on emergency loading at allowable copper temperatures that, for example, may be as high as 96 degrees centigrade at 15 kv (Association of Edison Illuminating Companies). Under such conditions a sheath temperature of the order of 80 degrees centigrade would not appear to be unduly high. The thermal volume resistivity values given in Table II of the paper are essentially independent of temperature over the entire operating range, but the surface resistivities increase as the cable temperature is lowered. The surface resistivity values given in Table II of the paper were obtained by extrapolating the curves of Figure 14 of the paper to a heat flow of ten watts per foot. The β values thus obtained represent a sheath temperature near 70 degrees centigrade for ducts in air and below 50 degrees centigrade for ducts in concrete. Extrapolation of these values to lower sheath temperatures, assuming a linear relationship over the operating range, is justifiable as a good approximation. The departure from a linear relationship when extrapolating to a lower temperature should yield resistivity values slightly on the conservative side, since it is expected from theoretical considerations that the slope of a β versus sheath temperature curve would approach zero as the temperature is lowered.

Mr. Neher has pointed out that Figure

Table I. Thermal Resistivity Factor β As a Function of Sheath Temperature and Cable Diameter

β in Degrees Centigrade Square Centimeter Per Watt

Temperature, Degrees Centigrade	Fiber in Air: Cable Diameter, Inches					Transite in Air: Cable Diameter, Inches					Fiber in Concrete: Cable Diameter, Inches					Transite in Concrete: Cable Diameter, Inches				
	1.5	2.0	2.5	3.0	3.5	1.5	2.0	2.5	3.0	3.5	1.5	2.0	2.5	3.0	3.5	1.5	2.0	2.5	3.0	3.5
40	1,160	1,230	1,290	1,320	1,350	970	1,030	1,070	1,110	1,130	1,030	1,090	1,140	1,180	1,200	840	880	920	950	970
45	1,140	1,210	1,270	1,300	1,330	960	1,010	1,060	1,090	1,110	1,020	1,070	1,120	1,160	1,180	820	860	910	930	950
50	1,120	1,180	1,240	1,270	1,300	950	1,000	1,050	1,080	1,100	990	1,040	1,090	1,130	1,150	810	850	890	920	940
55	1,100	1,160	1,220	1,250	1,280	930	980	1,030	1,060	1,080	960	1,020	1,070	1,100	1,120	790	840	870	900	920
60	1,080	1,140	1,190	1,230	1,250	910	960	1,010	1,040	1,060	940	990	1,040	1,070	1,090	780	820	860	880	900
65	1,060	1,120	1,170	1,200	1,230	900	950	990	1,020	1,040	920	970	1,020	1,050	1,070	770	810	840	870	890
70	1,030	1,090	1,140	1,180	1,200	870	920	980	990	1,010	900	950	990	1,020	1,040	750	790	820	850	870
75	1,010	1,060	1,110	1,150	1,170	850	900	940	970	990	880	930	970	1,000	1,020	730	770	800	820	840
80	990	1,040	1,090	1,130	1,150	830	870	910	940	960	850	900	940	970	990	710	750	780	800	820

14 of the paper does not give a complete picture of the variation in β since the effect of the cable diameter is not included. It is unfortunately true that β varies in a rather complicated manner with sheath temperature, cable diameter, and the many other factors in the over-all heat flow path from conductor to ambient that affect the heat field and, therefore, determine the temperature drop from the sheath to the duct. On the fortunate side, however, all of these variables produce only small changes in β over normal operating ranges; and so β can be treated as a constant for a given duct material or pipe for most practical thermal calculations.

The authors of this paper do not agree with Mr. Neher that as a best approximation β is dependent upon the fourth root of the heat flow, and this divergence has been elaborated upon by Mr. Greebler in his discussion of a paper by Mr. Buller and Mr. Neher.[3] The value of β is determined by the temperature distribution in the air space between the cable and the duct, and in relationship to the duct variables, this is most intimately associated with the sheath temperature. The cable diameter is probably the second most important of the variables that affect the value of β. In order to take into consideration variations in β with sheath temperature and cable diameter, values of β are tabulated in Table I of the discussion for sheath temperatures ranging from 40 to 80 degrees centigrade and for cable diameter from 1.5 to 3.5 inches. Four cases are considered separately, namely, fiber duct in air, Transite duct in air, fiber duct in concrete, and Transite duct in concrete. Thermal surface resistivity values shown in the table for a 3.5-inch diameter cable are the averages of the two series of tests, with the assumption that linear extrapolation to 40 degrees centigrade is justifiable. Approximate values of β for cable diameters smaller than 3.5 inches were obtained by assuming that convection contributes 30 per cent of the total heat transfer from the cable to the duct and that variations in the cable diameter produce an appreciable effect only upon the convection component of the thermal conductivity of the air space from the cable to the duct. These calculations are based in principle upon an analysis given by Mr. Buller and Mr. Neher[2] in which the convection component of thermal conductivity is shown to vary inversely with the square root of the cable diameter. The thermal conductivity is just the reciprocal of the resistivity factor β, and the resistivity value for a cable diameter D_s is computed from that experimentally determined for a 3.5-inch diameter $\beta_{3.5}$ employing formula

$$\beta = \frac{\beta_{3.5}}{0.70+0.56D_s^{-1/2}}$$

This formula is consistent with the assumption mentioned earlier in this paragraph, and the value of β thus obtained is associated with the same sheath temperature as the $\beta_{3.5}$ that is used in the formula. Table I of the discussion shows that a decrease in the sheath temperature of 20 degrees centigrade coupled with a 100 per cent increase in cable diameter produces an increase in β of the order of magnitude of 20 per cent for each of the four cases considered. These data substantiate the previous statement that for most practical calculations β may be considered as a constant for a given duct material in air or in concrete over the normal operating range of sheath temperatures. The values given in Table I of the discussion indicate, however, that averaging β values for duct in air and duct in concrete, as was done in Figure 14 and Table II of the paper, is not entirely justifiable.

The various discussions indicate that more experimental data are desired for a more complete investigation of the duct heating problem, and the authors heartily concur in this opinion. If our paper encourages additional experimental investigations and serves as a guide toward obtaining reliable data from such further work, it will have succeeded in its major objective.

References

1. See reference 3 of the paper.
2. The Thermal Resistance Between Cables and a Surrounding Pipe or Duct Wall, F. H. Buller, J. H. Neher. *AIEE Transactions*, volume 69, part I, 1950, pages 342–49.
3. Discussion by Paul Greebler of reference 2, page 348.
4. Discussion by Frank V. Smith of The Temperature Rise of Cables in a Duct Bank, J. H. Neher. *AIEE Transactions*, volume 68, part I, 1949, pages 547–48.

The Thermal Resistance Between Cables and a Surrounding Pipe or Duct Wall

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ONE step in the calculation of underground cable temperatures involves the determination of the temperature rise of the cable surface above the immediately surrounding inclosure such as a duct structure or a gas- or oil-filled pipe. Since the intervening medium is a fluid, the mode of heat transfer simultaneously involves convection, conduction, and radiation.

The semiempirical methods now in use for this determination in the case of cables in duct are not entirely satisfactory, and with the advent of gas- or oil-filled pipe-type cables there has arisen a definite need for a method of evaluation for these cable types as well.

Because of the complex nature of the problem and the number of independent variables which are present, it is impractical to cover completely all possible combinations which may be met within practice solely by tests. By developing a theoretical relationship between the variables, however, it is possible to develop procedures by which the test data available may be analyzed in such a way that relatively simple working expressions may be derived which may be applied with sufficient accuracy over the entire working range.

The theoretical relationship for the case of cables in duct was recently presented in a paper by one of the authors.[1] In the present paper this relationship has been extended to cover oil and gas pipe systems as well, and from the test data presented the requisite working expressions for thermal resistance or surface resistivity factors have been obtained.

Theoretical Considerations

The theoretical relationships given in Appendix II of reference 1 for the case of cables in duct have been expressed more completely to account for the physical characteristics of the media involved in Appendix I of this paper. The resulting equations for the the thermal conductivity between cable and duct or pipe with air or gas as the intervening medium are

$$\frac{Q}{\Delta T}(\text{gas}) = \frac{0.092\ D_s'^{3/4}\Delta T^{1/4}p^{1/2}}{1.39+D_s'/D_d} + \frac{0.0213}{\log_{10} D_d/D_s'} + 0.102 D_s'\epsilon(1+0.0167T_m)$$

watts per degree centigrade foot (1)

and with oil as the medium

$$\frac{Q}{\Delta T}(\text{oil}) = \frac{0.053 D_s'^{3/4}\Delta T^{1/4}T_m^{1/4}}{1.39+D_s'/D_d} + \frac{0.116}{\log_{10} D_d/D_s'}$$

watts per degree centigrade foot (2)

For a single cable $D_s' = D_s$, the diameter of the cable. For three cables in the pipe or duct it is customary to base D_s' on the circumscribing circle of the cables in triangular configuration, $D_s' = 2.15\ D_s$. For two cables the relationship $D_s' = 1.65\ D_s$ is satisfactory.

It will be noted that the primary variable in equation 1 is D_s'. As a result, subsequent analysis and development will be facilitated if this equation is written in the equivalent form

$$\frac{Q}{D_s'\Delta T} = \frac{0.092\Delta T^{1/4}p^{1/2}}{D_s'^{1/4}(1.39+D_s'/D_d)} + \frac{0.0213}{D_s'\log D_d/D_s'} + 0.102\epsilon(1+0.0167T_m)$$

watts per degree centigrade foot inch (1A)

From the method of derivation which assumes a coaxial arrangement of the cable within the duct or pipe, the numerical constants of the first two terms of equations 1, 1(A), and 2 must be considered as being approximate only. They will serve, however, to evaluate the relative magnitudes of the terms, and the corresponding values finally employed will be based on test data.

As a practical matter, a high degree of accuracy is not required since the thermal resistivity between cable and duct or pipe represents a relatively small part of the total thermal circuit, and we are justified in materially simplifying these equations. From the standpoint of analysis of the test data and the subsequent development of working expressions, it is desirable to utilize the simple linear relationship

$$y = ax + b \qquad (3)$$

where y and x are variables and a and b are constants. Equations 1 and 2 are of this form provided that the second (conduction) and third (radiation) terms may be considered as constants within the desired accuracy of the final result. Considering equation 1A the conduction term constitutes about 14 per cent of the total in the case of a typical cable in duct installation, and about 8 per cent for a typical gas-filled pipe-type installation at 200 pounds per square inch. The corresponding values for the radiation term are 63 and 43 per cent.

Normal variations in D_s'/D_d may produce considerable variation in the conduction term, but the effect on the overall picture is small, because conduction is such a small part of the total heat flow. Variations of T_m can affect the radiation term by as much as 20 per cent over a sufficiently wide operating range; however, when calculating a cable rating, with a fixed copper temperature of the order of 70 degrees to 80 degrees centigrade, the range of this variable is very small, and an accuracy of the order of 3 per cent to 5 per cent may be expected.

In the case of equation 2, the conduc-

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Table I. Test Data on Gas-Filled Pipe-Type Cable Systems

Test Number	Source	D_s'	D_d	P	Q	ΔT	T_m	$\frac{Q}{D_s'\Delta T}$	$\frac{\Delta T^{1/4}P}{D_s'^{1/4}}$
1	Detroit Edison Company	3.42	6.07	1	23.4	20	52	0.34	1.56
				7.8	27.3	15.6	51	0.51	4.08
				14.6	28.6	13.1	51	0.64	5.34
					28.9	17.1	50	0.49	5.71
					28.9	14.4	51	0.59	5.48
					27.5	14.0	51	0.58	5.43
2	General Electric Company	3.92	6.07	1.7	7.3	6.2	39	0.30	1.64
					11.4	9.7	45	0.30	1.63
					15.2	12.4	50	0.31	1.7[illegible]
				7.8	6.8	4.7	39	0.37	2.92
					11.5	7.0	43	0.42	3.22
					14.9	8.9	45	0.43	3.42
				14.6	6.8	3.7	35	0.46	3.77
					11.2	5.8	40	0.49	4.23
					15.9	8.0	45	0.51	4.55
3	General Cable Corporation	4.90	6.07	14.6	25.9	9.2	56	0.57	4.47
4	General Electric Company	4.90	6.07	14.6	23.1	11.8	44	0.40	4.77

Table II. Test Data on Cables in Fiber and Transite Ducts Encased in Concrete

Test Number	Source	Duct	D_s'	D_d	Q	ΔT	$\frac{Q}{D_s'\Delta T}$	$\frac{\Delta T^{1/4}P^{1/2}}{D_s'^{1/4}}$
5	Barenscher	Fiber	0.69	3.5	1.0	6.4	0.226	1.74
					1.7	11.8	0.203	2.03
					2.5	15.1	0.235	2.17
					4.4	24.6	0.257	2.44
					6.6	34.2	0.281	2.65
					8.1	39.7	0.296	2.76
					12.3	56.1	0.318	3.00
			1.13	3.5	1.0	4.5	0.204	1.41
					1.7	7.1	0.207	1.59
					4.5	16.3	0.246	1.95
					8.0	30.4	0.233	2.28
					11.0	32.6	0.300	2.32
					14.6	48.5	0.268	2.82
					16.8	52.4	0.285	2.61
					18.4	58.7	0.278	2.59
			3.13	3.5	0.9	1.6	0.194	0.8[illegible]
					1.7	3.2	0.173	1.00
					2.4	3.8	0.203	1.05
					4.5	7.7	0.188	1.75
					8.1	12.1	0.213	1.40
					14.8	21.9	0.217	1.63
6	Johns-Manville	Fiber	3.38	3.88	12.5	16.8	0.220	1.50
					14.9	19.2	0.230	1.56
					17.5	21.7	0.238	1.59
7	Johns-Manville	Transite	3.38	3.88	16.7	16.9	0.292	1.50
					19.6	19.4	0.299	1.55
					23.3	22.0	0.314	1.60
					26.4	24.5	0.318	1.64

tion term constitutes about 24 per cent of the total for a typical oil pipe installation. Variation is more important than is the case with the gas-pipe cable, but is still within tolerable limits.

One peculiar phenomenon has been observed. The ratio of D_d/D_s', which appears in the conduction term also, appears in the first (convection) term of equations 1 and 2 but in such a way that a change in this ratio produces an opposite, though lesser, effect on the total value of these equations. A minimum error should, therefore, prevail when the conduction term is treated as a constant if the denominator of the convection term also is treated as a constant. This procedure will simplify the convection term but it will have the effect of approximately halving its numerical constant as compared with equations 1 and 2 since the numerical value of the denominator omitted is in the order of two. Actually the test data was analyzed both with and without this simplification, and no apparent change in consistency in the results was observed.

Analysis of Test Data

It follows from the preceding discussion that the test data for cables in duct and for gas-filled pipe-type installations may be analyzed by plotting the observed values of

$$y=\frac{Q}{D_s'\Delta T}\quad \text{against}\quad x=\frac{\Delta T^{1/4}P^{1/2}}{D_s'^{1/4}} \qquad (4)$$

The data given in Table I were compiled from tests on gas-filled pipe-type cable systems by The Detroit Edison Company,[2] the General Electric Company, and the General Cable Corporation. These data are plotted in Figure 1 and the values of a and b in equation 3 are established as $a=0.070$; $b=0.20$.

Table II presents similar data for cables in single dry fiber and Transite ducts in concrete taken from the Barenscher[4] and Johns Manville tests discussed in reference 1. These data also are plotted in Figure 1 where it will be seen that the Transite duct points fall on the gas in pipe curve, but the fiber duct points result in a different curve having the same value of $a=0.07$ but $b=0.10$. This difference may be explained by the fact that the duct wall departs from an isothermal as a result of the relatively high thermal resistance of the materials used, that of the dry fiber being considerably higher than that of the transite.[1]

The test data for oil-filled pipe-type cable systems from tests by The Detroit Edison Company,[3] the General Electric Company, and the Okonite Company are presented in Table III and plotted in Figure 2. In this case, the analysis has been made by plotting the observed values of

$$y=\frac{Q}{\Delta T}\quad \text{against}\quad x=D_s'^{3/4}\Delta T^{1/4}T_m^{1/4} \qquad (5)$$

and results in the values of $a=0.025$ $b=0.60$ in equation 3.

It will be seen from the analysis of the test data that the agreement between theoretical and observed numerical constants of the simplified convection term is extremely good in the case of oil as the medium, but in the case of gas, the observed value of 0.07 is somewhat higher than the expected value of about 0.046. This is rather surprising since tests number 2 (with gas) and number 9 (with oil) which are consistently close to the established curves in Figures 1 and 2 were made with the same physical setup which remained unchanged throughout the tests except for the change in the media employed. Therefore, we should expect the ratio of values obtained to be the same as the ratio of the numerical constants of the convection terms in equations 1 and 2.

This discrepancy seems to be due to the fact that in the case of several cables within the pipe, a condition of the majority of test data, there is an additional circulation of the gas between the cables themselves which is not properly accounted for by the use of an equivalent diameter for the three cables, but which is apparently not effective when a more viscous medium such as oil is employed.

As indicated before, however, a high degree of accuracy is not required, and it is

Table III. Test Data on Oil-Filled Pipe-Type Cable Systems

Test Number	Source	D_s'	D_d	Q	ΔT	T_m	$\frac{Q}{\Delta T}$	$D_s^{3/4}\Delta T^{1/4}T_m^{3/4}$
8	Detroit	4.83	6.07	26.2	8.9	49	2.94	104
9	General Electric Company	3.92	6.07	6.6	3.0	37	2.19	55
				11.4	4.5	44	2.55	69
				16.5	5.8	48	2.88	79
10	Okonite Company	4.50	6.13	4.1	2.5	25	1.65	43
				9.4	4.4	31	2.14	58
				9.4	5.4	26	1.75	53.5
				1.1	7.5	38	2.81	79
				21.5	8.8	41	2.45	88.5
				35.2	11.4	50	3.09	106
				34.9	11.7	48	2.98	133

felt that a working expression based on the foregoing analysis will be sufficiently accurate.

Working Expressions

In formulating the thermal resistance between cable and duct, it is customary to express this resistance in terms of an equivalent surface resistivity factor, assuming that the entire resistance was concentrated at the cable surface, according to the expression

$$H_{sd}=0.00411\,\frac{\beta}{D_s'}\ \text{thermal ohm feet} \qquad (6)$$

in which β is expressed in degrees centigrade square centimeters per watt. Since $H_{sd}=\Delta T/Q$ it follows from equation 6 that

$$\beta=243\,\frac{D_s'\Delta T}{Q}\ \text{degree centigrade centimeter per watt} \qquad (7)$$

and

$$\Delta T^{1/4}=0.253\frac{\beta^{1/4}Q^{1/4}}{D_s'^{1/4}}\ (\text{degrees centigrade})^{1/4} \qquad (8)$$

It is thus possible to develop working expressions in terms of β in the case of cables in duct- or gas-filled pipe by substituting equations 7 and 8 in equations 3 and 4 with the appropriate values of a and b. In the case of oil-filled pipe a simpler expression is obtained in terms of H_{sd}.

For cables in single dry fiber ducts

$$\beta=\frac{13{,}700}{\beta^{1/4}\left(\dfrac{Q^{1/2}P}{D_s'}\right)^{1/2}+5.7}\ \text{degrees centigrade square centimeters per watt} \qquad (9)$$

For cables in other types of single dry ducts and in gas-filled pipe

$$\beta=\frac{13{,}700}{\beta^{1/4}\left(\dfrac{Q^{1/2}P}{D_s'}\right)^{1/2}+11.3}\ \text{degrees centigrade square centimeters per watt} \qquad (10)$$

For cables in oil-filled pipe

$$H_{sd}=\frac{40}{H_{sd}^{1/4}(QD_s'^{3}T_m^{3})^{1/4}+24}\ \text{thermal ohm feet} \qquad (11)$$

The value of β from equations 9 and 10 is plotted in Figure 3 as a function of $(Q^{1/2}P/D_s')^{1/2}$ and the value of H_{sd} from equation 11 appears in Figure 4 as a function on $(QD_s'^{3}T_m^{3})^{1/4}$. Also indicated on these figures are the values of these parameters for typical conditions.

In the case of cable in fiber duct, the thermal resistance of the duct wall is appreciable and should be accounted for. This is most readily accomplished by modifying equation 6 to include this resistance. Thus

$$H_{sd}'(\text{fiber})=0.00411\frac{\beta}{D_s'}+0.33\ \text{thermal ohm feet} \qquad (12)$$

in which the second term represents the difference in thermal resistance between a 4-inch fiber duct and the corresponding section of concrete which it replaces.

Discussion of Values for Cables in Duct

It will be seen that the method of determining the thermal resistance between cable and duct presented herein differs somewhat from the method given in reference 1, although the results are substantially the same for terra cotta and fibre ducts. For Transite ducts, the values of thermal resistance derived in a more fundamental manner in the present paper, are slightly lower than those appearing in the reference, being equal to those assumed for terra cotta.

It will be recalled that the reasoning used in developing algebraic expressions for these values assumes an isothermal duct wall. The test data presented in

Figure 1. Analysis of test data for cables in duct- and gas-filled pipes

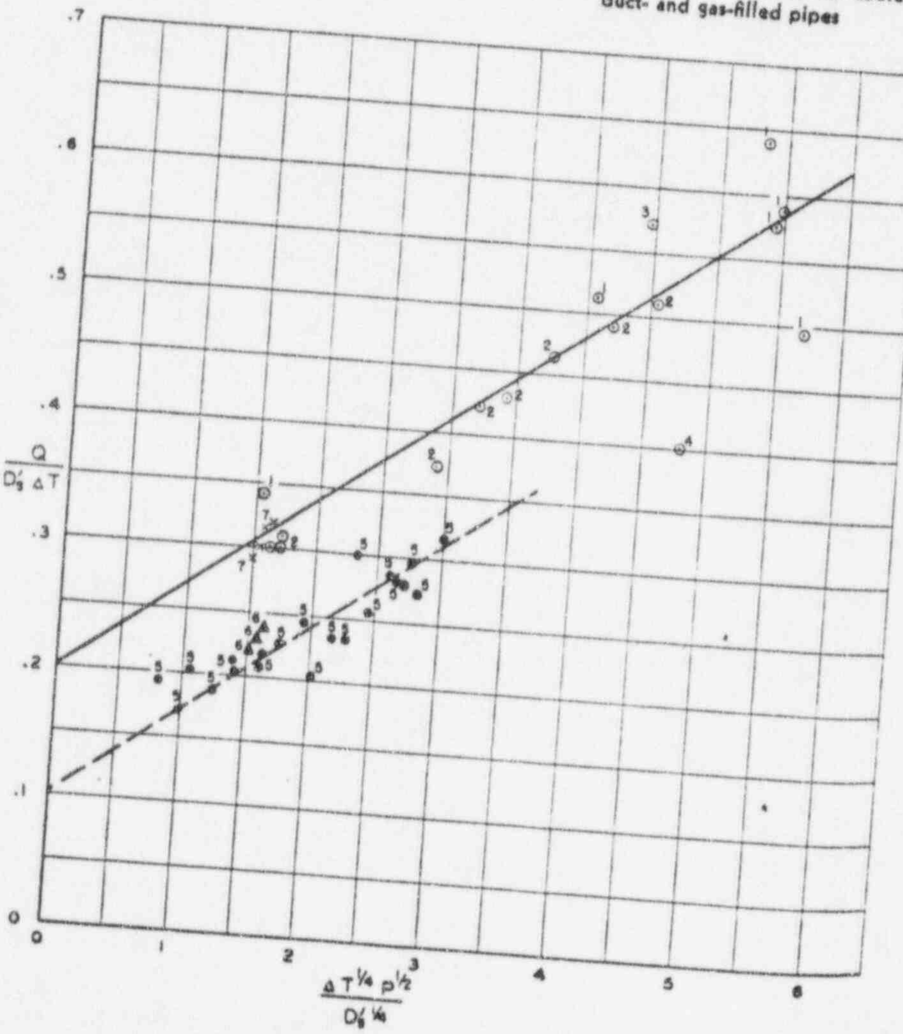


Table II and plotted in Figure 1, however, indicate a good correlation even where there is substantial deviation from the assumed isothermal as indicated by the basic data on which the table is based. Within the range covered by the data, increasing the departure from the isothermal changes the resulting constants somewhat but does not invalidate the method of analysis.

It follows therefore that a considerable variation in β for cables in single-fibre ducts may be expected depending upon the relative thermal resistivities of the duct wall and the surrounding medium, and other data which has come to the authors' attention confirms this. Thus the curve of Figure 3 for fibre duct should be considered as an upper limit.

Similarly, the application of the values given for single ducts to the case of cables in a multiduct structure, depends upon the effect which the total heat field has in further changing the temperature gradients around the individual duct walls. The data given by Smith in his discussion of reference 1 indicates a value of β for multiple-fiber ducts in concrete corresponding closely to the curve for cable in pipe.

As indicated in reference 1, additional test data taken on multiple-duct assemblies are desirable to definitely establish the limits under these conditions.

For reasons also indicated in reference 1 these values are not directly comparable to the values adopted by the Insulated Power Cable Engineers Association[5] and are not directly adaptable to their calculation procedure.

Conclusions

1. The theoretical relationships between the various quantities involved in the effective thermal resistance between cables and a surrounding single duct or pipe have been developed in a manner which properly accounts for the simultaneous modes of heat transfer by convection, conduction, and radiation.

2. By means of these relationships certain test data on cables in duct and in gas- and oil-filled pipes have been analyzed and working curves are presented for determining the thermal resistance for any particular case which may be encountered in practice.

3. Under typical conditions representative values of the equivalent surface resistivity factor β for use in equation 6 are 800 degree centigrade square centimeters per watt for cables in pipe, single dry terra cotta or transite ducts at atmospheric pressure, 450 for cables in gas-filled pipe-type installations at 200 pounds per square inch, and 350 for cables in oil-filled pipe-type installations. Representative values of β for cables in single dry fiber ducts will vary from 850 to 1,100.

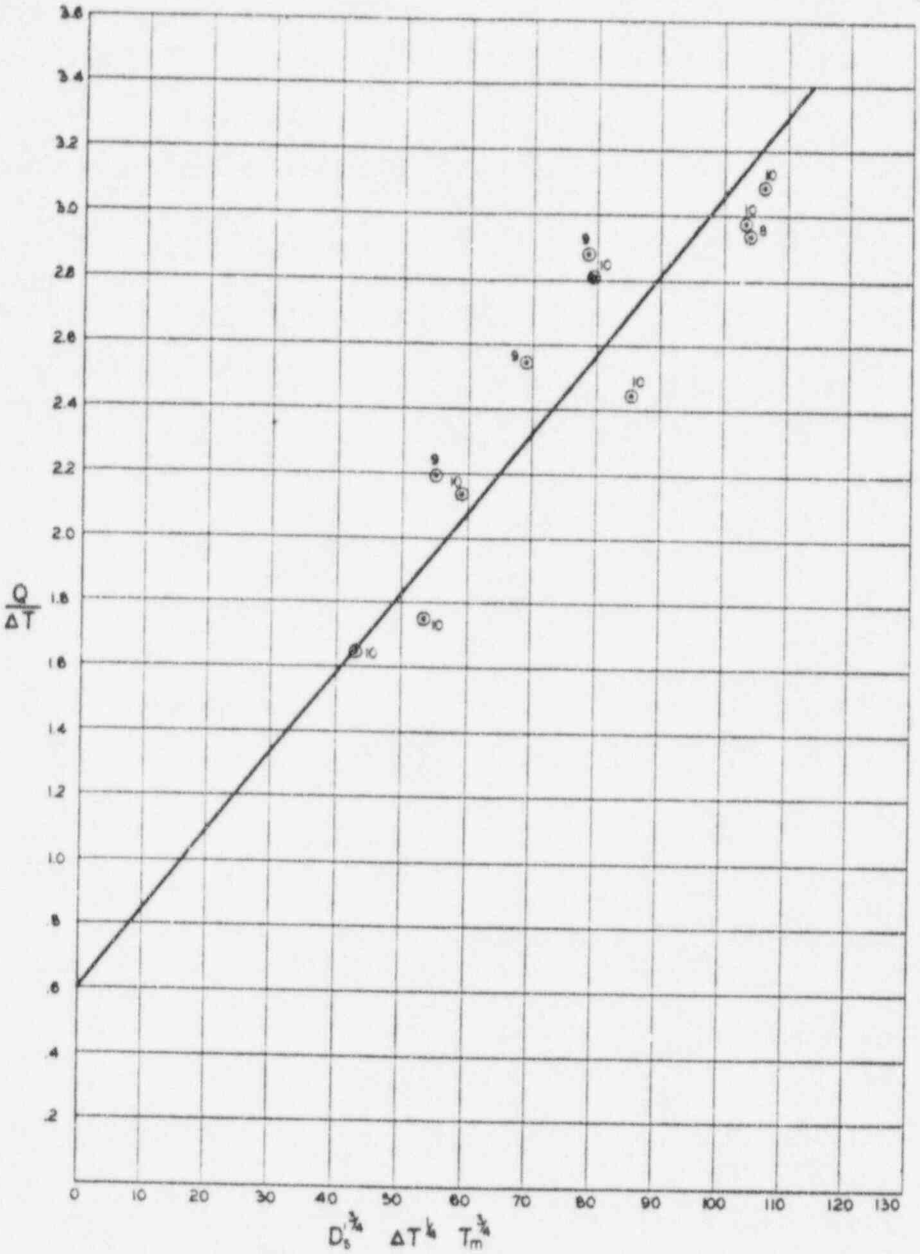


Figure 2. Analysis of test data for cables in oil-filled pipe

Appendix I. Theoretical Development of Thermal Conductivity between Concentric Isothermal Cylinders with Gas or Oil as the Intervening Medium

The mechanism of heat transfer between a cylindrical radiator and an enveloping isothermal enclosure through an intervening fluid medium is such that a portion of the total heat flow Q is carried by convection Q_{cv}, a portion by conduction Q_{cd}, and the remainder by radiation Q_r. In formulating the components of the thermal circuit, therefore, it is more convenient to work in terms of thermal conductances rather than thermal resistances since the former quantities are directly additive. Thus, if ΔT is the temperature drop in degrees centigrade across the circuit

$$\frac{Q}{\Delta T}=\frac{Q_{cv}}{\Delta T}+\frac{Q_{cd}}{\Delta T}+\frac{Q_r}{\Delta T} \text{ watts per degree centigrade foot} \quad (13)$$

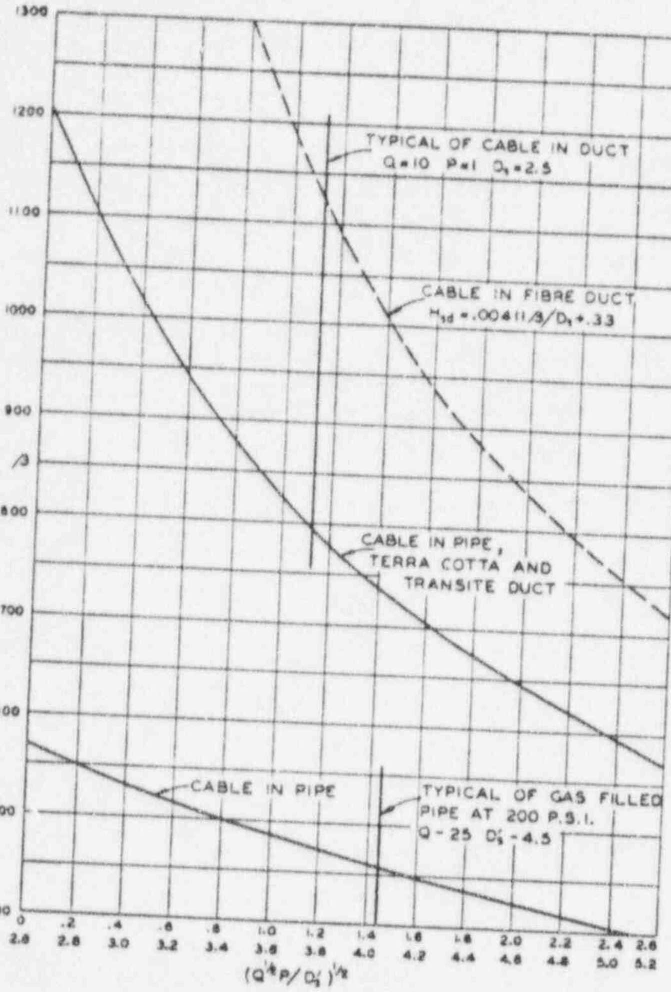
The phenomenon of convection involves the conception of the temperature drop being concentrated in two films, one at the surface of the cylindrical radiator of diameter substantially equal to the diameter of the radiator D_s in inches, and one at the surface of the enclosing isothermal surface which will be considered also being cylindrical of diameter D_d. The following formula based on McAdams[3] (equation 42, page 251, 1st edition only) is applicable to either film.

$$Q_{cv}=122D_f^{3/4}\Delta T_f^{5/4}K \text{ watts per foot} \quad (14)$$

in which D_f is in inches, and

$$K=\left(\frac{\delta^2 C_p g\, c}{\mu\ \ \rho^3}\right)^{1/4} \text{ watts per centimeter}^{1/4} \text{ degrees centigrade}^{5/4} \quad (15)$$

The significance of the components of equation 15 and representative values for gas (air or nitrogen) and Suniso number 6 oil are given in Table IV.



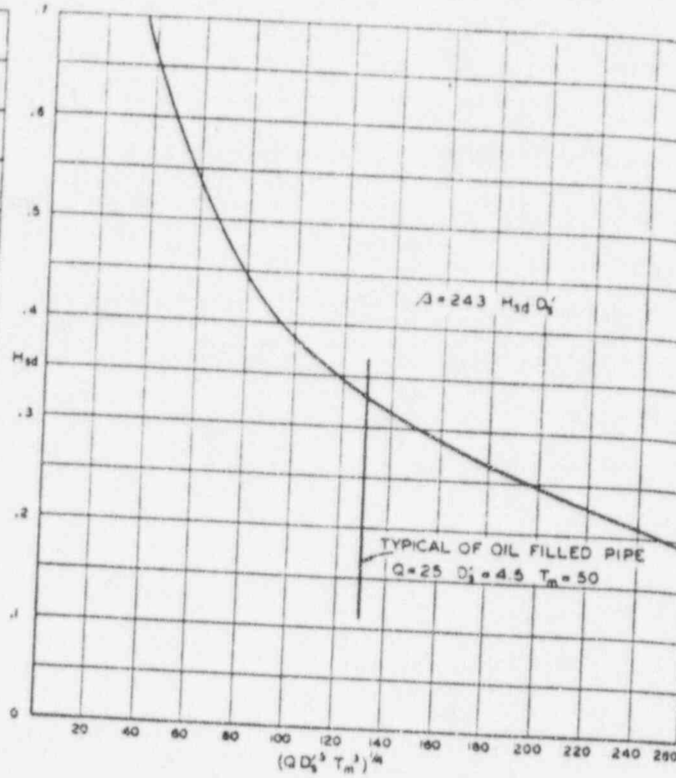


Figure 3 (left). Values of β for cables in dry single ducts and gas-filled pipe

Figure 4 (above). Values of H_{sd} for cables in oil-filled pipe

In the case of air or inert gas, these physical properties are substantially independent of temperature over the working range but the density is a direct function of the pressure. Thus, if P represents the pressure in atmospheres, from equation 15

$$K_g = 0.000755P^{1/4} \text{ watts per centimeter}^{1/4} \text{ degrees centigrade}^{5/4} \quad (16)$$

When oil is employed as the medium the physical constants are substantially independent of pressure and temperatures with the exception of the viscosity which for the type of oil commonly employed (Suniso number 6) may be taken as varying inversely as the cube of the temperature according to the relationship

$$\mu_0 = \frac{94{,}000}{T_m^3} \text{ grams per centimeter second} \quad (17)$$

The value of K for oil thus becomes

$$K_o = 0.000434T_m^{3/4} \text{ watts per centimeter}^{1/4} \text{ degrees centigrade}^{5/4} \quad (18)$$

The solution of equation 14 for the two films in series and with equation 16 or 18 substituted therein is given with sufficient accuracy by the expressions

$$\frac{Q_{cs}}{\Delta T}(\text{gas}) = 0.092\frac{D_s^{3/4}\Delta T^{1/4}P^{1/2}}{1.39+D_s/D_d} \text{ watts per degree centigrade foot} \quad (19)$$

$$\frac{Q_{cs}}{\Delta T}(\text{oil}) = 0.053\frac{D_s^{3/4}\Delta T^{1/4}T_m^{3/4}}{1.39+D_s/D_d} \text{ watts per degree centigrade foot} \quad (20)$$

From a theoretical standpoint the expression for the conduction component should take into account any eccentricity between the cylindrical radiator and the enveloping isothermal enclosure. In the practical case of cables in duct or pipe the cables will not rest uniformly on the bottom of the duct, and also in the case of a nonmetallic duct the duct wall is not strictly maintained as an isothermal. Since these effects cannot be evaluated, the familiar expression for the resistance between two concentric cylinders in terms of the dimensions of the cylinders and the thermal resistivity of the medium will be used. Thus

$$\frac{Q_{cd}}{\Delta T}(\text{gas}) = \frac{0.0213}{\log_{10} D_d/D_s} \text{ watts per degree centigrade foot} \quad (21)$$

$$\frac{Q_{cd}}{\Delta T}(\text{oil}) = \frac{0.116}{\log_{10} D_d/D_s} \text{ watts per degree centigrade foot} \quad (22)$$

The radiation component with gas as the medium is given with sufficient accuracy by the following expression based on McAdams'[8] equation 5, page 61, first edition

$$\frac{Q_r}{\Delta T}(\text{gas}) = 0.102D_s\epsilon(1+0.0167T_m) \text{ watts per degree centigrade foot} \quad (23)$$

in which ϵ is the emissivity coefficient of the surface of the cable and T_m is the average temperature of the medium. The radiation term is ineffective when oil is the medium.

The over-all thermal conductivity is obtained by substituting equations 19, 21, and 23 or equations 20 and 22 in equation 13.

Table IV

Symbol	Quantity	Units	Gas at 50 C	Oil at 50 C
ρ	Thermal resistivity	C cm/watt	3,900	715
μ	Average absolute viscosity	grams/cm sec	0.000195	0.75
δ	Density	grams/cm³	0.00110 P	0.906
C_p	Specific heat at constant pressure	watt sec/C gram	0.995	2.10
g	Acceleration due to gravity	cm/sec²	980	980
β	Thermal coefficient of expansion	1/C	0.00310	0.00068

Appendix II. List of Symbols

Q = total heat flow from equivalent sheath to duct wall or pipe in watts per foot
ΔT = temperature drop in degrees centigrade
P = pressure in atmospheres
D_s = diameter of the sheath in inches
D_s' = equivalent diameter of a group of cables in inches
D_d = inside diameter of the duct wall or pipe in inches

T_m = average temperature of the medium in degrees centigrade
ϵ = coefficient of emissivity of the cable surface
x and y = rectangular coordinates
a and b = experimentally determined constants
H_{sd} = thermal resistance between equivalent sheath and duct wall or pipe in thermal ohm feet
H_{sd}' = equivalent thermal resistance between equivalent sheath and fibre duct wall[1] including the increased thermal resistivity of the duct wall over that of the surrounding medium in thermal ohm feet
β = equivalent surface resistivity factor in degrees centigrade square centimeters per watt
ρ = thermal resistivity in degrees centigrade centimeters per watt
μ = average absolute viscosity in grams per centimeters second
δ = density in grams per cubic centimeter
C_p = specific heat at constant pressure in watt seconds per degree centigrade gram
g = acceleration due to gravity in centimeters per second squared
ϵ = thermal coefficient of expansion in centimeters per centimeter degree centigrade
K = a factor dependent upon the physical constants of the medium in watts per centimeter $^{7/4}$ degrees centigrade$^{5/4}$.

References

1. The Temperature Rise of Cables in a Duct Bank. J. H. Neher. *AIEE Transactions*, volume 68, part I, 1949, pages 540-49.

2. Heat Transmission (book). W. H. McAdams. McGraw-Hill Book Company, New York, N. Y., first edition, 1933.

3. Thermal Characteristics of a 120-Kv High-Pressure Gas-Filled Cable Installation. W. D. Sanderson, J. Sticher, M. H. McGrath. *AIEE Transactions*, volume 67, Part I, 1948, pages 487-98.

4. A Study of the Temperature Distribution in Electric Cables in Underground Ducts. P. J. Barenscher. *Thesis*, Department of Electrical Engineering, University of Wisconsin (Madison, Wis.), 1925.

5. Current Carrying Capacity of Impregnated Paper, Rubber and Varnished Cambric Insulated Cables. *Publication Number P-29-226*, Insulated Power Cable Engineers Association (New York, N. Y.), first edition, 1943.

Discussion

R. H. Norris and **Mrs. B. O. Buckland** (General Electric Company, Schenectady, N. Y.): Efficient work in the heat-transfer field on a variety of applications requires awareness of the definitions and units, in order to avoid confusion and misunderstanding. In this paper and other papers written by cable engineers, confusion arises as to the exact meaning of the expression "thermal resistivity." Resistivity as normally defined (by the American Standards Association (ASA) for example) is a property of a substance and is not affected by its geometry; for example, the resistivity of copper has a constant value at any specified temperature, while its resistance depends on its size and shape. Then the use of the word "resistivity" for surface phenomena is a misuse of the term.

To show how the distinction between resistance and resistivity enters into the picture, the thermal circuit for a single-conductor cable in air is given in Figure 1 of the discussion.

In this figure, t_{cu}, t_{sh}, and t_a are temperatures of copper, sheath, and ambient, respectively, δ is insulation thickness, ρ is thermal resistivity of the insulating material, A_L is the log mean area of the insulation for heat flow, A_{sh} is sheath area, and β_c and β_r are the cable engineers' terms for "surface resistivity" for free convection and radiation. Each fraction in the Figure is the thermal resistance; and when resistances and temperatures are known, the heat dissipation of the cable is known. But in order for the resistances to be dimensionally consistent, the dimensions of ρ must be different from the dimensions of β, and therefore ρ and β should not be called by the same name.

Since the definition of ρ as thermal resistivity conforms to ASA standards, it might be better to denote β as thermal resistance of a unit surface. Its reciprocal h, is defined as surface heat transfer coefficient, or alternatively as surface film conductance. The concept of conductance is particularly applicable here, as the total film conductance is the sum of h_r and h_c, and therefore numerically easier to handle.

The units of length used in the paper seem to be a mixture of metric and engineering units. A combination of square centimeters with feet has no logical basis. If any cable dimensions were expressed in centimeters, the mixture would be logical although not standard; but since dimensions are not so expressed, it seems time to abandon this practice and use the engineering system of units throughout.

It is therefore proposed that the AIEE Committee on Insulated Conductors take steps to persuade its adherents to become familiar with ASA standards and to use them where they apply.

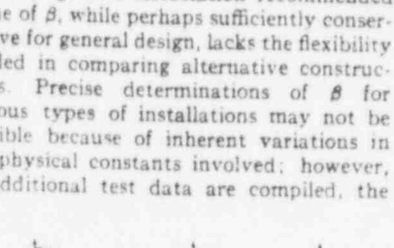


Figure 1. Thermal circuit for single conductor in air

R. W. Burrell (Consolidated Edison Company of New York, Inc., New York, N. Y.): The authors have presented a desirable elaboration of Appendix II of a previous paper by Mr. Neher.[1] Although the approach to the problem is not changed, the material presented in the Appendix referred to is of sufficient importance to justify a more detailed presentation.

It is apparent to those engaged in the field of cable heating that the Insulated Power Cable Engineers Association recommended value of β, while perhaps sufficiently conservative for general design, lacks the flexibility needed in comparing alternative constructions. Precise determinations of β for various types of installations may not be possible because of inherent variations in the physical constants involved; however, as additional test data are compiled, the probable range of β, for a particular case, will be better understood, thereby making possible more realistic comparisons. The authors clarify our conception of the effect of the various parameters involved in the temperature drop between cable surface and duct or pipe wall. For a given system of cables in duct or pipe, the thermal resistance will decrease sensibly with increasing watts loss.

W. B. Kirke[2] introduced this modification which is taken into account in determining cable ratings for the Consolidated Edison system.

As one follows the assumptions made in this paper, there appear various points to which exception might be taken on the ground that they are not substantiated, for example: the assumption of the same constant in the expression for the convection film at the cable surface and at the inner duct wall, the treatment of conduction on the basis of a concentric system, and the arbitrary assumption of an emissivity coefficient of the cable surface of 1.0. Yet, the important point is that putting all of these various assumptions together in the particular form given in the paper, the overall end result does produce expressions which are reasonably satisfactory.

It is unfortunate that, while the basic equations and the selection of parameters have a reasonably sound theoretical basis, the final working expressions given are essentially empirical and do not allow an accurate determination of the separate effect of the three modes of heat transfer. On the average, the calculated values of $Q/\Delta T$ for the oil-filled pipes, gas-filled pipes, and cable in duct are about 5 per cent, 15 per cent, and 55 per cent higher, respectively, than the measured values given in Tables I, II, and III of the paper. Specialists in the field of cable heating would be interested in knowing which component or components are responsible for these discrepancies so that extrapolation into new fields could be made with confidence.

It is stated in the paper that the agreement between theoretical and empirical numerical constants of the simplified convection term is close for the case of an oil medium, but is off appreciably for the case of a gas medium. It also can be said that the conduction-radiation constant agrees with theory for the case of a gas medium; however, for the case of an oil medium, the constant theoretically appears to range from 0.60, as given in the paper, to nearly twice that value, depending upon the values of D_s' and D_d involved.

From the over-all standpoint, it nevertheless appears that the expressions for β and H_{sd}, as given in equations 9, 10, and 11 of the paper are quite workable and agree with test data as well as could reasonably be expected. A high degree of accuracy in the calculation of allowable current ratings of cables is not yet to be expected but important work has been done in the past few years in clarifying our understanding of heat flow through duct structures and the earth, and this paper is an important contribution to such understanding.

References

1. See reference 1 of the paper.

2. The Calculation of Cable Temperatures in Subway Ducts. W. B. Kirke. *AIEE Journal*, volume 49, 1930, page 855.

R. J. Wiseman (The Okonite Company, Passaic, N. J.): I like the author's paper very much. It explains the three methods of heat flow from a cable to a surrounding medium, namely, conduction, convection, and radiation. Also, they give the various parameters which influence each factor namel , cable diameter, temperature, and temperature difference, and viscosity of the medium. The various formulas look quite formidable when we note terms raised to fractional powers. It is not easy to obtain the constants for each formula as they are dependent on conditions not easily calculable so it is necessary to get test data and work back to numerics which will give the desired results. It so happens that as all three modes of heat transfer are functioning at the same time, a change in dimensioning tends to work in opposite directions, reducing thereby the effect of diameter. Also the range in temperature is not great and as we take the one-fourth power of temperature difference and three-fourths power of temperature, the variation with temperature is not great.

About two years ago we decided to restudy the thermal constants we obtained when we originally set up the Oilostatic cable system. At that time we used the cylindrical log formula of ratio of internal pipe diameter to circumscribed circle over the assembled conductors, and also a constant which was a function of the temperature.

Our more recent tests showed that the thermal resistance was almost independent of temperature (a variation of about 10 per cent between 30 and 61 degrees centigrade) for an oil pressure zone and a very few per cent for a gas pressure zone at 200 pounds per square inch. We also noted that within the accuracy of testing we could safely assume the thermal resistance to vary as inversely as the diameter of the shielding tape over the insulation. As a result, we have set up two simple formulas for the determination of the thermal resistance of the pressure zone for three cables in a pipe, namely, for oil-pressure system $H=1.60/D$ thermal ohms per foot per conductor where, D is the diameter in inches over the shielding tape; and $H=2.58/D$ thermal ohms per foot per conductor for a gas pressure zone operating at 200 pounds per square inch. You will find these values of thermal resistance for the pressure zones amply accurate.

As the authors refer to the surface resistivity factor β, the values of β comparable to the above constants in $H=0.00411\ \beta/D$ are $\beta=390$ for an oil-pressure system as compared to 350 given by the authors and $\beta=627$ for a gas-pressure system at 200 pounds per square inch as compared to 450 given by the authors. We are quite confident in our values and have been using them for over a year.

Paul Greebler (Johns-Manville Corporation, Manville, N. J.): In this paper the authors have contributed immensely toward an understanding of the mechanisms of heat transfer from the cable to its surrounding pipe or duct wall. The theoretical analysis was necessarily based upon the simplifying assumption of a coaxial cable in duct arrangement. This does not, however, detract from the value of the analysis given in the paper, since this analysis gives the order of magnitude contributed by each of the three mechanisms of heat transfer.

The authors have assumed for cable in duct that the component of the thermal conductivity due to radiation can be treated as a constant in the range of normal operating temperatures. Only the component due to convection was considered as variable with changing cable diameter and heat flow. This assumption does not lead to a true picture of the variation in thermal resistivity with heat flow, or more fundamentally, with cable temperature. Mr. Barnett and I have stated in our paper[1] that the increase in thermal resistivity with incr sheath temperature is caused prima ria- tion in the radiation compc ne. at transfer, and that the effect of te e variations on convection are negl o r the normal operating range. is sta e- ment is verified by calculations based upon equation 1A of the Buller-Neher paper, which is repeated here:

$$\left(\frac{Q}{D_s'\,\Delta T}\right)=\underset{\text{(convection)}}{\frac{0.092\,\Delta T^{1/4}p^{1/2}}{D_s'^{1/4}(1.39+D_s'/D_d)}}+ \qquad \textbf{(1A)}$$

$$\underset{\text{(conduction)}}{\frac{0.0213}{D_s'\log D_d/D_s'}}+\underset{\text{(radiation)}}{0.102e(1+0.0167T_m)}$$

in watts per degree centigrade foot inch. The emissivity factor, e, is assumed to be unity at atmospheric pressure.

Table I of the discussion lists two representative sheath temperatures from our test data on fiber duct in concrete, and these temperatures might very well be representative of the operating range of a cable. The term $(Q/D_s'\,\Delta T)$ evaluated in equation 1A is inversely proportional to the surface resistivity factor, β.

The three terms in the equation give the thermal conductivity components due to convection, conduction, and radiation respectively. As we increase the sheath temperature over the range shown, the increase in the radiation term produced by substituting our experimental data in the Buller-Neher equation is five times greater than that of the convection term. This shows that the experimentally observed decrease in β over this range is due almost entirely to the increase in the radiation term. These calculations are based, of course, on the rather large cable size that we employed in our tests. A smaller cable size will increase the effect of the convection term only slightly, however, and not nearly enough to make its variation with temperature equal to that of the radiation term. Identical calculations with our data on Transite in concrete, Transite in air, and fiber in air, show similar relative variations in the radiation and convection terms.

The authors have neglected the variation in radiation component of conductivity with temperature, pointing out that these variations are quite small. This is justifiable from a practical standpoint. However, the variations in the convection component with temperature also should be neglected for practical considerations, since, as is shown in Table I of the discussion this factor is even smaller than the change in the radiation term. This would considerably simplify the Buller-Neher equations for the surface resistivity factor. In their equations 9 and 10, the surface resistivity factor, β, depends upon the fourth root of the heat flow. This does not have much significance since it is based upon the variation in the convection term, a second order effect compared with the radiation term. Similarly the dependence of β upon the square root of the sheath diameter is doubtful, since the change from a fourth root to a square root dependence in the convection term also was based on the very small change in convection conductivity with temperature.

The foregoing discussion was confined to cable in duct with air as the intervening fluid. Its applicability to cable in gas-filled pipe at high pressures, where convection becomes the principal mechanism of heat transfer, requires further study.

The authors have done an excellent job in helping to establish the theoretical groundwork necessary to both encourage and guide experimental workers in the duct heating problem.

Reference

1. Heat Transfer Study on Power Cable Ducts and Duct Assemblies, Paul Greebler, Guy F. Barnett. *AIEE Transactions*, volume 69, part I, 1950, pages 357–67.

A. H. Kidder (Philadelphia Electric Company, Philadelphia, Pa.): This paper by Buller and Neher, together with two previous papers by Mr. Neher,[1,2] completes presentation of the steady-state considerations involved in a project which was started about four years ago when Philadelphia Electric Company interested Mr. Neher in undertaking an investigation of fundamental relationships, as necessary to determine approximately what pipe-type cable circuit load ratings would be accurately comparable with the load ratings of conventional cable circuits in ducts.

The thermal resistance through the spaces between the cable sheaths and the pipe or duct wall inclosures is an important link in the thermal circuit. It had been hoped that a general relationship could be developed in such a form that all of the differences be-

Table I. Greebler-Barnett Data

Lead Sheath Temperature	Inside Duct Wall Surface Temperature*	Mean Temperature T_m	Temperature Drop ΔT	Buller-Neher Equation 1A: Convection Term	Buller-Neher Equation 1A: Radiation Term	Greebler-Barnett Data β in °C(cm)/w
66.2	46.8	56.5	19.4	0.062	0.198	990
77.2	53.6	65.4	23.6	0.065	0.213	930
			increase →	0.003	0.015	60 ← decrease

*Temperatures are in degrees centigrade. The inside duct wall surface temperature is an average value.

tween cables in air in ducts and cables in high-pressure gas or oil-filled pipes could be explained in terms of the physical constants which characterize the respective fluids and the pertinent geometrical relationships.

The method presented by Buller and Neher has approximately achieved this result, at least to the extent of permitting the correlation of data obtained by various investigators at various times in various constructions. It does not disturb me particularly to find that there is some apparent difference between the effects of Transite and fiber duct walls, respectively, under the conditions which prevailed at the time the tests were made. I think we should hesitate to attach much significance to these apparent differences because there was no attempt to control the moisture content in the fiber or the Transite, or even to make the tests under conditions comparable to those to be expected in the usual exposures to natural but variable moisture conditions to be encountered in underground structures. The significant point is that Buller and Neher have obtained a correlation which now permits estimating the thermal resistance from cable to pipe or duct wall with sufficient accuracy, so that little, if any, practical improvement in cable load ratings can be gained by introducing further refinements in their analysis of this part of the thermal circuit.

REFERENCES

1. The Temperature Rise of Buried Cables and Pipes. J. H. Neher. *AIEE Transactions*, volume 68, part I, 1949, pages 9–17.

2. See reference 1 of the paper.

F. H. Buller and **J. H. Neher:** Mr. Norris and Mrs. Buckland have taken us somewhat to task for our apparent inconsistency in expressing our physical units in one system and our geometric units in another. For better or worse it has long been the custom in cable rating procedure to express the physical units involved in the watt-second-centimeter-gram system, and to express lengths in feet and diameters in inches. In developing our equations it would have been more consistent to have expressed the latter quantities also in centimeters, and then to have converted the final expressions to the system of measurement used in practice. We chose to use the mixed system throughout, however, in order that the reader might be able to use any equation in the development, directly, without encountering the uncertainty which inevitably arises as to whether you multiply or divide by the transformation constants.

The use of the term "surface resistivity factor" is a slightly different matter, and as our mentors have pointed out, it has dimensions which are not those of true, or volumetric, "resistivity." Here again, this nomenclature has been hallowed by time and is thoroughly understood by cable engineers, for whom this paper was written. It should be stressed, however, that this "surface resistivity" is not a fundamental physical quantity, in the sense that volumetric resistivity is; but as pointed out, is the resistance of a unit surface of a film which, purely for purposes of convenience, is assumed arbitrarily to represent the entire thermal resistance of the composite heat transfer effects operating in the region between cable sheath and duct wall. It is unfortunate that we do not have a more distinctive name for it.

Mr. Burrell has presented a thoughtful discussion of the assumptions which we have made in developing the theory used for correlating the test data. In this respect, a book by Prof. McAdams[1] gives a constant for the convection film on the outside of a cylindrical surface in a free medium which is about 20 per cent lower than that for the inside of a pipe and which we have used for both films. We have not distinguished between the two constants because no information is given as to the values of these constants when the cylinder is placed within the pipe. While a formula for the conduction component in a non-concentric system is given by Whitehead and Hutchings[2] it is far too complicated to use in this analysis, and it reduces substantially to the concentric formula which we have employed except for extremely small separations between the cylinders at one point. Further there is considerable experimental evidence to support the assumption that the emissivity constant is substantially unity for the types of cable surfaces employed.

Discrepancies were expected, because of the assumptions which had to be made, and because the physical location of the cables within the pipe cannot be controlled. We have used assumptions and theory only to obtain a sensible understanding of the problem with which we have to deal and to determine what simplifications can justifiably be made in order to obtain practical working expressions. These working expressions were then developed directly from actual tests rather than from theory. We do not share Mr. Burrell's desire for working expressions of sufficient complexity to identify the separate effects of the three modes of heat transfer.

Dr. Wiseman's simplified formulas for calculating h_{ds} (on a per cable basis) for three cables in an oil-filled pipe or in a gas-filled pipe at 200 pounds per square inch are very interesting and similar formulas may be derived from Figures 3 and 4 of the paper assuming that Q, P, and T_m have fixed typical values. Unfortunately Dr. Wiseman's derivation of the equivalent β in his formulas gives values which are not comparable to β as defined in this paper. The corresponding relationship for β as defined in the paper is

$$\frac{h_{sd}}{3} = \frac{0.00411\beta}{2.15D_s}$$

and this yields $\beta = 290$ for the oil-pressure system and $\beta = 450$ for the gas-pressure system.

We cannot accept his formula for the oil system since its corresponding value on a total heat flow basis is $H_{sd} = 1.15/D_s'$ which is equivalent to $Q/\Delta T' = 3.9$ for $D_s' = 4.5$. None of the tests cited in Table III of the paper give any support for so high a value.

Dr. Wiseman also assumes that the overall thermal resistance varies inversely with the diameter whereas we believe that a more representative variation may be deduced from the slope of the curves of Figures 3 and 4 in the vicinity of the typical operating points. Thus for $Q = 25$ watts per foot and $T_m = 50$ degrees centigrade, we derive the simplified expressions

$$H_{sd}\text{ (oil)} = 0.70/D_s'^{1/2}\text{ thermal ohm feet} \quad (1)$$

$$H_{sd}\text{ (gas at 200 psi)} = 1.20/(D_s')^{0.7}\text{ thermal ohm feet} \quad (2)$$

The corresponding equations on a per cable basis and with three cables in the pipe are

$$h_{sd} = \frac{1.44}{\sqrt{D_s}}\text{ and } h_{sd} = \frac{2.07}{(D_s)^{0.7}}\text{ respectively}$$

Figures 3 and 4 are intended to give practical working values of β or H_{sd} over a wide range of operating conditions. Mr. Greebler is right in pointing out that the effect of temperature variations upon the radiation component is considerably greater than the effect of variations in the convection term which is the essential variant in Figure 3. The inclusion of the temperature of the medium in the working expressions would vastly complicate them, however, and as a practical matter this is unnecessary.

In all of the Greebler-Barnett data[3] it will be observed that β varies inversely as $Q^{1/4}$ within the accuracy of measurement. The dependence of β upon D_s cannot be evaluated from this data since only a single value of D_s was employed, but since the convection term theoretically varies directly as $Q^{1/4}/D^{1/4}$ we believe that the temperature variation in the radiation term which Greebler has mentioned will be accounted for with sufficient accuracy by expressing the Greebler-Barnett data for fiber and Transite ducts in the form

$$\beta\text{(fiber)} = 1120D_s^{1/4}/Q^{1/4}\text{ degrees centigrade square centimeters per watt} \quad (3)$$

$$\beta\text{(Transite)} = 990D_s^{1/4}/Q^{1/4}\text{ degrees centigrade square centimeters per watt} \quad (4)$$

This will have the effect of changing the slope of the curves when plotted in accordance with Figure 3.[4]

The corresponding values of H_{sd} assuming a working value of $Q = 10$ watts per foot are

$$H_{sd}\text{ (fiber)} = (2.59/D_s^{3/4}) + 0.33\text{ thermal ohm feet} \quad (5)$$

$$H_{sd}\text{ (Transite)} = 2.29/D_s^{3/4}\text{ thermal ohm feet} \quad (6)$$

While further theoretical and experimental work may well be undertaken in order to clear up some of the apparent discrepancies between theory and practice and to yield more factual data on the performance of cables in duct; we agree with Mr. Kidder that little of any practical improvement in cable load ratings will result. We do not wish to discourage further efforts in this direction, but we feel that it is sufficient to base cable ratings on Figures 3 and 4 of the paper or more simply on equations (1, 2, 5, and 6) just given.

REFERENCES

1. See reference 2 of the paper.

2. Current Rating of Cables for Transmission and Distribution. S. Whitehead, E. E. Hutchings. *Journal*, Institution of Electrical Engineers (London, England), volume 83, 1938, equation 10.3, page 531.

3. Heat Transfer Study on Power Cable Ducts and Duct Assemblies. Paul Greebler, Guy F. Barnett. *AIEE Transactions*, volume 69, part I, 1950, pages 357–67.

4. Discussion by J. H. Neher of reference 3 above, pages 365–66.

Calculation of the Electrical Problems of Underground Cables

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Calculation of the Electrical Problems of Underground Cables

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Fundamental Circuit Constants

SEVEN years ago an article on this same subject was printed in The Electric Journal. Since that time the art of manufacture of underground cables has moved steadily forward; the type H construction for high-voltage, multi-conductor cables has been generally adopted, the oil-filled cable has been introduced, theoretical investigations and experimental studies of many cable problems have been made.

To bring this subject up to date and to collect the mathematical data and equations relating to the electrical problems of underground cables scattered throughout the literature this group of articles is presented. The data is assembled, in a somewhat more limited way but in the general fashion of that in the masterly series of articles on overhead transmission line problems by Nesbit[1], so as to assist in calculating any of the ordinary electrical problems in underground transmission. Equations are included which considered best adapted to the practical calculations of the problems met in cable engineering. The derivations of the various formulas and treatment of certain problems which occur but infrequently are not discussed. Many of the refinements included here, but not present in the original article, need not be worked out fully as the simpler formulas still apply in many cases; thus some of the complications are more apparent than real. Specifically these articles discuss, first, the fundamental constants of the circuit, and, second, the question of current-carrying capacity as limited by temperature rise. The problems of feeder sizes as affected by economics will not be discussed.

THE calculation of the performance of an underground cable, such as regulation at full load and no load, is not essentially different from that for overhead transmission when certain constants are known. For underground transmission the simpler methods are usually applicable, as the lines are relatively shorter, though, for any given length of line, the distributed capacity and its effects are greater in an insulated cable than in a bare wire line. The calculation of some of the line constants is, however, different. Moreover, a fundamental difference between the two cases is that for cables, the allowable current is more often limited by the permissible temperature of the insulating material.

Insulation Thickness

Knowledge of the usual insulation thicknesses may be helpful in some cases, especially in preliminary calculations where definite dimensions of cables are not available. In multi-conductor belted cables, there are two thicknesses to be considered, as shown by the cable cross-sections in Fig. 2: each conductor is insulated separately with a thickness T; and after the conductors have been twisted or cabled together and the interstices rounded out with the fillers, the group is insulated as a whole with insulation called the "belt", of thickness t. The insulation thicknesses may be given in terms of the conductor insulation thickness and the belt (T and t), or in terms of total thickness of insulation between conductors and between conductors and ground ($2T$ and $T + t$). In the multi-conductor type H cable each conductor is separately insulated as before but each core has a thin layer of metallic tape or metallized paper immediately surrounding the individual conductor insulation. The three cores are cabled together and rounded out with fillers as in the belted type but the belt insulation is omitted. For multi-conductor type H or single-conductor cable, there is of course only one thickness of insulation to be considered.

There is no general agreement on exactly what thickness should be used for a given voltage, since there are so many factors involved. In most cases, continuity of service is the most important consideration, and relatively conservative thicknesses are chosen; in other cases, some sacrifice has been made in the factor of assurance by using thinner layers of insulation so that there will be room for more copper in a given diameter of cable. Another case in point is in Europe where, in general, slightly less insulation is used than in this country. This is due chiefly to the lower permissible operating temperature allowed, the practice of basing cable ratings on 100% load-factor, and the fact that cables abroad are usually buried in the ground without being subjected to the strains involved in pulling a cable into a duct. Furthermore, while in this country, most companies make temperature measurements in idle ducts and regulate cable loadings by actual conditions, in Europe usually no such provisions are made for measuring temperatures, so that with ordinary load-factors even the low permissible temperature values are not approached. It is not to be expected, therefore, that there are or will be any hard and fast rules for insulation thickness. The recommended thicknesses of insu-

TABLE I—RECOMMENDED MINIMUM AVERAGE THICKNESS OF INSULATION[1] FOR SINGLE-CONDUCTOR CABLE AND THREE-CONDUCTOR TYPE H CABLE

Rated Voltage[2]	Size of Conductor AWG Number or 1000 Cir. Mils	Paper				Varnished Cambric[7]				Rubber[8]			
		Grounded Neutral		Ungrounded Neutral		Grounded Neutral		Ungrounded Neutral		Grounded Neutral		Ungrounded Neutral	
		64ths	Mils	64ths	Mils	64ths	Mils	64ths	Mils	64ths	Mils	64ths	Mils
600	14-9	—	—	—	—	3	47	3	47	3	47	3	47
	8	4	63	4	63	3	47	3	47	4	63	4	63
	7-2	4	63	4	63	4	63	4	63	4	63	4	63
	1-4/0	4	63	4	63	5	78	5	78	5	78	5	78
	225-500	4	63	4	63	6	94	6	94	6	94	6	94
	525-1000	5	78	5	78	7	109	7	109	7	109	7	109
	Over 1000	6	94	6	94	8	125	8	125	8	125	8	125
1000	14-8	[illegible] 4	63	4	63	4	63	4	63	4	63	4	63
	7-2	4	63	4	63	4	63	4	63	5	78	5	78
	1-4/0	4	63	4	63	5	78	5	78	6	94	6	94
	225-500	4	63	4	63	6	94	6	94	7	109	7	109
	525-1000	5	78	5	78	7	109	7	109	8	125	8	125
	Over 1000	6	94	6	91	8	125	8	125	9	141	9	141
2000	14-8	(8) 5	78	5	78	(8) 5	78	5	78	5	78	5	78
	7-2	5	78	5	78	5	78	5	78	6	94	6	94
	1-4/0	5	78	5	78	6	94	6	94	7	109	7	109
	225-500	5	78	5	78	6	94	6	94	8	125	8	125
	525-1000	6	91	6	91	7	109	7	109	9	141	9	141
	Over 1000	7	109	7	109	8	125	8	125	9	141	9	141
3000	14-8	(8) 5	78	5	78	(8) 7	109	7	109	7	109	7	109
	7-4/0	5	78	5	78	7	109	7	109	8	125	8	125
	225-500	5	78	5	78	8	125	8	125	9	141	9	141
	525-1000	6	94	6	94	8	125	8	125	9	141	9	141
	Over 1000	7	109	7	109	9	141	9	141	10	156	10	156
4000	14-4/0	(8) 6	94	6	94	(8) 8	125	8	125	9	141	9	141
	225-1000	6	94	6	94	9	141	9	141	10	156	10	156
	Over 1000	7	109	7	109	10	156	10	156	11	172	11	172
5000	14-4/0	(8) 6	94	6	94	(8) 9	141	10	156	10	156	10	156
	225-1000	6	94	6	94	10	156	11	172	11	172	11	172
	Over 1000	7	109	7	109	10	156	11	172	12	188	12	188
6000	14-4/0	(8) 7	109	7	109	(8) 10	156	11	172	10	156	12	188
	225-1000	7	109	7	109	10	156	12	188	11	172	12	188
	Over 1000	8	125	8	125	10	156	12	188	12	188	13	203
7000	8-1000	8	125	9	141	11	172	13	203	(7) 11	172	14	219
	Over 1000	8	125	9	141	11	172	13	203	12	188	15	234
8000	7-1000	(6) 9	141	10	156	(6) 12	188	14	219	12	188	16	250
	Over 1000	9	141	10	156	12	188	14	219	13	203	17	266
9000	7-1000	(6) 9	141	11	172	(6) 13	203	16	250	13	203	17	266
	Over 1000	9	141	11	172	13	203	16	250	14	219	18	281
10000[4]	7-1000	(6)[6] 10	156	12	188	(6) 14	219	18	281	14	219	18	281
	Over 1000	10	156	12	188	14	219	18	281	15	234	19	297
11000	7-1000	(6) 10	156	12	188	(6) 15	234	19	297	15	234	20	313
	Over 1000	10	156	12	188	15	234	19	297	16	250	21	328
12000	6-1000	11	172	13	203	(4) 16	250	20	313	(1) 16	250	22	344
	Over 1000	11	172	13	203	16	250	20	313	17	266	23	359
13000	6-1000	11	172	14	219	(4) 17	266	22	344	(1) 17	266	23	359
	Over 1000	11	172	14	219	17	266	22	344	18	281	24	375
14000	6-1000	12	188	15	234	(4) 18	281	24	375	(1) 18	281	25	391
	Over 1000	12	188	15	234	18	281	24	375	19	297	26	406
15000	6-1000	13	203	16	250	(4) 19	297	26	406	(1) 19	297	27	422
	Over 1000	13	203	16	250	19	297	26	406	20	313	28	438
16000	4 and larger	13	203	17	266	20	313	27	422	..	...	..	...
17000	4 and larger	14	219	18	281	21	328	28	438	..	...	..	...
18000	4 and larger	15	234	18	281	22	344	..	...	..	...	..	...
19000	4 and larger	15	234	19	297	23	359	..	...	..	...	..	...
20000	2 and larger	16	250	19	297	(4) 24	375	..	...	..	...	..	...
21000	2 and larger	16	250	20	313	25	391	..	...	..	...	..	...
22000	2 and larger	17	266	21	328	26	406	..	...	..	...	..	...
23000	2 and larger	17	266	22	344	27	422	..	...	..	...	..	...
24000	2 and larger	18	281	22	344	28	438	..	...	..	...	..	...
25000	2 and larger	18	281	23	359	29	453	..	...	..	...	..	...
26000	2 and larger	19	297	24	375	30	469	..	...	..	...	..	...
27000	2 and larger	19	297	25	391	..	...	..	...	..	...	..	...
28000	1 and larger	20	313	26	406	..	...	..	...	..	...	..	...
29000	1 and larger	21	328	27	422	..	...	..	...	..	...	..	...
30000	1 and larger	21	328	27	422	..	...	..	...	..	...	..	...
31000	1/0 and larger	22	344	28	438	..	...	..	...	..	...	..	...
32000	1/0 and larger	22	344	28	438	..	...	..	...	..	...	..	...
33000	1/0 and larger	23	359	29	453	..	...	..	...	..	...	..	...
34000	1/0 and larger	24	375	30	469	..	...	..	...	..	...	..	...
35000[4]	1/0 and larger[6]	24	375	31	484	..	...	..	...	..	...	..	...
36000	2/0 and larger	25	391										
37000	2/0 and larger	26	406										
38000	2/0 and larger	26	406										
39000	2/0 and larger	27	422										
40000[5]	2/0 and larger	27	422										
41000	3/0 and larger	28	438										
42000	3/0 and larger	28	438										
43000	3/0 and larger	29	453										
44000	3/0 and larger	29	453										
45000	3/0 and larger	30	469										
46000	3/0 and larger	30	469										
47000	4/0 and larger	31	484										
48000	4/0 and larger	32	500										
49000	4/0 and larger	32	500										
50000	4/0 and larger	33	516										
51000	4/0 and larger	34	531										
52000	4/0 and larger	34	531										
53000	250 and larger	35	547										
54000	250 and larger	36	563										
55000	250 and larger	36	563										
56000	250 and larger	37	578										
57000	250 and larger	37	578										
58000	300 and larger	38	594										
59000	300 and larger	38	594										
60000	300 and larger	39	609										
61000	300 and larger	39	609										
62000	300 and larger	40	625										
63000	300 and larger	40	625										
64000	300 and larger	41	641										
65000	350 and larger	42	656										
66000	350 and larger	42	656										
67000	350 and larger	43	672										
68000	350 and larger	44	688										
69000	350 and larger	44	688										

[1]The insulation thicknesses for paper and varnished cambric are I.P.C.E.A. Standards. The rubber walls, though not I.P.C.E.A. Standards, are believed to be representative of modern practice under average conditions.

[2]All cables are rated on conductor to conductor basis for three-phase circuits. All cables have an operating tolerance of 5% above the rated voltage except those rated at 15 000 volts and below, which have no operating tolerance.

[3]For intermediate voltages take the wall for the next higher listed voltage. Minimum round conductor size (AWG) indicated in parentheses under respective insulation when differing from size shown under "Size of Conductor".

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[4]For voltages 10 000 to 35 000 the thicknesses given apply to both single-conductor and three-conductor type H cable.

[5]For voltages 40 000 and higher, type H single-conductor cable is recommended.

[6]Recommended minimum sector conductor sizes for three conductor type H cable are 1/0, 2/0, 3/0, and 4/0 (AWG) corresponding to voltage ranges of 10 000 to 20 000, 21 000 to 25 000, 26 000 to 30 000 and 31 000 to 35 000 respectively.

VARNISHED CAMBRIC

[7]For braided or special designs consult I.P.C.E.A. specifications or manufacturer.

RUBBER

[8]These thicknesses also apply to multi-conductor cables. For multi-conductor lead covered cables for voltages 10 000 and higher, shielding is advisable. For braided or special designs consult the manufacturer.

TABLE II—RECOMMENDED MINIMUM AVERAGE THICKNESS OF INSULATION FOR THREE-CONDUCTOR BELTED CABLES

Recommended by the Insulated Power Cable Engineers Association Standards

Rated Voltage[1]	Size of Conductor AWG Number or 1000 Cir. Mils	PAPER Conductor 64ths	PAPER Conductor Mils	PAPER Belt Grounded Neutral 64ths	PAPER Belt Grounded Neutral Mils	PAPER Belt Ungrounded Neutral 64ths	PAPER Belt Ungrounded Neutral Mils	VARNISHED CAMBRIC[4] Conductor 64ths	VARNISHED CAMBRIC[4] Conductor Mils	VARNISHED CAMBRIC[4] Belt Grounded Neutral 64ths	VARNISHED CAMBRIC[4] Belt Grounded Neutral Mils	VARNISHED CAMBRIC[4] Belt Ungrounded Neutral 64ths	VARNISHED CAMBRIC[4] Belt Ungrounded Neutral Mils
600	14-8	(8)[2] 4	63	2	31	2	31	3	47	0	0	0	0
	7-2	4	63	2	31	2	31	4	63	0	0	6	0
	1-4/0	4	63	2	31	2	31	5	78	0	0	0	0
	225-500	5	78	2	31	2	31	6	94	0	0	0	0
	525-1000	5	78	2	31	2	31	6	94	2	31	2	31
	Over 1000	5	78	2	31	2	31	7	109	2	31	2	31
1000	14-2	(8) 4	63	2	31	2	31	4	63	0	0	0	0
	1-4/0	4	63	2	31	2	31	5	78	0	0	0	0
	225-500	5	78	2	31	2	31	6	94	0	0	0	0
	525-1000	5	78	2	31	2	31	6	94	2	31	2	31
	Over 1000	5	78	2	31	2	31	7	109	2	31	2	31
2000	8-2	5	78	2	31	2	31	5	78	0	0	0	0
	1-4/0	5	78	2	31	2	31	6	94	0	0	0	0
	225-500	5	78	3	47	3	47	6	94	0	0	0	0
	525-1000	5	78	3	47	3	47	6	94	2	31	2	31
	Over 1000	5	78	3	47	3	47	7	109	2	31	2	31
3000	8-2	5	78	3	47	3	47	5	78	2	31	2	31
	1-500	5	78	3	47	3	47	6	94	2	31	2	31
	525-1000	5	78	3	47	3	47	6	94	3	47	3	47
	Over 1000	5	78	3	47	3	47	7	109	3	47	3	47
4000	8-500	6	94	3	47	3	47	6	94	3	47	3	47
	525-1000	6	94	3	47	3	47	6	94	4	63	4	63
	Over 1000	6	94	3	47	3	47	7	109	4	63	4	63
5000	8-4/0	6	94	4	63	4	63	6	94	4	63	4	63
	225-1000	6	94	4	63	4	63	7	109	4	63	4	63
	Over 1000	6	94	4	63	4	63	7	109	5	78	5	78
6000	8-4/0	6	94	4	63	4	63	6	94	5	78	5	78
	225-1000	6	94	4	63	4	63	7	109	5	78	5	78
	Over 1000	6	94	4	63	4	63	7	109	5	78	6	94
7000	8 and larger	7	109	4	63	6	94	7	109	5	78	6	94
8000	6 and larger	7	109	4	63	7	109	7	109	6	94	7	109
9000	6 and larger	8	125	4	63	8	125	8	125	6	94	8	125
10000[3]	6 and larger	8	125	4	63	8	125	9	141	6	94	9	141
11000	6 and larger	8	125	5	78	8	125	10	156	6	94	10	156
12000	6 and larger	9	141	5	78	9	141	(4) 10	156	7	109	10	156
13000	6 and larger	9	141	5	78	9	141	(4) 11	172	7	109	11	172
14000	6 and larger	10	156	5	78	10	156	(4) 12	188	7	109	12	188
15000[3]	6 and larger	10	156	5	78	10	156	(4) 13	203	7	109	13	203
16000	4 and larger	...	...	...	...	...	...	14	219	7	109	14	219
17000	4 and larger	...	...	...	...	...	...	14	219	7	109	14	219

NOTES

GENERAL:

[1]All cables for three-phase circuits are rated on conductor to conductor basis. All cables have an operating tolerance of 5% except those rated at 15 000 volts and below which have no operating tolerance.
For intermediate voltage take wall for next higher listed voltage.

[2]Minimum round conductor size (AWG) indicated in parentheses under respective insulation when differing from size shown under "Size of Conductor". Minimum sector conductor size for paper cables of all voltages listed is 1/0 AWG.

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[3]For voltages 10 000 to 15 000 type H Cable is recommended where applicable, for voltages 16 000 and higher type H cable only is recommended.

VARNISHED CAMBRIC:

[4]For braided or special designs consult I.P.C.E.A. Specifications or manufacturer.

RUBBER:

For rubber multi-conductor lead-covered cables use thicknesses given in Table I.

lation adopted by the Insulated Power Cable Engineers Association for paper and varnished cambric are given in Tables I and II. The thicknesses listed are generally considered as representative minimums for modern cables in this country installed and operated under average conditions.

The ungrounded neutral thicknesses listed in Tables I and II are intended to apply to systems in which the neutral is ungrounded or grounded through a resistance which has a value in excess of the criterion proposed by the American Standards Association:—

"An underground cable system is classified as having the neutral grounded when resistance in the neutral ground (or grounds) is of such value that the lagging component of short-circuit current is always equal to or greater than the total charging current of the system at the point of fault. When the neutral resistance exceeds this value, the system should be classified as being ungrounded, since the conditions are such that cumulative oscillation may be caused during a failure."

Cable Diameters

If the number of conductors in a cable, the diameter of the conductor (given in Table IV), and insulation thicknesses are known, it is possible to calculate the core diameter and outside diameter by elementary geometry. For convenience, however, the following formulas are given:

$D_s = d + 2T$ for one-conductor cables .... (1)

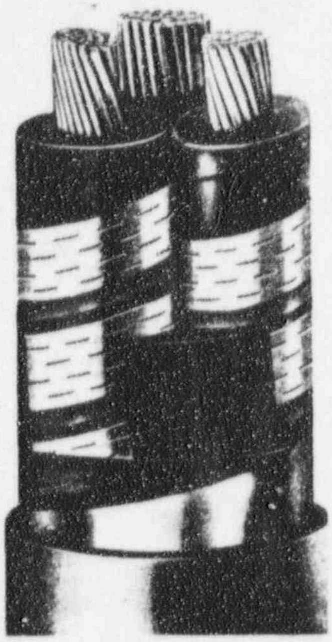
$D_s = 2(d + 2T) + 2t$ for two-conductor cables (round duplex) .... (2)

$D_s = \left(1 + \frac{2}{\sqrt{3}}\right)(d + 2T) + 2t$ for three-conductor cables .... (3)

$D_s = (1 + \sqrt{2})(d + 2T) + 2t$ for four-conductor cables .... (4)

In which D_s is the inside diameter of the lead sheath, d the conductor diameter, T the conductor insulation thickness, t the belt thickness—all in inches.

These formulas refer to cables with round conductors. For the case of sector cables, there is no hard and fast rule, because the term "sector" is not a mathematical description of shape, and there is a large variety



Three-conductor type H cable

of sector shapes. In general, the saving in cable diameter is approximately proportional to the size of conductor and thus the saving relative to the diameter of the cable is less with thick insulation. The reduction in diameter due to sector conductors also diminishes as the number of conductors increases. In other words, it is a maximum for two-conductor cables, with the so-called D-shaped sector.

From a practical point of view, it is of course necessary to know the diameter of a cable in order to determine whether or not it can be installed in a given duct line. For the purpose of calculating cable characteristics, however, great accuracy in calculating the diameter is not of real importance. Most transmission problems with multi-conductor cables deal with three-conductor cables. A close approximation to the core diameter of a sector three-conductor cable can be obtained by calculating the diameter by eq. (3) as if the conductors were round, and then subtracting 0.3 to 0.4 times the conductor diameter d, depending on the shape of the sector. For the case of two-conductor sector cables, the manufacturers have information available for the usual sizes, or close results can be obtained by laying off the cable graphically.

TABLE III—THICKNESS OF LEAD SHEATH
Recommended by The Insulated Power Cable Engineers' Association

Paper Lead Power Cables

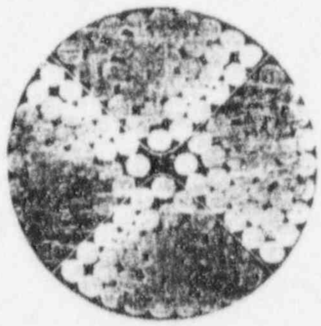
Core Diameter Under Lead—Mils	Lead Thickness 64ths	Lead Thickness Mils
0— 400	5	78
401—1000	6	94
1001—1800	7	109
1801—2800	8	125
2801—3200	9	141
3201 and over	10	156

Varnished Cambric* Lead Power Cables

Core Diameter Under Lead—Mils	Lead Thickness 64ths	Lead Thickness Mils
0— 425	3	47
426— 700	4	63
701—1050	5	78
1051—1500	6	94
1501—2000	7	109
2001—3000	8	125
3001 and over	9	141

*Usually considered representative for rubber lead power cables also.

The diameter over the lead sheath is figured by adding twice the lead thickness to the core diameter. The lead thickness in general varies with the core diameter, the larger cables having thicker lead. The thickness of the lead sheath varies somewhat with conditions, but in general will be close to that given in Table III:



Segmental Conductor

Resistance and Reactance

To permit bending the cable, and especially because of the large conductors now ordinarily used in underground transmission, only stranded conductors are used, and only such will be considered here. The dimensions and resistance of the usual sizes of conductor are shown in Table IV. The values of resistance are based on the International Annealed Copper Standard 0.15328 ohms (meter-gram) at 20° C. which figure is increased two percent to allow for the effect of stranding.

These figures can be used directly for single-conductor cables. For multi-conductor cables, where the conductors are twisted around each other, there will be an additional increase of resistance. This will vary with the makeup of the cable, but can be taken as an increase of two percent above the values in Table IV for most practical purposes.

The dimensions and resistance of stranded aluminum conductors can also be readily determined from Table IV with the aid of the following ratios:

	al.	cu.
Relative conductance for equal volume	0.63	1.00
Relative volume for equal conductance	1.59	1.00
Relative diameter for equal conductance	1.26	1.00
Relative weight for equal volume	1.00	3.29
Relative weight for equal conductance	1.00	2.07
	0.483	1.00

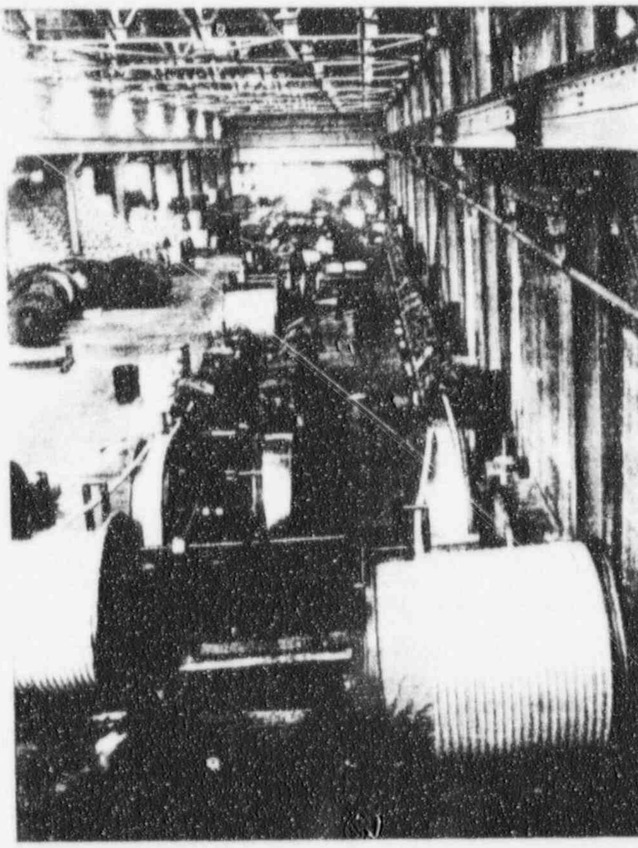
It is necessary to correct the resistance for temperature in most problems. The resistance of copper is proportional to $234 + T$ where T is in deg. C., and the resistance at any temperature T can therefore be calculated from the 25° C. value in Table IV by the following equation:

$$R_T = \frac{234 + T}{259} \times R_{25} \qquad (5)$$

R_{25} is the table value, the number in the denominator being equal to 234 + 25.

The skin effect, or ratio of alternating to direct-current resistance, is quite appreciable even at 60 cycles, especially for conductors larger than 1 000 000 circular mils. A novel method for reducing the skin effect of large sizes of single-conductor cable, which avoids the usual rope core construction and resultant diameter increase, has been recently developed. The construction is illustrated on p. 4. It will be seen that the "conductor" consists of four sectors, which are separately stranded and then cabled together to form a round conductor with a layer of insulation between sectors. The reduction in skin effect is due to the equalized reactance of the various layers, resulting from the transposition of the individual strands. By this means, the total skin effect of the conductor can be reduced to a value approximately that of one round conductor, with one-third the cross-section of the segmental conductor.

The values of skin effect ratio at 60 cycles is tabulated in Table V for the ordinary range of size of conductor, both in the usual form of strand and for the



Applying paper tapes to cables. This machine applies 192 tapes simultaneously, making it possible to insulate cable for voltages up to 220-kv service in one operation.

case of tubular conductors. The temperature is taken as 65° C., which is an average value of maximum permissible operating temperature. To find the alternating-current resistance, multiply the direct-current resistance by the skin effect.**

The skin effect or ratio of alternating to direct-current resistance varies with temperature. However, if the skin effect at 65° C. is used, no very large error will be introduced in an ordinary problem, especially with the sizes of conductor usual in three-conductor cables. For instance, the change in skin effect between 20 and 65° C. for 1 000 000 c.m. conductors is only 2.7%, the change being less for the smaller sizes. For large conductors, such as in single-conductor cables, it may be necessary to calculate the skin effect at the operating temperature if a high degree of accuracy is

**The skin effect in the usual form of strand has been calculated by the Bessel function formula which can be conveniently obtained from Table XXII of reference 2 of the bibliography. The skin effect of the stranded conductors is calculated as if for solid conductors of the same cross-section, which is the proper procedure both theoretically and experimentally, as shown in reference 3. For the case of tubular conductors, the calculations have been based on Dwight's work, reference 4. For this case the actual inside and outside dimensions of the stranded tube have been taken, but the fact that it is a stranded conductor and not a solid tube has been taken care of by using an equivalent resistivity calculated from the known direct-current resistance and the actual cross-sectional area of the tube.

desired, as the change with temperature is greater though such precision is rarely required.

The skin effect is a measure of the increase in resistance because there is a tendency for the current density to decrease toward the center or axis of the conductor and for the phase of the current in the different layers to change, due to an increase of inductance towards the center. The skin effect assumes an isolated conductor; that is, it is an increase of resistance due to what takes place in the conductor itself. If two conductors are close together, however, there will be a tendency for an increase in current density at the points where the two conductors are closest together, which is another cause of increase in resistance. This is called the proximity effect. For the case of stranded conductors it is not only a function of the spacing, but is greatly influenced by the strand lay, cable lay, and contact resistance between strands. In general it is considerably less than indicated by theoretical considerations[4 5 6 7] based on solid conductors. Tests made in the laboratories of the company with which the writer is associated, on its standard stranded sector cables show the value of the combined skin and proximity effect at 60 cycles to be approximately 1.35* times the theoretical skin effect for a round isolated single-conductor of the same size.

Several other features must be considered in calculating conductor resistance. Any induced currents in the cable sheath and the losses due to them can be most conveniently considered mathematically by assuming that there is an additional effective conductor resistance of such value that when this resistance is multiplied by the square of the actual current flowing through the conductor (and by the number of conductors for multi-conductor cables), the loss in the sheath will be obtained. The ratio of the conductor loss plus the sheath loss to the conductor loss is then given by:

$$\frac{\textit{Conductor Loss} + \textit{Sheath Loss}}{\textit{Conductor Loss}} = \frac{R + R_o}{R} \qquad (6)$$

where R = actual conductor resistance in ohms per 1000 feet at a given temperature and frequency. R_o is the added effective resistance due to sheath losses and might be greater than the conductor resistance itself for larger sizes of single-conductor cables.

For two single-conductor cables in a single-phase circuit or three single-conductor cables in a three-phase circuit with equilateral spacing and with negligible sheath proximity effect (which is usually the case except for very close spacings) R_o is accurately given by the following:

$$R_o = \frac{X^2_M R_s}{X^2_M + R^2_s} \text{ ohms per 1000 feet} \qquad (7)$$

where X_M is equal to $2\pi fM$ in which M is mutual inductance of conductor and sheath of a single-conductor cable obtained by eq. (13), and R_s is the sheath resistance and is given in eq. (10).

For the other usual arrangements of one and two circuits of single-conductor cables, Halperin and Miller[8] in their admirable work on sheath losses have given readily applicable formulas for the sheath losses, currents, and voltages in the individual sheaths[9 10 13] Table VI is taken directly from their work with numerical magnitudes substituted for the vector ratios. From Table VI, R_o for the individual sheaths of the different arrangements may be found by multiplying eqs. (4), (5), (6), (7) (Table VI) by R_s.

For the case of multi-conductor cables, the induced

TABLE IV—RESISTANCE OF STRANDED COPPER CONDUCTORS**

Size of Cable		Ohms per 1000 Feet		Pounds per 1000 Feet	Standard Concentric Stranding		
Circular Mils	AWG. No.	25° C. (= 77° F.)	65° C. (= 149° F.)		No. of Wires	Diameter of Wires in Mils	Outside Dia. in Mils
2 000 000		0.00539	0.00622	6180	127	125.5	1631
1 900 000		0.00568	0.00655	5870	127	122.3	1590
1 800 000		0.00599	0.00692	5560	127	119.1	1548
1 700 000		0.00634	0.00732	5250	127	115.7	1504
1 600 000		0.00674	0.00778	4940	127	112.2	1459
1 500 000		0.00719	0.00830	4630	91	128.4	1412
1 400 000		0.00770	0.00889	4320	91	124.0	1364
1 300 000		0.00830	0.00958	4010	91	119.5	1315
1 200 000		0.00889	0.0104	3710	91	114.8	1263
1 100 000		0.00981	0.0114	3400	91	109.9	1209
1 000 000		0.0108	0.0124	3090	61	128.0	1152
950 000		0.0114	0.0131	2930	61	124.8	1123
900 000		0.0120	0.0138	2780	61	121.5	1093
850 000		0.0127	0.0146	2620	61	118.0	1062
800 000		0.0135	0.0156	2470	61	114.5	1031
750 000		0.0144	0.0166	2320	61	110.9	998
700 000		0.0154	0.0178	2160	61	107.1	964
650 000		0.0166	0.0192	2010	61	103.2	929
600 000		0.0180	0.0207	1850	61	99.2	893
550 000		0.0196	0.0226	1700	61	95.0	855
500 000		0.0216	0.0249	1540	37	116.2	814
450 000		0.0240	0.0277	1390	37	110.3	772
400 000		0.0270	0.0311	1240	37	104.0	728
350 000		0.0308	0.0356	1080	37	97.3	681
300 000		0.0360	0.0415	926	37	90.0	630
250 000		0.0431	0.0498	772	37	82.2	575
212 000	0000	0.0509	0.0587	653	19	105.5	528
168 000	000	0.0642	0.0741	518	19	94.0	470
133 000	00	0.0811	0.0936	411	19	83.7	418
106 000	0	0.102	0.117	326	19	74.5	373
83 700	1	0.129	0.149	258	19	66.4	332
[illegible]	2	0.162	0.187	205	7	97.4	292
[illegible]	3	0.205	0.237	163	7	86.7	260
41 700	4	0.259	0.299	129	7	77.2	232
33 100	5	0.326	0.376	102	7	68.8	206
26 300	6	0.410	0.473	81	7	61.2	184
20 800	7	0.519	0.599	64.3	7	54.5	164
16 500	8	0.654	0.755	51	7	48.6	146

*On a type of very compact sector, developed since the writing of this article, this value becomes 0.90.

**Taken from Table XII of Bulletin 31 of the Bureau of Standards.

TABLE V—DIMENSIONS AND 60-CYCLE SKIN EFFECT RATIO OF STRANDED COPPER CONDUCTORS AT 65° C.

Size Circular Mils	Inside Diameter of Tubular Conductor in Inches							
	0		0.25		0.50		0.75	
	d	Ratio	d	Ratio	d	Ratio	d	Ratio
3 000 000	1.998	1.439	2.02	1.39	2.08	1.36	2.15	1.29
2 500 000	1.825	1.336	1.87	1.28	1.91	1.24	2.00	1.20
2 000 000	1.631	1.239	1.67	1.20	1.72	1.17	1.80	1.12
1 500 000	1.412	1.145	1.45	1.12	1.52	1.09	1.63	1.06
1 000 000	1.152	1.068	1.19	1.05	1.25	1.03	1.39	1.02
800 000	1.031	1.046	1.07	1.04	1.16	1.02	1.28	1.01
600 000	0.893	1.026	0.94	1.02	1.04	1.01	....	....
500 000	0.814	1.018	0.86	1.01	0.97	1.01	....	....
400 000	0.728	1.012	0.78	1.01	....	....	....	....
300 000	0.630	1.006	....	....	....	....	....	....

Size Circular Mils	Inside Diameter of Tubular Conductor in Inches							
	1.00		1.25		1.50		2.00	
	d	Ratio	d	Ratio	d	Ratio	d	Ratio
3 000 000	2.27	1.23	2.39	1.19	2.54	1.15	2.87	1.08
2 500 000	2.12	1.16	2.25	1.12	2.40	1.09	2.75	1.05
2 000 000	1.94	1.09	2.09	1.06	[illegible]	1.05	2.61	1.02
1 500 000	1.75	1.04	1.91	1.03	2.07	1.02	2.47	1.01
1 000 000	1.53	1.01	1.72	1.01	....	....	....	....
800 000	1.45	1.01	....	....	....	....	....	....
600 000	....	....	....	....	....	....	....	....
500 000	....	....	....	....	....	....	....	....
400 000	....	....	....	....	....	....	....	....
300 000	....	....	....	....	....	....	....	....

This table gives the conductor diameter in inches d, and the 60-cycle skin effect or ratio of alternating to direct-current resistance, both for the ordinary form of stranding (inside diameter = 0) and for tubular conductors, based on references 3 and 4.

TABLE VI—FORMULAS FOR SHEATH VOLTAGES CURRENTS, AND LOSSES FOR SINGLE-CONDUCTOR CABLES

Equation Number	Cable Arrangement Number and Diagram	I One Phase	II Equilateral	III Rectangular	IV Flat	V Two Circuit	VI Two Circuit
		B A ←S→	A B C ←S→	A S B C ←S→	A B C ←S→←S→	A B C S ←S→←S→ A B C	A B C S ←S→←S→ C B A
	Sheaths Open Circuited						
1	$\frac{E_{s1}}{I} =$	X_M	X_M	$\frac{1}{2}\sqrt{3Y^2+\left(X_M-\frac{a}{2}\right)^2}$	$\frac{1}{2}\sqrt{3Y^2+(X_M-a)^2}$	$\frac{1}{2}\sqrt{3Y^2+\left(X_M-\frac{b}{2}\right)^2}$	$\frac{1}{2}\sqrt{3Y^2+\left(X_M-\frac{b}{2}\right)^2}$
2	$\frac{E_{s2}}{I} =$	X_M	X_M	X_M	X_M	$\left(X_M+\frac{a}{2}\right)$	$\left(X_M+\frac{a}{2}\right)$
3	$\frac{E_{s3}}{I} =$		X_M	$\frac{1}{2}\sqrt{3Y^2+\left(X_M-\frac{a}{2}\right)^2}$	$\frac{1}{2}\sqrt{3Y^2+(X_M-a)^2}$	$\frac{1}{2}\sqrt{3Y^2+\left(X_M-\frac{b}{2}\right)^2}$	$\frac{1}{2}\sqrt{3Y^2+\left(X_M-\frac{b}{2}\right)^2}$
	Sheaths Solidly Bonded						
4	$\frac{I_{s1}^2}{I^2} = \frac{W_{s1}\times 10^3}{I^2R_s} = \frac{R_{01}}{R_s} =$	$\frac{X_M^2}{R_s^2+X_M^2}$	$\frac{X_M^2}{R_s^2+X_M^2}$	$\frac{(P^2+3Q^2)+2\sqrt{3}(P-Q)+4}{4(P^2+1)(Q^2+1)}$			
5	$\frac{I_{s2}^2}{I^2} = \frac{W_{s2}\times 10^3}{I^2R_s} = \frac{R_{02}}{R_s} =$	$\frac{X_M^2}{R_s^2+X_M^2}$	$\frac{X_M^2}{R_s^2+X_M^2}$	$\frac{1}{(Q^2+1)}$			
6	$\frac{I_{s3}^2}{I^2} = \frac{W_{s3}\times 10^3}{I^2R_s} = \frac{R_{03}}{R_s} =$		$\frac{X_M^2}{R_s^2+X_M^2}$	$\frac{(P^2+3Q^2)-2\sqrt{3}(P-Q)+4}{4(P^2+1)(Q^2+1)}$			
7	$\frac{W_{sT}\times 10^3}{3I^2R_s} = \frac{R_{0\,aver.}}{R_s}$		$\frac{X_M^2}{R_s^2+X_M^2}$	$\frac{P^2+Q^2+2}{2(P^2+1)(Q^2+1)}$			
	Where	$Y =$	X_M	$\left(X_M+\frac{a}{2}\right)$	(X_M+a)	$\left(X_M+a+\frac{b}{2}\right)$	$\left(X_M+a-\frac{b}{2}\right)$
	$P=\frac{R_s}{Y}$ $Q=\frac{R_s}{Z}$	$Z =$	X_M	$\left(X_M-\frac{a}{6}\right)$	$\left(X_M-\frac{a}{3}\right)$	$\left(X_M+\frac{a}{3}-\frac{b}{6}\right)$	$\left(X_M+\frac{a}{3}-\frac{b}{6}\right)$

In ohms to neutral per 1000 feet. $X_M = 2\pi f\left(0.1404 \log_{10}\frac{S}{r_m}\right)10^{-3}$; $a = 2\pi f(0.1404 \log_{10}2)10^{-3}$; $b = 2\pi f(0.1404 \log_{10}5)10^{-3}$

In ohms to neutral per 1000 feet at 60 cycles. $X_M = 0.05292 \log_{10}\frac{S}{r_m}$; $a = 0.01593$; $b = 0.03699$

All cases II to VI, inclusive, are three-phase, phase rotation A, B, C. Subscripts 1, 2, 3 correspond to phases A, B, C, respectively.

To form the equations, set the quantities in the vertical column headed "Cable Arrangement, . . . " equal to the quantities under the column which is headed by the arrangement being investigated.

sheath currents are usually small, inasmuch as the sheath surrounds all the conductors and the inductive effects of the current in one conductor usually are almost completely neutralized by the effect of the currents in the other conductors. However, with large conductors and large currents, such neutralization is not complete and there is an appreciable loss in the sheath comparable in magnitude to the dielectric loss or skin effect.

Miller and Dwight have both published rigid theoretical equations* involving convergent series for the sheath losses in round three-conductor cables. Fortunately, for practical cables at 60 cycles, the convergence is so rapid that values of R_o within a few percent of those given by the rigid equations may be obtained from the following approximate equation based on the simplifying assumptions suggested by Atkinson[12],

$$R_o = \frac{396\, S_1^2}{R_s r_m^2} \times 10^{-6} \text{ ohms per 1000 feet per conductor} \qquad (8)$$

R_s is given by eq. (10) and r_m is the mean sheath radius in inches. S_1 is the distance between the effective current center of each conductor and the cable center, and is accurately given for round conductors by

$$S_1 = \frac{1}{\sqrt{3}}(d + 2T) \qquad (9)$$

For sector conductors an approximate value of S_1 can be found by using eq. (9) but taking d from 0.82 to 0.86 times the round conductor diameter depending on the shape of the sector, or by measurement from the center of the sector to the center of the cable.

While theoretically at least some allowance should be made for the losses which occur in the shielding tape on the individual conductors of type H cable, actual measurements indicate that for all practical purposes these are negligible with slotted foil. The losses that occur in the usual non-magnetic binders of brass or bronze have also been found to be quite small, although allowance can be made for these if desired by assuming a portion of the tape to be in parallel with the sheath. On type H cables having a magnetic binder tape, the available test data appears to show that the distribution rather than the actual magnitude of the losses is altered so that eq. (8) may be used at least for current-carrying capacity calculations with reasonable accuracy when no more accurate data is obtainable.

The apparent conductor resistance for a three-conductor cable, (also two single-conductor cables in a single-phase line, or three single-conductor cables in a three-phase line with equilateral spacing, in which there are induced sheath losses) is the actual conductor resistance R at a given temperature and frequency plus the added effective resistance R_o from eq. (7) or (8). For the other usual arrangements of single-conductor cables that are not perfectly symmetrical, the apparent conductor resistance when the sheaths are solidly bonded and grounded at least at both ends is found from the master equations developed by Miller[8,9] and tabulated for convenience in Table VII.

The added effective resistance given in Table VI should be carefully distinguished from the apparent conductor resistance. The added effective resistance, when added to the conductor resistance and multiplied by the square of the conductor current, gives the actual loss dis-

*Compare eq. (81) reference 9 and eq. (7) reference 11.

sipated in heat in the conductor and sheath of the phase considered. The apparent conductor resistance, when multiplied by the square of the conductor current, gives a fictitious loss representing that portion of the total conductor and sheath losses in all phases that are supplied by the phase considered. While the respective sums of these loss values for all three phases are always equal, the values for any one phase are not necessarily equal, the difference representing the power transferred between phases due to their mutual inductances. Furthermore, the apparent resistance when properly combined with the apparent reactance to form an apparent impedance and multiplied by the conductor current, gives the voltage drop in the phase considered. The added effective resistance is used in determining the sheath losses and in all current-carrying capacity calculations, while the apparent resistance along with the apparent reactance as given in Table VII is employed in the calculation of line impedance, regulation, etc. See example 5 for an illustration.

TABLE VII–EQUATIONS FOR LINE CONSTANTS OF SINGLE-CONDUCTOR CABLES
(Sheaths Solidly Bonded)

	Apparent Resistance	Apparent Reactance
Phase A	$R + \frac{R_s}{4}\left[\frac{\sqrt{3}(\sqrt{3}+P)}{(P^2+1)} + \frac{(1-\sqrt{3}Q)}{(Q^2+1)}\right]$	$X_L - X_M + \frac{R_s}{4}\left[\frac{\sqrt{3}(\sqrt{3}P-1)}{(P^2+1)} + \frac{(Q+\sqrt{3})}{(Q^2+1)}\right]$
Phase B	$R + \frac{R_s}{Q^2+1}$	$X_L - X_M + \frac{R_s\,Q}{Q^2+1}$
Phase C	$R + \frac{R_s}{4}\left[\frac{\sqrt{3}(\sqrt{3}-P)}{(P^2+1)} + \frac{(1+\sqrt{3}Q)}{(Q^2+1)}\right]$	$X_L - X_M + \frac{R_s}{4}\left[\frac{\sqrt{3}(\sqrt{3}P+1)}{(P^2+1)} + \frac{(Q-\sqrt{3})}{(Q^2+1)}\right]$
Average	$R + R_s\left[\frac{P^2+Q^2+2}{2(P^2+1)(Q^2+1)}\right]$	$X_L - X_M + R_s\left[\frac{Q(P^2+1)+P(Q^2+1)}{2(P^2+1)(Q^2+1)}\right]$

Phase Rotation A, B, C.
$X_L = 2\pi fL$ where L is found from eq's. (11) or (12).
$X_M = 2\pi fM$ where M is found from eq. (13).
R_s is given by eq. (10).
R = actual conductor resistance at given temperature and frequency.
P and Q are determined for given arrangement from Table VI.
All quantities are in ohms per 1000 feet.

The sheath resistance may be calculated by the following equation:

$$R_s = \frac{1503\ \rho_s}{r_5^2 - r_4^2} \times 10^{-4} \text{ ohms per 1000 feet} \qquad (10)$$

where ρ_s = resistivity of the lead sheath in microhm-cm. units, which according to the Smithsonian Physical Tables may be taken as 24.2 at 40°C., 25.2 at 50°, 26.1 at 60°, and 27.1 at 70°, 25.2 being a value corresponding to an average value of maximum permissible currents. r_4 and r_5 are the inner and outer radii respectively of the lead sheath in inches.

The denominator can be most conveniently calculated by writing this in the form $(r_5 + r_4)(r_5 - r_4)$, noting that the second factor is equal to the lead thickness.

Protective armor of steel wire or steel tape such as employed on submarine or buried cables further modify conductor resistance calculations. When the magnetic characteristics of the steel used are known, it is possible to calculate with a good degree of accuracy the resistance and reactance effects[41 42]. Although in preliminary work such data is usually not to be had, it is often important in making current-carrying capacity calculations to be able at least to estimate in advance the armor loss to be expected.

With three-conductor cables, the losses occasioned by either type of armor are not of serious consequence, being of the same order of magnitude as the sheath losses. With single-conductor cables on alternating-current circuits, however, the armor loss is so pronounced that except for very small cables, such as those used for series street lighting, steel-tape armor is impractical. With steel-wire armor, the loss, while large, is not in most cases prohibitive.

An exceedingly simple method for approximating the losses in single-conductor cables with steel-wire armor at spacings ordinarily employed in submarine installations is to assume that the combined sheath and armor current is equal to the conductor current and is divided between the armor and sheath in proportion to their alternating-current conductances. The sheath resistance is found from eq. (10), and the 60-cycle alternating-current resistance of the armor circuit may be estimated by increasing the direct-current resistance of all the armor wires in parallel (corrected for lay) by from 30 to 60%. The parallel combination of the sheath and armor resistance thus found represents directly the added effective resistance due to sheath and armor losses, and if multiplied by the square of the conductor current, gives these losses. The method just described is admittedly approximate, and may involve errors as large as 20%, but the error in the final current-carrying capacity will be much smaller than this.

Reactance

As far as inductance and inductive reactance in multi-conductor cables are concerned, there is no essential difference between cables and overhead transmission lines, and the same equation* applies, namely:

*Equations (11) and (12) apply strictly to a straight solid round wire or tube and assume uniform current distribution over the conductor area. The corrections in inductance due to cable lay, stranding, skin effect, etc., while usually small are not always negligible in precise work. When very accurate

$$L = \left(0.1404\, log_{10} \frac{S}{r} + 0.01525\right) \times 10^{-3} \text{ henries to neutral per 1000 ft...(11)}$$

S being the distance between centers of conductors and r the radius of the conductor all in inches.

For sector-shaped conductors, complete data is not available; however, a reasonable approximation may be had if S in eq. (11) is taken as the distance between the centers of the small diameters of the sectors. (Refer to sector correction given with eq. (9).)

In three-conductor type H cables in which a magnetic binder tape is employed, there is an appreciable increase over the theoretical reactance. A series of unpublished tests made by Electrical Testing Laboratories working in collaboration with the Insulated Power Cable Engineers Association and the Association of Edison Illuminating Companies indicates that at 60 cycles the steel binder will increase the reactance from 10 to 20%, depending on the steel.

There are additional complications in the calculation of the reactance of cables not ordinarily found in overhead cases. If a tubular conductor is used, such as Type H H cable, in the illustration below, that is, a conductor whose inner region is composed of non-conducting materials, the following equation should be used for inductance, which reduces to eq. (11) if the inner radius r_o = zero:

$$L = \left[0.1404\, log_{10} \frac{S}{r} + \frac{0.1404\, r_o^4}{(r^2 - r_o^2)^2} \times log_{10} \frac{r}{r_o} + 0.01525 \frac{(r^2 - 3r_o^2)}{(r^2 - r_o^2)}\right] 10^{-3} \text{ henries to neutral per 1000 feet.........(12)}$$

There are still further considerations for the case of single-conductor cables, inasmuch as the lead sheath surrounds the conductor, and the effect of induced currents in the sheath must be taken into consideration. The mutual inductance between conductor and sheath can be calculated by the following equation, X_M of course being given by $2\pi f M$, f being the frequency.

$$M = 0.1404\, log_{10} \frac{S}{r_m} \times 10^{-3} \text{ henries to neutral per 1000 feet.........(13)}$$

This equation for mutual inductance assumes that the current is uniformly distributed around the lead sheath, that is, that there is no proximity effect in the sheath due to an adjacent cable. The error is a small one, though it is a maximum for the case of cables with sheaths touching. Miller and Dwight have worked out the ratio of sheath losses to conductor losses[9 10 11] in a rigid manner, so that the calculation can be made accurately, even if the sheaths are touching. For accurate work with sheaths in contact, the equations found in these references should be used.

values are required, as for example in connection with the paralleling of dissimilar cables, the formulas found in references 3, 4, 15, and 16 should be consulted. In three-conductor cables in which there are no magnetic materials, the reduction in reactance resulting from induced sheath losses may also be normally neglected, although at 60 cycles, this reduction may be readily computed from the following approximate formula obtained by adjusting the initial term of an unpublished convergent series formula developed by Miller (Compare formula (8) above).

$$\text{Reduction in reactance} = \frac{4.55\, S_1^2}{R_s^2\, r_m^2} \times 10^{-4} \text{ ohms per 1000 ft. per conductor}$$

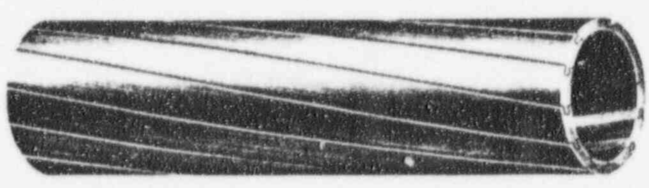
Equation (13) in effect states that the mutual inductance is closely dependent upon the average diameter of the lead sheath. S is actually the distance between axes of adjacent conductors if the conductors are symmetrically located in the vertices of an equilateral triangle. This will be the condition in three-conductor cables, and may be the condition in single-conductor cables. If the configuration in a three-phase transmission line is such that the conductors are not on the vertices of an equilateral triangle, then as far as sheath losses are concerned, there is no "effective spacing" independent of sheath resistance. This will be apparent if an attempt is made to solve for an "effective X_M" in eq. (7) of Table VI. However, a geometric average of the three distances between the three conductors taken pair by pair can usually be used with reasonable accuracy for the purpose of estimating current-carrying capacity and average temperature rise, in place of using the accurate equations given in Table VI. This is strictly true only for the impractical operating case in which the conductors and sheaths are transposed together, the sheaths being solidly bonded at the end points only[9].

For the case of single-conductor cables in a single-phase line or three-phase line with equilateral spacing and in which the sheaths are not interconnected at more than one point, so that there are induced voltages along the sheath but no flow of current, the field, exclusive of a very slight effect of sheath proximity currents, is the same as in an open-wire line of the same size of conductor and the same separation. The self-inductance is therefore the same as that given by eq. (11) the reactance being, of course, $X_L = 2\pi f L$.* With other configurations the equations given by many authors for overhead lines may be used directly in obtaining either the apparent resistance or reactance.

If the sheaths of a single-phase or three-phase line with equilateral spacing are interconnected at the two ends so that sheath currents will circulate, there is an apparent reduction of reactance according to:

$$\text{Apparent Reactance} = X_L - \frac{X_M^2}{X_M^2 + R_s^2} \text{.........(14)}$$

The apparent reduction in reactance due to induced

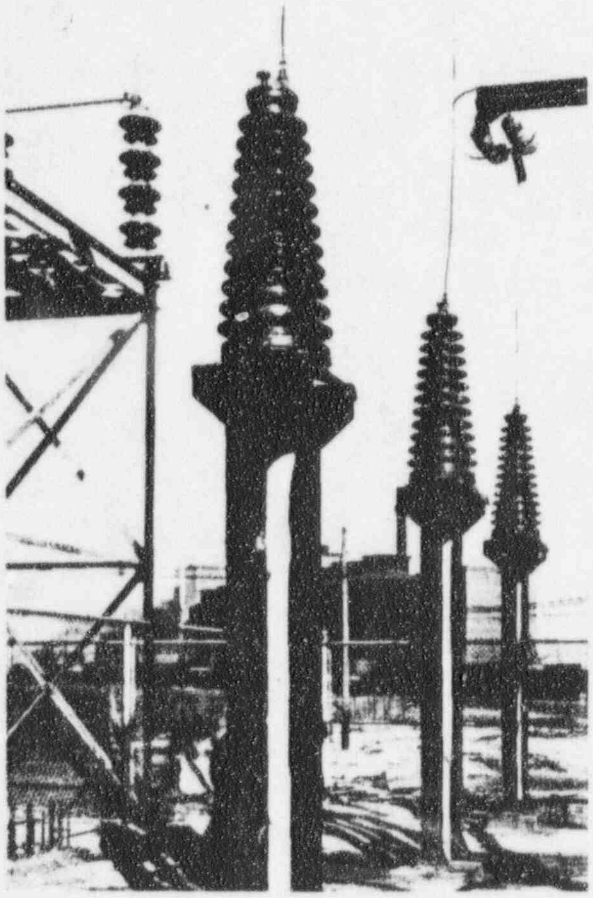


Type H H hollow-core transmission conductor

*For a complete investigation of the impedance characteristics of the individual sheaths of various arrangements, references 8, 9, and 14 should be consulted. These references also discuss residual sheath voltage and related effects existing with asymmetrical arrangements. In the present article, in line with the field data and theoretical considerations discussed in 9, it has been assumed that with two or more ground points, the residual voltage becomes zero without appreciably altering sheath currents as computed for the ideal case of a single ground point.

sheath currents is usually not great, by no means as great as the apparent increase in resistance from the same cause which is often of material amount. For the other usual configurations of single-conductor cable shown in Table VI, the apparent conductor reactance when the sheaths are solidly bonded and grounded at the end points is obtained from the equations given in Table VII.

Geometric Factor, Capacity, and Leakage



Termination of 132-kv oil-filled cable line. Within the porcelain bushing, a spun copper cone joined to the sheath supports a series of concentric insulating tubes

THE equations thus far deal with the conductor and resistance and reactance, which are independent of the shape and quality of the dielectric, except as they affect separation between conductors. There is a series of other characteristics of cables which depend upon the shape and specific qualities of the dielectric. These are primarily the capacity, the thermal resistance, and the electrical resistance of the insulation, and the dependent quantities, charging current, temperature rise, dielectric loss, etc.

For the case of single-conductor cables, the capacity is proportional to the permittivity or specific inductive capacity and inversely proportional to $log_e\ (D_1/d)$. The electrical and thermal resistances are proportional to the electrical and thermal specific resistances of the material and $log_e\ (D_1/d)$. The dimensions of the cable occur only in $log_e\ (D_1/d)$ in these three expressions and in the others derived from them, such as dielectric loss and charging current. $Log_e\ (D_1/d)$ therefore plays the role of a geometric factor or shape modulus, and has been defined as the "geometric factor".

For the case of multi-conductor type H cable with round conductors, it is apparent that as the layers of conducting material are in contact with one another and with the sheath, the electrical field within such a cable will be identical to that within a single-conductor cable. Obviously, therefore, the calculation of the electrical characteristics of each conductor such as capacity, electrical resistance of the insulation, and the dependent quantities will be the same as for a single-conductor cable having the same dimensions as one of the individually-insulated conductors of the shielded cable. While theoretically the single-conductor geometric factor does not strictly hold for sector shape, yet with ordinary practical sector shapes the difference is entirely negligible.

If the metal layer which covers the surface of the individual conductor insulation is so thick that there is no appreciable difference in temperature in this metal around the periphery, then the metal can be considered as an isothermal as well as an equipotential surface and the thermal problem is also exactly the same as for single-conductor cable. Actually, however, the metal foil around the insulation is so thin that there is a rise of temperature in it. A mathematical solution to this problem has been presented elsewhere[17] and results in the following equation for the reciprocal of the thermal resistance (i.e., thermal conductance) between conductors and lead of a three-conductor type H cable with two mils or more copper foil:

$$K = 183\sqrt{\frac{2t_1}{\rho\rho_2 G(d+2T)}} \times \tanh\left(\sqrt{\frac{\rho_2(d+2T)}{2\rho t_1 G}} \times \phi\right) + \frac{183\,(\pi - \phi)}{\rho\, G} \text{ thermal mhos per foot......(15)}$$

From this equation the following expression for G_1 (i.e., the geometric factor to be used in determining the thermal characteristics of three-conductor type H cable) is readily obtained.

$$G_1 = \pi \left[\sqrt{\frac{2\alpha}{\beta \log_e \beta}} \times \tanh \left(\sqrt{\frac{\beta}{2\alpha \log_e \beta}} \times \phi \right) + \frac{\pi - \phi}{\log_e \beta} \right]^{-1} \quad (16)$$

where $\alpha = \frac{\rho t_1}{\rho_2 d}$ and $\beta = \left(1 + \frac{2T}{d} \right)$

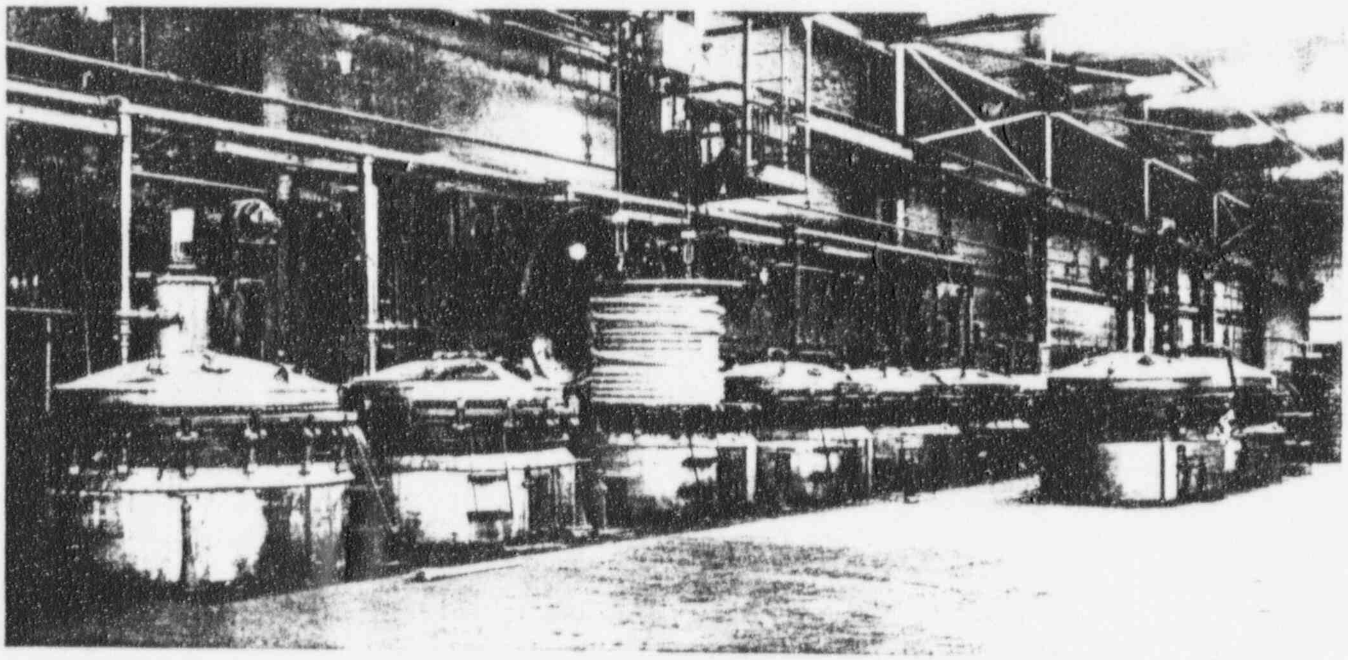
Table VIII gives the geometric factors for a large range of conductor sizes and voltages calculated from eq. (16).

Equation (16) shows that G_1 for a type H cable is not strictly a shape modulus. It is dependent upon two parameters, namely α and β. The former includes the ratio of the thermal resistivities of the foil and insulation. It would be possible to plot G_1 much the same as has been done in Fig. 2 for multi-conductor belted cables using α in place of t/T. This has not been done because the above equation, neglecting as it does heat conduction through the filler spaces, is subject to some error in cases of very thin foils or metallized papers. Dr. Konstantinowsky and his associates[18 19 20] have already published results of an experimental determination of G_1 by an electrolytic method for round conductors and have well under way similar experiments on sector conductors. Using this data it will be possible to plot a family of curves covering the usual practical range of α and β.

For the case of multi-conductor belted cables the quantities, capacity, and the electrical and thermal resistance of the insulation can easily be expressed in accurate mathematical equations except for the element introducing the dimensions of the cable, or, in other words, the geometric factor. There are two approximate equations for the geometric factor due to Russell, both of which, while applicable to many cables without large error, approach an infinite error for large conductors and thin insulation. An equation developed by Mie is much more accurate, though this has errors likely to exceed ten percent*. There have been, however, two experimental and two graphical determinations of the geometric factor, and, subsequently, Dwight and his associates have worked out rigid equations for the geometric factors of two and three-conductor belted cables[23 40]. With these equations the geometric factor can be computed to even a higher degree of accuracy than is warranted by the facts, since there is an essential non-uniformity of cables in view of their construction with conductor insulation, fillers, and belt. For all practical purposes, however, the graphical and experimental solutions are in such close agreement that our data will be based on them.

The geometric factors of a single-conductor cable and a multi-conductor type H cable have been defined. For a multi-conductor belted cable such as a three-conductor cable, capacity, insulation resistance, etc., are not definite quantities characteristic of a cable, but are quantities which depend on the connections. For three-conductor belted cables there are nine different capacities depending on how the measurement is made, and the same with the other quantities, i.e., capacity of one conductor against the other two and the sheath, of three conductors versus the sheath, etc., etc. There will therefore be a geometric factor corresponding to each of these connections. The geometric factor for the three conductors connected as one electrode and the sheath as the other will be defined as G_1, and the geometric factor under normal three-phase working conditions will be indicated as G_2. All of these various capacities and geometric factors for the other conditions, such as one conductor versus the other two and the sheath, etc., are interrelated, however, and may be

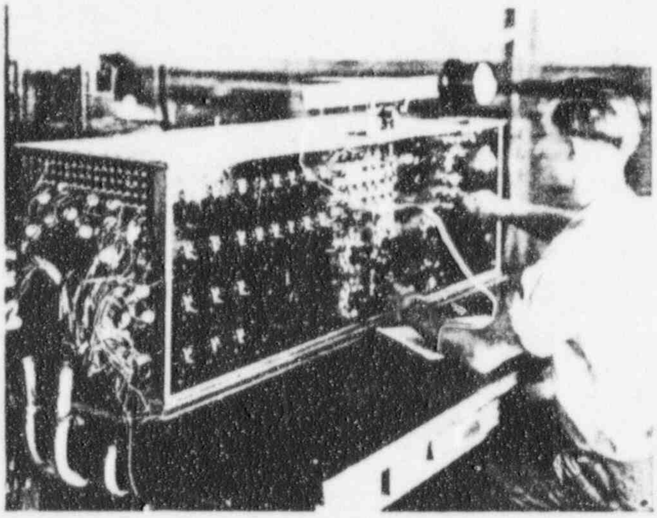
*See references 21 and 22 for a full discussion of these features.



A bank of impregnating tanks. A battery of carefully constructed and maintained tanks operated with strict temperature and electrical control during drying and impregnation is a vital factor in cable manufacture

expressed for three-conductor belted cables in terms of G_1 and G_2 as follows, A, B, and C representing the three conductors:

$$\left.\begin{array}{ll}
G_1 = & \text{geometric factor, } A, B, \text{ and } C \text{ vs. sheath, from Fig. 2} \\
G_2 = & \text{geometric factor, three-phase operation, from Fig. 2} \\
G_3 = & \text{geometric factor, } A \text{ vs. } B \text{ (} C \text{ and sheath floating or connected to midpoint of transformer)} = 2G_2 \\
G_4 = & \text{geometric factor, } A \text{ vs. } B \text{ and } C = 1.5\,G_2 \\
G_5 = & \text{geometric factor, } A \text{ vs. sheath (} B \text{ and } C \text{ insulated)} \\
 = & \dfrac{3G_1 + 2G_2}{3} \\
G_6 = & \text{geometric factor, } A \text{ vs. } B \text{ and sheath (} C \text{ insulated)} \\
 = & \dfrac{G_2\,(6G_1 + G_2)}{3G_1 + 2G_2} \\
G_7 = & \text{geometric factor, } A \text{ vs. } B, C \text{ and sheath} \\
 = & \dfrac{9G_1\,G_2}{6G_1 + G_2} \\
G_8 = & \text{geometric factor, } A \text{ and } B \text{ vs. sheath (} C \text{ insulated)} \\
 = & \dfrac{6G_1 + G_2}{6} \\
G_9 = & \text{geometric factor, } A \text{ and } B \text{ vs. } C \text{ and sheath} \\
 = & \dfrac{4.5\,G_1G_2}{3G_1 + 2G_2}
\end{array}\right\} \quad (17)$$



Accurate measurement of dielectric loss on cable samples from two or three inches long to full-reel lengths at power-factors as low as 0.01% and at voltages as high as 220-kv are all within the range of this high-tension bridge

In Fig. 2, G_1 is given for 1, 2, 3, and 4-conductor cables, and G_2 for three-conductor belted cables. The data for G_1 are based on the graphical solution[21] of the writer. The values of G_2 for three-conductor cables are based on Atkinson's experiments, no corresponding values being available for two-conductor and four-conductor cables. For the single-conductor cables there is only one geometric factor, namely $G = \log_e (D_1/d)$, given for convenience in Fig. 2.

The geometric factor for a three-conductor belted cable with sector conductors is smaller than the geometric factor for a round-conductor cable with the same size of conductor and insulation thicknesses. The sector geometric factor for multi-conductor belted cables can be obtained by finding the geometric factor for a cable with the same makeup with round conductors, and multiplying it by the ordinate obtained from the bottom curve of Fig. 2[24], due to Atkinson. The accuracy of the sector correction factor is believed to be fairly satisfactory, although there are wide differences in sector shape. The original experimental data does not extend below the ratio of 0.6; the extrapolation below is, however, justified by the fact that the theoretical maximum and minimum limits of geometric factor are not very far apart, and the curve falls directly in the range.

Having the geometric factors available from Fig. 2 and Table VIII, it is immediately possible to calculate the various quantities referred to above by the following equations, for cables having from one to four conductors:

$$\text{Capacity, } C = \frac{0.0169nk}{G} \text{ microfarads per 1000 feet} \quad (18)$$

$$\text{Thermal Resistance, } R_{th} = \frac{0.00522\rho G_1}{n} \text{ thermal ohms per foot} \quad (19)$$

$$\text{Insulation Resistance, } R_i = \frac{0.989\rho_1 G \times 10^{-6}}{n} \text{ megohms per mile} \quad (20)$$

$$\text{Charging Current, } I = \frac{0.106Efnk}{G} \text{ milliamperes per 1000 feet} \quad (21)$$

$$\text{Three-Phase Dielectric Loss, } W_{p_3} = \frac{0.000106e^2fn^2k\ \cos\phi}{G_2} \text{ watts per foot of cable} \quad (22)$$

In these equations n is the number of conductors in a cable, k is the specific inductive capacity of the insulation (see Appendix A), ρ is the thermal resistivity of the insulation in watt-cm. units, ρ_1 is the electrical resistivity of the insulation in megohm-cm. units, E and e are the voltages between conductors and to neutral in kilovolts respectively, and $\cos\phi$ is the power-factor of the insulation for a given temperature and frequency.

In the above equations any of the quantities can be found for any method of connection for three-conductor belted cables by substituting the correct G. The dielectric loss has been shown for three-phase voltage only, G_2 having been shown. The same equation, however, can be used for calculating the losses in three-conductor belted cables in other connections if the proper G is used and the constant is divided by 3. It is important to note that in the above equations, if the constants under three-phase voltage are desired, G_2 being used, E must be the voltage e to neutral, and the resistance capacity, etc., will be to neutral.

In using eqs. (18), (20), (21), (22), or (23) for handling three-conductor type H cable, the geometric factor for three conductors against the sheath is taken equal to G, and the geometric factor under three-phase voltage is taken equal to $3G$ where G is the geometric factor for a single-conductor cable of the same size of conductor and an insulation thickness equal to that on each conductor of the type H cable. This manner of dealing with type H cable avoids the necessity of interpreting n for the connection employed and is readily shown to be identical to a single-conductor method of treatment. Equation (19) is used as given for type H three-conductor cable, the proper G_1 being obtained directly from Table VIII.

It will be noted that the geometric factor G_1 has been used in the expression for thermal resistance. This is the correct geometric factor to use because heat is generated in the three conductors and flows .....

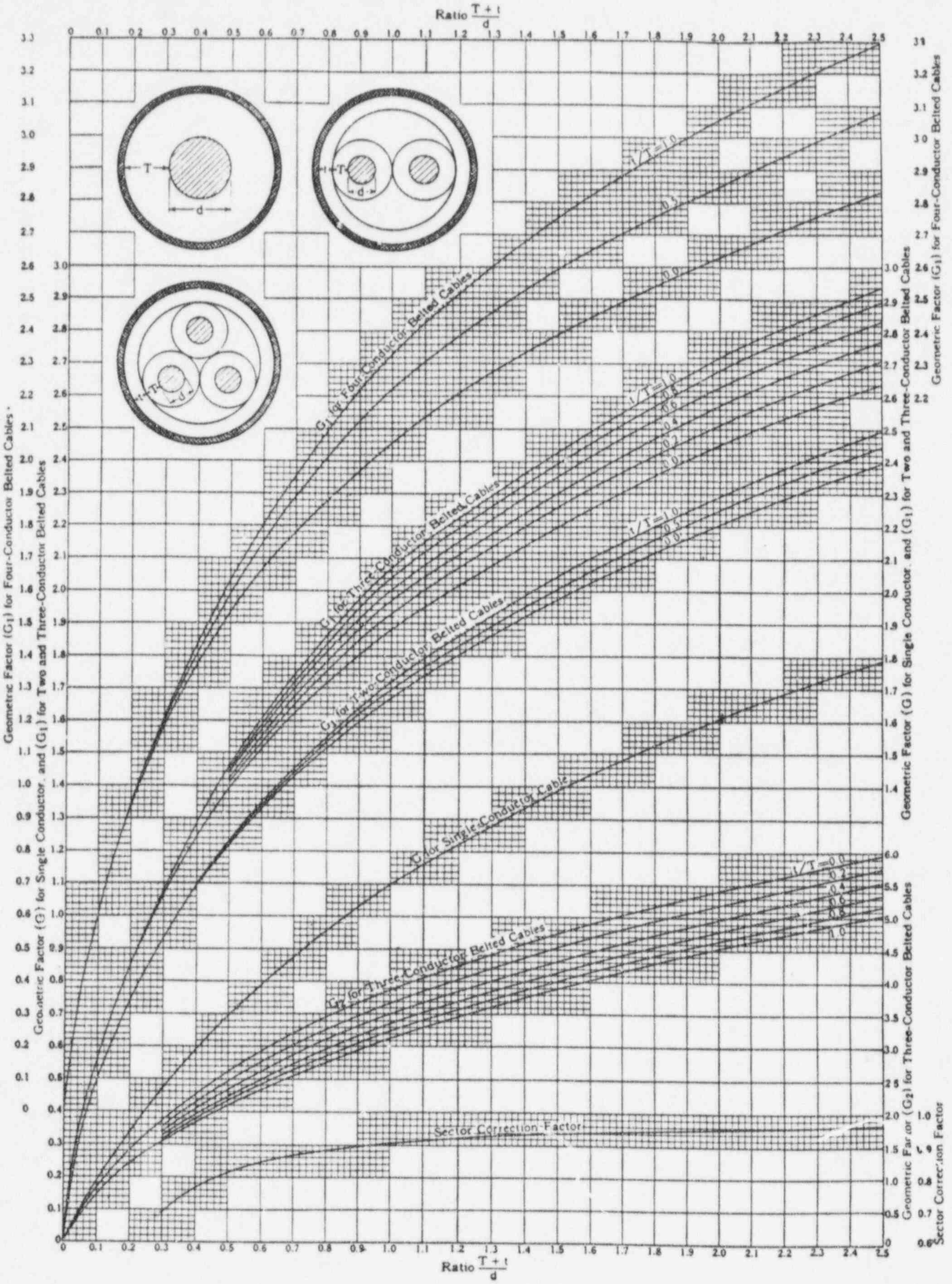


Fig. 2—Geometric factors for single-conductor cable and multi-conductor belted cable with round or sector conductors

Geometric factors can be obtained by calculating the ratios $(T + t)/d$ and t/T (d being defined for sector cables as the diameter of a round conductor of the same area as the sector), and then reading the required value of geometric factor from a curve above. The value thus obtained will be the correct geometric factor for a round-conductor cable. For sector conductors the values so obtained should be multiplied by the sector correction factor. In cables of the non-type H form without belts, such as multi-conductor rubber cables, the ratio becomes T/d, and $t/T = 0$.

TABLE VIII—GEOMETRIC FACTOR (G_1) BETWEEN CONDUCTORS AND SHEATH OF THREE-CONDUCTOR TYPE H CABLE

This geometric factor is to be used in calculating current-carrying capacity and is based on insulation of thermal resistivity of 700* watt-cm. units with wrappings over the insulation of copper tape 3 mils thick.

SECTOR CONDUCTORS

Size of Cond. AWG or C.M.	Insulation Thickness in 32nd Inches																
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1/0	0.54	0.63	0.72	0.81	0.88	0.96	1.03	1.09	1.15	1.21	1.26	1.31	1.36	1.40	1.45	1.49	1.53
2/0	0.50	0.58	0.67	0.75	0.82	0.89	0.96	1.02	1.07	1.13	1.18	1.23	1.28	1.32	1.37	1.41	1.45
3/0	0.45	0.54	0.62	0.69	0.76	0.83	0.89	0.95	1.00	1.05	1.10	1.15	1.20	1.24	1.29	1.33	1.36
4/0	0.41	0.49	0.57	0.63	0.70	0.76	0.82	0.87	0.93	0.98	1.03	1.08	1.12	1.16	1.20	1.24	1.28
250 000	0.39	0.46	0.53	0.60	0.66	0.72	0.78	0.83	0.89	0.93	0.98	1.03	1.07	1.11	1.15	1.19	1.23
300 000	0.37	0.44	0.50	0.56	0.62	0.68	0.73	0.78	0.84	0.88	0.93	0.97	1.01	1.05	1.09	1.13	1.17
350 000	0.35	0.41	0.48	0.54	0.60	0.65	0.70	0.75	0.80	0.84	0.89	0.93	0.97	1.01	1.05	1.08	1.12
400 000	0.33	0.40	0.46	0.51	0.57	0.62	0.67	0.72	0.76	0.80	0.85	0.89	0.93	0.97	1.01	1.04	1.08
450 000	0.31	0.38	0.44	0.49	0.55	0.60	0.64	0.69	0.74	0.78	0.82	0.86	0.90	0.94	0.97	1.01	1.04
500 000	0.30	0.37	0.43	0.48	0.53	0.58	0.63	0.67	0.72	0.76	0.80	0.84	0.87	0.91	0.94	0.98	1.01
600 000	0.29	0.34	0.40	0.45	0.50	0.54	0.58	0.63	0.67	0.71	0.75	0.79	0.82	0.86	0.89	0.93	0.96
700 000	0.27	0.32	0.37	0.42	0.47	0.51	0.55	0.60	0.64	0.68	0.72	0.75	0.79	0.82	0.85	0.88	0.91
800 000	0.26	0.31	0.36	0.41	0.45	0.49	0.53	0.57	0.61	0.65	0.68	0.72	0.75	0.79	0.82	0.85	0.88

ROUND CONDUCTORS

Size of Cond. AWG or C.M.	Insulation Thickness in 32nd Inches																
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1/0	0.61	0.71	0.81	0.90	0.98	1.07	1.14	1.21	1.28	1.34	1.41	[illegible]	1.52	1.57	[illegible]	1.67	1.71
2/0	0.57	0.67	0.76	0.85	0.93	1.00	1.07	1.14	1.20	1.26	1.32	1.38	1.44	1.49	1.51	1.59	1.64
3/0	0.53	0.63	0.71	0.79	0.87	0.94	1.01	1.08	1.14	1.20	1.25	1.31	1.36	1.41	[illegible]	1.51	1.56
4/0	0.50	0.59	0.67	0.74	0.82	0.89	0.95	1.02	1.07	1.13	1.18	1.24	1.29	1.34	[illegible]	1.44	1.49
250 000	0.48	0.56	0.64	0.71	0.78	0.85	0.91	0.97	1.03	1.08	1.14	1.19	1.24	1.29	1.34	1.38	1.43
300 000	0.46	0.54	0.61	0.68	0.75	0.81	0.87	0.93	0.98	1.04	1.09	1.14	1.19	1.24	1.29	1.33	1.38
350 000	0.44	0.52	0.59	0.65	0.72	0.78	0.84	0.90	0.95	1.01	1.06	1.11	1.15	1.20	1.24	1.29	1.33
400 000	0.43	0.50	0.57	0.63	0.70	0.76	0.82	0.87	0.92	0.98	1.03	1.07	1.11	1.16	1.20	1.25	1.29
450 000	0.42	0.49	0.55	0.62	0.68	0.74	0.79	0.85	0.90	0.95	1.00	1.05	1.09	1.13	1.17	1.22	1.26
500 000	0.41	0.48	0.54	0.60	0.66	0.72	0.78	0.83	0.88	0.93	0.98	1.02	1.06	1.11	1.15	1.19	1.23

*While not strictly so, the thermal resistance of type H cable is closely proportional to the thermal resistivity, so that the above geometric factors may be used for other resistivities with a reasonable degree of accuracy.

formly out toward the lead sheath, the lines of heat flow being identical with the lines of current flow if voltage is impressed between the three conductors as one electrode and the sheath as the other, for which condition G_1 has been defined as the geometric factor.

The other two of the four fundamental quantities, namely, capacity and leakage, can now be calculated. The capacity may be calculated directly by eq. (18), it being noted that the 60-cycle capacity will vary only slightly with voltage or temperature and can practically be considered as independent of both.

The insulation resistance is given by eq. (20). This, of course, means the insulation resistance as measured by direct-current which is of such a high value that leakage is ordinarily negligible. The alternating-current conductance or leakage, however, is the quantity g generally used in transmission-line calculations. This can be calculated by considering that the dielectric loss is due to a hypothetical resistance in parallel with the capacity of the cable. It can be considered that in each phase there is a dielectric loss equal to the square of the voltage times the conductance. In the n phases, there will be n times this loss, which will equal the dielectric loss, and from the above relationship the leakage can be determined:

$$\text{Leakage } g = \frac{0.106 f k n \cos \phi}{G_2} \times 10^{-6} \text{ mhos per 1000 feet to neutral} \quad (23)$$

The calculation of resistance, inductance, capacity, and leakage has been shown. In calculating the performance of a line, such as regulation and efficiency, and the usual problems of power transmission, after these four quantities have been determined, the cable problem is a straight transmission problem no different from that of an overhead line. Voltage drop, however, is rarely a determining factor in limiting the allowable current in cable circuits for transmission, though, of course, regulation is the important criterion in the case of low-tension mains and feeders. For transmitting alternating current, a cable system has much better regulation than an open-wire circuit because the conductors are so close together, as compared with an aerial line, that the reactive drop is much less.

It is sometimes interesting to calculate the voltage rise of the cable system at no load, which can be done by using the above constants, though most cable lines are so short that this effect is not great. As a matter of fact, the ordinary transmission line calculation is not the usual one made with cables. The above quantities are used not so much in the normal way as in connection with the allowable current as limited by temperature rise. The capacity and charging current, however, are often of practical importance, not so much from their effect on the voltage characteristics of the cable itself as from the effect of the leading kv-a upon the reactors, transformers, generators, and other station equipment.

In addition, the apparent alternating-current resistance and reactance are of considerable importance in determining whether the current will divide as expected between parallel cables of different types and sizes. It is in this connection particularly that refinements in reactance calculations are justified.