ATTACHMENT 3
ST-HL-AE-1003

South Texas Project

Probabilistic Evaluation of Tornado Missile Hazard to the Containment Isolation Valve Compartment Equipment

14926-001

Risk/Reliability Group
Los Angeles Power Division
Bechtel Power Corporation

August 1983

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure		Page

SOUTH TEXAS PROJECT TORNADO MISSILE EVALUATION REPORT

I. Introduction

This study uses the Probabilistic Risk Assessment (PRA) methodology to evaluate the probability of damage to equipment located in the containment isolation valve cubicle (IVC) from tornado-generated missiles. Tornado-generated missiles include objects, on or near the plant site, that could become airborne during a tornado and be transported to the top of the IVC.

The study includes an evaluation of the likelihood of a tornado occurrence, as well as the probability of tornado-generated missiles entering through the top of the IVC. The extent of damage is not evaluated, but is conservatively assumed to be certain and total for all missile strikes.

II. Acceptance Criteria

The NRC's acceptance criteria are contained in the "General Design Criteria (GDC) for Nuclear Plants" [1]. Specifically, GDC 2 and 4 apply to this evaluation and are summarized below:

a. GDC 2 requires that "Structures, systems, and components important to safety shall be designed to withstand the effects of natural phenomena such as - tornadoes - without loss of capability to perform their safety functions . . ."

b. GDC 4 requires that ". . . structures, systems, and components shall be appropriately protected against dynamic effects, including the effects of missiles,. . . from events and conditions outside the nuclear power unit."

The Standard Review Plan (SRP) [2] Section 3.5.1.4 and NRC Regulatory Guide 1.76 [3] provide further guidance in meeting GDC 2 and 4 requirements. Specifically, SRP Section 3.5.1.4 refers to the acceptance criteria of SRP 2.2.3 which states ". . . design basis events include each postulated type of accident for which the expected rate of occurrence of potential exposures in excess of the 10 CFR Part 100 guidelines is estimated to exceed the NRC staff objective of approximately 10^{-7} per year . . . expected rate of occurrence of potential exposures in excess of the 10 CFR 100 guidelines of approximately 10^{-6} per year is acceptable if, when combined with reasonable qualitative arguments, the realistic probability can be shown to be lower. . ."

III. Summary

The probability of failure of the equipment located in the IVC to perform its safety function in the event of a tornado is evaluated using the PRA methodology. The study quantifies the probability of tornado-generated

missiles entering through the top of the IVC. Since there is considerable uncertainty in all factors, probability distributions are propagated throughout the analysis. The results are compared to the NRC acceptance criteria. The results indicate that tornado-generated missiles are not a significant threat to the IVC equipment. The results further indicate that no physical barriers are required at the top of the IVC.

IV. Analysis Approach

The probability of damage to equipment located in the IVC depends on three factors:

A. The tornado occurrence rate at the plant site,

B. The conditional probability of one or more tornado-generated missiles striking the top of any one of four IVCs, given the tornado occurrence, and

C. The conditional probability of IVC equipment being damaged, given that the tornado-generated missile or missiles have entered the IVC.

The tornado occurrence rate is based on the National Weather Service record of tornado strikes for the site region between 1953 and 1982 [4]. The probability of tornado occurrence at the plant site and its contributors are shown in Table I. The distributions of annual frequency, ν, of tornado occurrences in area, $S = 10000 \text{ mi}^2$, and tornado path area, a, are based on plant specific historical data. These data are in good agreement with nationwide and regional assessments.

The conditional probability of the missile strike, given the tornado occurrence, depends on the following subfactors:

A. The number of potential missiles,

B. The conditional probability of the potential missile becoming airborne (or injected), given the occurrence of a tornado,

C. The conditional probability of missiles being transported from their origin to the target, given that they become airborne, and

D. The target area.

The number of potential missiles is based on data from Electric Power Research Institute (EPRI) surveys at seven nuclear power plant sites [5]. The probability of the potential missiles becoming airborne is calculated using a missile model developed at Jet Propulsion Laboratory (JPL) [6]. The conditional probability of missiles being transported from their origin to the target is based on a statistical mechanics model [7], [8]. This approach develops a modified Green's function to quantify the probability of the airborne missile striking a unit target area at some distance and elevation from its origin. This probability is then

multiplied by the area of the target to get the total probability of strike. The area of the top of each of the four IVCs is 745 square feet (total target area is 2980 square feet). The IVC height is 55 feet above the grade and the grade elevation does not vary significantly within 300 feet of the IVC. The number of potential missiles and the missile density incorporated into this study are shown in Table II.

The conditional probability of IVC equipment being damaged, given that the tornado-generated missile or missiles have entered the IVC, is conservatively taken to be certain and total. That is, the conditional probability is taken as unity.

Each of the above factors has a considerable amount of uncertainty associated with it. For some factors, the uncertainty is associated with the statistical nature of the data, and in others, it is associated with the modeling techniques. For this reason, a probability distribution is used for each factor. These uncertainties are propagated throughout the analysis. Therefore, the final results are not a single value for probability of damage, but a distribution of values. The median (50th) and 95th percentile values are reported in Table III. The median is then compared to the NRC acceptance criteria.

The NRC acceptance criteria values require careful consideration because they are given as point values (10^{-7} and 10^{-6} per year). However, as mentioned above, there are significant uncertainties associated with the probability of damage, ranging over orders of magnitude. The median is often compared to the acceptance criteria value because the median is interpreted as the "best estimate" or "recommended" value [9].

V. Assumptions and Conservatisms

The following assumptions are used in this study:

A. The IVC roof area is assumed to be transparent to tornado missiles. That is, the top of the IVC is assumed to be open and without missile protection of any kind.

B. A tornado missile strike in the open top of any one IVC compartment represents failure (see conservatisms A and D, below).

C. The distribution of potential tornado missiles by number and length are based on an EPRI survey of seven nuclear plants [5].

Conservatisms incorporated in this study are:

A. The comparison of the strike probability to the activity release frequency acceptance criteria, assumes;

1. Missile inflicted damage is certain and total and

2. Damage leads directly to activity releases in excess of 10CFR100.

B. The potential missile model assumes

1. A missile distribution based on EPRI survey maximum,

2. A missile density increased by factor of 2.5 over EPRI survey,

3. One half of missiles are distributed up to 20 feet above grade, remainder at grade, and

4. The number of unrestrained missiles postulated for this study is equal to the total number of missiles (restrained and unrestrained) in the EPRI survey.

C. The tornado frequency is based on a 30-year historical record fitted with a more conservative lognormal distribution having a larger mean and spread than the empirical distribution.

D. Geometric factors that result in further conservatisms are:

1. Sheltering by other structures is neglected,

2. A missile strike in any IVC opening results in failure (i.e, no credit is taken for the existence of redundant components or for separation between safety-related trains), and

3. Safety related target areas are less than IVC open area.

VI. Results and Conclusions

The results of the analysis is a probability distribution for tornado missile damage to the IVC equipment. The median (50th percentile) and upper bound (95th percentile) values are reported in Table III. The median value is 2×10^{-10} and the upper bound is 6×10^{-6} per year. The median or "best estimate" value of 2×10^{-10} per year is very small compared to the NRC acceptance criteria value of 10^{-7} to 10^{-6} per year.

The above results indicate that tornado-generated missiles are not a significant threat to the IVC equipment. These results further indicate that no physical barriers are required at the IVC top opening to protect IVC equipment from potential tornado-generated missiles.

VII. REFERENCES

[1] 10 CFR Part 50, Appendix A, "Design Basis for Protection Against Natural Phenomena."

[2] Standard Review Plan, U.S. Nuclear Regulatory Commission, NUREG-75087.

[3] Nuclear Regulatory Commission, "Design Basis Tornado for Nuclear Power Plants," Regulatory Guide 1.76, April 1974.

[4] "U.S. Tornado Breakdown by Counties 1953-1982," U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service National Severe Storms Forecast Center, Room 1728, Federal Building, 601 E. 12th Street, Kansas City, Missouri 64106.

[5] Twisdale, L. A., et al., "Tornado Missile Risk Analysis," EPRI NP-768, May 1978, EPRI NP-769, May 1978.

[6] Redmann, G. M., et al., "Wind Field and Trajectory Models for Tornado-Propelled Objects," EPRI 308, Technical Report 1, February 1976.

[7] Goodman, J. and Koch., J. E., "Conditional Probability of the Tornado Missile Impact Given a Tornado Occurrence," Proceedings of the International ANS/ENS Topical Meeting on Probabilistic Risk Assessment, Port Chester, New York, September 20-24, 1981, pp. 419-424.

[8] Goodman, J. and Koch, J. E., "The Probability of a Tornado Missile Hitting a Target," Nuclear Engineering and Design 74, (1983).

[9] Apostolakis, G., et al., "Data Specialization for Plant Specific Risk Studies," Nuclear Engineering and Design 56 (1980), pp. 321-329.

Table I

Probability of Tornado Occurrence at Plant Site

	Median (50th Percentile)	Upper Limit (95th Percentile)
Annual Frequency v of Tornado Occurrence in Area S = 10,000 mi^2	5.25	37.39
Tornado Path Area (sq. mi.), a	2.22×10^{-2}	1.39×10^{-1}
Probability of Tornado Occurrence at Plant Site, per Year, P_o	1.17×10^{-5}	1.71×10^{-4}

Table II

Distribution of Potential Missiles

	Median (50th Percentile)	Upper Limit (95th Percentile)
Number of Potential Missiles on Site and Vicinity (2.5×10^7 ft^2), N_p	6,000	6,196
Average Surface Density of Potential Missiles on Site (ft^{-2})	2.40×10^{-4}	2.48×10^{-4}
Local Surface Density of Potential Missiles Near the IVC Compartments (ft^{-2}), n_p	2.40×10^{-4}	6.13×10^{-4}
Effective Number of Potential Missiles on Site and Vicinity, N_{ef}	6,000	15,325

Table III

Probability of Damage to IVC from Tornado-Generated Missiles per Year

Median (50th Percentile)	Upper Limit (95th Percentile)
2×10^{-10}	6×10^{-6}

APPENDIX A

CONDITIONAL PROBABILITY OF HITTING A TARGET

A.1 The Equation of Motion of a Tornado Missile

The Newton equation of motion of the center-of-mass of a tornado missile is:

$$m\dot{\vec{v}} = \vec{F}_D + \vec{F}_L + \vec{F}_S + \vec{F}_g \tag{A.1}$$

where:

m = mass of missile,

$\vec{v}$ = velocity vector of missile,

$\dot{\vec{v}}$ = acceleration vector of missile,

$\vec{F}_D$ = aerodynamic drag force,

$\vec{F}_L$ = aerodynamic lift force,

$\vec{F}_S$ = aerodynamic side force,

$\vec{F}_g$ = gravity force.

The expression for the gravity force is:

$$\vec{F}_g = - mg\,\vec{k} \tag{A.2}$$

where g is the gravitational acceleration constant and k is a unit vector oriented upward. The drag force $\vec{F}_D$ acts in the direction of the relative wind-missile velocity vector $\vec{u}$ (see Figure A-1). The velocity ($\vec{u}$) can be expressed as the vector difference of wind velocity ($\vec{w}$) and missile velocity ($\vec{v}$) as follows:

$$\vec{u} = \vec{w} - \vec{v} \tag{A.3}$$

The absolute value of the vector $\vec{u}$ is denoted as u. The lift force $\vec{F}_L$ is perpendicular to $\vec{F}_D$ and in the plane formed by the vector $\vec{u}$ and the unit vector $\vec{\mu}_o$ along the main missile axis. The side force $\vec{F}_S$ is perpendicular to the vectors $\vec{F}_D$ and $\vec{F}_L$.

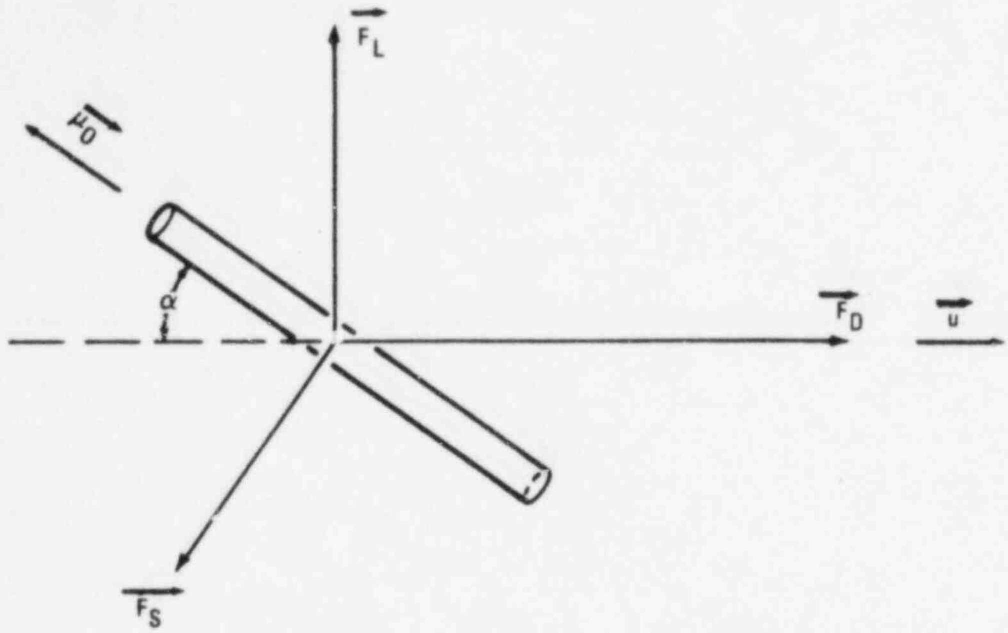

Figure A-1. Aerodynamic Forces Acting Upon a Tornado Missile

The empirical expressions for these aerodynamic forces are:

$$\vec{F}_D = C_D \frac{\rho_a A}{2} u\vec{u} \tag{A.4}$$

$$\vec{F}_L = C_L \frac{\rho_a A}{2} \left[\vec{u} \times \left[\vec{\mu}_0 \times \vec{u}\right]\right] \tag{A.5}$$

$$\vec{F}_S = C_S \frac{\rho_a A}{2} \left[\vec{\mu}_0 \times \vec{u}\right] \tag{A.6}$$

where:

ρ_a = air density

A = missile cross-sectional area (see Figure A-2)

C_D = aerodynamic drag coefficient

C_L = aerodynamic lift coefficient

C_S = aerodynamic side coefficient

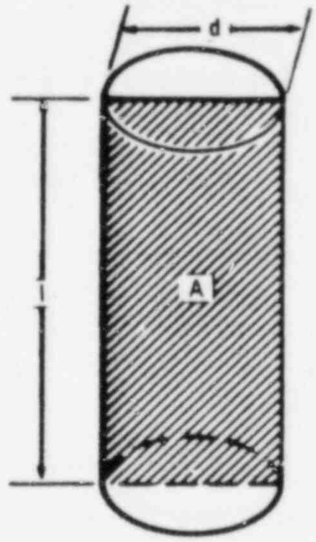

Figure A-2. Geometrical Parameters of a Tornado Missile

The empirical aerodynamic coefficients C_D, C_L, and C_S depend on certain aerodynamic parameters and the angle of attack (α) (see Figure A-1). For cylindrical missiles, they are m/A and ℓ/d, where A, ℓ, and d are shown in Figure A-2. Expressions for C_D, C_L, and C_S for several body shapes are given by Redmann et al. [A.1]. For some symmetrical bodies (for example, cylinders), the coefficient $C_S = 0$.

The missile trajectory depends on six external parameters: the wind speed (w), the angle of attack (α), the angle (β) between $\vec{w}$ and $\vec{v}$, two angles θ and ϕ which give the orientation of force $\vec{F}_D$ relative to laboratory system of coordinates, and the angle (ψ) between $\vec{F}_L$ and the vertical axis (OZ). These parameters are very convenient for a tornado missile problem but the standard mechanical approach uses other parameters: three quasi-external parameters (Euler angles, which can be determined by using the angular momentum equations) and three truly external parameters (the components of the wind velocity $\vec{w}$).

The wind velocity vector field ($\vec{w}(x, y, z, t)$) consists of a regular part which depends on tornado parameters (Fujita scale, length and width of tornado, translational velocity vector, etc.) and an irregular or stochastic part which is due to turbulent fluctuations.

In probabilistic risk analysis, the distribution of the possible tornado parameters is considered. Therefore, the entire vector field $\vec{w}(x, y, z, t)$ is a random vector function. Hence, equation (A.1) can be rewritten in the form:

$$m\dot{\vec{v}} = \vec{R} - mg\,\vec{k} \qquad \text{(A.7)}$$

where $\vec{R}$ is a random force, equal to $\vec{F}_D + \vec{F}_L + \vec{F}_S$.

For cylindrical missiles, equation (A.7) can be written in the form:

$$m\ddot{x} = f_D \sin\theta \cos\phi + f_L (\cos\theta \cos\phi \cos\psi - \sin\phi \sin\psi) \quad (A.8)$$

$$m\ddot{y} = f_D \sin\theta \sin\phi + f_L (\cos\theta \sin\phi \cos\psi + \cos\phi \sin\psi) \quad (A.9)$$

$$m\ddot{z} = f_D \cos\theta - f_L \sin\theta \cos\psi - mg \quad (A.10)$$

where:

$$f_D = C_D \frac{\rho_a Au^2}{2} \quad (A.11)$$

$$f_L = C_L \frac{\rho_a Au^2}{2} \quad (A.12)$$

The angles θ and ϕ determine the orientation of the drag force $\vec{F}_D$ in the spherical system of coordinates relative to the earth's surface. The angle between the lift force $\vec{F}_L$ and the plane containing the vertical axis and the drag force $\vec{F}_D$ is denoted as ψ.

Air density is denoted as ρ_a and missile cross-section area as A. The aerodynamic coefficients C_D and C_L for a cylindrical missile are considered in Redmann [A.1]. For the standard missile, the approximate expression is:

$$C_D = 0.98 \sin^3\alpha \quad (A.13)$$

$$C_L = 0.98 \sin^2\alpha \cos\alpha \quad (A.14)$$

where the attack angle α has the range:

$$0 \leq \alpha \leq \pi \quad (A.15)$$

The system of equations (A.8) through (A.10) can be rewritten in the form:

$$\ddot{x} = a_x \quad (A.16)$$

$$\ddot{y} = a_y \quad (A.17)$$

$$\ddot{z} = a_z - g \quad (A.18)$$

or, in the vector notation, as

$$\ddot{\vec{r}} = \vec{a} - g\,\vec{k} \quad (A.19)$$

where $\vec{a}$ is a random vector of acceleration with components:

$$a_x = a_o [\sin^3\alpha \sin\theta \cos\phi + \sin^2\alpha \cos\alpha$$
$$(\cos\theta \cos\phi \cos\psi - \sin\phi \sin\psi)] \quad (A.20)$$

$$a_y = a_o \, [\sin^3 \alpha \sin \theta \sin \phi + \sin^2 \alpha \cos \alpha$$

$$(\cos \theta \sin \phi \cos \psi + \cos \phi \sin \psi)] \qquad (A.21)$$

$$a_z = a_o \, [\sin^3 \alpha \cos \theta - \sin^2 \alpha \cos \alpha \sin \theta \cos \psi] \qquad (A.22)$$

and

$$a_o = 0.003727 \, (w^2 + v^2 - 2 \, w \, v \cos \beta) \qquad (A.23)$$

where β is an angle between wind velocity $\vec{w}$ and missile velocity $\vec{v}$. Speeds w and v are measured in metric units.

The random vector $\vec{a}$ depends on six random parameters: w, β, α, θ, ϕ, and ψ. These parameters are functions of other independent random parameters: three components of wind field $\vec{w}$ and three Euler's angles determining the orientation of the missile in the space. Even randomness of some independent random parameter (for example, randomness only of the wind field $\vec{w}$ in the case of nontumbling missiles) can create the randomness of all parameters, w, α, β, θ, ϕ and ψ. The number of independent random parameters determines the number of degrees of freedom of randomness. The maximum number of degrees of freedom is six.

For a large number of degrees of freedom of randomness, the distribution of components of acceleration a_x, a_y, and a_z is practically normal according to the central limit theorem. The means of random accelerations are zeros:

$$\overline{a_x} = 0 \qquad (A.24)$$

$$\overline{a_y} = 0 \qquad (A.25)$$

$$\overline{a_z} = 0 \qquad (A.26)$$

A.2 Tornado Missile Motion As a Markovian Process

Every solution to the Newton equation with a random force constituent represents a Markovian process [A.2]. An example is a Brownian movement.

To show this, let us discretize the random acceleration vector $\vec{a}$. We assume that a random vector $\vec{a}$ is constant during the small interval of time τ. At the end of every time period τ, this vector is assumed to change suddenly and randomly.

Consider some tornado missile trajectory with an initial point $\vec{r}_o$, a velocity $\vec{v}_o$, a final point $\vec{r}$, and a velocity $\vec{v}$. Dividing the missile transportation time $T = t - t_o$ by n equal intervals of duration τ each, we can calculate all intermediate points and velocities of the missile trajectory:

$$\begin{aligned}
\vec{r}_1 &= \vec{r}_o + \vec{v}_o \tau + \frac{\vec{a}_1 \tau^2}{2} \\
\vec{v}_1 &= \vec{v}_o + \vec{a}_1 \tau \\
\vec{r}_2 &= \vec{r}_1 + \vec{v}_1 \tau + \frac{\vec{a}_2 \tau^2}{2} \\
\vec{v}_2 &= \vec{v}_1 + \vec{a}_2 \tau \\
&\cdots\cdots\cdots\cdots \\
\vec{r}_i &= \vec{r}_{i-1} + \vec{v}_{i-1} \tau + \frac{\vec{a}_i \tau^2}{2} \\
\vec{v}_i &= \vec{v}_{i-1} + \vec{a}_i \tau \\
&\cdots\cdots\cdots\cdots \\
\vec{r}_n &= \vec{r}_{n-1} + \vec{v}_{n-1} \tau + \frac{\vec{a}_n \tau^2}{2} \\
\vec{v}_n &= \vec{v}_{n-1} + \vec{a}_n \tau
\end{aligned} \tag{A.27}$$

where

$$\begin{aligned}
\vec{r}_n &= \vec{r} \\
\vec{v}_n &= \vec{v}
\end{aligned} \tag{A.28}$$

and $\vec{a}_i$ is a random acceleration of the missile during time period t:

$$t_{i-1} \leq t < t_i \tag{A.29}$$

where

$$t_i = t_o + i \cdot \tau \tag{A.30}$$

It follows from equation (A.27) that the location $\vec{r}_i$ and velocity $\vec{v}_i$ of the tornado missile at the moment t_i depend on $\vec{r}_{i-1}$, $\vec{v}_{i-1}$ and $\vec{a}_i$. It is clear from equations (A.8) through (A.10) that a probability distribution function for a random vector $\vec{a}_i$ could depend only on $\vec{v}_{i-1}$. Therefore, the probability density that a missile at moment t_i will have location $\vec{r}_i$ and velocity $\vec{v}_i$ depends on $\vec{r}_{i-1}$ and $\vec{v}_{i-1}$ only:

$$dP\ (\vec{r}_{i-1}, \vec{v}_{i-1}, \vec{r}_i, \vec{v}_i) = f_i\ (\vec{r}_i, \vec{v}_i \mid \vec{r}_{i-1}, \vec{v}_{i-1})\ d^3\vec{r}_i\ d^3\vec{v}_i \tag{A.31}$$

where:

$dP\ (\vec{r}_{i-1}, \vec{v}_{i-1}, \vec{r}_i, \vec{v}_i)$ = probability that a tornado missile having at moment t_{i-1} the location $\vec{r}_{i-1}$ and velocity $\vec{v}_{i-1}$ will have at moment t_i the location in the range $(\vec{r}_i, \vec{r}_i + d\vec{r}_i)$ and velocity in the range $(\vec{v}_i, \vec{v}_i + d\vec{v}_i)$;

$f_i\ (\vec{r}_i, \vec{v}_i \mid \vec{r}_{i-1}, \vec{v}_{i-1})$ = conditional probability density corresponding to the probability $dP\ (r_{i-1}, v_{i-1}, r_i, v_i)$;

$$d^3\vec{r}_i = dx_i\ dy_i\ dz_i$$

$$d^3\vec{v}_i = dv_{x(i)}\ dv_{y(i)}\ dv_{z(i)}$$

We see that tornado missile motion is a Markov chain $(\vec{r}_o, \vec{v}_o; \vec{r}_1, \vec{v}_1; \vec{r}_2, \vec{v}_2; \ldots \vec{r}_i, \vec{v}_i; \ldots \vec{r}_n, \vec{v}_n)$ because the probability (A.31) that a tornado missile will occur in the i^{th} state described by $\vec{r}_i$ and $\vec{v}_i$ given being in the $(i-1)^{th}$ state before depends only on the preceded $(i-1)^{th}$ state and does not depend on all other previous steps.

If we approach $\tau \to 0$ and $n \to \infty$ then our Markov chain will tend to the continuous Markov process.

A.3 Green's Function of the Tornado Missile in the Phase Space

A continuum of tornado missile locations forms the real space (R-space). A continuum of tornado missile velocities forms the velocity space (V-space). A continuum of locations and velocities together forms the phase (or configuration) space (P-space).

A radius-vector of missile location $\vec{r}$ is a point in the R-space. A missile velocity $\vec{v}$ is a point in the V-space. A combination of missile location $\vec{r}$ and velocity $\vec{v}$ is a point in the P-space.

Sequential missile locations $\vec{r}(t)$ and velocities $\vec{v}(t)$ putting in the time-ascending order form a phase trajectory or phase path.

In our model with discrete random accelerations, every phase trajectory can be represented by the set of phase points: $\vec{r}_o, \vec{v}_o; \vec{r}_1, \vec{v}_1;$ $\ldots \vec{r}_i, \vec{v}_i, \ldots \vec{r}_{n-1}, \vec{v}_{n-1}; \vec{r}, \vec{v}$. Trajectories with the same initial point $\vec{r}_o, \vec{v}_o$ and final point $\vec{r}, \vec{v}$ can be distinguished by the sets Γ of intermediate points:

$$\Gamma\ (\vec{r}_1, \vec{v}_1; \vec{r}_2, \vec{v}_2; \ldots . \vec{r}_i, \vec{v}_i; \ldots \vec{r}_{n-1}, \vec{v}_{n-1}) \tag{A.32}$$

The probability density $f(\vec{r}_o, \vec{v}_o; \vec{r}, \vec{v}; \Gamma)$ that a tornado missile follows the phase trajectory Γ can be estimated as:

$$f(\vec{r}_o, \vec{v}_o; \vec{r}, \vec{v}; \Gamma) = \prod_{i=1}^{n} f_i(\vec{r}_i, \vec{v}_i\ ; \vec{r}_{i-1}, \vec{v}_{i-1}) \tag{A.33}$$

where condition (A.28) is assumed.

The total probability density $G\ (\vec{r}_o, \vec{v}_o; \vec{r}, \vec{v})$ that a tornado missile from the initial phase point $\vec{r}_o, \vec{v}_o$ will reach the final phase point $\vec{r}, \vec{v}$ is the G(n-1)-fold integral of the expression (A.33) over all possible phase trajectories Γ between points $\vec{r}_o, \vec{v}_o$ and $\vec{r}, \vec{v}$:

$$G\ (\vec{r}_o, \vec{v}_o; \vec{r}, \vec{v}) = \int_\Gamma f\ (\vec{r}_o, \vec{v}_o; \vec{r}, \vec{v}; \Gamma)\ d\Gamma \tag{A.34}$$

where

$$d\Gamma = d^3\vec{r}_1\ d^3\vec{v}_1\ d^3\vec{r}_2\ d^3\vec{v}_2 \ldots d^3\vec{r}_i\ d^3\vec{v}_i \ldots d^3\vec{r}_{n-1}\ d^3\vec{v}_{n-1} \tag{A.35}$$

The quantity $G(\vec{r}_o, \vec{v}_o; \vec{r}, \vec{v})$ is the so-called propagation or Green's function.

In the previous section, we assumed that a propagation time $T = t - t_o$ is fixed. If we drop this limitation (for a discrete model, it means that the number n of steps can vary) then the Green's function will have a more general form:

$$G\ (t_o,\ \vec{r}_o,\ \vec{v}_o;\ t,\ \vec{r},\ \vec{v};\ F,\ a,\ \gamma) \tag{A.36}$$

We explicitly show all parameters that Green's function depends on (F is a Fujita scale, a is a tornado path area, and γ is a collective index which stands for all other parameters).

The Green's function (A.36) has the following properties:

1. $$G\ (t_o,\ \vec{r}_o,\ \vec{v}_o;\ t,\ \vec{r},\ \vec{v};\ F,\ a,\ \gamma) \equiv 0\ ,$$

   $$\text{if} \quad t < t_o \quad \text{or} \quad z < 0 \tag{A.37}$$

2. $$G\ (t_o,\ \vec{r}_o,\ \vec{v}_o;\ t,\ \vec{r},\ \vec{v};\ F,\ a,\ \gamma) \rightarrow 0$$

   $$\text{if} \quad \left|\vec{r} - \vec{r}_o\right| \rightarrow 0 \quad \text{or} \quad \left|\vec{v} - \vec{v}_o\right| \rightarrow 0 \tag{A.38}$$

Property 1 reflects the principle of causality (the missile can hit a target only after it is airborne) and boundary condition (no missiles below the ground level $z = 0$).

Property 2 means that there is some finite radius of missile transportation and maximum missile speed beyond which the probability of missile occurrence is infinitesimally small.

A.4 Properties of the Averaged Green's Function

For a given F and a, we have a number of other parameters beyond our control. These are the direction of tornado movement, translational velocity, ratio of tornado width to tornado length, number of vortices, exact location of trajectories of the centers of every vortex relative to the target, detailed distribution of wind field, etc. All sets of these parameters are denoted as γ.

Because it is impossible to develop the detailed distribution for all sets of parameters γ, we average our probability over all γ.

Introduce the averaged Green's function $G\ (t_o, \vec{r}_o, \vec{v}_o;\ t, \vec{r}, \vec{v};\ F, a)$:

$$G\ (t_o, \vec{r}_o, \vec{v}_o;\ t, \vec{r}, \vec{v};\ F, a) =$$

$$\frac{1}{N} \sum_{\gamma} G\ (t_o, \vec{r}_o, \vec{v}_o;\ t, \vec{r}, \vec{v};\ F, a, \gamma) \tag{A.39}$$

where N is a total number of different sets of parameters γ.

The averaged Green's function retains all properties of the ordinary Green's function discussed in the previous sections. It also contains some new properties.

These properties are:

(1) Time uniformity

(2) Uniformity in the plane x0y

(3) Axial symmetry around axis 0z

(4) Uniformity in V-space

Time uniformity means that the probability distribution for tornado missile propagation does not depend on initial moment of injection. It depends only on transportation time $t - t_o$.

Uniformity in the plane x0y means that the probability distribution for tornado missile propagation does not depend on the location of the potential missile at the plane x0y. Therefore, the Green's function depends on differences $x - x_o$ and $y - y_o$.

Axial symmetry means that there is no preferable direction for tornado missile distribution in the plane x0y. Thus the Green's function depends on $\sqrt{(x - x_o)^2 + (y - y_o)^2}$ rather than $x - x_o$ and $y - y_o$ separately.

Uniformity in V-space means that the probability distribution for tornado missile propagation does not depend on initial velocity $\vec{v}_o$ and depends only on the difference, $\vec{v} - \vec{v}_o$.

Therefore, the averaged Green's function has the structure:

$$G\ (t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) \equiv$$

$$G\ (z_o\ ; t - t_o\ ,\ \sqrt{(x - x_o)^2 + (y - y_o)^2}\ , \vec{v} - \vec{v}_o; F, a) \tag{A.40}$$

A.5 Equations for the Green's Function

The Green's function for a Markovian process satisfies the Chapman-Kolmogorov-Smoluchovski equation [A.2]:

$$G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) =$$

$$\int_{V'} \int_{\vec{v}'} G(t_o, \vec{r}_o, \vec{v}_o; t', \vec{r}', \vec{v}'; F, a)\, G(t', \vec{r}', \vec{v}'; t, \vec{r}, \vec{v}; F, a)$$

$$d^3\vec{r}'\, d^3\vec{v}' \tag{A.41}$$

where $G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a)$ is an averaged Green's function (A.40).

The Markovian process is a diffusion Markovian process if

$$G(t_o, \vec{r}_o, \vec{v}_o; t_o, \vec{r}, \vec{v}; F, a) \equiv \delta\,(\vec{r} - \vec{r}_o)\ \delta\,(\vec{v} - \vec{v}_o) \tag{A.42}$$

and there exist continuous and differentiable functions $a_i\ (t, \vec{r}, \vec{v}; F, a)$, $b_i\ (t, \vec{r}, \vec{v}; F, a)$, $c_{ik}\ (t, \vec{r}, \vec{v}; F, a)$, $d_{ik}\ (t, \vec{r}, \vec{v}; F, a)$ and $f_{ik}\ (t, \vec{r}, \vec{v}; F, a)$ satisfying the equations:

$$a_i\ (t, \vec{r}, \vec{v}; F, a) =$$

$$\lim_{t' \to t} \left\{ \frac{1}{t' - t} \int_{V'} \int_{\vec{v}'} (x_i' - x_i)\, G(t, \vec{r}, \vec{v}; t', \vec{r}', \vec{v}'; F, a)\, d^3\vec{r}'\, d^3\vec{v}' \right\} \tag{A.43}$$

$$b_i\ (t, \vec{r}, \vec{v}; F, a) =$$

$$\lim_{t' \to t} \left\{ \frac{1}{t' - t} \int_{V'} \int_{\vec{v}'} (v_i' - v_i)\, G(t, \vec{r}, \vec{v}; t', \vec{r}', \vec{v}'; F, a)\, d^3\vec{r}'\, d^3\vec{v}' \right\} \tag{A.44}$$

$$c_{ik}\ (t, \vec{r}, \vec{v}; F, a) =$$

$$\lim_{t' \to t} \left\{ \frac{1}{t' - t} \int_{V'} \int_{\vec{v}'} (x_i' - x_i)\, (x_k' - x_k)\, G(t, \vec{r}, \vec{v}; t', \vec{r}', \vec{v}'; F, a) \right.$$

$$\left. d^3\vec{r}'\, d^3\vec{v}' \right\} \tag{A.45}$$

$$d_{ik}(t, \vec{r}, \vec{v}; F, a) =$$

$$\lim_{t' \to t} \left\{ \frac{1}{t' - t} \int_{V'} \int_{\vec{v}'} (x_i' - x_i)(v_k' - v_k)\, G(t, \vec{r}, \vec{v}; t', \vec{r}', \vec{v}'; F, a) \right.$$

$$\left. d^3\vec{r}'\, d^3\vec{v}' \right\} \tag{A.46}$$

$$f_{ik}(t, \vec{r}, \vec{v}; F, a) =$$

$$\lim_{t' \to t} \left\{ \frac{1}{t' - t} \int_{V'} \int_{\vec{v}'} (v_i' - v_i)(v_k' - v_k)\, G(t, \vec{r}, \vec{v}; t', \vec{r}', \vec{v}'; F, a) \right.$$

$$\left. d^3\vec{r}'\, d^3\vec{v}' \right\} \tag{A.47}$$

where $i,k = 1, 2, 3$ and x_1, x_2, x_3 stand for x, y, z and v_1, v_2, v_3 for v_x, v_y, v_z.

The Green's function of a diffusion Markovian process satisfies the Fokker-Planck equation:

$$\frac{\partial\, G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a)}{\partial t} -$$

$$\sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left[a_i(t, \vec{r}, \vec{v}; F, a)\, G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) \right] -$$

$$\sum_{i=1}^{3} \frac{\partial}{\partial v_i} \left[b_i(t, \vec{r}, \vec{v}; F, a)\, G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) \right] -$$

$$\frac{1}{2} \sum_{i=1}^{3} \sum_{k=1}^{3} \frac{\partial^2}{\partial x_i\, \partial x_k} \left[c_{ik}(t, \vec{r}, \vec{v}; F, a)\, G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) \right] -$$

$$\sum_{i=1}^{3} \sum_{k=1}^{3} \frac{\partial^2}{\partial x_i\, \partial v_k} \left[d_{ik}(t, \vec{r}, \vec{v}; F, a)\, G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) \right] -$$

$$\frac{1}{2} \sum_{i=1}^{3} \sum_{k=1}^{3} \frac{\partial^2}{\partial v_i\, \partial v_k} \left[f_{ik}(t, \vec{r}, \vec{v}; F, a)\, G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}; F, a) \right] = 0 \tag{A.48}$$

Conditions (A.43) through (A.47) mean that the radius-vector of the missile r(t) and the velocity v(t) change more or less smoothly without jumps and discontinuities in the P-space. These conditions are satisfied for tornado missile motion because of inertia of movement.

A.6 Hitting Function

Let us find the probability that a missile injecting at point $\vec{r}_o$ with velocity $\vec{v}_o$ at moment t_o will hit the unit area of a target at moment t near point $\vec{r}$ with velocity $\vec{v}$.

Consider the small volume

$$dV = dA \cdot \left| \vec{v} \cdot \vec{\Omega} \right| dt \tag{A.49}$$

shown in Figure A-3. This is a skew cylinder with base dA, slant height vdt, and altitude $\left| \vec{v} \cdot \vec{\Omega} \right|$ dt.

All missiles with center-of-mass inside volume dV given by equation (A.49) and velocity vector ($\vec{v}$) will hit the area dA during time dt.

Thus, the probability of hitting area dA during time dt given velocity $\vec{v}$ is:

$$G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}) \cdot \left| \vec{v} \cdot \vec{\Omega} \right| \cdot dA \cdot dt \tag{A.50}$$

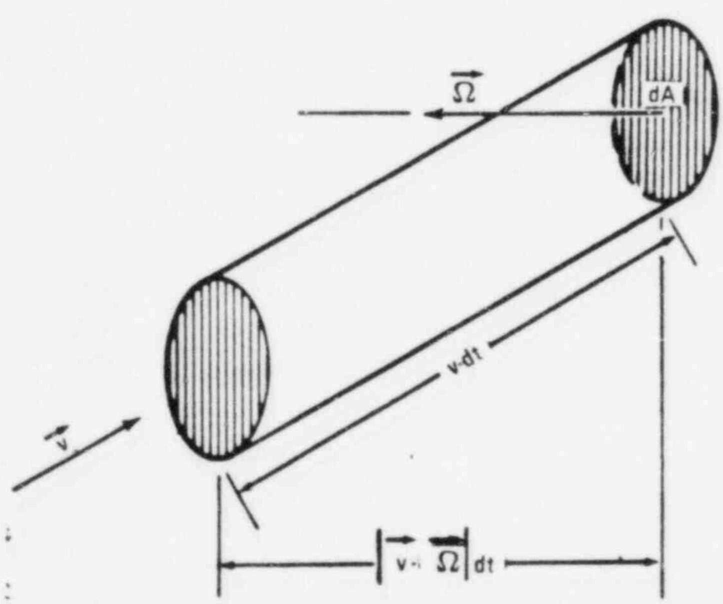

Figure A-3. Illustration to Formula (A.49)

Dividing expression (A.50) by dA dt, the probability of hitting the unit area oriented in the direction $\vec{\Omega}$ per unit time given velocity $\vec{v}$ is found:

$$G(t_o, \vec{r}_o, \vec{v}_o; t, \vec{r}, \vec{v}) \cdot \left| \vec{v} \cdot \vec{\Omega} \right| \qquad \text{(A.51)}$$

Let $\eta(F)$ be the probability of injection of a potential missile, and $\rho_p(\vec{r}_o, \vec{v}_o, t_o)$ be the density of potential missiles at moment t_o near point $\vec{r}_o$ with initial velocity at the moment of injection $\vec{v}_o$. Then the probability of hitting the unit area near point $\vec{\Omega}$ oriented in the direction $\vec{\Omega}$, or hitting function h $(\vec{r}, \vec{\Omega}, F, a)$, is:

$$h\ (\vec{r}, \vec{\Omega}; F, a) =$$

$$\eta(F)\int_{t_1}^{t_2} dt_o \int_{t_1}^{t_2} dt \int_{V_o} d^3\vec{r_o} \int_{\vec{v}_o} d^3\vec{v_o} \int_{(\vec{v}\cdot\vec{\Omega} < 0)} \rho_p(\vec{r_o}, \vec{v_o}, t_o)\ G(t_o, \vec{r_o}, \vec{v_o};$$

$$t, \vec{r}, \vec{v}) \left| \vec{v} \cdot \vec{\Omega} \right| d^3\vec{v} \qquad \text{(A.52)}$$

The integration over velocity $\vec{v}$ satisfies the condition:

$$\vec{v} \cdot \vec{\Omega} < 0 \qquad \text{(A.53)}$$

which selects the missiles intersecting the unit area only from one side.

Assuming that all potential missiles are constrained and uniformly distributed in the plan x0y (actually, we need the uniformity in the area of radius 300 ft corresponding to the 95th percentile of missile transportation distribution), we obtain the density of potential missiles in the form:

$$\rho_p(\vec{r}_o, \vec{v}_o, t_o) \equiv n_p\ \rho(z_o) \qquad \text{(A.54)}$$

where n_p is the surface density of potential missiles and $\rho(z_o)$ is a dimensionless function reflecting the height distribution of potential missiles.

If all missiles are on the ground then:

$$\rho(z_o) = \delta(z_o) \qquad \text{(A.55)}$$

where $\delta(z_o)$ is the delta-function.

If all missiles are uniformly distributed up to elevation h_o then

$$\rho(z_o) = \begin{cases} 1 & , \quad z_o \leq h_o \\ 0 & , \quad z_o > h_o \end{cases} \tag{A.56}$$

The expression for the hitting function $h(z, \vec{\Omega}; F, a)$ takes the form:

$$h(z, \vec{\Omega}; F, a) =$$

$$\eta(F)\, n_p \int_{t_1}^{t_2} dt_o \int_{t_1}^{t_2} dt \int_{V_o} d^3\vec{r}_o \int_{\vec{v}_o} d^3\vec{v}_o \int_{(\vec{v}\cdot\vec{\Omega} < 0)} \rho(z_o)\, G(z_o;\, t-t_o,$$

$$\sqrt{(x - x_o)^2 + (y - y_o)^2},\, \vec{v} - \vec{v}_o,\, F,\, a) \cdot \left| \vec{v} \cdot \vec{\Omega} \right| d^3\vec{v} \tag{A.57}$$

A.7 Height Distribution of Airborne Missiles

Present the hitting function $h\ (z, \vec{\Omega}; F, a)$ in the form:

$$h\ (z, \vec{\Omega}; F, a) = n_p\ \eta(F)\ \psi(z, \vec{\Omega}; F, a) \tag{A.57}$$

where $\psi\ (z, \vec{\Omega}; F, a)$ is the so-called height distribution of airborne missiles given by the formula:

$$\psi\ (z, \vec{\Omega}; F, a) =$$

$$\int_{t_1}^{t_2} dt_o \int_{t_1}^{t_2} dt \int_{V_o} d^3 r_o \int_{\vec{v}_o} d^3 v_o \int_{(\vec{v}\cdot\vec{\Omega}<0)} \rho(z_o)\ G(z_o;\ t-t_o,\ \sqrt{(x-x_o)^2 + (y-y_o)^2},\ \vec{v}-\vec{v}_i,\ F,\ a)$$

$$\times \left|\vec{v} \cdot \vec{\Omega}\right| d^3\vec{v} \tag{A.58}$$

To find the equation for the height distribution, we have to multiply the equation (A.48) by $\rho(z_o)$ and $|\vec{v} \cdot \vec{\Omega}|$ and integrate over t_o, t, $\vec{r}_o$, $\vec{v}_o$ and $\vec{v}$. Using the properties of averaged Green's function, Gauss' theorem in the V-space and the condition that Green's function is equal to zero before and after the tornado strike, we obtain:

$$- \frac{\partial}{\partial z} [a(z, \vec{\Omega}; F, a)\ \psi(z, \vec{\Omega}; F, a)] =$$

$$\frac{\partial^2}{\partial z^2} [\mathcal{D}(z, \vec{\Omega}; F, a)\ \psi(z, \vec{\Omega}; F, a)] \tag{A.59}$$

where:

$$a(z, \vec{\Omega}; F, a) = a_3(z, \vec{\Omega}; F, a) \tag{A.60}$$

$$\mathcal{D}(z, \vec{\Omega}; F, a) = \frac{1}{2}\ c_{33}(z, \vec{\Omega}; F, a) \tag{A.61}$$

Now we can make some conservative assumptions. Consider the direction $\vec{\Omega}$ which gives the maximum value for ψ given all other parameters. Assuming this value for all other directions, we make the function ψ isotropic and conservative.

If the tornado missile lands before it leaves the tornado wind field, the Green's function will not depend on tornado path area, a. This is true for all tornadoes except some very small ones. Premature leaving of the tornado wind field only reduces the height and distance of missile transportation. Therefore, the assumption that Green's function does not depend on tornado path area a will be conservative.

Hence, the equation (A.59) takes the form:

$$- \frac{\partial}{\partial z} [a(z, F)\ \psi(z, F)] = \frac{\partial^2}{\partial z^2} [\mathcal{D}(z, F)\ \psi(z, F)] \tag{A.62}$$

If all potential missiles have the same original elevation z_o = const (for example, ground distribution of potential missiles with $z_o = 0$) the coefficients $a(z, F)$ and $\mathcal{D}(z, F)$ do not depend on z according to their definitions (A.43) and (A.45).

Therefore, the solution to equation (A.62) satisfying the boundary condition is:

$$\psi(z, F) = e^{-\alpha_o(F)z} \tag{A.63}$$

where

$$\alpha_o(F) = \frac{a(F)}{\mathcal{D}(F)} \tag{A.64}$$

This solution is good for cases when all potential missiles are at ground elevation.

If all potential missiles are uniformly distributed from the ground to the elevation h_o, we assume that coefficients $a(z, F)$ and $\mathcal{D}(z, F)$ are constant but different in the areas $z \leq h_o$ and $z > h_o$:

$$a(z, F) = \begin{cases} a_1(F) & , \quad z \leq h_o \\ a_2(F) & , \quad z > h_o \end{cases} \tag{A.65}$$

$$\mathcal{D}(z, F) = \begin{cases} \mathcal{D}_1(F) & , \quad z \leq h_o \\ \mathcal{D}_2(F) & , \quad z > h_o \end{cases} \tag{A.66}$$

The solution to the equation (A.62) for this case is:

$$\psi(z, F) = \begin{cases} \dfrac{B(z)}{B(0)} & , \quad 0 \leq z \leq h_o \\ \dfrac{\alpha_1(F)}{B(0) \cdot \alpha_2(F)} e^{\alpha_2(F)[h_o - z] - \alpha_1(F) \cdot h_o} & , \quad z > h_o \end{cases} \tag{A.67}$$

where

$$B(z) = e^{-\alpha_1(F) \cdot z} + \left[\frac{\alpha_1(F)}{\alpha_2(F)} - 1\right] e^{-\alpha_1(F) h_o} \tag{A.68}$$

$$\alpha_1(F) = \frac{a_1(F)}{\mathcal{D}_1(F)} \tag{A.69}$$

$$\alpha_2(F) = \frac{a_2(F)}{\mathcal{D}_2(F)} \tag{A.70}$$

If $h_o \to 0$, then $\alpha_1(F) = \alpha_2(F) \equiv \alpha_o(F)$ and the expression (A.67) coincides with (A.63).

According to definitions (A.43) and (A.45), parameters $\alpha_1(F)$ and $\alpha_2(F)$ can be found from the formula:

$$\alpha = - \lim_{\Delta\tau \to 0} \left[\frac{\overline{\Delta z}}{\frac{1}{2} \overline{(\Delta z^2)}} \right] \tag{A.71}$$

where $\overline{\Delta z}$ and $\overline{(\Delta z^2)}$ are average displacement and squared displacement in the vertical direction for time $\Delta\tau$.

The differential equation of tornado missile motion in the vertical direction is:

$$\ddot{z} = a_z - g \tag{A.72}$$

where g is the gravitational constant and

$$a_z = \frac{R_z}{m} \tag{A.73}$$

where m is the missile mass and R_z is the z-component of the random aerodynamic force.

For a small increment of time $\Delta\tau$:

$$z = z_o + v_{oz} \Delta\tau + (a_z - g) \frac{(\Delta\tau)^2}{2} \tag{A.74}$$

Because

$$\overline{v_{oz}} = 0, \text{ and} \tag{A.75}$$

$$\overline{a_z} = 0, \tag{A.76}$$

due to random distribution of velocity and acceleration directions, the average increment in elevation is:

$$\overline{\Delta z} = \frac{g(\Delta\tau)^2}{2} \tag{A.77}$$

Similarly

$$\overline{(\Delta z^2)} \doteq \overline{v_{oz}^{\,2}} \cdot (\Delta\tau)^2 + \ldots \tag{A.78}$$

where higher degrees of $\Delta\tau$ are omitted.

Putting equations (A.77) and (A.78) into equation (A.71), we obtain:

$$\alpha = \frac{g}{\overline{v_z^2}} \qquad (A.79)$$

where v_{oz}^2 is replced by v_z^2 because the average $\overline{v_z^2}$ does not depend on time.

In the region

$$0 \leq z \leq h_o \qquad (A.80)$$

the number of horizontally injected missiles is dominant because the probability of horizontal injection is much higher.

For horizontally injected missiles, it can be assumed, at the moment of injection, the vertical velocity of the missile is equal to zero. Multiplying equation (A.72) by $v_z = \dot{z}$ and integrating from the moment of injection to the moment of striking the ground yields the following:

$$\frac{v_g^2}{2} = \overline{a_z v_z} \cdot t + gz_o \qquad (A.81)$$

where

z_o = initial elevation of missile,

t = flight time of missile,

v_g = vertical velocity at the ground

Because

$$\overline{a_z v_z} = 0 \qquad (A.82)$$

due to equation (A.75) and equation (A.76) and independence of a_z and v_z:

$$v_g^2 = 2\ gz_o \qquad (A.83)$$

It is clear that

$$\overline{v_z^2} = \frac{1}{2} v_g^2 \qquad (A.84)$$

where $\overline{v_z^2}$ is an average squared velocity in a vertical direction for a missile falling from elevation z_0.

Therefore,

$$\overline{v_z^2} = gz_o \tag{A.85}$$

For uniform distribution of initial elevation z_o, from $z_o = 0$ to $z_o = h_o$ the average for all missiles is:

$$\overline{v_z^2} = \frac{1}{2} gh_o \tag{A.86}$$

Putting (A.86) into (A.79) yields:

$$\alpha_1 = \frac{2}{h_o} \tag{A.87}$$

In this approximation, the parameter $\alpha_1(F)$ does not depend on Fujita scale F.

Now, consider the region

$$z > h_o \tag{A.88}$$

In this region there are only vertically injected missiles. Let w be the damaging wind speed. According to [A.3] and [A.4], the range for the maximum missile velocity is:

$$\frac{1}{4} w \leq v_{max} \leq \frac{1}{2} w \tag{A.89}$$

Because

$$\overline{\vec{v}^2} = \frac{1}{2} v_{max}^2 \tag{A.90}$$

and

$$\overline{v_z^2} = \frac{1}{3} \overline{\vec{v}^2} \tag{A.91}$$

The following expression is obtained:

$$\overline{v_z^2} = \frac{1}{C} w^2 \tag{A.92}$$

Hence,

$$\alpha_2(F) = \frac{Cg}{w^2} \tag{A.93}$$

where coefficient C is in the range:

$$24 \leq C \leq 96 \tag{A.94}$$

Coefficient C is assumed to have lognormal distribution with a median value of 48 and an error factor of 2.

If we consider any Fujita scale, F, a corresponding middle value from the intervals given in Table 1, then this velocity w can be found as:

$$w = 6.30\ (F + 2.5)^{1.5} \tag{A.95}$$

putting equation (A.95) into equation (A.93) we obtain:

$$\alpha_2(F) = \frac{C_o}{(F + 2.5)^3} \tag{A.96}$$

where

$$C_o = \frac{9.81 \cdot C}{6.30^2} \tag{A.97}$$

A.8 Conditional Probability of Hitting the Horizontal Target Given a Tornado Strike to the Nuclear Power Plant

Multiplying the expression (A.57) for the conditional probability of hitting the unit area of a target given a tornado strike to the nuclear power plant by the horizontal area of target at the elevation z, we obtain the conditional probability of hitting the horizontally oriented target:

$$P_H = n_p \, A \, \eta(F) \, \psi(z, F) \tag{A.98}$$

In the formula (A.98), we took into account the above-mentioned conservative assumptions which eliminated the dependence of height distribution $\psi(z, F)$ on Ω and a.

The formula (A.98) can be applied to the IVC roof.

REFERENCES

[A.1] Redmann, G. H., et al., "Wind Field and Trajectory Model for Tornado-Propelled Objects," EPRI 308, Technical Report 1, February 1976.

[A.2] Papoulis, A., "Probability, Random Variables, and Stochastic Processes," McGraw-Hill Book Company, New York, 1965.

[A.3] Standard Review Plan, U.S. Nuclear Regulatory Commission, NUREG-75/087.

[A.4] Twisdale, L. A., et al., "Tornado Missile Risk Analysis," EPRI NP-768, May 1978, EPRI NP-769, May 1978.

APPENDIX B

ADJUSTMENT FOR REPORTING EFFICIENCY

To determine the reporting efficiency (C_{RE}), defined as the ratio of reported number of tornadoes (N_r) to the real number of tornadoes (N_n) that have occurred, the following expression is used:

$$C_{RE} = \frac{N_r}{N_n} \tag{B.1}$$

C_{RE} is meaningful only when the number N_n is quite large. Therefore, C_{RE} can be determined for small area and a long period of time or a short period of time and a large area.

Because a population bias that depends on time is being analyzed, the number of tornadoes per year must be considered. Therefore, the largest area for estimation of N_n (or observed number N_r) has to be considered. In this analysis, N_n is taken to be the total number of tornadoes per year in the U.S.A. The numbers of reported tornadoes per year N_r and the population density, $\mathcal{D}$, are shown in Table B-1. The numbers N_r are taken from [B.1] and $\mathcal{D}$ from [B.2]. Corresponding graphs are shown in Figures B-1 and B-2.

Actually, the reporting efficiency C_{RE} depends on many factors but all of them are assumed to be statistically correlated with population density. The time trend of N_r is assumed to depend on reporting efficiency C_{RE}, which depends on population density, $\mathcal{D}$. Fluctuations of N_r depend on climatology.

Following the work of Abbey and Fujita [B.3], the relationship of C_{RE} and population density ($\mathcal{D}$) takes the form:

$$C_{RE} = 1 - e^{-c\,(\mathcal{D} + \mathcal{D}_o)} \tag{B.2}$$

where c and $\mathcal{D}_o$ are constants.

Abbey and Fujita applied the equation (B.2) to the relatively small area (10,000 square miles) and found a low saturation density $\mathcal{D}_s$, which is determined from the condition:

$$C_{RE}\ (\mathcal{D}_s)\ (\simeq)1 \tag{B.3}$$

According to [B.3], $\mathcal{D}_s$ = 1 person per square mile. So, the formula (B.2) provides no correction to the STP data with original coefficients assumed by Abbey and Fujita.

In this study the nationwide trend is checked. Therefore, the coefficients c and $\mathscr{D}_o$ have to be found from national data, shown in Table B-1 [B.1,B.2].

Using the least square method, the curve shown in Figure B-1 is fit to the data and is given by the following equation:

$$\overline{N_r} = a - be^{-c\mathscr{D}} \tag{B.4}$$

where:

$\overline{N_r}$ = smoothed number of reported tornadoes

a,b, and c = constants

$\mathscr{D}$ = population density

Constant (c) from formulas (B.2) and (B.4) are identical and constant $_o$ is:

$$\mathscr{D}_o = -\frac{1}{c}\ln\frac{b}{a} \tag{B.5}$$

Let index i numerate the year. Index i=1 corresponds to 1950 and index i=30 corresponds to 1979. Let N_i and $\mathscr{D}_i$ be the reported number of tornadoes and population density for i^{th} year, from Table B-1. The coefficients a, b, and c have to be chosen to minimize the expression S:

$$S\ (a,b.c) = \sum_{i=1}^{30}\left(a - be^{-c\ \mathscr{D}_i} - N_i\right)^2 \tag{B.6}$$

The minimum conditions are satisfied when:

$$\frac{\partial\ S\ (a,b,c)}{\partial a} = 0 \tag{B.7}$$

$$\frac{\partial\ S\ (a,b,c)}{\partial b} = 0 \tag{B.8}$$

$$\frac{\partial\ S\ (a,b,c)}{\partial c} = 0 \tag{B.9}$$

Using (B.7), (B.8) and (B.9), the constants are found as follows:

$$a = \bar{N} + b\, E_1(c) \tag{B.10}$$

$$b = \frac{\bar{N}\, E_1(c) - N_1(c)}{E_2(c) - E_1^{\,2}(c)} \tag{B.11}$$

$$b = \frac{\bar{N}\, D_1(c) - F(c)}{D_2\,(c) - D_1\,(c)\, E_1(c)} \tag{B.12}$$

where:

$$\bar{N} = \frac{1}{30} \sum_{i=1}^{30} N_i \tag{B.13}$$

$$E_1(c) = \frac{1}{30} \sum_{i=1}^{30} e^{-c\mathscr{D}_i} \tag{B.14}$$

$$E_2(c) = \frac{1}{30} \sum_{i=1}^{30} e^{-2c\mathscr{D}_i} \tag{B.15}$$

$$D_1\,(c) = \frac{1}{30} \sum_{i=1}^{30} \mathscr{D}_i\, e^{-c\mathscr{D}_i} \tag{B.16}$$

$$D_2(c) = \frac{1}{30} \sum_{i=1}^{30} \mathscr{D}_i e^{-2c\mathscr{D}_i} \tag{B.17}$$

$$N_1(c) = \frac{1}{30} \sum_{i=1}^{30} N_i e^{-c\mathscr{D}_i} \tag{B.18}$$

$$F(c) = \frac{1}{30} \sum_{i=1}^{30} N_i \mathscr{D}_i e^{-c\mathscr{D}_i} \tag{B.19}$$

Equating (B.11) and (B.12), an equation for constant (c) is obtained. Once constant (c) is evaluated, constant (a) is found using (B.10) and constant (b) found using (B.11) or (B.12).

Using a computer code for least square fit, the coefficients are determined to be:

$$\begin{aligned} a &= 917.584091 \\ b &= 104264.8753 \\ c &= 0.118188864 \end{aligned} \tag{B.20}$$

Hence,

$$\mathscr{D}_o = -40.04561369 \tag{B.21}$$

NOTE: High precision is required to obtain the absolute minimum for expression S(a,b,c), EQN (B-6). Once the constants are determined, fewer significant figures will be used.

The formula (B.4) is applicable if

$$\mathscr{D} > -\mathscr{D}_o \tag{B.22}$$

The approximate saturated density ($\mathscr{D}_s$) is:

$$\mathscr{D}_s \simeq 100 \text{ persons/per square mile} \tag{B.23}$$

The smoothed function $\bar{N}_r(\mathscr{D})$ is shown in Figure B-1 and the reporting efficiency (C_{RE}) in Figure B-3. The corrected number of tornadoes (N_c) is determined by the following equation:

$$N_c = \frac{N_r}{1 - e^{-c(\mathscr{D}+\mathscr{D}_o)}} \tag{B.24}$$

The graph of N_c is shown in Figure B-4.

REFERENCES

[B.1] U.S. Tornado Breakdown by Counties 1950-1980, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service, National Severe Storms Forecast Center.

[B.2] "The World Almanac and The Book of Facts 1981," Newspaper Enterprise Association, Inc., New York, 1981

[B.3] Abbey, R. F., Jr. and Fujita T. T., "The Dapple Method for Computing Tornado Hazard Probabilities: Refinements and Theoretical Considerations," 11th Conference on Severe Local Storms, Oct. 1979, Kansas City.

TABLE B-1

Reported Number of Tornadoes N_r and Population Density $\mathcal{D}$ in the U.S.A.

Year	N_r	$\mathcal{D}$
1950	202	42.6
1951	260	43.5
1952	265	44.3
1953	427	45.1
1954	557	45.9
1955	626	46.8
1956	509	47.4
1957	861	48.2
1958	568	49.0
1959	607	49.8
1960	617	50.6
1961	695	51.5
1962	661	52.3
1963	458	53.1
1964	759	53.9
1965	909	54.7
1966	590	55.3
1967	931	55.9
1968	666	56.5
1969	609	56.9
1970	660	57.4
1971	901	57.9
1972	743	58.4
1973	1105	59.0
1974	950	59.6
1975	920	60.2
1976	829	60.6
1977	856	61.1
1978	780	61.7
1979	855	62.2

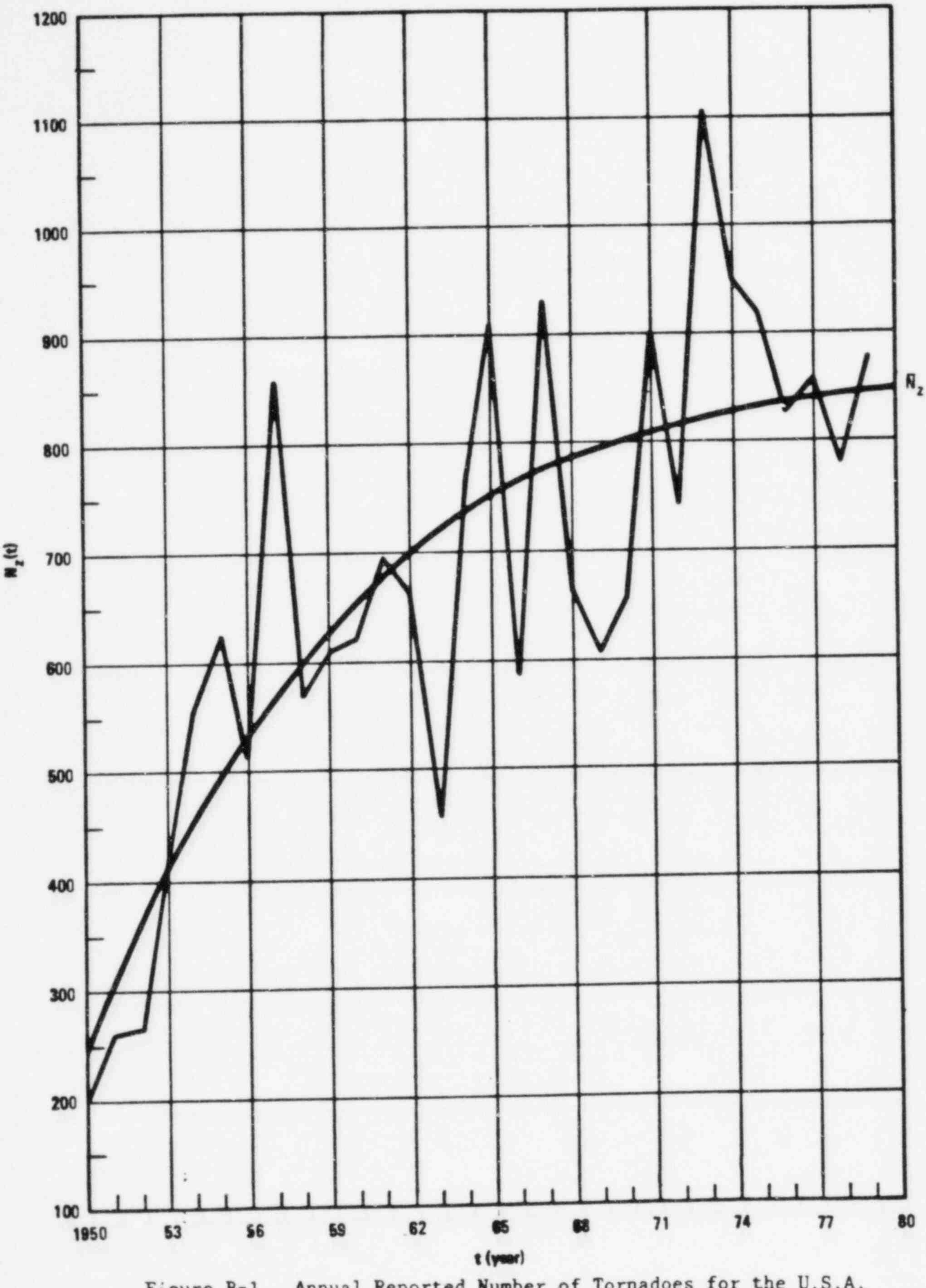

Figure B-1. Annual Reported Number of Tornadoes for the U.S.A.

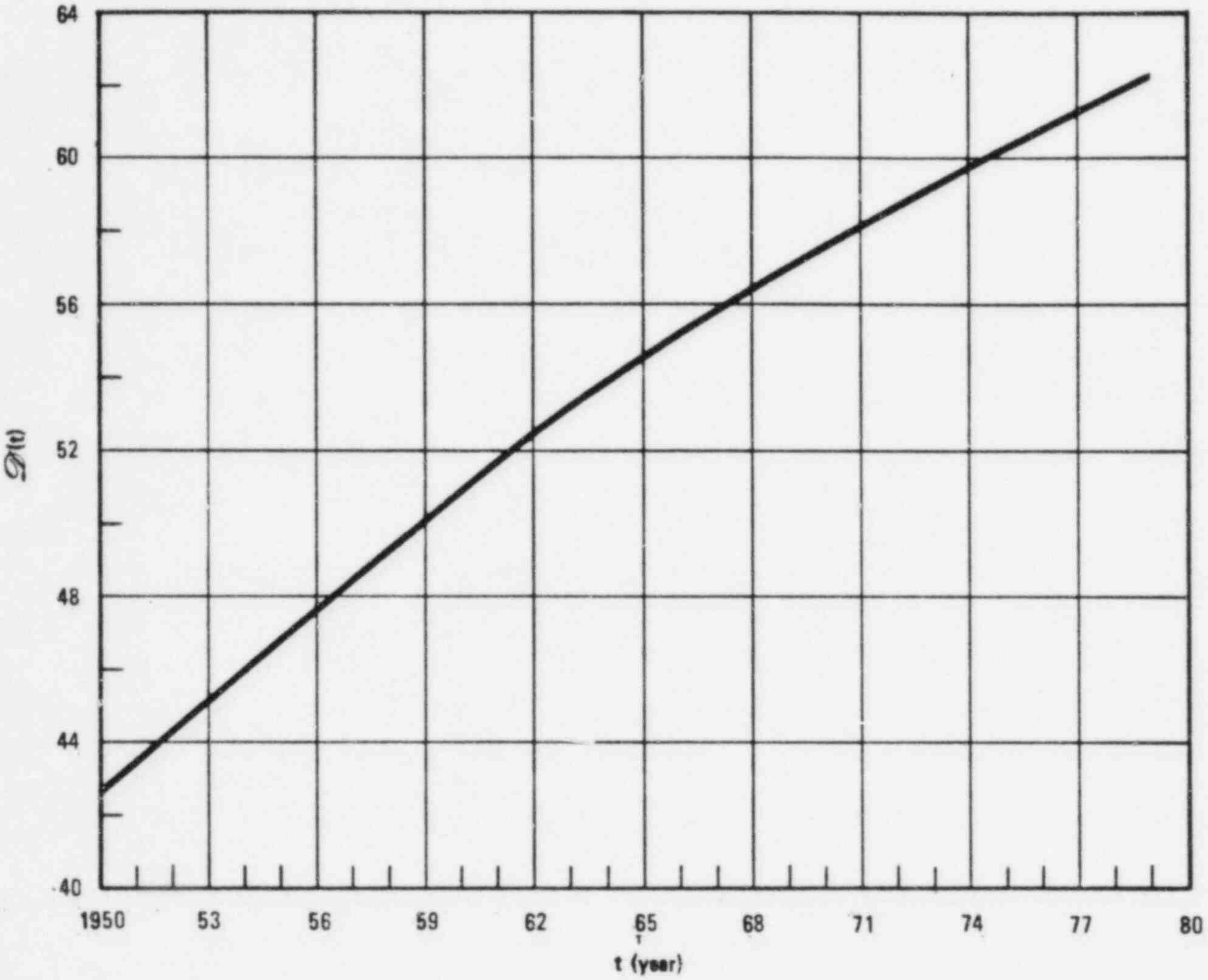

Figure B-2. Population Density in the U.S.A.

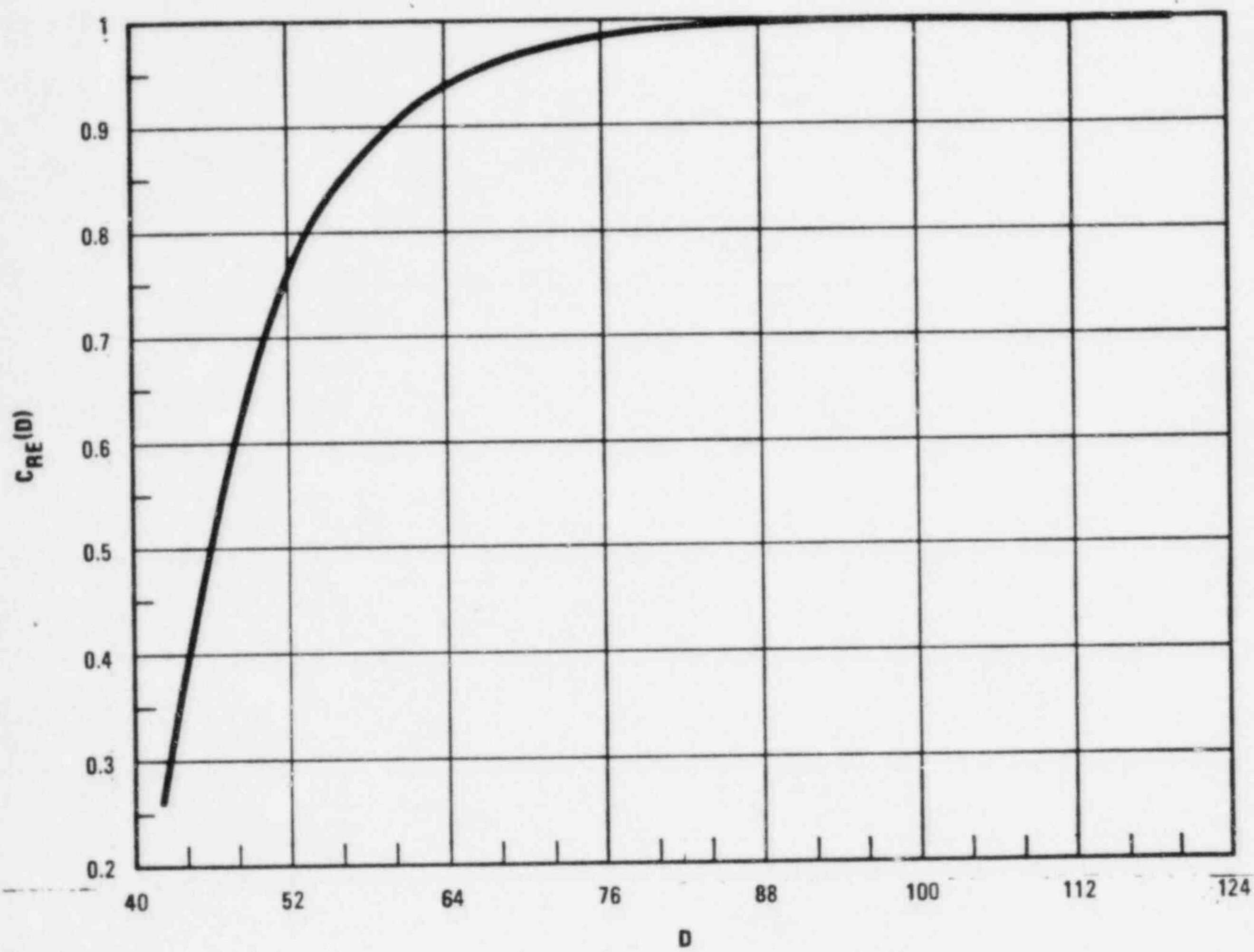

Figure B-3. Reporting Efficiency Curve

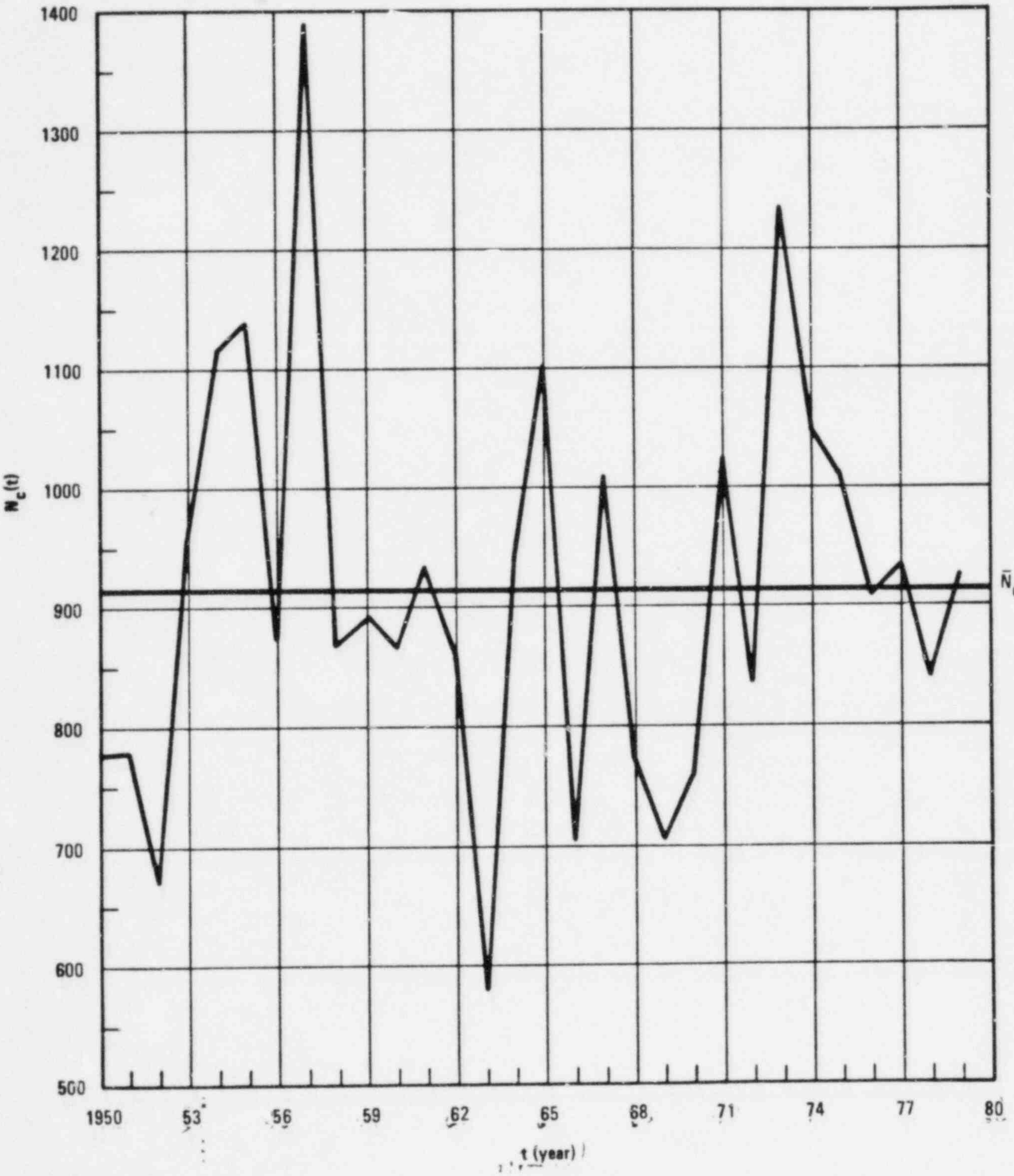

Figure B-4. Annual Adjusted Number of Tornadoes for the U.S.A.

APPENDIX C

THE PROBABILITY OF INJECTION OF POTENTIAL TORNADO MISSILES

Introduction

In this appendix, the probability of injection of potential missiles or the probability that a potential missile will become airborne is considered.

The model of missile injection is similar to that developed in Twisdale [C.1]. Because Twisdale did not calculate the injection probability in an explicit form, this study develops an explicit expression and presents the numerical results for the injection probability.

For tornado missile injection, the restraining forces ($\vec{F}_R$) must be overcome by aerodynamic forces before motion is possible. These aerodynamic forces are the lift and drag forces discussed in Appendix A. The expressions for the aerodynamic forces are:

$$F_{Ax} = f_D \sin\theta \cos\phi + f_L (\cos\theta \cos\phi \cos\psi - \sin\phi \sin\psi) \qquad (C.1)$$

$$F_{Ay} = f_D \sin\theta \sin\phi + f_L (\cos\theta \sin\phi \cos\psi + \cos\phi \sin\psi) \qquad (C.2)$$

$$F_{Az} = f_D \cos\theta - f_L \sin\theta \cos\psi \qquad (C.3)$$

The angles θ and ϕ give the orientation of the drag force $\vec{F}_D$ in the spherical system of coordinates and the angle between the lift force $\vec{F}_L$ and plane containing the axis OZ and the force $\vec{F}_D$ is denoted as ψ.

The angles θ, ϕ and ψ are the Euler's angles (sometimes other definitions are used, for example, $\phi \to \pi - \psi$, $\psi \to \pi - \phi$). The ranges for these angles are:

$$0 \leq \theta \leq \pi \qquad (C.4)$$

$$0 \leq \phi < 2\pi \qquad (C.5)$$

$$0 \leq \psi < 2\pi \qquad (C.6)$$

The coefficients f_D and f_L are determined by the formulas:

$$f_D = C_D \frac{\rho_a A w^2}{2} \tag{C.7}$$

$$f_L = C_L \frac{\rho_a A w^2}{2} \tag{C.8}$$

where:

ρ_a = air density,

A = missile cross-section area,

w = tornado wind speed,

C_D = aerodynamic drag coefficient,

C_L = aerodynamic lift coefficient.

The aerodynamic coefficients C_D and C_L for a cylindrical missile are considered in Redmann [C.2]. For the "standard missile" (see Appendix B), the approximate expression is:

$$C_D = 0.98 \sin^3 \alpha \tag{C.9}$$

$$C_L = 0.98 \sin^2 \alpha \cos \alpha \tag{C.10}$$

where the attack angle α has range:

$$0 \leq \alpha \leq \pi \tag{C.11}$$

The restraining forces $\vec{F}_R$ include gravity force and frictional, structural, and interlocking forces which tend to resist motion. The expression for restraining forces can be written in the following form:

$$F_{Rx} = -K_x \, mg \tag{C.12}$$

$$F_{Ry} = -K_y \, mg \tag{C.13}$$

$$F_{Rz} = -K_z \, mg \tag{C.14}$$

where m is the missile mass, g is the gravitational constant and K_x, K_y and K_z are restraint coefficients which show how many times greater (or less) the restraining forces are than pure gravity (mg).

Because F_{Rz} includes the gravity force, the coefficient K_z satisfies the inequality:

$$K_z \geq 1 \tag{C.15}$$

The coefficients K_x and K_y satisfy the following inequalities:

$$K_x > 0 \tag{C.16}$$

$$K_y > 0 \tag{C.17}$$

For potential missiles lying on the ground, the injection condition is

$$F_{Az} \geq \left|F_{Rz}\right| \tag{C.18}$$

or

$$f_D \cos\theta - f_L \sin\theta \cos\psi \geq K_L\, mg \tag{C.19}$$

where:

$K_L \equiv K_z$ is the lift restrain coefficient satisfying the inequality:

$$K_L \geq 1 \tag{C.20}$$

The expression (C.19) is the condition of vertical injection.

For the case where the potential missiles are located at some elevated height, the potential missile can become airborne due to horizontal displacement. In this case, the condition for vertical injection is:

$$F_{Ax}^{2} + F_{Ay}^{2} \geq F_{Rx}^{2} + F_{Ry}^{2} \tag{C.21}$$

or

$$F_{Ax}^{2} + F_{Ay}^{2} \geq K_D\, mg \tag{C.22}$$

where

$$K_D = K_x^{2} + K_y^{2} \tag{C.23}$$

The drag restrain coefficient K_D satisfies the inequality:

$$K_D > 0 \tag{C.24}$$

Using results of Appendix A, the variables w, K_D, K_L, α, θ, ϕ, ψ are treated as random functions with uniform distribution in the ranges:

$$w_1(F) \leq w \leq w_2(F) \tag{C.25}$$

$$K_1 \leq K_D \leq K_1 + \Delta K \tag{C.26}$$

$$K_2 \leq K_L \leq K_2 + \Delta K \tag{C.27}$$

$$0 \leq \alpha \leq \pi \tag{C.28}$$

$$0 \leq \theta \leq \pi \tag{C.29}$$

$$0 \leq \phi < 2\pi \tag{C.30}$$

$$0 \leq \psi < 2\pi \tag{C.31}$$

where $w_1(f)$ and $w_2(F)$ are lower and upper levels of wind speed corresponding to some Fujita scale of tornado intensity.

For angles α, θ, ϕ, and ψ, the uniform distribution is a good approximation. For wind speed (w), the uniform distribution is conservative because it overestimates the contribution of higher wind speeds. The real distributions of restrain coefficients K_D and K_L are not known, but, if the interval ΔK is quite narrow, then the uniform distribution is appropriate.

In the seven-dimension space of variables w, K_D, K_L, α, θ, ϕ, and ψ, the expressions (C.25) through (C.31) specify a right parallelepiped of volume V_F:

$$V_F = 4\pi^4 (\Delta K)^2 \left[w_2(F) - w_1(F)\right] \tag{C.32}$$

The equation

$$F_{Az} = K_L mg \tag{C.33}$$

is determined by a seven-dimension surface which divides the volume V_F into two parts $V_F^{(V)}$ and $V_F^{(\bar{V})}$. In the volume $V_F^{(V)}$, the condition of vertical injections is satisfied. It is obvious that

$$V_F^{(V)} + V_F^{(\bar{V})} = V_F \tag{C.34}$$

:

:

The equation

$$F_{Ax}^2 + F_{Ay}^2 = K_D \, mg \tag{C.35}$$

is determined on another seven-dimension surface which divides the volume V_F into another two parts $V_F^{(H)}$ and $V_F^{(\bar{H})}$. The condition of horizontal injection (C.22) is satisfied in the volume $V_F^{(H)}$.

In the same manner, the volume $V_F^{(T)}$ can be introduced where either of the conditions (C.19) or (C.22) are satisfied. It is clear that

$$V_F^{(H)} + V_F^{(\bar{H})} = V_F \tag{C.36}$$

and

$$V_F^{(T)} + V_F^{(\bar{T})} = V_F \tag{C.37}$$

but, generally

$$V_F^{(H)} + V_F^{(V)} \neq V_F^{(T)} \tag{C.38}$$

because in some subvolume both conditions (C.19) and (C.22) can be satisfied simultaneously.

Because of the uniform distribution of all parameters, the probability of horizontal injection $\eta^{(H)}(F)$, vertical injection $\eta^{(V)}(F)$, and total injection $\eta^{(T)}(F)$ are calculated according to the formulas:

$$\eta^{(H)}(F) = \frac{V_F^{(H)}}{V_F} \tag{C.39}$$

$$\eta^{(V)}(F) = \frac{V_F^{(V)}}{V_F} \tag{C.40}$$

$$\eta^{(T)}(F) = \frac{V_F^{(T)}}{V_F} \tag{C.41}$$

For calculation of the probabilities $\eta^{(H)}(F)$, $\eta^{(V)}(F)$, and $\eta^{(T)}(F)$, the Monte Carlo method is used. For this purpose, it is convenient to use the scaled variables:

$$x_1 = \frac{K_D - K_1}{\Delta K} \tag{C.42}$$

$$x_2 = \frac{K_L - K_2}{\Delta K} \tag{C.43}$$

$$x_3 = \frac{w - w_1}{w_2 - w_1} \tag{C.44}$$

$$x_4 = \frac{\alpha}{\pi} \tag{C.45}$$

$$x_5 = \frac{\theta}{\pi} \tag{C.46}$$

$$x_6 = \frac{\phi}{2\pi} \tag{C.47}$$

$$x_7 = \frac{\psi}{2\pi} \tag{C.48}$$

All variables x_i (i=1, 2, ... 7) are random variables with uniform distribution in the range from 0 to 1.

Computer program [3] generated the random vector $\vec{x}$ (x_1, x_2, ... x_7), calculated the variables w, K , K_L, α, θ, ϕ, ψ and checked conditions (C.19) and (C.22).

After N_F trials for F-scale tornado, the condition (C.22) is met $N_F^{(H)}$ times, the condition (C.19) is met $N_F^{(V)}$ times, and either of conditions (C.19) and (C.22) is met $N_F^{(T)}$ times.

Because for a large number of trials N_F, the number $N_F^{(H)}$ is proportional to $V_F^{(H)}$, the number $N_F^{(V)}$ is proportional to $V_F^{(V)}$, the number $N_F^{(T)}$ is proportional to $V_F^{(T)}$ and the number N_F is proportional to V_F.

Hence, the probability in question can be calculated by formulas:

$$\eta^{(H)}(F) = \frac{N_F^{(H)}}{N_F} \tag{C.49}$$

$$\eta^{(V)}(F) = \frac{N_F^{(V)}}{N_F} \tag{C.50}$$

$$\eta^{(T)}(F) = \frac{N_F^{(T)}}{N_F} \tag{C.51}$$

In further considerations, index T for total injection is omitted, but indices H and V (for horizontal and vertical) are retained.

The results of the calculation for $\Delta K = 0.5$, $K_D = 0, 1, 2, 3, 4, 5$, $K_L = 1, 2, 3, 4, 5$ and $F = 0, 1, 2, 3, 4, 5, 6$ are shown in Tables C-1 to C-7. In these tables, the letter H stands for horizontal injection, V for vertical injection, and T for total injection. The number of trials for every case is taken as 10,000. This corresponds to an accuracy of about 1%.

In Table C-8, the sensitivity of the results as a function of the number of simulation trials is shown. The result, rounded to two digits in parenthesis, stabilizes between 10,000 and 100,000 trials. This suggests that 10,000 trials is a good approximation.

The upper limit for the probability of injection corresponds to the case when the restraining force for horizontal injection is friction and for vertical injection is gravity. The lower limit for $\eta(F)$ is quite uncertain because it depends on maximum values of K_D and K_L which we assign to the potential missiles. The number of potential missiles derived from [4] was based on maximum values for $K_D = 5$ and $K_L = 5$.

The maximum and minimum values of $\eta(F)$ extracted from Tables C-1 to C-7 are shown in Table C-9.

Assuming a uniform distribution for K_D and K_L in the range

$$0 < K_D \leq 5 \tag{C.52}$$

$$1 \leq K_L \leq 5 \tag{C.53}$$

we will find mean values for $\eta(F)$ given in Table C-10.

Now the distribution for $\eta(F)$ is fit with a lognormal distribution, which has the same upper limit as that given in Table C-9 and the mean and median are as close as possible to the data in Tables C-1 through C-7.

The results are shown on Table C-11.

For $\eta(3)$, $\eta(4)$, $\eta(5)$, and $\eta(6)$, the distribution generated by uniform distributions of K_D and K_L is exactly lognormal. For $\eta(0)$, $\eta(1)$, and $\eta(2)$, the lognormal distribution is a conservative approximation.

The same methodology is applied to the vertical injection probability, $\eta^{(V)}(F)$. The lower and upper limits and means are extracted from Tables C-1 through C-7 and are shown in Table C-12. The data for the lognormal fit is shown in Table C-13.

The results, shown in Tables C-11 and C-13, indicate that the probability of potential missiles being injected (i.e., becoming airborne) is rather small, except for the larger F-scale tornadoes.

During the operation period, there are no unrestrained potential missiles at any elevation above the ground. Although the equation (C.52) is valid for the ground potential missiles, for the elevated potential missiles we should use the range for K_D:

$$1 \leq K_D \leq 5 \qquad (C.54)$$

The lower and upper limits and means for this case are shown in Table C-14. The lognormal fit is shown in Table C-15.

References

[C.1] Twisdale, et. al., "Tornado Missile Risk Analysis", EPRI NP-768, NP-769, May (1978).

[C.2] Redmann, G. M. et. al., "Wind Field and Trajectory Models for Tornado - Propelled Objects", EPRI 308, Technical Report 1, Feb. (1976).

[C.3] IMSL Library Reference Manual, Edition 8, IMSL LIB-0008, International Mathematical and Statistical Libraries, Inc., June, 1980.

Table C-1

Probability of Injection η (0)

K_D \ K_L	Type	1	2	3	4	5
0	H V T	.1756 .0000 .1756	.1756 .0000 .1756	.1756 .0000 .1756	.1756 .0000 .1756	.1756 .0000 .1756
1	H V T	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
2	H V T	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
3	H V T	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
4	H V T	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
5	H V T	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000

Table C-2

Probability of Injection η (1)

K_D \ K_L	Type	1	2	3	4	5
0	H	.4274	.4274	.4274	.4274	.4274
	V	.0000	.0000	.0000	.0000	.0000
	T	.4274	.4274	.4274	.4274	.4274
1	H	.0000	.0000	.0000	.0000	.0000
	V	.0000	.0000	.0000	.0000	.0000
	T	.0000	.0000	.0000	.0000	.0000
2	H	.0000	.0000	.0000	.0000	.0000
	V	.0000	.0000	.0000	.0000	.0000
	T	.0000	.0000	.0000	.0000	.0000
3	H	.0000	.0000	.0000	.0000	.0000
	V	.0000	.0000	.0000	.0000	.0000
	T	.0000	.0000	.0000	.0000	.0000
4	H	.0000	.0000	.0000	.0000	.0000
	V	.0000	.0000	.0000	.0000	.0000
	T	.0000	.0000	.0000	.0000	.0000
5	H	.0000	.0000	.0000	.0000	.0000
	V	.0000	.0000	.0000	.0000	.0000
	T	.0000	.0000	.0000	.0000	.0000

Table C-3

Probability of Injection η (2)

K_D \ K_L	Type	1	2	3	4	5
0	H V T	.6474 .0305 .6528	.6474 .0000 .6474	.6474 .0000 .6474	.6474 .0000 .6474	.6474 .0000 .6474
1	H V T	.0647 .0305 .0941	.0647 .0000 .0647	.0647 .0000 .0647	.0647 .0000 .0647	.0647 .0000 .0647
2	H V T	.0000 .0305 .0305	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
3	H V T	.0000 .0305 .0305	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
4	H V T	.0000 .0305 .0305	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000
5	H V T	.0000 .0305 .0305	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000	.0000 .0000 .0000

Table C-4

Probability of Injection η (3)

K_D \ K_L	Type	1	2	3	4	5
0	H V T	.7591 .1230 .7657	.7591 .0281 .7610	.7591 .0001 .7592	.7591 .0000 .7591	.7591 .0000 .7591
1	H V T	.2895 .1230 .3654	.2895 .0281 .3104	.2895 .0001 .2896	.2895 .0000 .2895	.2895 .0000 .2895
2	H V T	.0592 .1230 .1771	.0592 .0281 .0873	.0592 .0001 .0593	.0592 .0000 .0592	.0592 .0000 .0592
3	H V T	.0004 .1230 .1234	.0004 .0281 .0285	.0004 .0001 .0005	.0004 .0000 .0004	.0004 .0000 .0004
4	H V T	.0000 .1230 .1230	.0000 .0281 .0281	.0000 .0001 .0001	.0000 .0000 .0000	.0000 .0000 .0000
5	H V T	.0000 .1230 .1230	.0000 .0281 .0281	.0000 .0001 .0001	.0000 .0000 .0000	.0000 .0000 .0000

Table C-5

Probability of Injection η (4)

K_D \ K_L	Type	1	2	3	4	5
0	H V T	.8184 .1943 .8218	.8184 .1100 .8206	.8184 .0466 .8198	.8184 .0102 .8188	.8184 .0000 .8184
1	H V T	.4871 .1943 .5428	.4871 .1100 .5230	.4871 .0466 .5065	.4871 .0102 .4934	.4871 .0000 .4871
2	H V T	.2554 .1943 .3798	.2554 .1100 .3321	.2554 .0466 .2935	.2554 .0102 .2651	.2554 .0000 .2554
3	H V T	.1046 .1943 .2751	.1046 .1100 .2063	.1046 .0466 .1504	.1046 .0102 .1148	.1046 .0000 .1046
4	H V T	.0188 .1943 .2102	.0188 .1100 .1285	.0188 .0466 .0654	.0188 .0102 .0290	.0188 .0000 .0188
5	H V T	.0000 .1943 .1943	.0000 .1100 .1100	.0000 .0466 .0466	.0000 .0102 .0102	.0000 .0000 .0000

Table C-6

Probability of Injection η (5)

K_D \ K_L	Type	1	2	3	4	5
0	H	.8584	.8584	.8584	.8584	.8584
	V	.2449	.1728	.1192	.0751	.0377
	T	.8602	.9598	.9596	.8594	.8593
1	H	.6208	.6208	.6208	.6208	.6208
	V	.2449	.1728	.1192	.0751	.0377
	T	.6545	.6442	.6396	.6359	.6313
2	H	.4300	.4300	.4300	.4300	.4300
	V	.2449	.1728	.1192	.0751	.0377
	T	.5212	.4926	.4773	.4652	.4514
3	H	.2784	.2784	.2784	.2784	.2784
	V	.2449	.1728	.1192	.0751	.0377
	T	.4260	.3840	.3566	.3339	.3088
4	H	.1715	.1715	.1715	.1715	.1715
	V	.2449	.1728	.1192	.0751	.0377
	T	.3584	.3074	.2709	.2399	.2074
5	H	.0827	.0827	.0827	.0827	.0827
	V	.2449	.7728	.1192	.0751	.0377
	T	.3036	.2424	.1964	.1564	.1204

Table C-7

Probability of Injection η (6)

K_D \ K_L	Type	1	2	3	4	5
0	H V T	.8857 .2847 .8865	.8857 .2214 .8864	.8857 .1737 .8863	.8857 .1360 .8862	.8857 .1034 .8862
1	H V T	.6999 .2847 .7206	.6999 .2214 .7148	.6999 .1737 .7122	.6999 .1360 .7107	.6999 .1034 .7093
2	H V T	.5623 .2847 .6210	.5623 .2214 .6041	.5623 .1737 .5954	.5623 .1360 .5903	.5623 .1034 .5862
3	H V T	.4346 .2847 .5423	.4346 .2214 .5135	.4346 .1737 .4964	.4346 .1360 .4860	.4346 .1034 .4773
4	H V T	.3208 .2847 .4774	.3208 .2214 .4404	.3208 .1737 .4153	.3208 .1360 .3979	.3208 .1034 .3841
5	H V T	.2384 .2847 .4285	.2384 .2214 .3863	.2384 .1737 .3568	.2384 .1360 .3352	.2384 .1034 .3167

Table C-8

Sensitivity of η to the Number of Trials

N	$\eta^{(H)}(1)$ for K = 0, K_L = 1	$^1/\sqrt{N}$	$\delta = \frac{\eta_N - \eta_{10^5}}{\eta_{10^5}}$
100	.47000 (.47)	.1000	.0830
1000	.44600 (.45)	.0316	.0277
10,000	.42740 (.43)	.0100	.0151
100,000	.43397 (.43)	.0032	-

Table C-9

Maximum and Minimum Values for $\eta(F)$

F	Maximum $\eta(F)$	Minimum $\eta(F)$
0	0.1756	0.0000
1	0.4274	0.0000
2	0.6528	0.0000
3	0.7657	0.0000
4	0.8218	0.0000
5	0.8602	0.1204
6	0.8865	0.3167

Table C-10

Means for $\eta(F)$

F	Mean
0	0.0293
1	0.0712
2	0.1239
3	0.2082
4	0.3281
5	0.4708
6	0.5817

Table C-11

Lognormal Distribution for $\eta(F)$

F	Lower Limit	Median	Mean	Upper Limit
0	0.0008	0.0119	0.0454	0.1756
1	0.0020	0.0292	0.1105	0.4274
2	0.0029	0.0435	0.1687	0.6528
3	0.0098	0.0866	0.2083	0.7657
4	0.0789	0.2546	0.3282	0.8218
5	0.2160	0.4310	0.4708	0.8602
6	0.3529	0.5593	0.5817	0.8865

Table C-12

Lower and Upper Limits and Means for $\eta^{(V)}(F)$

F	Lower Limit	Mean	Upper Limit
0	0	0	0
1	0	0	0
2	0	0.0061	0.0305
3	0	0.0302	0.1230
4	0	0.0722	0.1943
5	0.0377	0.1199	0.2449
6	0.1034	0.1838	0.2847

Table C-13

Lognormal Distribution for $\eta^{(V)}(F)$

F	Lower Limit	Median	Mean	Upper Limit
2	0.0001	0.0017	0.0079	0.0305
3	0.0006	0.0086	0.0318	0.1230
4	0.0143	0.0527	0.0722	0.1943
5	0.0450	0.1050	0.1199	0.2449
6	0.1089	0.1761	0.1838	0.2847

Table C-14

Lower and Upper Limits and Means for $\eta(F)$ for Restrained Potential Missiles

F	Lower Limit	Mean	Upper Limit
0	0	0	0
1	0	0	0
2	0	0.0141	0.0941
3	0	0.0977	0.3654
4	0	0.2297	0.5428
5	0.1204	0.3930	0.6545
6	0.3167	0.5207	0.7206

Table C-15

Lognormal Distribution for $\eta(F)$ for Restrained Potential Missiles

F	Lower Limit	Median	Mean	Upper Limit
2	0.0002	0.0043	0.0249	0.0941
3	0.0038	0.0375	0.0976	0.3654
4	0.0635	0.1857	0.2297	0.5428
5	0.2093	0.3701	0.3930	0.6545
6	0.3598	0.5092	0.5207	0.7206

Appendix D. General Methods

D.1 Preamble

The purpose of the PRA analysis of a tornado missile's hazard is to evaluate the probability, P_T, of damaging some target per year. However, due to randomness of natural factors and uncertainty of our knowledge about many parameters, we have an estimate for the probability, P_T, which has uncertainty associated with it.

The best approach is to develop a distribution function, $f(P_T)$, for the damage probability, P_T. The typical curve of the distribution of the damage probability (in logarithmic scale) is shown in Figure D-1.

The distribution function, $f(P_T)$, allows us to evaluate the "best" estimate for P_T, which is a median value, and a confidence interval that shows the spread of the most likely values for the probability, P_T, around the median.

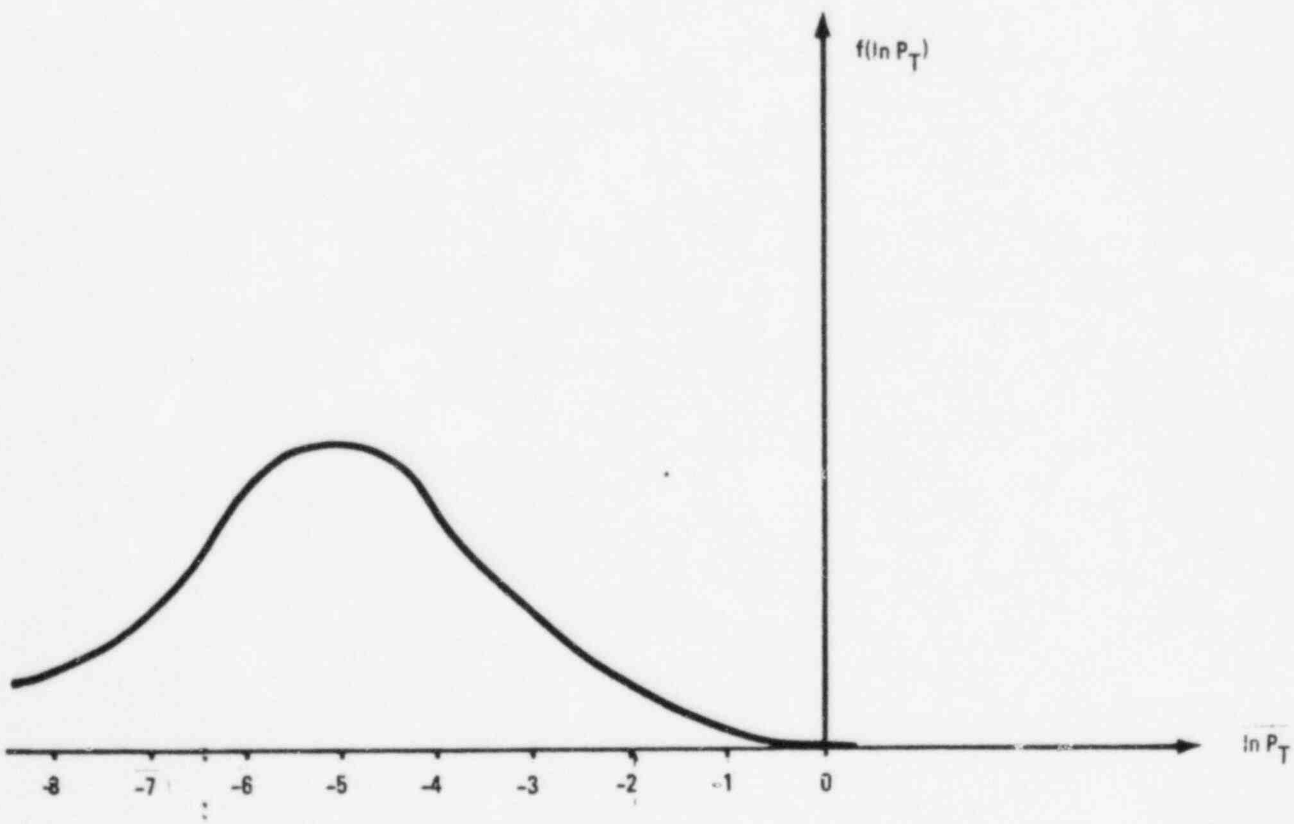

Figure D-1. Typical Distribution Function

Therefore, we can specify three major steps of PRA analysis:

A. Development of the model for calculating the damage probability, P_T, as a function of some random parameters.

B. Development of the distributions for all random parameters that the damage probability, P_T, depends on.

C. Calculation of the final distribution of the damage probability, $f(P_T)$, using the distribution for all random parameters (propagation of uncertainties).

There are three sources of randomness and uncertainty:

A. Natural phenomena are essentially random.

B. Data are incomplete.

C. Mathematical models or solutions for these models are approximate.

Classical statisticians believe that a complete set of data is always available and the uncertainty could be reduced to zero. Classical statistics has developed very well-established methods of dealing with randomness.

Recently, some analysts have started to recognize (see [D.1]-[D.2]) that in many areas of research the uncertainty cannot be completely eliminated. The reason is not based only on difficulty of collecting data.

In early PRA analyses, classical statistics was applied with poor data bases. Naturally, all these results were questionable and only compromised the credibility of PRA methods.

The Bayesian approach improved the practice of dealing with uncertainties but the major flaw contained in the arbitrary a priori distribution is still present.

We have to recognize that classical statistics was developed for problems dealing with randomness but not with uncertainty. In cases when uncertainty is not removable, a new approach has to be developed. This approach has some features in common with the Bayesian approach, but it is much broader.

The major features of the new approach are:

A. Every random parameter can be described in a way that reflects both randomness and uncertainty.

B. Distribution of the random parameter has to satisfy the principle of maximum entropy given the knowledge.

The entropy is a measure of the expected uncertainty. If the entropy of the probability distribution given some knowledge is maximal, it means that the shape of the probability curve exactly reflects what we know.

For example, the continuous distribution having the largest entropy for a given variance, σ^2, is the normal distribution [D.3]. However, if we know that a random parameter is positive, then a maximum entropy distribution is presumably lognormal. The entropy of the lognormal distribution is less than for the normal one because we have some additional information (a parameter could be only positive). However, the entropy of the lognormal distribution is believed to be the highest in the class of distribution functions with a positive domain of definition of a random parameter and a given variance.

If the distribution satisfies the principle of maximum entropy, we are guaranteed that a confidence interval for a damage probability can only be reduced when additional information becomes available.

The principles discussed are incorporated in this study.

D.2 Tornado Characteristics

The main characteristics of tornadoes that determine the probability of damage are the tornado path area, a, and the Fujita scale, F.

The tornado path area, a, is a product of the length, L, and width, W, of the tornado's destructive track on the earth's surface:

$$a = L \times W \qquad (D.1)$$

The classification of tornadoes according to their damage areas was proposed by Fujita [D.4]. He used a decimal logarithm of the tornado path area, a, measured in square miles. However, a computerized file at the National Severe Storms Forecast center keeps information about tornado length, L, in tenths of miles and tornado width, W, in tens of feet. Because all data are rounded, it will be more accurate to classify all tornadoes by areas estimated in miles-feet.

The computerized record naturally groups all tornadoes at the decimally logarithmic scale of tornado area, a, measured in miles-feet. If we translate miles-feet to square miles and calculate the number of tornadoes into new intervals, we have to somehow divide the number of tornadoes in old intervals between the new ones. This procedure requires us to use some hypothesis of tornado distribution by areas and introduces additional sources of error.

Therefore, in our classification of tornadoes by areas, we will use tornado path area, a, measured in miles-feet. We will call this classification an A-scale (see Table D.1).

The upper and lower limits of tornado area belonging to some scale, A, can be determined by formulae:

$$a_{up} = 10^{A} \tag{D.2}$$

$$a_{low} = 10^{A-1} \tag{D.3}$$

The median value assuming a lognormal distribution is:

$$a_{med} = 10^{A-0.5} \tag{D.4}$$

where A = 1, 2, ... 7.

Fujita also proposed classifying tornadoes by intensity [D.5]. The Fujita scale, F, is a characteristic of tornado intensity that depends on the damaging wind speed according to Table D.2.

The relationships between upper, lower, and median wind speeds with corresponding Fujita scale, F, are given by the following approximate formulae:

$$w_{up} = 14.1 \times (F + 3)^{3/2} \text{ (mph)} \tag{D.5}$$

$$w_{low} = 14.1 \times (F + 2)^{3/2} \text{ (mph)} \tag{D.6}$$

$$w_{med} = 14.1 \times (F + 2.5)^{3/2} \text{ (mph)} \tag{D.7}$$

For wind speed, w, given in meters per second (m/s), the coefficient in formulae (D.5) through (D.7) should be 6.30.

D.3 Tornado Missile Description

The spectrum of potential tornado missiles at the plant site and in the nearby area is described in the Standard Review Plan, section 3.5.1.4 [2]. A more detailed spectrum, based on seven-plant survey data, is given in an EPRI study [5].

From the point of view of missile injection and transportation, the classification of potential missiles should be based on parameters determining the missile acceleration. The equations of missile motion can be written in the form:

$$\dot{v}_x = C_x \frac{\rho_a u^2}{2 \left(\frac{m}{A}\right)} \tag{D.8}$$

$$\vdots$$

$$\dot{v}_y = C_y \frac{\rho_a u^2}{2\left(\frac{m}{A}\right)} \tag{D.9}$$

$$\dot{v}_z = C_z \frac{\rho_a u^2}{2\left(\frac{m}{A}\right)} - g \tag{D.10}$$

Where m is the missile mass; v_x, v_y, and v_z are components of the missile velocity; C_x, C_y, and C_z are empirical aerodynamic coefficients; ρ_a is the air density; A is the missile cross-section area; u is the relative wind-missile speed; and g is the gravitational acceleration near the earth's surface.

Missile acceleration components depend on aerodynamic coefficients C_x, C_y, and C_z, and parameter m/A. Aerodynamic coefficients depend on missile orientation, wind velocity, and missile shape.

For a cylindrical missile with length, ℓ, and diameter, d, the shape parameter is ℓ/d (see Figure D-2).

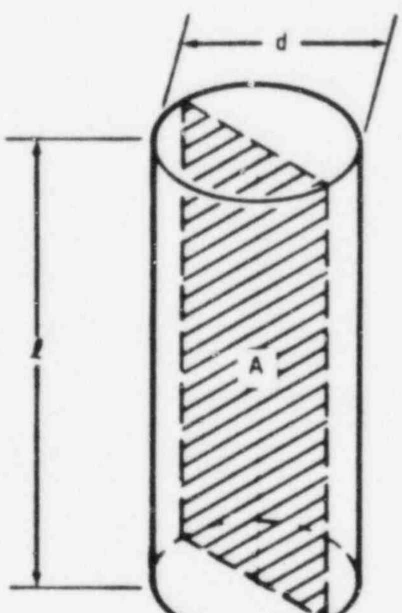

Figure D-2. Cylindrical Missile

Consider a simplified case when the wind velocity, $\vec{w}$, is directed along axis Ox and a cylindrical missile is located in the plane xOz with the angle of attack, α, (see Figure D-3). For this case, we have:

$$C_x = C_D \tag{D.11}$$

$$C_y = 0 \tag{D.12}$$

$$C_z = C_L \tag{D.13}$$

Where C_D is the drag coefficient and C_L is the lift coefficient (the side coefficient for a cylinder is equal to zero). In general, these equations are considered in Appendix A.

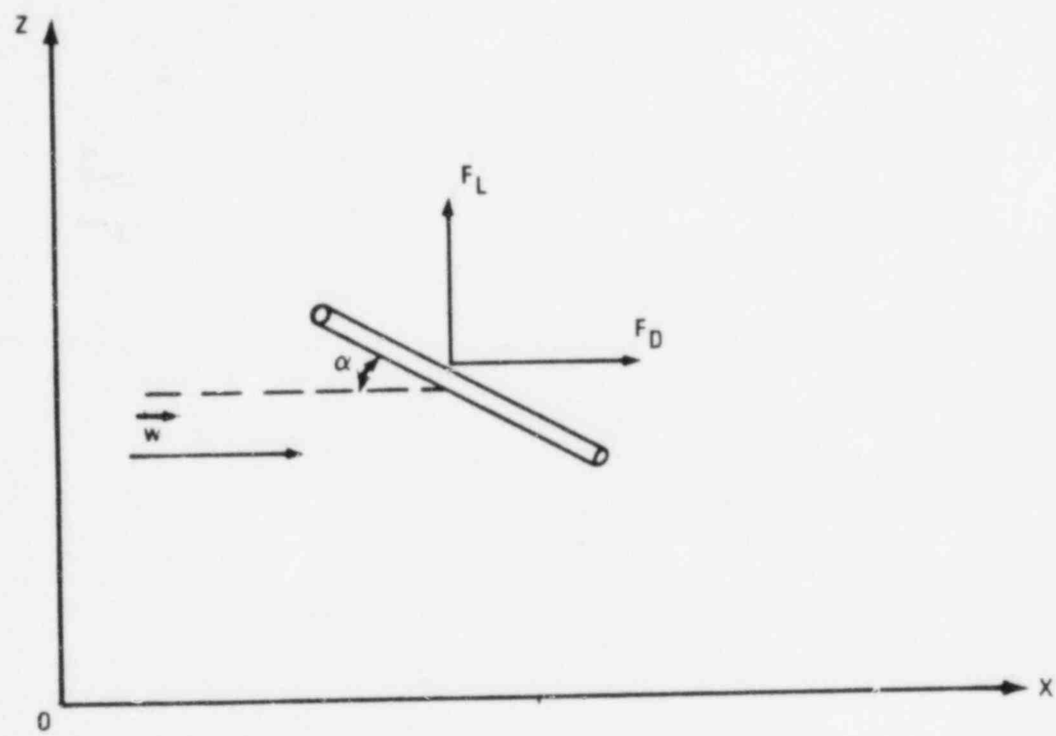

Figure D-3. Missile Orientation

According to [6], the coefficients C_L and C_D can be determined by formulae:

$$C_L = C_1 \sin^2\alpha \cos\alpha - C_2 \cos\alpha \left|\cos\alpha\right| \sin\alpha \tag{D.14}$$

$$C_D = C_1 \sin^3\alpha + C_2 \left|\cos^3\alpha\right| \tag{D.15}$$

where

$$C_1 = 1.2\ k \tag{D.16}$$

$$C_2 = \frac{\pi}{4} \cdot \frac{d}{\ell}\ C \tag{D.17}$$

and

$$K = 0.59 + 0.41 \exp\left(-20 \frac{d}{\ell}\right) \tag{D.18}$$

$$C = \begin{cases} 1.16, & \frac{\ell}{d} \leq 1 \\ 0.84 + 0.32 \exp\left[-2\left(\frac{\ell}{d} - 1\right)\right], & 1 \leq \frac{\ell}{d} < 4 \\ 0.79 + 0.0125 \cdot \frac{\ell}{d}, & \frac{\ell}{d} \geq 4 \end{cases} \tag{D.19}$$

For long missiles

$$\frac{d}{\ell} < 1 \tag{D.20}$$

we have approximately

$$C_L \simeq C_1 \sin^2\alpha \cos\alpha \tag{D.21}$$

$$C_D \simeq C_1 \sin^3\alpha \tag{D.22}$$

The aerodynamic coefficient, C_L, is maximal for the angle of attack $\alpha \simeq 55°$ and coefficient C_D for the angle $\alpha \simeq 90°$.

The specification of tornado-generated missiles is given in Table D.3.

We adopted the effective diameter for a wooden plank, d = 0.65 ft, and for an automobile, d = 6 ft.

According to equations (D.8) through (D.10), the missile acceleration depends on parameters $C_L/(m/A)$ and $C_D/(m/A)$. These parameters for different missiles are shown in Table D.4. Averaging these parameters over the spectrum of potential missiles given in the EPRI study [5], we find so-called "standard missile" parameters. The major contributors to the spectrum of potential missiles are: type C (46%) and types B, D, and E (21% altogether).

Our analysis assumes, for simplicity of explanation, that we have only one type of potential missiles: so-called "standard" missiles.

However, the final results are not sensitive to the particular values of standard missile parameters. Actually, the numerical values of the parameters under consideration will affect the parameters of height distribution, $\psi(z, F)$, (see section D.11) and the probability of injection, $\eta(F)$, (see section D.10).

The uncertainties of $\psi(z, F)$ and $\eta(F)$ adopted in our study are much broader than those created by dispersion of aerodynamic characteristics.

D.4 Probability of Damage Given Tornado Frequency ν, Path Area a, Fujita Scale F, Density of Potential Missiles n_p, Injection Probability $\eta(F)$ and Height Distribution $\psi(z, F)$

Let us assume that a tornado striking the plant site has a given path area, a, and Fujita scale, F. We assume also that the missile's characteristics are known. Therefore, the density of potential missiles near a target, n_p, injection probability, $\eta(F)$, and height distribution, $\psi(z, F)$ are certain values.

Under these conditions, the probability of damage, P_T, can be written in the form:

$$P_T = P_O \cdot P_H \cdot P_D \tag{D.23}$$

Where:

P_O = Probability per year that a tornado with given characteristics a and F strikes the plant site.

P_H = Conditional probability of hitting a target given that the tornado strikes the plant site.

P_D = Conditional probability of a target damage given a hit.

According to Thom [D.6], the probability P_0 is:

$$P_0 = \frac{\nu a}{S} \tag{D.24}$$

where ν is the annual frequency that a tornado having the same occurrence characteristics as a tornado at the plant site will strike the area S.

For the probability of damage, P_D, we conservatively assume:

$$P_D = 1 \tag{D.25}$$

This assumption will provide a conservatism of several orders of magnitude.

For the calculation of the conditional probability of hitting, P_H, a special statistical mechanics approach was developed. This approach is based on the following assumptions:

1. Tornado missile motion is a diffusion Markovian process.
2. Propagation or Green's function for a tornado missile is uniform in space and time and has axial symmetry.

The derivation of the expression for P_H and justification for the assumptions are given in Appendix A. The final expression for the hitting probability, P_H, is:

$$P_H = n_p \, A \, \eta(F) \, \psi(z, F) \tag{D.26}$$

Where A is a target area (in our case, it is the area of the IVC roof) and Z is a target elevation above the ground.

Putting (D.24) through (D.26) into (D.23), we obtain:

$$P_T = \frac{\nu a}{S} \; n_p \, A \, \eta(F) \, \psi(z, F) \tag{D.27}$$

The probability of damage, P_T, depends on tornado path area, a, tornado Fujita scale, F, and set of parameters, ξ, that determine the distributions of ν, n_p, $\eta(F)$ and $\psi(z, F)$.

The distributions of these parameters will be discussed in the next sections.

D.5 Distribution of Random Parameters

The probability of damage, P_T, depends on two groups of random parameters:

1. ν, a, F

2. n_p, $\eta(F)$, $\psi(z, F)$

The first group reflects the randomness of natural phenomena and depends on tornado characteristics.

The second group depends on tornado missile characteristics and reflects the uncertainty of data.

The distributions for the first group of parameters are based on historical data, and those for the second group are partially appealing to the state-of-knowledge, engineering judgment, and physical limitations.

The two groups of parameters are completely independent. Among the first group of parameters, we did not find any credible correlation between frequency, ν, and the other two parameters, a and F. Therefore, the distribution of these parameters can be written in the form:

$$f(\nu, a, F) = f_1(\nu)\, f_2(a, F) \tag{D.28}$$

It is convenient to write the joint distribution, $f_2(a, F)$ in the following way:

$$f_2(a, F) = f(a)\, \phi(F|a) \tag{D.29}$$

When f(a) is a marginal distribution of the tornado path area, and $\phi(F\ a)$ is a conditional distribution of Fujita scale given the path area.

The distribution, f(a), is different for different geographical areas. The typical area containing enough data to develop a reliable distribution is about the area of one or several states. In our case, the data for Texas are sufficient.

The distribution, f(a), depends on meteorological conditions which set the scale of tornado size. Therefore, every large meteorological zone requires its own distribution.

On the other hand, the conditional distribution, $\phi(F|a)$, depends on the physical model of the tornado. Essentially, the physical model of the tornado as a natural phenomenon is the same for all geographical areas. All varieties of tornado characteristics depend on ratios of fundamental parameters: total energy, momentum, and angular momentum of the tornado. Therefore, the nationwide data were used for the calculation of the conditional distribution, $\phi(F|a)$.

For the second group of parameters, we remove all possible correlations by using the principle of the superior estimate.

Let $\vec{\xi}$ $(\xi_1, \xi_2, \dots \xi_n)$ be the set of random parameters determining the distributions of n_p, $\eta(F)$ and $\psi(z, F)$. The joint distributions of all parameters are:

$$g(\xi_1, \xi_2, \dots \xi_n) = g_1(\xi_1)\, g_2(\xi_2|\xi_1) \cdots g_n(\xi_n|\xi_1\ \xi_2 \cdots \xi_{n-1}) \quad \text{(D.30)}$$

Define the set of superior functions:

$$\begin{aligned} \bar{g}_2(\xi_2) &\geq g_2(\xi_2|\xi_1) \text{ for all } \xi_1 \\ \bar{g}_3(\xi_3) &\geq g_2(\xi_3|\xi_1\ \xi_2) \text{ for all } \xi_1, \xi_2 \\ \bar{g}_n(\xi_n) &\geq g_n(\xi_n|\xi_1\ \xi_2 \cdots \xi_{n-1}) \text{ for all } \xi_1, \xi_2, \cdots \xi_{n-1} \end{aligned} \quad \text{(D.31)}$$

Then the conservative estimate of the joint probability $g(\xi_1, \xi_2, \dots \xi_n)$ (in the meaning of higher probability of damage) is:

$$g(\xi_1, \xi_2, \dots \xi_n) = g_1(\xi_1)\, \bar{g}_2(\xi_2) \cdots \bar{g}_n(\xi_n) \quad \text{(D.32)}$$

The specific details of all distributions will be discussed in the corresponding sections of this report. The total joint distribution of all parameters takes the form:

$$f(v, a, F, \vec{\xi}) = f_1(v) \cdot f(a) \cdot \phi(F|a) \cdot g_1(\xi_1)\, \bar{g}_2(\xi_2) \cdots \bar{g}_n(\xi_n) \quad \text{(D.33)}$$

The distribution (D.33) generates the distribution for damage probability P_T which will be considered in section D.13).

D.6 Annual Frequency v of tornado occurrence

The nuclear power plant at the South Texas Project site is located in Matagorda county with coordinates of units:

	Unit I	Unit II
Latitude	28° 47' 42"	28° 47' 42"
Longitude	96° 02' 53"	96° 02' 59"

Because both units are located near the coast, a 10,000-square-mile area with its center within the plant site will contain about 40% sea with low efficiency of tornado counting.

Therefore, we determine the annual frequency, v, using the historical record for six counties located near the plant site.

The list of selected counties is given in Table D.5.

Our computer code estimates the annual number of tornado occurrences, finds the 25th percentile, median, 75th percentile, mean, and standard deviation for the empirical distribution. It then fits this distribution by a lognormal distribution having the same median and ratio of 25th and 75th percentiles as the empirical distribution.

The annual number of tornado occurrences for six counties listed in Table D.5 are given in Table D.6. Tornado segments are conservatively treated as separate occurrences for purposes of determining frequency.

Parameters of the fitted lognormal distribution are given in Table D.7.

The comparison of the empirical and fitted lognormal distribution is shown in Table D.8.

We see that the lognormal fit is quite conservative.

The annual frequency, v_6, of tornado occurrences for six counties has to be adjusted to the area S = 10,000 square miles according to the formula:

$$v' = v_6 \cdot \frac{S'}{S_6} \qquad \text{(D.34)}$$

Where v' and v_6 are medians for the areas S and S_6, correspondly, and S_6 = 5880 square miles.

The annual frequency v' has to be adjusted for reporting efficiency (see Appendix B) according to the formula:

$$v = \frac{v'}{C_{RE}} \qquad \text{(D.35)}$$

Where v is an adjusted annual frequency and C_{RE} is the reporting efficiency:

$$C_{RE} = 1 - e^{-C(D + D_o)} \qquad \text{(D.36)}$$

Where:

D = Population density for six counties (69.97)

D_o = -40.04561369

C = 0.118188864

The calculation according to the formulae (D.35) and (D.36) for the median value gives:

$$v = 5.25 \qquad \text{(D.37)}$$

The parameters of distribution for the adjusted annual frequency ν are given in Table D.9.

The local data for Matagorda county are not sufficient to develop a reliable distribution. However, we can estimate the mean for Matagorda County and adjust it to the area S = 10,000 square miles. The adjusted mean annual frequency $\bar{\nu}$ for Matagorda County:

$$\bar{\nu} = 7.394 \tag{D.38}$$

This number is lower than the number 10.704 incorporated in the fitted lognormal distribution.

D.7 Distribution f(a) for Tornado Path Area

To collect sufficient data to develop the distribution, f(a), of the tornado path area, we use data for Texas for 30 years (1953-1982) containing 2730 records [D.9].

The distribution of the path area according to the A-scale is given in Table D.10. This distribution is compared with the nationwide distribution.

Mean tornado path area $\bar{a}$ for Texas is 3.5 times less than $\bar{a}$ for the U.S. However, this difference is significantly less than the difference between Thom data for Kansas and Iowa [D.6] and nationwide data. The mean path area $\bar{a}$ for these states is 20 times greater than for the U.S.

A thorough analysis of data presented in Table D.10 shows that a real number of tornado occurrences in each A-scale for Texas could vary in the range up to 15%. Therefore, these data should be corrected by using the appropriate analytical fit.

We assume that the distribution, f(a), of the tornado path area is exactly lognormal. The best parameters of lognormal distributions μ and σ are shown in Table D.11. These parameters were found with the least-squared method [D.3].

The empirical and lognormal distributions for A-scale for Texas are compared in Table D.12. We see that the lognormal distribution has a tendency to overestimate the probability of large tornadoes.

Therefore, the use of the lognormal distribution will be conservative because it will increase the estimated probability of a tornado strike to the nuclear power plant site.

D.8 Joint Distribution of Tornado Path Area a and Fujita Scale F

The correlation coefficient between the tornado path area and Fujita scale, F, is very sensitive to data and reflects some generic tornado characteristics rather than local peculiarities. Therefore, the nationwide statistics are used for its determination.

Data from the computerized file of National Severe Storms Forecast Center [D.9] contain 14,563 complete records for a 30-year period (1953-1982). The joint distribution of A- and F-scales is shown in Tables D.13 and D.14.

Following Wen and Chu [D.7], we will fit this discrete distribution by a continuous one:

$$f(a, w) = \frac{1}{2\pi \sigma_a \sigma_w a w} \exp - \left\{ \frac{1}{2(1 - \rho^2)} \left[\left(\frac{\ell n\ a - \mu_a}{\sigma_a} \right)^2 + \left(\frac{\ell n\ w - \mu_w}{\sigma_w} \right)^2 - 2\rho \left(\frac{\ell n\ a - \mu_a}{\sigma_a} \right) \left(\frac{\ell n\ w - \mu_w}{\sigma_w} \right) \right] \right\} \quad (D.39)$$

Where the tornado path area, a, is related to the A-scale according to Table D.1, and tornado wind speed, w, is related to the Fujita scale according to Table D.2.

The best fit for parameters of the distribution is:

$$\begin{aligned} \mu_a &= -2.8175 \\ \sigma_a &= 2.8759 \\ \mu_w &= 4.6390 \\ \sigma_u &= 0.4040 \\ \rho &= 0.59396 \end{aligned} \quad (D.40)$$

Now, present the distribution, f(a, w), in the form:

$$f(a, w) = f(a) \cdot \phi(w|a) \quad (D.41)$$

where:

$$f(a) = \frac{1}{\sqrt{2\pi}\ \sigma_a\ a} \exp\left[- \frac{1}{2}\left(\frac{\ell n\ a - \mu_a}{\sigma_a}\right)^2\right] \tag{D.42}$$

and

$$\phi(w|a) = \frac{1}{\sqrt{2\pi}\ \sigma_{w|a}\ w} \exp\left[- \frac{1}{2}\left(\frac{\ell n\ w - \mu_{w|a}}{\sigma_{w|a}}\right)^2\right] \tag{D.43}$$

$$\mu_{w|a} = \mu_w + \rho\ \frac{\sigma_w}{\sigma_a}\ (\ell n\ a - \mu_a) \tag{D.44}$$

$$\sigma_{w|a} = \sqrt{1 - \rho^2}\ \sigma_w \tag{D.45}$$

We assume that parameters μ_w, σ_w and ρ are generic. Parameters μ_a and σ_a are region-specific.

If we compare parameters μ_w, σ_w and ρ for Texas, we find that nationwide parameters are more conservative because the median wind speed distribution for the U.S. is higher than for Texas.

D.9 Surface Density of Potential Missiles n_p

In the EPRI study [D.10], the number of potential missiles in the missile origin zone (2.5×10^7 square feet including the plant site) is provided. For a two-unit plant with both units under operation, the possible range for the number, N_p, of potential missiles is:

$$5836 \leq N_p \leq 6196 \tag{D.46}$$

Dividing relationship (D.46) by the missile origin area, we find the range for the average density of potential missiles:

$$2.33 \times 10^{-4} \leq \bar{n}_p \leq 2.48 \times 10^{-4} \tag{D.47}$$

Assuming the lognormal distribution for average density n_p with a 90 percent confidence interval given by inequality (D.47) we can readily find the parameters of this distribution (see Table D.15).

An analysis of zone distribution of potential missiles in the area of 2.5×10^7 square feet based on data given in [D.10] shows that the maximum deviation from the average density for a plant under operation is 2.55.

The local density of potential missiles n_p can be presented in the form:

$$n_p = K_n\ \bar{n}_p \tag{D.48}$$

where K_n is the nonuniformity coefficient.

Assuming for coefficient K_n a lognormal distribution with upper limit 2.55 and median equal to 1, we find the distribution for n_p as a product of two lognormal distributions for n_p and K_n. The parameters of the distribution for n_p are given in Table D.15.

D.10 Probability of Injection η(F)

The probability that a potential tornado missile could become airborne or the probability of injection was considered in [D.11]. This probability is different for potential missiles located on the surface and at some elevation. For the surface potential missiles, the probability of injection is lower because the minimum restraining force is gravity. For elevated potential missiles, the minimum restraining force is friction, which is less than gravity. For both cases, the maximum restraining force is assumed five times greater than gravity. This restraint can be overcome only by tornadoes F5 and F6.

We assume that 50% of all potential missiles are lying on the ground and 50% are distributed uniformly up to elevation 20 ft. Twenty percent of the elevated missiles are restrained, (i.e., the minimum restraining coefficient is 1). Ten percent of the elevated missiles are unrestrained (i.e., the minimum restraining coefficient is equal to zero).

For the surface potential missiles (so-called vertically injected missiles) the distribution of η(F) is shown in Table D.16.

This distribution is created by random orientation of potential missiles and random distribution of restraining coefficients. (See Appendix C.) Tornadoes of Fujita scales F0 and F1 cannot lift the potential missiles specified in [D.12] from the surface.

For the elevated unrestrained potential missiles (so-called horizontally injected missiles), the distribution of injection probability η(F) is shown in Table D.17.

For the elevated restrained potential missiles, the distribution of injection probability η(F) is shown in Table D.18. These missiles also cannot be lifted by tornadoes of Fujita scales F0 and F1.

Actually, the assumption that 10% of elevated missiles are not restrained introduces some additional conservatisms.

D.11 Height Distribution of Airborne Missiles ψ (z, F)

The height distribution of airborne missiles ψ (z, F) for a uniformly spread source of potential missiles is addressed by [D.11]:

$$\psi(z, F) = \begin{cases} \dfrac{B(z)}{B(0)}, \; 0 \le z \le h_o \\ \\ \dfrac{1}{B(0)} \; \dfrac{\alpha_1(F)}{\alpha_2(F)} \, e^{\alpha_2 (F) \cdot (h_o - z) - \alpha_1(F) \cdot h_o}, \; z > h_o \end{cases} \tag{D.49}$$

where B(z), $\alpha_1(F)$ and $\alpha_2(F)$ are defined in section A.7. The median value of $\psi(z,F)$ for z = 55 ft and h_o = 20 ft is shown in Table D.19.

D.12 Conditional Probability of Damage Given a Hit

For the sake of simplicity and conservatism, we assumed that the conditional probability of damage P_D given a hit is equal to unity:

$$P_D = 1 \tag{D.50}$$

Because not every missile entering the IVC will hit the sensitive part of the equipment and because damage of at least two redundant elements is required to incapacitate the system, this assumption results in a safety margin of several orders-of-magnitude.

D.13 Distribution of Damage Probability P_T

The Monte Carlo simulation method was used for the propagation of uncertainty. Using distribution functions for all random parameters described in Sections D.6 through D.12, we developed the distribution for the damage probability P_T. Based on this distribution, the median (the best estimate) and 95% upper limit are reported. The best estimate should be compared with the acceptance criteria.

Our computer code uses the standard procedure generally accepted for this sort of problems with the best available "on-the-market" generator of random numbers and effective sorting procedure.

We used 10,000 simulations per run and reran the code 10 times with different seed numbers. It assures us that relative error for the best estimate is within the range 2-3%, which is very good for such spread distributions. For comparison, the best result that the SAMPLE code used in WASH-1400 can give is 12% accuracy [D.8].

The result of the calculation is shown in Table D.20.

D.14 Point Estimate of Median Value of Damage Probability P_T

To give some idea about the numerical value of every contributor to damage probability P_T and independently check our computer code, we provide an easy-to-follow manual point estimate of the median of damage probability P_T.

The median of frequency, ν, of tornado occurrence in area S = 10000 square miles is calculated according to the formula:

$$\nu = e^{\mu_\nu} \tag{D.51}$$

where parameter $\mu_\nu = 1.659$ is taken from Table D.9.

The median of tornado path area a is estimated according to a similar formula:

$$a = e^{\mu_a} \tag{D.52}$$

where parameter $\mu_a = -3.8076$ is taken from Table D.11.

Tornado freq`ncy, P_o, at the plant site is calculated using formula D.24.

The median wind velocity can be calculated as

$$w = e^{\mu_w|a} \tag{D.53}$$

where the parameter $\mu_w|_a$ is determined by formula D.44. However, for median value, a

$$\mu_{w\ a} = \mu_w \tag{D.54}$$

For the U.S., $\mu_w = 4.6390$ which yields w = 103.4 mph (for Texas, $\mu_w = 4.505$ and $\bar{w} = 90.5$ mph). It corresponds to Fujita Scale F1.

Tornadoes of Fujita Scale F1 cannot lift surface and restrained elevated missiles. Only unrestrained elevated missiles could be lifted. The fraction, f, of unrestrained missiles is 0.1. Therefore, the density of available missiles, n_a, can be estimated according to the formula:

$$n_a = f \cdot n_p \tag{D.55}$$

The median of density, n_p (D.55), of potential missiles is taken from Table D.15. The probability of injection, $\eta(F)$, and the height distribution, $\psi(z,F)$, for F1 tornadoes are taken from Tables C.11 and D.19, respectively.

The total probability of damage is estimated according to formula D.27. We have to note that the multiplication of medians is also the median only in the case when all multipliers are distributed lognormally. In our case, the height distribution $\psi(z,F)$ is not distributed lognormally. Therefore, we get some approximation of the median value of P_T.

The result of this calculation is shown in Table D.21. This estimate is close to the exact value for the median reported in Table D.20.

References

[D.1] Kaplan, S. and Garrick, B. J., "On the Quantitative Definition of Risk", Risk Analysis 1, 11-27 (1981).

[D.2] Kaplan, S., Apostolakis, G., Garrick, B. J., Bley, D. C., and Woodward, K., "Methodology for Probabilistic Risk Assessment of Nuclear Power Plants," Draft, PLG-0209 (June 1981).

[D.3] Korn, G. A. and Korn, J. M., Mathematical Handbook for Scientists and Engineers, McGraw-Hill Book Company, New York, 1968.

[D.4] Fujita, T. T., "Estimate of Areal Probability of Tornadoes from Inflationary Reporting of their Frequencies," SMRP Research Paper No. 89, the University of Chicago, 1970.

[D.5] Fujita, T. T., "Proposed Characterization of Tornadoes and Hurricanes by Area and Intensity," SMRP Research Paper No. 91, the University of Chicago, 1970.

[D.6] Thom, M. C. S., "Tornado Probabilities," Monthly Weather Review, Oct.-Dec., 730, (1963).

[D.7] Wen, Y. K. and Chu, S. L., "Tornado Risks and Design Wind Speed," Journal of the Structural Division, Proceedings, ASCE, Vol. 99, No. C. T. 12, Dec. 1973.

[D.8] Goodman, J., "Accuracy and Efficiency of Monte Carlo Method," Proceedings of International Conference on Numerical Methods in Nuclear Engineering, September 6-9, 1983, Montreal, Quebec, Canada.

[D.9] "U.S. Tornado Breakdown by Counties 1953-1982," U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Weather Service National Severe Storms Forecast Center, Room 1728, Federal Building, 601 E. 12th Street, Kansas City, Missouri 64106.

[D.10] Twisdale, L. A., et al., "Tornado Missile Risk Analysis," EPRI NP-768, May 1978, EPRI NP-769, May 1978.

[D.11] Goodman, J. and Koch, J. E., "The Probability of a Tornado Missile Hitting a Target," Nuclear Engineering and Design 74, (1983).

[D.12] Standard Review Plan, U.S. Nuclear Regulatory Commission, NUREG-75087.

Table D.1

Classification of Tornadoes According to Path Area a

Classification	Range of Tornado Path Area a (mi-ft)
A1	$1 < a \leq 10$
A2	$10 < a \leq 10^2$
A3	$10^2 < a \leq 10^3$
A4	$10^3 < a \leq 10^4$
A5	$10^4 < a \leq 10^5$
A6	$10^5 < a \leq 10^6$
A7	$10^6 < a \leq 10^7$

Table D.2

Relationship Between Fujita Scale F and Damaging Wind Speed w (mph)

Fujita Scale F	Range for the Damaging Wind Speed (mph)
F0	$40 < w \leq 72$
F1	$72 < w \leq 112$
F2	$112 < w \leq 157$
F3	$157 < w \leq 206$
F4	$206 < w \leq 260$
F5	$260 < w \leq 318$
F6	$318 < w \leq 380$

Table D.3

Specification of Tornado-Generated Missiles

Type	Description	$\frac{m}{A}$ (kg/m^2)	$\frac{\ell}{d}$	C_L (α=55°)	C_D (α=90°)
A.	Wood plank, 4 in. x 12 in. x 12 ft, weight 200 lb.	125	18.5	.335	0.87
B.	Steel pipe, 3 in. diameter, schedule 40, 10 ft long, weight 78 lb.	152	40	.389	1.01
C.	Steel rod, 1 in. diameter x 3 ft long, weight 8 lb.	156	36	.381	0.99
D.	Steel pipe, 6 in. diameter, schedule 40, 15 ft long, weight 285 lb.	186	30	.370	0.96
E.	Steel pipe, 12 in. diameter, schedule 40, 15 ft long, weight 743 lb.	242	15	.323	0.84
F.	Utility pole, 13-1/2 in. diameter, 35 ft long, weight 1490 lb.	179	31	.373	0.97
G.	Automobile, frontal area 20 ft^2, weight 4000 lb.	199	2.75	.273	0.71

Table D.4

Acceleration Parameters of Potential Missiles

Type of Missile	$\frac{m}{A}$	C_L	C_D	$\frac{C_L}{\left(\frac{m}{A}\right)}$	$\frac{C_D}{\left(\frac{m}{A}\right)}$
A	125	0.335	0.87	2.68×10^{-3}	6.96×10^{-3}
B	152	0.389	1.01	2.56×10^{-3}	6.64×10^{-3}
C	156	0.381	0.99	2.44×10^{-3}	6.35×10^{-3}
D	186	0.370	0.96	1.99×10^{-3}	5.16×10^{-3}
E	242	0.323	0.84	1.33×10^{-3}	3.47×10^{-3}
F	179	0.373	0.97	2.08×10^{-3}	5.42×10^{-3}
G	199	0.273	0.71	1.37×10^{-3}	3.57×10^{-3}
"Standard"	170	0.369	0.98	2.17×10^{-3}	5.76×10^{-5}

Table D.5

List of Counties Near the Plant Site of STP

County	Population	Area (mi^2)
Matagorda	37,828	1,127
Brazoria	169,587	1,407
Fort Bend	130,846	876
Wharton	40,242	1,086
Jackson	13,352	844
Calhoun	19,574	540
Total	411,429	5,880

Table D.6

Annual Number v_6 of Tornado Occurrences for Six Counties

Year	v_6	Year	v_6
1953	1	1968	5
1954	1	1969	4
1955	4	1970	8
1956	2	1971	0
1957	3	1972	15
1958	1	1973	5
1959	4	1974	4
1960	1	1975	5
1961	5	1976	8
1962	1	1977	4
1963	1	1978	3
1964	2	1979	2
1965	1	1980	3
1966	3	1981	7
1967	27	1982	3

Table D.7

Parameters of Lognormal Distribution for Annual Frequency of Tornado Occurrences In Six Counties

Lower Limit (5%)	Median (50%)	Mean	Upper Limit (95%)
0.42	3.00	7.39	21.35

μ	σ
1.099	1.193

Table D.8

Comparison of Empirical and Fitted Lognormal Distributions for v_6

Percentile	Empirical	Lognormal
25	1	1.34
50	3	3.00
75	5	6.71
90	8	13.87
95	15	21.35
98.33	27	38.10

Table D.9

Parameters of Distribution for the Adjusted Annual Frequency ν

Lower Limit (5%)	Median (50%)	Mean	Upper Limit (95%)
0.738	5.254	10.704	37.393

μ	σ
1.659	1.193

Table D.10

Distribution of Tornado Path Area

A-Scale	Number of Tornadoes for 30 years in Texas	Probability for Texas	Probability for U.S.A.
A1	354	.1297	.1100
A2	906	.3319	.2371
A3	844	.3092	.3028
A4	478	.1751	.2567
A5	144	.0527	.0907
A6	4	.0015	.0027
A7	0	.0000	.0000

Table D.11

Best Parameters for Lognormal Distribution of Tornado Path Area for Texas

μ	-3.8076
σ	2.5697

Table D.12

Comparison of Empirical and Lognormal Distributions of Tornado Path Area for Texas

	Empirical Distribution		Lognormal Distribution	
A-Scale	Probability	Cumulative Probability	Probability	Cumulative Probability
A1	.1297	.1297	.1371	.1371
A2	.3319	.4616	.3060	.4431
A3	.3092	.7708	.3224	.7655
A4	.1751	.9459	.1605	.9260
A5	.0527	.9986	.0377	.9637
A6	.0015	1.0000	.0041	.9678
A7	.0000	1.0000	.0002	.9680

Table D.13

Joint Number of Tornadoes
(A-Scale - F-Scale)

	F0	F1	F2	F3	F4	F5	F6	Total Number by Path Area
A1	930	571	94	7	0	0	0	1602
A2	1081	1736	586	48	2	0	0	3453
A3	531	2162	1429	259	28	0	0	4409
A4	131	1018	1645	732	202	11	0	3739
A5	24	171	413	419	242	52	0	1321
A6	1	1	10	16	10	1	0	39
A7	0	0	0	0	0	0	0	0

Total Number by Fujita Scale							Total Number of Cases
2698	5659	4177	1481	484	64	0	14563

Table D.14

<u>Joint Probability Distribution for A and F Scales</u>

	F0	F1	F2	F3	F4	F5	F6	Marginal Probability by Path Area
A1	.0639	.0392	.0065	.0005	.0000	.0000	.0000	.1100
A2	.0742	.1192	.0402	.0033	.0001	.0000	.0000	.2371
A3	.0365	.1485	.0981	.0178	.0019	.0000	.0000	.3028
A4	.0090	.0699	.1130	.0503	.0139	.0008	.0000	.2567
A5	.0016	.0117	.0284	.0288	.0166	.0036	.0000	.0907
A6	.0001	.0001	.0007	.0011	.0007	.0001	.0000	.0027
A7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Marginal Probability by Fujita Scale

.1853	.3886	.2868	.1017	.0332	.0044	.0000

Table D.15

Density of Potential Missiles

	Lower Limit (5th Percentile)	Median (50th Percentile)	Upper Limit (95th Percentile)
Average density (n_p)	2.33×10^{-4}	2.40×10^{-4}	2.48×10^{-4}
Nonuniformity (K_n) Coefficient	0.39	1.00	2.55
Local density (n_p)	9.42×10^{-5}	2.40×10^{-4}	6.13×10^{-4}

Table D.16

Probability of Injection $\eta(F)$ For Surface Potential Missiles

Fujita Scale	Lower Limit	Median	Upper Limit
F2	0.0001	0.0017	0.0305
F3	0.0006	0.0086	0.1230
F4	0.0143	0.0527	0.1943
F5	0.0450	0.1050	0.2449
F6	0.1089	0.1761	0.2847

Table D.17

Probability of Injection $\eta(F)$ For Elevated Potential Missiles

Fujita Scale	Lower Limit	Median	Upper Limit
F0	0.0008	0.0119	0.1756
F1	0.0020	0.0292	0.4274
F2	0.0029	0.0435	0.6528
F3	0.0098	0.0866	0.7657
F4	0.0789	0.2546	0.8218
F5	0.2160	0.4310	0.8602
F6	0.3529	0.5593	0.8865

Table D.18

Probability of Injection $\eta(F)$ for Restrained Elevated Potential Missiles

Fujita Scale	Lower Limit	Median	Upper Limit
F2	0.0002	0.0043	0.0941
F3	0.0038	0.0375	0.3654
F4	0.0635	0.1857	0.5428
F5	0.2093	0.3701	0.6543
F6	0.3598	0.5092	0.7206

Table D.19

Median Value for Height Distribution of Airborne Missiles (z = 55 ft, h_o = 20 ft)

Fujita Scale	$\psi(z, F)$ (Formula D-49, $z > h_o$)
F0	0.00002
F1	0.00818
F2	0.07053
F3	0.19531
F4	0.34255
F5	0.47867
F6	0.59131

Table D.20

Probability of Damage to IVC from Tornado-Generated Missiles per Year

Median (50th Percentile)	Upper Limit (95th Percentile)
2×10^{-10}	6×10^{-6}

Table D.21

<u>Point Estimate of Damage Probability</u>

Description	Notation	Value
Frequency of tornado striking area $S = 10000\ mi^2$ (per year)	ν	5.2341
Tornado path area (mi^2)	a	0.0222
Frequency of tornado striking the plant site (per year)	P_o	1.1664×10^{-5}
Local density of potential missiles (ft^{-2})	n_p	2.40×10^{-4}
Fraction of available missiles	f	0.1
Density of available missiles (ft^{-2})	n_a	2.40×10^{-5}
IVC target area (ft^2)	A	2980
Probability of injection ($F = 1$)	$\eta(F)$	0.0292
Height distribution ($z = 55$ ft, $F = 1$)	$\psi(z, F)$	0.00818
Probability of damage	P_T	[illegible]$.993 \times 10^{-10}$