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March 20, 1990

U. S. Nuclear Regulatory Commission
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Subject: McGuire Nuclear Station
Docket Numbers 50-369 and -370
Catawba Nuclear Station
Docket Numbers 50-413 and -414
Topical Report DPC-NE-3001-P;
"Safety Analysis Physics Parameters
and Multidimensional Reactor
Transients Methodology"

On January 29, 1990, Duke submitted the subject Topical Report for NRC review. Enclosed with the Topical Report were 3 EPRI documents which described the ARROTTA computer code.

As discussed in a telephone conversation between the NRC staff (Darl Hood, Stan Kirslis, and Dan Fieno) and Duke Power, (Scott Gewehr and Robert Van Namen), one of the documents which described the ARROTTA Theory was missing an appendix. That appendix is attached.

If we can be of further assistance in your review of this topical, please call Scott Gewehr at (704) 373-7581.

Very truly yours,

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Appendix 3

EVALUATION OF SPATIAL COUPLING MATRICES

Actual application of the Analytic Nodal Method requires evaluation of the matrices defined in Eq. A2-11. Each of these matrices is a $2G \times 2G$ matrix whose elements depend only on the properties of a single node. The essential difficulty in evaluating these matrices stems from the fact that the exponential of $[N_{\ell,m,n}]$, as defined in Eq. A2-1, must be evaluated. $[N_{\ell,m,n}]$ is block antidiagonal with its lower block being partially comprised of the $G \times G$ group-to-group scattering matrix. In the general multigroup case, it is not apparent how to obtain this exponential. If the matrix $[N_{\ell,m,n}]$ could be diagonalized in some fashion, the exponential of $[N_{\ell,m,n}]$ would, of course, be readily obtainable.

If the number of neutron energy groups is restricted to a small number, direct evaluation of the matrices becomes feasible. Since this paper is primarily concerned with light water reactor analysis, in which two-group diffusion theory is commonly used, the matrices will be evaluated directly for the two-group case.

The one-dimensional, source-free, two-group, diffusion equation for a nuclearly homogeneous region ($u_\ell < u < u_{\ell+1}$) can be expressed as

$$\begin{bmatrix} D_1 \frac{d^2}{du^2} - \Sigma_1 & \frac{1}{\gamma} \nu\Sigma_{f_2} \\ \Sigma_{21} & D_2 \frac{d^2}{du^2} - \Sigma_2 \end{bmatrix} \begin{bmatrix} \phi_1(u) \\ \phi_2(u) \end{bmatrix} = [0] \tag{A3-1}$$

where

D_g = group g diffusion coefficient (cm)

Σ_2 = group 2 macroscopic absorption cross section (cm^{-1})

$\nu\Sigma_{f_g}$ = group g macroscopic fission cross section times nu, the mean number of neutrons emitted per fission (cm^{-1})

Σ_1 = group 1 macroscopic removal (absorption plus outscatter) cross section minus $\frac{1}{\gamma}\nu\Sigma_{f_1}$ (cm^{-1})

Σ_{21} = macroscopic transfer cross section from group 1 to group 2 (cm^{-1})

ϕ_g = group g scalar neutron flux (cm^{-2} sec^{-1})

γ = critical eigenvalue of global static reactor problem,

and it has been assumed that there exists no upscatter and all fission neutrons are born in group 1 (i.e., $\Sigma_{12} = 0$, $\chi_1 = 1$, $\chi_2 = 0$). If a particular solution to Eq. A3-1 exists such that

$$\begin{bmatrix} \frac{d^2}{du^2} & 0 \\ 0 & \frac{d^2}{du^2} \end{bmatrix} \begin{bmatrix} \phi_1(u) \\ \phi_2(u) \end{bmatrix} = \begin{bmatrix} -B^2 & 0 \\ 0 & -B^2 \end{bmatrix} \begin{bmatrix} \phi_1(u) \\ \phi_2(u) \end{bmatrix} \tag{A3-2}$$

then B^2 must satisfy the equation

$$\begin{bmatrix} -D_1B^2 - \Sigma_1 & \frac{1}{\gamma}\nu\Sigma_{f_2} \\ \Sigma_{21} & -D_2B^2 - \Sigma_2 \end{bmatrix} \begin{bmatrix} \phi_1(u) \\ \phi_f(u) \end{bmatrix} = [0] \; . \qquad \text{(A3-3)}$$

For a nontrivial solution to exist to Eq. A3-3, the determinant of the coefficient matrix must be identically zero. This implies that B^2 must have very special values. If the two values of B^2 which satisfy Eq. A3-2 are designated κ^2 and $-\mu^2$, their values are given by

$$\kappa^2 = -\frac{1}{2}\left(\frac{\Sigma_1}{D_1} + \frac{\Sigma_2}{D_2}\right) + \left\langle\left(\frac{\Sigma_2}{2D_2} - \frac{\Sigma_1}{2D_1}\right)^2 + \frac{\nu\Sigma_{f_2}\Sigma_{21}}{\gamma D_1 D_2}\right\rangle^{1/2}$$

$$\mu^2 = \frac{1}{2}\left(\frac{\Sigma_1}{D_1} + \frac{\Sigma_2}{D_2}\right) + \left\langle\left(\frac{\Sigma_2}{2D_2} - \frac{\Sigma_1}{2D_1}\right)^2 + \frac{\nu\Sigma_{f_2}\Sigma_{21}}{\gamma D_1 D_2}\right\rangle^{1/2} . \qquad \text{(A3-4)}$$

μ^2 has been chosen such that it will always be positive, and κ^2 can be either positive or negative. With two "slow-to-fast flux ratios" defined to be

$$r = \frac{\Sigma_{21}}{D_2\kappa^2 + \Sigma_2}$$

$$s = \frac{\Sigma_{21}}{-D_2\mu^2 + \Sigma_2} , \qquad \text{(A3-5)}$$

the general solution to Eq. A3-3 is then given by

$$\begin{bmatrix} \phi_1(u) \\ \phi_2(u) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ r & s \end{bmatrix} \begin{bmatrix} a_1 \sin \kappa u + a_2 \cos \kappa u \\ a_3 \sinh \mu u + a_4 \cosh \mu u \end{bmatrix} . \qquad \text{(A3-6)}$$

Likewise, the current vector is given by

$$\begin{bmatrix} J_1(u) \\ J_2(u) \end{bmatrix} = \begin{bmatrix} -D_1 & -D_1 \\ -rD_2 & -sD_2 \end{bmatrix} \begin{bmatrix} a_1\kappa \cos \kappa u - a_2\kappa \sin \kappa u \\ a_3\mu \cosh \mu u + a_4\mu \sinh \mu u \end{bmatrix} . \quad \text{(A3-7)}$$

With the definitions

$$[\Phi(u)] = \begin{bmatrix} \phi_1(u) \\ \phi_2(u) \\ J_1(u) \\ J_2(u) \end{bmatrix}$$

$$[P] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ r & s & 0 & 0 \\ 0 & 0 & -D_1 & -D_1 \\ 0 & 0 & -rD_2 & -sD_2 \end{bmatrix}$$

(A3-8)

$$[Q(u)] = \begin{bmatrix} \sin \kappa u & \cos \kappa u & 0 & 0 \\ 0 & 0 & \sinh \mu u & \cosh \mu u \\ \kappa \cos \kappa u & -\kappa \sin \kappa u & 0 & 0 \\ 0 & 0 & \mu \cosh \mu u & \mu \sinh \mu u \end{bmatrix}$$

$$[R] = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} ,$$

Equations A3-6 and A3-7 can be expressed as

$$[\Phi(u)] = [P][Q(u)][R] \,. \tag{A3-9}$$

The inverses of $[P]$ and $[Q]$ both exist and are given by

$$[P]^{-1} = \begin{bmatrix} s & -1 & 0 & 0 \\ -r & 1 & 0 & 0 \\ 0 & 0 & -\frac{s}{D_1} & \frac{1}{D_2} \\ 0 & 0 & \frac{r}{D_1} & -\frac{1}{D_2} \end{bmatrix} \frac{1}{s - r} \tag{A3-10}$$

and

$$[Q(u)]^{-1} = \begin{bmatrix} \sin \kappa u & 0 & \frac{1}{\kappa} \cos \kappa u & 0 \\ \cos \kappa u & 0 & -\frac{1}{\kappa} \sin \kappa u & 0 \\ 0 & -\sinh \mu u & 0 & \frac{1}{\mu} \cosh \mu u \\ 0 & \cosh \mu u & 0 & \frac{1}{\mu} \sinh \mu u \end{bmatrix} . \tag{A3-11}$$

Hence, the unknown coefficients of the general solution are

$$[R] = [Q(u)]^{-1}[P]^{-1}[\Phi(u)] \,. \tag{A3-12}$$

For a homogeneous region which extends from u_ℓ to $u_{\ell+1}$, $[\Phi(u_\ell)]$ can be expressed in terms of $[\Phi(u_{\ell+1})]$ by applying Eq. A3-9 at $u = u_\ell$ and Eq. A3-12 at $u = u_{\ell+1}$ and eliminating $[R]$ to obtain

$$[\Phi(u_\ell)] = [P][Q(u_\ell)][Q(u_{\ell+1})]^{-1}[P]^{-1}[\Phi(u_{\ell+1})] \,. \tag{A3-13}$$

With the further definitions, $h = u_{\ell+1} - u_\ell$ and $[Q] = [Q(u_\ell)][Q(u_{\ell+1})]^{-1}$, $[Q]$ can be expressed as

$$[Q] = \begin{bmatrix} \cos \kappa h & 0 & -\frac{1}{\kappa} \sin \kappa h & 0 \\ 0 & \cosh \mu h & 0 & -\frac{1}{\mu} \sinh \mu h \\ \kappa \sin \kappa h & 0 & \cos \kappa h & 0 \\ 0 & -\mu \sinh \mu h & 0 & \cosh \mu h \end{bmatrix}, \tag{A3-14}$$

and Eq. A3-13 becomes

$$[\Phi(u_\ell)] = [P][Q][P]^{-1}[\Phi(u_{\ell+1})] . \tag{A3-15}$$

In Appendix 2, an expression was derived which also related $[\Phi(u_\ell)]$ to $[\Phi(u_{\ell+1})]$. This expression, from Eq. A2-6, is

$$[\Phi_{\ell,m,n}(u_\ell)] = e^{+[N_{\ell,m,n}]h_u^\ell}[\Phi_{\ell,m,n}(u_{\ell+1})] ,$$

which (with the node subscripts dropped since each node is treated separately) becomes

$$[\Phi(u_\ell)] = e^{[N]h}[\Phi(u_{\ell+1})] . \tag{A3-16}$$

Comparison of Eqs. A3-15 and A3-16 indicates that

$$e^{[N]h} = [P][Q][P]^{-1} . \tag{A3-17}$$

The matrices $[P]$ and $[Q]$ depend only on the nuclear properties and mesh spacing of each node; hence, the exponential of $[N]h$ is completely specified by Eq. A3-17. With this expression for $e^{[N]h}$,

identities for each of the spatial coupling matrices of Eq. A2-11 can be derived. Making use of the definitions of the hyperbolic functions, one finds that

$$(\sinh [N]h)^{-1} = [P] \begin{bmatrix} 0 & 0 & \frac{1}{\kappa} \csc \kappa h & 0 \\ 0 & 0 & 0 & -\frac{1}{\mu} \operatorname{csch} \mu h \\ -\kappa \csc \kappa h & 0 & 0 & 0 \\ 0 & -\mu \operatorname{csch} \mu h & 0 & 0 \end{bmatrix} [P]^{-1}$$

(A3-18a)

$$[I] - \cosh [N]h = [P] \begin{bmatrix} 1 - \cos \kappa h & 0 & 0 & 0 \\ 0 & 1 - \cosh \mu h & 0 & 0 \\ 0 & 0 & 1 - \cos \kappa h & 0 \\ 0 & 0 & 0 & 1 - \cosh \mu h \end{bmatrix} [P]^{-1} .$$

(A3-18b)

The matrices defined in Eq. A2-11 involve only certain blocks of the full (4×4) matrices; hence, only certain blocks need to be evaluated. Several identities which prove very useful in simplifying the matrices are

$$[N]^{-1}_{12} = h^2 \begin{bmatrix} \frac{1}{\kappa^2 h^2} & -\frac{1}{\mu^2 h^2} \\ \frac{r}{\kappa^2 h^2} & -\frac{s}{\mu^2 h^2} \end{bmatrix} [P]^{-1}_{22} \qquad \text{(A3-19a)}$$

$$(\sinh [N]h)_{12}^{-1} [N]_{21}^{-1} (\sinh [N]h)_{12} = [N]_{12}^{-1} \tag{A3-19b}$$

$$(\sinh [N]h)_{21}^{-1} [N]_{12}^{-1} ([I] - \cosh [N]h)_{22} = -(\tanh [N] \tfrac{h}{2})_{21} [N^{-1}]_{12} \ . \tag{A3-19c}$$

The latter two identities are easily proven from the fact that

$$f([N])g([N]) = g([N])f([N])$$

for any functions f and g which can be expressed as series involving powers of $[N]$.

Evaluation of the matrices in Eq. A2-11 is by no means a trivial exercise, but once the algebra has been performed, the following simple expressions are obtained:

$$[A]_{12} = \begin{bmatrix} -\alpha & -\beta \\ -r\alpha & -s\beta \end{bmatrix} [P]_{22}^{-1} h$$

$$[B]_{11} = \begin{bmatrix} \gamma & \delta \\ r\gamma & s\delta \end{bmatrix} \begin{bmatrix} s & -1 \\ -r & 1 \end{bmatrix} \frac{1}{s-r}$$

$$[C^{\pm}]_{12} = \begin{bmatrix} -\varepsilon & \zeta \\ -r\varepsilon & s\zeta \end{bmatrix} [P]_{22}^{-1} h^2$$

$$[D^{+}]_{12} = \begin{bmatrix} -\xi & o \\ -r\xi & so \end{bmatrix} [P]_{22}^{-1} h^2$$

$$[D^{-}]_{12} = \begin{bmatrix} -\eta & \theta \\ -r\eta & s\theta \end{bmatrix} [P]_{22}^{-1} h^2$$

$$[E^+]_{12} = \begin{bmatrix} -\sigma & \tau \\ -r\sigma & s\tau \end{bmatrix} [P]_{22}^{-1} h^2$$

$$[E^-]_{12} = \begin{bmatrix} -\pi & \rho \\ -r\pi & s\rho \end{bmatrix} [P]_{22}^{-1} h^2 \qquad \text{(A3-20)}$$

where

$$\alpha = \frac{1}{\kappa h} \tan(\kappa h/2); \quad \kappa^2 > 0 \qquad = \frac{1}{\sqrt{|\kappa^2|}\,h} \tanh\left(\sqrt{|\kappa^2|}\,h/2\right); \quad \kappa^2 < 0$$

$$\beta = \frac{1}{\mu h} \tanh(\mu h/2)$$

$$\gamma = \kappa h \csc(\kappa h); \quad \kappa^2 > 0 \qquad = \sqrt{|\kappa^2|}\,h \,\mathrm{csch}\left(\sqrt{|\kappa^2|}\,h\right); \quad \kappa^2 < 0$$

$$\delta = \mu h \,\mathrm{csch}(\mu h)$$

$$\varepsilon = \frac{1}{\kappa^2 h^2} (\gamma - 1)$$

$$\zeta = \frac{1}{\mu^2 h^2} (\delta - 1)$$

$$\eta = \frac{1}{\kappa^2 h^2} \left(\frac{\gamma}{2} - \alpha\right)$$

$$\theta = \frac{1}{\mu^2 h^2} \left(\frac{\delta}{2} - \beta\right)$$

$$\xi = \frac{1}{\kappa^2 h^2} \left(\frac{\gamma}{2} + \alpha - 1\right)$$

$$o = \frac{1}{\mu^2 h^2} \left(\frac{\delta}{2} + \beta - 1\right)$$

$$\pi = \frac{1}{\kappa^2 h^2} \left(\frac{\gamma}{3} - 2\varepsilon\right)$$

$$\rho = \frac{1}{\mu^2 h^2}\left(\frac{\delta}{3} + 2\zeta\right)$$

$$\sigma = \pi + \frac{1}{\kappa^2 h^2}(2\alpha - 1)$$

$$\tau = \rho + \frac{1}{\mu^2 h^2}(2\beta - 1) \quad . \tag{A3-21}$$

When $\nu\Sigma_{f_2}$ is identically zero, s is infinite, and l'Hospitol's rule must be used to obtain,

$$[A]_{12} = \begin{bmatrix} \frac{\alpha}{D_1} & 0 \\ \frac{r}{D_1}(\alpha - \beta) & \frac{\beta}{D_2} \end{bmatrix} h$$

$$[B]_{11} = \begin{bmatrix} \gamma & 0 \\ r(\gamma - \delta) & \delta \end{bmatrix}$$

$$[C^{\pm}]_{12} = \begin{bmatrix} \frac{\varepsilon}{D_1} & 0 \\ \frac{r}{D_1}(\varepsilon + \zeta) & -\frac{\zeta}{D_2} \end{bmatrix} h^2$$

$$[D^{+}]_{12} = \begin{bmatrix} \frac{\xi}{D_1} & 0 \\ \frac{r}{D_1}(\xi + o) & -\frac{o}{D_2} \end{bmatrix} h^2$$

$$[D^-]_{12} = \begin{bmatrix} \frac{\eta}{D_1} & 0 \\ \frac{r}{D_1}(\eta+\theta) & -\frac{\theta}{D_2} \end{bmatrix} h^2$$

$$[E^+]_{12} = \begin{bmatrix} \frac{\sigma}{D_1} & 0 \\ \frac{r}{D_1}(\sigma+\tau) & -\frac{\tau}{D_2} \end{bmatrix} h^2$$

$$[E^-]_{12} = \begin{bmatrix} \frac{\pi}{D_1} & 0 \\ \frac{r}{D_1}(\pi+\rho) & -\frac{\rho}{D_2} \end{bmatrix} h^2 \; , \qquad \text{(A3-22a)}$$

and κ^2, μ^2 are given by the simple expressions

$$\kappa^2 = \max\left[-\frac{\Sigma_1}{D_1}, -\frac{\Sigma_2}{D_2}\right]$$

$$\mu^2 = \max\left[\frac{\Sigma_1}{D_1}, \frac{\Sigma_2}{D_2}\right] . \qquad \text{(A3-22b)}$$

When $\kappa^2 h^2$ or $\mu^2 h^2$ approach zero, many of the leading terms in the Taylor's series expansions of α, β, . . . τ cancel, and it becomes important to use the expansions rather than Eq. A2-21. The expansions for small κh and μh are

$$\alpha = \left(\frac{1}{2} + \frac{(\kappa h)^2}{24} + \frac{(\kappa h)^4}{240} + \ldots\right)$$

$$\beta = \left(\frac{1}{2} - \frac{(\mu h)^2}{24} + \frac{(\mu h)^4}{240} + \ldots\right)$$

$$\gamma = \left(1 + \frac{(\kappa h)^2}{6} + \frac{7(\kappa h)^4}{360} + \ldots\right)$$

$$\delta = \left(1 - \frac{(\mu h)^2}{6} + \frac{7(\mu h)^4}{360} + \ldots\right)$$

$$\varepsilon = \left(\frac{1}{6} + \frac{7(\kappa h)^2}{360} + \frac{31(\kappa h)^4}{15120} + \ldots\right)$$

$$\zeta = \left(-\frac{1}{6} + \frac{7(\mu h)^2}{360} - \frac{31(\mu h)^4}{15120} + \ldots\right)$$

$$\eta = \left(\frac{1}{24} + \frac{4(\kappa h)^2}{720} + \frac{73(\kappa h)^4}{120960} + \ldots\right)$$

$$\theta = \left(-\frac{1}{24} + \frac{4(\mu h)^2}{720} - \frac{73(\mu h)^4}{120960} + \ldots\right)$$

$$\xi = \left(\frac{1}{8} + \frac{(\kappa h)^2}{72} + \frac{175(\kappa h)^4}{120960} + \ldots\right)$$

$$o = \left(-\frac{1}{8} + \frac{(\mu h)^2}{72} - \frac{175(\mu h)^4}{120960} + \ldots\right)$$

$$\pi = \left(\frac{1}{60} + \frac{36(\kappa h)^2}{15120} + \frac{478(\kappa h)^4}{1814400} + \ldots\right)$$

$$\rho = \left(-\frac{1}{60} + \frac{36(\mu h)^2}{15120} - \frac{478(\mu h)^4}{1814400} + \ldots\right)$$

$$\sigma = \left(\frac{1}{10} + \frac{162(\kappa h)^2}{15120} + \frac{2008(\kappa h)^4}{1814400} + \ldots\right)$$

$$\tau = \left(-\frac{1}{10} + \frac{162(\mu h)^2}{15120} - \frac{2008(\mu h)^4}{1814400} + \ldots\right) \quad . \qquad \text{(A3-23)}$$

Equations A3-20 - A3-23 completely specify the spatial coupling matrices. From the definitions of κ^2 and μ^2 in Eq. A3-4, it is apparent that all of the matrices depend on the eigenvalue of the global static reactor problem.

The matrices required for a one-group model are equal to the (1,1) elements of the matrices in Eq. A3-22a, with $D_1 = D$.