SECOND SYMPOSIUM
ON
EARTHQUAKE ENGINEERING

Sponsored by the

UNIVERSITY GRANTS COMMISSION

SCHOOL OF RESEARCH AND TRAINING
IN
EARTHQUAKE ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE (U.P.) INDIA

(November 10-12, 1962)

EARTHQUAKE RESISTANT DESIGN OF RETAINING WALLS

I. P. Kapila *

SYNOPSIS

This paper relates the determination of active and passive pressures exerted by cohesionless meterial under earthquake conditions. The introduction of earthquake inertia forces to Coulomb's sliding wedge yields the Mononobe-Okabe formula. Modifications of known graphical methods to include the effect of earthquake forces are presented in this paper.

INTRODUCTION

The pressure, both active and passive, exerted by cohesionless material against a retaining structure in the absence of earthquake conditions may be determined by the well known Coulomb's Wedge Theory. The earthquake problem introduces an inertia force corresponding to the acceleration imparted to the mass of the sliding wedge. The solution of the resulting force diagram yields the pressure exerted against the the structure which can be expressed by the Mononobe-Okabe Formula (1, 2). Several graphical methods are available for pressure determination from Coulomb's Wedge Theory. Modification of two such methods are presented in this paper to include the effect of eathquake force.

ACTIVE PRESSURE

Coulomb's Formula : In Fig 1 (a). AB is the fill-side face of the retaining structure, inclined at an angle α to the vertical. The surface of the fill AD is inclined to the horizontal at an angle δ. If the retaining structure is removed, the material of the fill will settle finally at the angle of repose which is close to the angle of internal friction, represented by the plane BD. According to the Coulomb's Wedge Theory, however, when the retaining structure moves forward only slightly, the surface of rupture will not form at plane BD immediately. Instead, the plane of rupture will lie somewhere between AB and BD. It may be assumed to correspond to the plane BC in Fig. 1 (a). The active pressure against the retaining structure is caused by the weight of the wedge ABC (assuming earthquake force to be absent) and it is desired to determine the location of the plane BC which will cause the maximum reaction against the face AB.

* Director, Bhakra Beas Dam Design Directorate, New Delhi.

The reaction against the face AB will be inclined at an angle ϕ_1 to the normal, where ϕ_1 is the angle of wall friction between the cohesionless material and the face of the retaining structure. The reaction against the plane BC will be inclined at an angle ϕ to the normal.

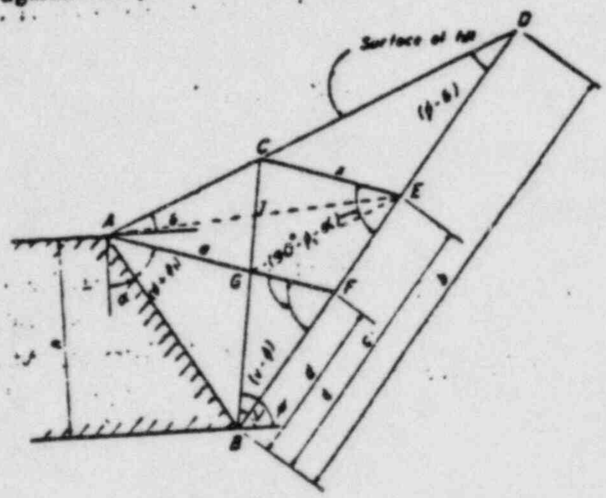


Fig. 1 (a)

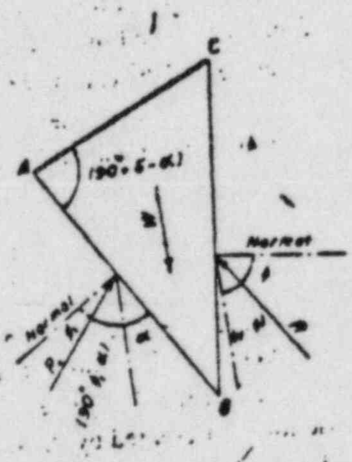


Fig. 1 (b)

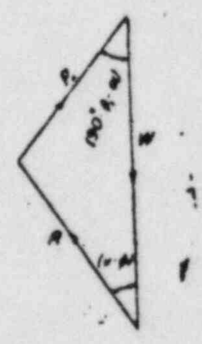


Fig. 1 (c)

If these two reactions are designated P_A and R respectively and the weight of wedge is W, P_A may be determined from the triangle of forces Fig. 1 (c). If AF and CE in Fig. 1 (a) are drawn such that each makes an angle $(90° - \phi_1 - \alpha)$ with BD, it has been shown (3, 4) that the maximum active pressure would result when

$$\triangle ABC = \triangle BCE \tag{1}$$

and $$x = \frac{a\sqrt{b}}{\sqrt{b} + \sqrt{d}} \tag{2}$$

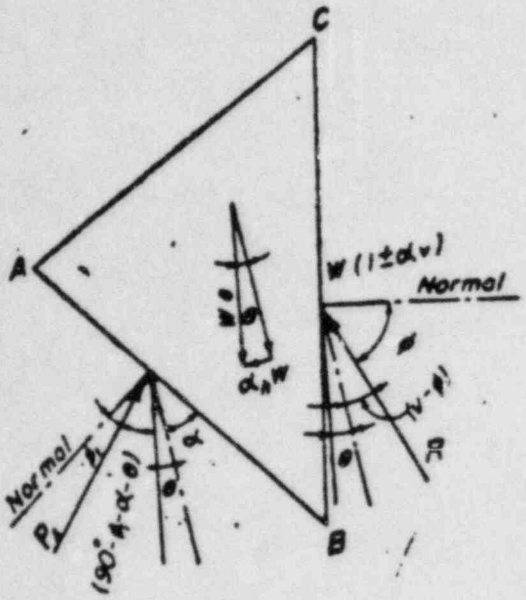


Fig. 2 (b)

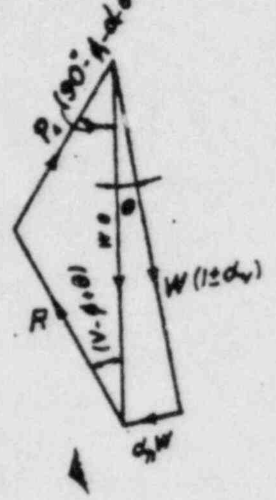


Fig. 2 (c)

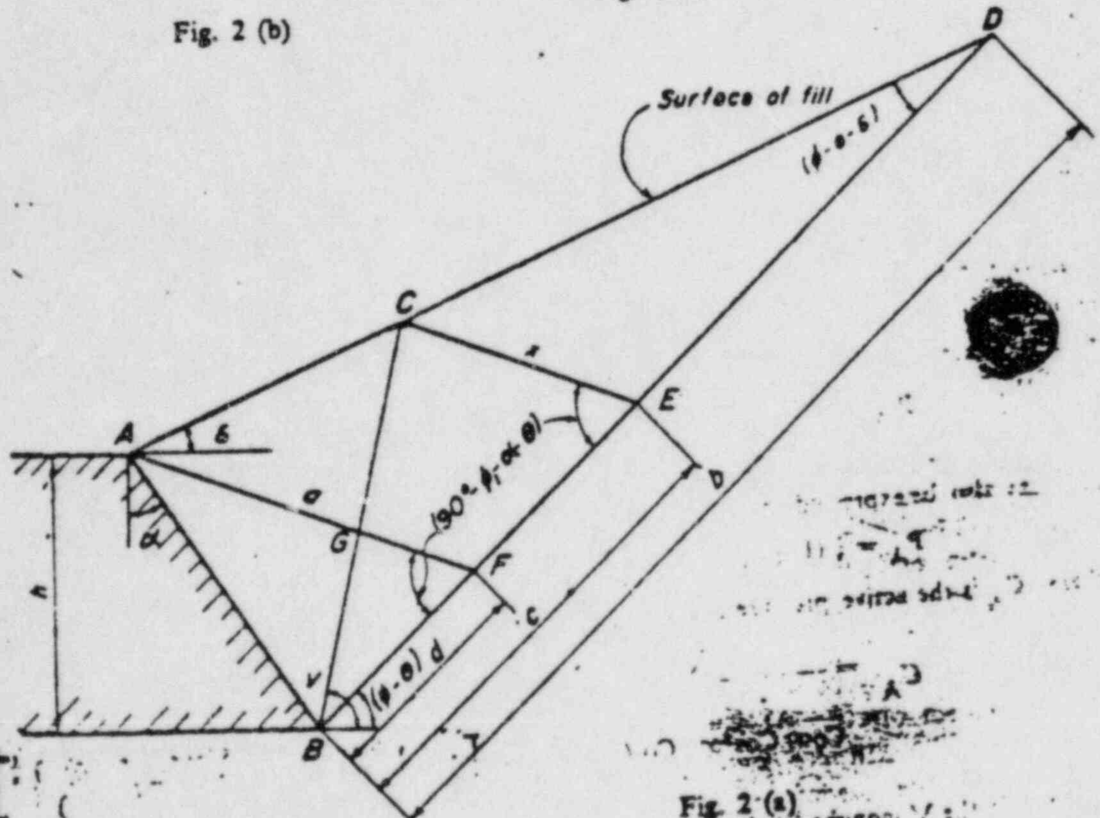


Fig. 2 (a)

the mazimum active pressure would be

$$= \frac{1}{2}\gamma x^2 \,\text{Sin}\,[90-(\alpha + \varnothing_1 \quad +) \tag{3}$$

where γ is the unit weight of the cohesionless material. This may also be expressed as

$$P_A = \frac{\frac{1}{2}\gamma . h^2 \;\text{Cos}^2\,(\varnothing-\alpha)}{\text{Cos}^2\alpha .\; \text{Cos}(\varnothing_1+\alpha)\left[1+\left\{\frac{\text{Sin}(\varnothing+\varnothing_1)\,\text{Sin}(\varnothing-\delta)}{\text{Cos}(\varnothing_1+\alpha)\,\text{Cos}(\delta-\alpha)}\right\}^{\frac{1}{2}}\right]^2} \tag{4}$$

which is the Coulomb's Formula for active pressure.

The Mononobe-Okabe Formula : For earthquake conditions, additional inertia forces $\alpha_h W$ and $\pm\,\alpha_v W$ would be imposed on the mass of the sliding wedge (Fig. 2) where α_h and α_v are the horizontal and vertical seismic coefficients respectively. The resultant force, W_e, is thus

$$W_e = \frac{(1\pm\alpha_v)\,W}{\text{Cos}\,\theta} \tag{5}$$

where $$\theta = \tan^{-1}\frac{\alpha_h}{1\pm\alpha_v} \tag{6}$$

According to the Mononobe's theory, the effect of the accelerations due to an earthquake is to modify the direction of the gravity force, which would be equivalent to a rotation of the vertical and horizontal planes of reference through an angle θ in the same direction. This effect may be introduced by plane BD making an angle $(\varnothing-\theta)$ with the horizontal as indicated in Fig. 2(a). The resulting force polygon would be as shown in Fig. 2 (c)

If AF and CE in Fig. 2 (a) are drawn such that each makes an angle $(90-\varnothing_1-\alpha-\theta)$ with BD, it can be shown that the maximum active pressure would result when the requirements of Eq. (1) and Eq. (2) as for the non-seismic condition are satisfied. The maximum active pressure would be

$$P_A = \frac{1}{2}\,\frac{(1\pm\alpha_v).\gamma}{\text{Cos}\,\theta}\,x^2 .\,\text{Sin}\,[\,90-(\alpha+\varnothing_1+\theta\,)\,] \tag{7}$$

This may also be expressed as

$$P_A = \frac{1}{2}\,(1\pm\alpha_v)\,\gamma h^2\;C_A \tag{8}$$

where C_A is the active pressure coefficient

$$C_A = \frac{\text{Cos}^2\,(\varnothing-\alpha-\theta)}{\text{Cos}\theta\;\text{Cos}^2\alpha .\;\text{Cos}(\varnothing_1+\alpha+\theta)\left[1+\left\{\frac{\text{Sin}(\varnothing+\varnothing_1)\;\text{Sin}(\varnothing-\delta-\theta)}{\text{Cos}(\varnothing_1+\alpha+\theta)\;\text{Cos}(\delta-\alpha)}\right\}^{\frac{1}{2}}\right]^2}$$

This is the Mononobe-Okabe Formula for active pressure.

5

GRAPHICAL METHOD

Culmann's Method

Culmann's graphical method for the determination of active pressure is of more general application than sevral others described in text-books on Soil Mechanics. It essentially comprises the construction of the triangle of forces for each of the several assumed planes of rupture. The vector representing the weight of the wedge is plotted on plane BD, Fig. (1). The vector representing the active pressure is then drawn so as to make an angle $(90-\phi_1-\alpha)$ with the plane BD. The triangle of forces is thus that which is formed by the assumed plane of rupture, the plane BD and the active pressure vector drawn as indicated above. The locus of the point of intersection of the active pressure vector with the corresponding plane of rupture is designated as the "Culmann Line". The maximum distance between plane BD and the tangent to the Culmann Line drawn parallel to the plane BD, measured parallel to the active pressure vector is the active pressure to the force scale adopted in the construction.

The modifications necessary in Culmann's graphical construction to include the effect of earthquake forces would be readily apparent from the preceding discussion regarding the Mononobe—Okabe Formula. These are:—

i) The plane BD is to be drawn such that it is inclined at an angle $(\phi-\theta)$ to the horizontal.

ii) The active pressure vector is to be drawn at an angle $(90-\phi_1-\alpha-\theta)$ with the plane BD.

It would be seen that the resulting triangle of forces conforms to that shown in Fig. 2(c) This modification is illustrated in the example on Fig. 3.

Melbye's Method

Another graphical method based on Coulomb's Wedge Theory has been proposed by A. Melbye (4). This is directed towards drawing the plane of rupture such that the requirments of Eq. (1) are met, and then scaling off the length 'x' to the linear scale of the construction. In Fig. 1, if EG is drawn parallel to AD, then N must lie on the plane of rupture for Eq. 1 to be satisfied. Furthermore, it can be shown that the locus of the point 'I' (which is the intersection of the diagonals GC and AE of the parallelogram ACEG) will be parallel to BD and will bisect AB. The graphical construction, therefore, merely requires drawing a line which bisects AB and is parallel to BD, and then determinining by trial the plane of rupture BC which yields GI = IC. The modifications necessary to include the effect of earthquake forces would be,

(i) The plane BD is to be drawn so as to make an angle $(\phi-\theta)$ wit the horizontal

(ii) AF is to be drawn to make an angle $(90-\phi_1-\alpha-\theta)$ with BD.

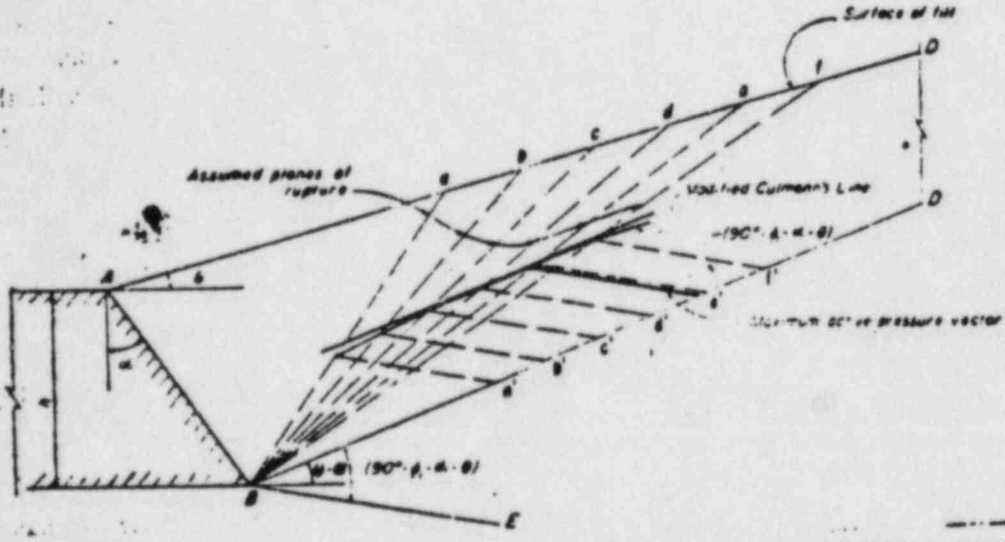
It would be seen that the requirements of Eq. (1) remain unchanged under the earthquake conditions. The value of 'x' as determined by this construction can be substituted into Eq. (7) to yeild maximum active pressure. The application of this method is illustrated in Fig. 4.

DATA :

$h = 100$ ft.
$\alpha = 35°$
$\phi = 30°$
$\phi_1 = 15°$
$\delta = 15°$
$\alpha_h = 0.15$ (acceleration away from retaining structure)
$\alpha_v = 0.05$ (acceleration downwards)
$\theta = 9°$
$\gamma = 132$ lbs./cft.

Maximum active pressure vector = 930 kips

$$P_A = \frac{(1-0.05)}{\cos 9°} \cdot 930 \text{ kips}$$
$$= 894.5 \text{ kips}$$



PROCEDURE

1. Draw BD to make an angle $(\phi - \theta)$ with the horizontal.
2. Assume planes of rupture Ba, Bb etc., and compute weight of wedge ABa, ABb etc. and plot on any convenient force scale Ba′, Bb′ on BD.
3. Draw active pressure vectors from a′, b′ etc. at an angle $(90° - \phi_1 - \alpha - \theta)$ with BD to intersect corresponding assumed planes of rupture.
4. Draw the locus of the intersection of assumed planes of rupture and corresponding active pressure vector (The Culmann Line) and determine the active pressure vector parallel to BE.

Fig. 3—Modified Culmann's Construction for Active Pressure.

DATA:

$h = 100$ ft.

$\alpha = 35°$

$\varnothing = 30°$

$\varnothing_1 = 15°$

$\delta = 15°$

α_h = Acceleration away from retaining structure = 0.15

α_v = Acceleration downwards = 0.05

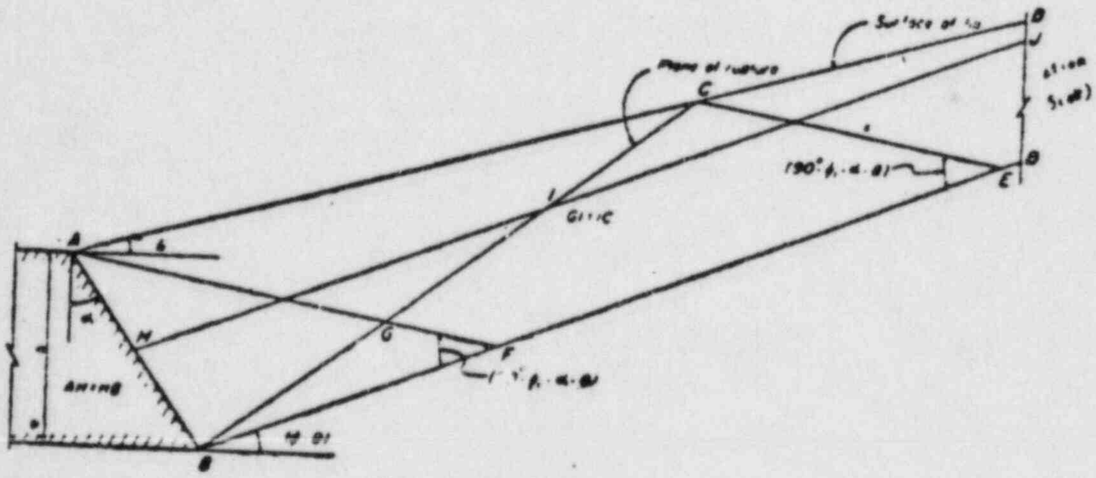
$\theta = 9°$

γ = 132 lbs./cft.

$$P_a = \tfrac{1}{2}(1 \pm \alpha_v)\gamma x^2 \frac{\text{Cos.}(\varnothing_1 + \alpha + \theta)}{\text{Cos}\,\theta}$$

$$= \tfrac{1}{2}(1-0.05).\ 132.\ (165.5)^2 \frac{\text{Cos. } 59°}{\text{Cos. } 9°} \text{ lbs.}$$

$$= 895.6 \text{ kips}$$



PROCEDURE

1. Draw BD to make an angle $(\varnothing - \theta)$ with the horizontal.
2. Bisect AB, AH = BH.
3. Draw HJ parallel to BD.
4. Draw AF to make an angle $(90° - \varnothing_1 - \alpha - \theta)$ with BD.
5. Determine plane of rupture BC by trial such that GI = IC.
6. Draw CE parallel to AF and measure length to linear scale of construction.

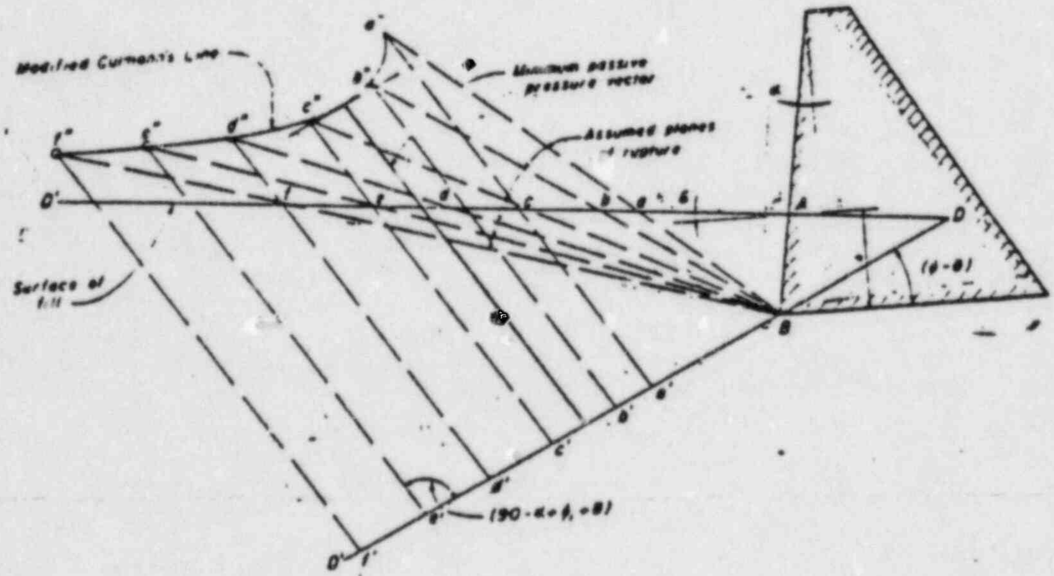
Fig. 4*—Modified Melbye's Construction for Active Pressure.

* Based on paper "Determining Active Thrust on the back of a Wall Retaining Cohesion-less Material" by A Melbye, Civil Engineering & Public Works Review Vol. 56, No. 660, July 1961.

PASSIVE PRESSURE

Coulomb's Formulae:—The determination of the passive pressure follows the same basic considerations as for active pressure, with the main point of difference being that the plane of rupture be such as will result in the minimum pressure against the retaining structure. The plane BD shown in Fig. 1 and 2 to be above the horizontal plane for the active pressure condition would be below the horizontal plane for passive pressure and horizontal earthquake inertia forces would have to be assumed acting away from the structure and not towards the structure.

$$P_p = \frac{1}{\cos\theta} \text{ (Minimum Passive Pressure Vector) } (1 \pm \alpha_v)$$

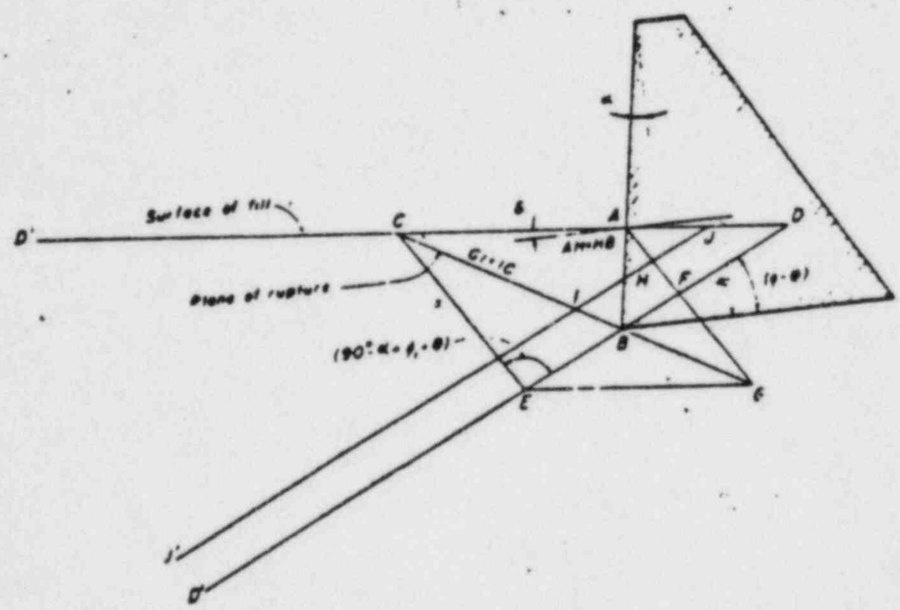


PROCEDURE

1. Draw DBD′ to make angle $(\varnothing - \theta)$ with horizontal.
2. Assume planes of rupture Ba, Bb, etc. and compute weight of wedge ABa, ABb, etc. and plot Ba′, Bb′ to any convenient force scale on DBD′
3. Draw passive pressure vectors from a′, b′ etc. at an angle $(90 - \alpha + \varnothing_1 + \theta)$ to intersect corresponding assumed planes of rupture.
4. Draw the locus of the intersection of the assumed rupture planes and the corresponding passive pressure vectors (The Culmann Line) and determine the minimum passive pressure vector.

Fig. 5—Modified Culmann's Construction for Passive Pressure.

$$P_p = \frac{1}{2} \gamma . x^2 . \frac{\text{Cos.} (\varnothing_1 - \alpha + \theta)(1 \pm \alpha_v)}{\text{Cos}\, \theta}$$



PROCEDURE

1. Draw DBD′ to make an angle $(\varnothing - \theta)$ with the horizontal.
2. Bisect AB, AH = BH.
3. Draw JHJ parallel to DBD′.
4. Draw AFG to make angle $(90° + \varnothing_1 - \alpha + \theta)$ with DBD′
5. Determine plane of rupture BC by trial such that GI = IC.
6. Draw CE parallel to AG and measure length to linear scale of construction.

Fig. 6—Modified Melbye's Construction for Passive Pressure.

The analytical solution for passive pressure due to cohesionless material would show that the requirements of Eq. (1) continue to hold and that whether or not earthquake forces are included in the analysis. For normal conditions, without earthquake, the passive pressure is

$$P_p = \frac{1}{2} \gamma x^2 \text{Cos} (\alpha - \varnothing_1) \qquad (10)$$

* Based on paper, "Determining Active Thrust on the back of a Wall Retaining Cohesion-less Material" by A. Melbye, Civil Engineering & Public Works Review Vol. 56, No. 660, July 1961.

This may also be expressed as

$$P_p = \frac{\frac{1}{2}\,\gamma . h^2 \, Cos^2 (\alpha + \emptyset)}{Cos^2 \alpha \; Cos(\emptyset_1 - \alpha)\left[1 - \left\{\frac{Sin(\emptyset - \emptyset_1)\; Sin(\emptyset + \delta)}{Cos(\emptyset_1 - \alpha)\; Cos(\alpha - \delta)}\right\}^{\frac{1}{2}}\right]^2} \qquad (11)$$

The Mononobe–Okabe Formula

When earthquake inertia forces are included, the passive pressure is

$$P_p = \tfrac{1}{2}(1 \pm \alpha_v)\frac{\gamma . x^2}{Cos\,\theta}\; Cos.\,(\alpha - \emptyset_1 - \theta) \;\ldots \quad \ldots \quad \ldots \quad \ldots \qquad (12)$$

Where θ is as defined in Eq. (6).

This may also be expressed as

$$P_p = \tfrac{1}{2}\,(1 \pm \alpha_v)\,\gamma . h^2 . C_p \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (13)$$

where C_p is the passive pressure coefficient

$$C_p = \frac{Cos^2(\emptyset + \alpha - \theta)}{Cos\theta . \; Cos^2\alpha . \; Cos(\emptyset_1 - \alpha + \theta)\left[1 - \left\{\frac{Sin(\emptyset + \emptyset_1)\; Sin(\emptyset + \delta - \theta)}{Cos(\delta - \alpha)\; Cos(\emptyset_1 - \alpha + \theta)}\right\}^{\frac{1}{2}}\right]^2} \qquad (14)$$

This is the Mononobe–Okabe Formula for passive pressure.

GRAPHICAL METHODS

The Culmann and Melbye graphical methods for passive pressure determination follow the same basis described earlier, and the modifications considered necessary for active pressure would apply for including the effects of earthquake forces. The application of these two graphical methods for earthquake resistant design is indicated in Figs. 5 and 6.

REFEENCES;

1. Mononobe, N. and H. Matsuo, (1929). "On Determination of Earth Pressure During Earthquakes" Proceedings World Engineering Congress, Tokyo, 1929.

2. Okamoto, S. (1956) "Bearing Capacity of Sandy Soils and Lateral Earth Pressure during Earthquake". Proceedings of the World Conference on Earthquake Engineering, Berkeley, Calif. June 1956.

3. Kapila, I.P., and S.S. Maini, (1962). "Active Pressure Exerted by Cohesionless Materials under Earthquake Conditions." Indian Concrete Journal, Vol. 36, No. 5, May 1962.

4. Capper, P.L. and W.F. Cassie, (1962). "The Mechanics of Engineering Soils" E and F.N. Spon, London.

5. Melbye, A., (1961) "Determining the Maximum Active Thrust on the Back of a Wall Retaining Cohesionless Material" Civil Engineering and Public Works Review Vol. 56. No. 660. July 1961.