



TEXAS UTILITIES SERVICES INC.

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Mr. Spottswood Burwell
Licensing Project Manager
U. S. Nuclear Regulatory Commission
Office of Nuclear Reactor Regulation
Washington, D.C. 20555

SUBJECT: COMANCHE PEAK STEAM ELECTRIC STATION
RESPONSES TO Q251.1 THRU Q251.4

Dear Mr. Burwell:

Attached is a rough draft of the responses to NRC Questions 251.1 thru Q251.4 for your early review. These responses will be included in Amendment 24, scheduled to be submitted on July 15, 1981.

Sincerely,

J. S. Marshall

BSD:tls

Attachment

cc: J. T. Merritt - 1L, OA
H. C. Schmidt - 1L, OA
R. D. Calder - 1L, OA
R. E. Ballard - 1L, OA

Boo/

Ques-251.1
Q-251.1

Identify the specific model (s) of Allis-Chalmers low pressure turbine (s) installed in Comanche Peak SES Units 1 and 2. Provide tables showing, for each of the unique wheels in a Comanche Peak low pressure turbine, the weight and location of the wheels relative to the turbine center, and the a) dimensions, b) shapes, c) weights and d) initial energy (or velocity) ranges of missiles postulated to be representative of missile-producing turbine wheel rupture at:

- i. design overspeed, and
- ii. destructive overspeed.

Show how values for the exit energies are obtained or reference an available document which contains an adequate description.

Response 251.1
R-251.1 See revised Section 3.5.1.3.1.

Q 251.2 With regard to reporting quantitative analyses, in general, and information such as contained in FSAR Section 3.5.1.3.2, in particular, the following applies:

- All equations are to be numbered and, except for those which express fundamental physical laws, they must be either derived or a reference to an available document (i.e. one in the open literature) provided.
- Constants and variables appearing in these equations are to be defined, showing the assumed values for constants and the value ranges for variables.

R 251.2 See new ^{Appendix} ~~Section~~ 3.5A.

U 251.3
The analysis of the damage probability is not sufficiently detailed. The applicants are to show the formulas or constants which constitute quantitative definitions of the probabilities presented and how they are used.

The values of P shown in Table 3.5-7 are unacceptable; the values currently accepted by the NRC staff are 5×10^{-5} and 4×10^{-5} per turbine year for design and destructive overspeed failures, respectively (see Standard Review Plan Section 3.5.1.3).

As stated in Regulatory Guide 1.115, with regard to low trajectory missiles, the protection of an essential or safety related system or component is acceptable if the system or component is located outside the low trajectory missile strike zones. If the applicant has examined the strike zones for the turbine trains at the site and found no safety related targets (see Regulatory Guide 1.117) within the zones, it must be specifically so stated. Otherwise, the determination of P must include an analysis for the low trajectory missile contribution.

There is no description in the FSAR of the method used to analyze P for high trajectory missiles. The P probability equation employed must either be derived or an available document referenced which contains an adequate derivation.

To be conservative, and due to lack of information, the values of P are generally taken to be one. If other values are used, such as those shown in Table 3.5-7, appropriate equations must be derived and/or the values obtained for each target must be justified to the NRC staff.

Note, the targets should be specific components (e.g. steam generators 1 and 2, reactor vessel, feedwater lines, main steam lines, auxiliary feedwater pumps 1 and 2, etc.)

R 251.3

See revised Section 3.5.1.3.3, revised Table 3.5-7
and ^{Appendix} Section 3.5B.

Q 251.4

The $P_2 \times P_3$ probabilities in Table 3.5-7 appear to be too small.

Re-calculate the $P_2 \times P_3$ probabilities associated with each safety related target for design overspeed, and destructive overspeed,

and show that the total $P_2 \times P_3$ for each of these failures conditions is less than 10^{-3} per failure, as stipulated in Regulatory Guide 1.115.

Prepare a table showing the $P_2 \times P_3$ probabilities for each target.

Section 3.5.1.3.3 and

R 251.4

See revised Table 3.5-7.

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 JULY 31, 1980

3.5.1.3 Turbine Missiles

3.5.1.3.1 Turbine Placement and Orientation

The turbine placement and orientation of the twin unit are indicated in the plant layout drawing, Figure 3.5-3. *The orientation of the turbine axis is radial to the containment. There are no essential systems or structures located inside the low trajectory missile zones.*

3.5.1.3.2 Missile Identification and Characteristics

Guide 1.115; Therefore, Only high trajectory missiles need be considered

The worst case accident requires a turbine overspeed of 180 percent of rated rpm to produce a failure of a last stage disc, which is the source of the most dangerous missile. Such a missile is produced from a 133.6 degree section of the turbine rotor. The missile has a weight of 6990 lb with a minimum projected area of 2.43 ft². The translational velocity of the missile after leaving the outer casing is 410 ft/sec. The missile can emerge at any angle above the horizontal joint of the low-pressure turbine and at any axial angle from zero degrees (radial exit) to about 16.8 degrees tilted toward the exhaust end of the turbine. The most likely axial exit angle is considered to be 8.40 degrees with an energy equal to 19×10^6 ft-lb.

~~These quantities are in accordance with Reference [5]; however, calculations and figures included in this analysis use the value of +25 degree-angle in accordance with NRC Regulatory Guide 1.115.~~

The mathematical model for selecting the missile size and its sector angle assumes maximization of the translational energy *is described in 3.5A, B*

$$E_{MT} = (1/2) M_M \cdot V_{MT}^2$$

delete

In terms of missile sector angle θ , the energy equation can be written as follows:

(B)

Each of the two CPSES low pressure turbines ~~stages~~ is an Allis-Chalmers Power Systems, Inc. (ACPSI) 1800 rev/min steam turbine-generator with 44-inch last stage blades designed for Light Water Reactor (LWR) applications. Each rotor is made from a stepped shaft with a total of 10 shrunk-on blade ~~disks~~ arranged in symmetrical groups of five.

Figures (a) provide the weight, dimensions and location of each disk relative to the turbine center.

one half The turbine-generator is designed and tested for safe operation in the range up to 120 percent of design speed. At speeds up to 120 percent of rated, the maximum disk stress at the shrink fit is less than $\frac{1}{2}$ of the burst strength of the material. Rotor failure could only occur ^{because of} ~~due to~~ defective material, inadequate quality assurance, or design error. ^{because of} ~~due to~~ the high reliability provided by design, manufacture and quality assurance of LP disks and rotors, the probability of such a failure is extremely low; even if one should occur, conservative analysis indicates that the disk missiles would be contained by the LP turbine casings. Based on these considerations, the case of failure at or near design speed can be eliminated from detailed study for purposes of turbine missile analysis. [8]

LP disk failure due to excessively high overspeed could produce external turbine missiles. Characteristics of LP disk missiles leaving the outer casing are provided on ~~Figure 35.10~~ ^{rev/min} LP disk 5 is the last stage disk with 44-inch end blades, and the other disks are for the other stages of a typical 1800 ~~rev/min~~ LP turbine. All missiles are wedge-shaped sectors of the disks having a sector angle of 134 degrees which yields the maximum translational missile energy. [8]

The burst speed of disks 4 and 5 is taken as the speed at which the average tangential stress in the disk equals 85 percent of the ultimate tensile strength of the material. It is assumed that disks 1, 2 and 3 will burst at least at the burst speed of disk 5. The net missile energy leaving the outer LP casing is determined by analysis of energy dissipation as the disk missile crashes through the casings. [8] Due to the conservative calculations of energy dissipation, the net energies and missile velocities given should be considered as maximum values.

The largest potential missile from a hypothetical LP turbine disk failure is a missile of the last stage disk. The missile deflection angles ~~will be 10 to 25 degrees for the end disk.~~ [9]
postulated for the last stage disc were degrees

$$E_{MT} = (1/2) A_M Y_0 \theta \rho \left[\frac{I_M}{S} \cdot \frac{\sin \theta/2}{\theta/2} W_M \right]^2$$

After combining the constants (A_M , Y_0 , ρ , W_M , I_M , S) into a single constant K , the previous equation can be written as follows:

$$E_{MT} = K \left[\frac{\sin^2 \theta/2}{\theta/2} \right]$$

Taking its first derivative with respect to θ and setting it equal to zero we obtain the following:

$$\frac{dE_{MT}}{d\theta} = K \frac{(\sin \theta/2) (\cos \theta/2) (\theta) - \sin^2 \theta/2}{\theta^2} = 0$$

The translational energy is maximum when $\frac{d^2 E_{MT}}{d\theta^2} = 0$

This condition occurs when

$$\sin \theta/2 [(\cos \theta/2) \theta - \sin \theta/2] = 0$$

Using trigonometric manipulation, this reduces to

$$\tan \theta/2 = \theta$$

This is true and only true at $\theta = 133.6$ degrees. This sector angle defines the shape and size of the missile possessing the maximum energy. Penetration of the outer casing of the turbine is based on the Stanford formula [2].

$$\Delta E_M = D_M \sigma U L T_0 (0.344 T^2 + 8.06 \times 10^{-3} W T)$$

$$B_M = 4 \frac{A_{min}}{U}$$

The missile trajectory path is shown on Figure 3.5-4.

The assumption made for determining the missile characteristics is that all protective and control devices failed one after the other and overspeed of 180 percent of the normal rotating speed had been reached.

W_M = Angular velocity of the rotor at the moment of bursting

E = Maximum translational energy

A_{MT} = Area of cross section of missile

Y_0 = Distance from turbine axis to the center of gravity of cross section of missile

I_m = Moment of inertia of cross section of missile with respect to turbine axis

S = Static moment of cross section of missile with respect to turbine axis

ρ = Density of the missile material

D_m = Equivalent diameter of the missile

~~A_{min} = Minimum projected area of missile~~

~~U = Perimeter of minimum projected area of missile~~

~~T = Thickness of outer casing~~

~~W = Distance between supports of outer casing wall~~

~~θ = Missile sector angle~~

~~$U.T._0$ = Ultimate missile stress, outer casing~~

3.5.1.3.3 Strike Probability Analysis

an essential

This analysis uses theories and assumptions which yield the strike probability of a missile on a vital component, ~~whose damage or destruction would release a large amount of radioactivity.~~

~~Results of this analysis are tabulated in Table 3.5-7 and indicate that the summation of all strike probabilities due to the failure of one rotor is very small and need not be considered a major factor in the design of the plant. The assumptions made in this analysis are as follows:~~

1. Only one missile is released at a time
2. Strike probability due to low trajectory ~~has not been~~ ^{is not} considered because of the orientation of the plant's structures with respect to the turbine generators

as shown in Figure 3.5-3

3.5-16

3. ~~Because~~ Since the low pressure turbine disk missile can only penetrate the turbine casings at a turbine-generator speed in excess of about 160% ^{percent}, the probability of significant turbine disk missiles within the design speed range of 120% ^{percent} is considered zero. [5,8]

- 4 The probability of an overspeed incident of the low pressure turbine disk missile with the highest energy level (180% of rated speed) CPSES/FSAR approximately percent has been analyzed.

Insert (A) here X. Results Calculations in Table 3.5-7 are shown for the failure of only one turbine, the one closer to the structures or components and thus having the higher strike probability; this is considered the worst case.

The overall probability (P4) is given by:

$$P4 = P1 * P2 * P3$$

where:

P1 = Probability of occurrence of turbine missile per turbine year
 $= 4 \times 10^{-5}$ [10]

P2 = Strike probability

P3 = Probability of damage due to the strike

The value of P1 has been determined by Allis-Chalmers Power System Inc. and is equal to 3.1×10^{-5} per unit year for a four flow turbine generator [5]. The method of analysis used in the calculations is described in Reference [8], and it follows the numerical example, 3.1 of the same reference.

The combined strike and damage probability (P2 x P3) for all targets is 3.6×10^{-4} and it is within the NRC criterion of 10^{-3} per turbine failure [10].

3.5.1.3.4 Turbine Overspeed Protection

For a detailed operational mechanism for the overspeed protection components of the turbine, refer to Section 10.2.2.

The components reliability analysis has been based on the following assumptions:

1. Failure rates for the turbine valves and controls derived primarily from actual operating experience and calculated with a statistical confidence level of 95 percent

(A)

Turbine missiles ejected within a few degrees of the vertical are characterized as high trajectory missiles (HTM). Their strike distribution, consisting for the most part of "lob shot" hits on horizontal surfaces, is relatively isotropic.

An assessment of the damage probability by HTM's ~~usually~~ involves a calculation of the missile strike probability with respect to each critical target. ^{Our} ~~One~~ approach is to determine the strike probability distribution on a unit area basis. Then the strike probability for a selected target can be estimated ~~by integrating~~ ^{by averaging} over the area of the target.

In formulating the technique presented ^{in 3.5B} ~~here~~ for determining strike probability distribution, ~~we have adopted~~ ^{we have} the following assumptions:

a. All turbine missile deflections are limited by the ~~inner~~ ^{outer} disk ~~deflection~~ angles.

b. The initial missile flight directions are distributed uniformly.

c. The initial missile exit speeds are distributed uniformly between some minimum and maximum values.

d. Missile trajectories are constrained by the classical ballistic equations, and air resistance is neglected.

e. All target areas are horizontal and coplanar with the turbine axis.

2. A 2-week test interval for the turbine valves and the overspeed trip system, and a yearly maintenance inspection of the complete overspeed prevention system
3. An average of one true load rejection per year
4. Consideration of the turbine extraction of common mode failure possibilities in addition to random failures

Thus, the overspeed failure probability has been estimated to be very small (2.10×10^{-7} per ^{Turbine} unit-year for a four flow turbine generator), and subsequently a high degree of component reliability exists. ⁵

~~CPS/FSAR~~ turbine valve analysis is based on a conservative ⁵ of 4×10^{-5} per ~~unit~~ year.

3.5.1.3.5 Turbine Valve Testing Periodic testing of the stop and control valves is necessary to insure reliability and continuity of service; therefore, a biweekly valve testing for all nuclear units is recommended with the aid of an automatic turbine tester (ATT). If an ATT is not available, monthly test intervals are recommended.

For high reliability of the electrohydraulic control (EHC), one primary AC power source for valve testing is provided from safeguard bus; two separate DC sources for control circuits are provided from one ± 24 VDC system with common neutral. For more details of inservice inspection, refer to Section 10.2.3.6.

3.5.1.3.6 Turbine Characteristics

The turbine is a multicasing, tandem compound, four flow, reaction type, 1800-rpm unit with 44-in. last stage blades. For more detailed characteristics, refer to Section 10.2.2.

6. Bush, S.H., "Probability of Damage to Nuclear Components Due to Turbine Failure" presented at Topical Meeting on Water Reactor Safety, March 26-28, 1973, CONF-730304, USAEC.
7. Burnell, J., "The Flow of Boiling Water Through Nozzles, Orifices and Pipes," The Institute of Engineers, Australia, March, 1946.
- ~~X~~ ~~Methods of Determining the Probability of a Turbine Missile Hitting a Particular Plant Region, Topical Report WCAP-7861, February 1972.~~
8. Allis - Chalmers P. S. Inc., Engineering Report No. ER-503 "Turbine Missile Analysis for 1800 r/min Nuclear Steam Turbine-Generators with 44-inch Last Stage Blades", July 1975.
9. USNRC, Regulatory Guide 1.115, "Protection Against Low-Trajectory Turbine Missiles" Rev. 1, July 1977

CPSES / FSAR
TABLE 3.5-7

High Trajectory Missile Strike and Damage Probability
(Missile Exits From Unit 1 Low Pressure Turbine)

Target Component Contained Within:	P_1	P_2	P_3	$P_2 \times P_3$	P_4
Unit 1 Safeguard Bldg	4×10^{-5}	1.1×10^{-5}	1.	1.1×10^{-5}	4.4×10^{-10}
Unit 1 Diesel Generator Bldg	4×10^{-5}	1.1×10^{-5}	1.	1.1×10^{-5}	4.3×10^{-10}
Unit 1 Reactor Containment	4×10^{-5}	1.8×10^{-5}	1.	1.8×10^{-5}	7.2×10^{-10}
Auxiliary Bldg	4×10^{-5}	4.4×10^{-6}	1.	4.4×10^{-6}	1.8×10^{-10}
Fuel Bldg	4×10^{-5}	3.0×10^{-5}	1.	3.0×10^{-5}	1.2×10^{-9}
Unit 2 Reactor Containment	4×10^{-5}	1.8×10^{-5}	1.	1.8×10^{-5}	7.2×10^{-10}
Unit 2 Safeguard Bldg	4×10^{-5}	1.1×10^{-5}	1.	1.1×10^{-5}	4.4×10^{-10}
Unit 2 Diesel generator Bldg	4×10^{-5}	1.1×10^{-5}	1.	1.1×10^{-5}	4.3×10^{-10}
Electrical and Control Building	4×10^{-5}	2.5×10^{-4}	1.	2.5×10^{-4}	9.9×10^{-9}
Service Water Intake Structure	4×10^{-5}	2.8×10^{-6}	1.	2.8×10^{-6}	1.1×10^{-10}

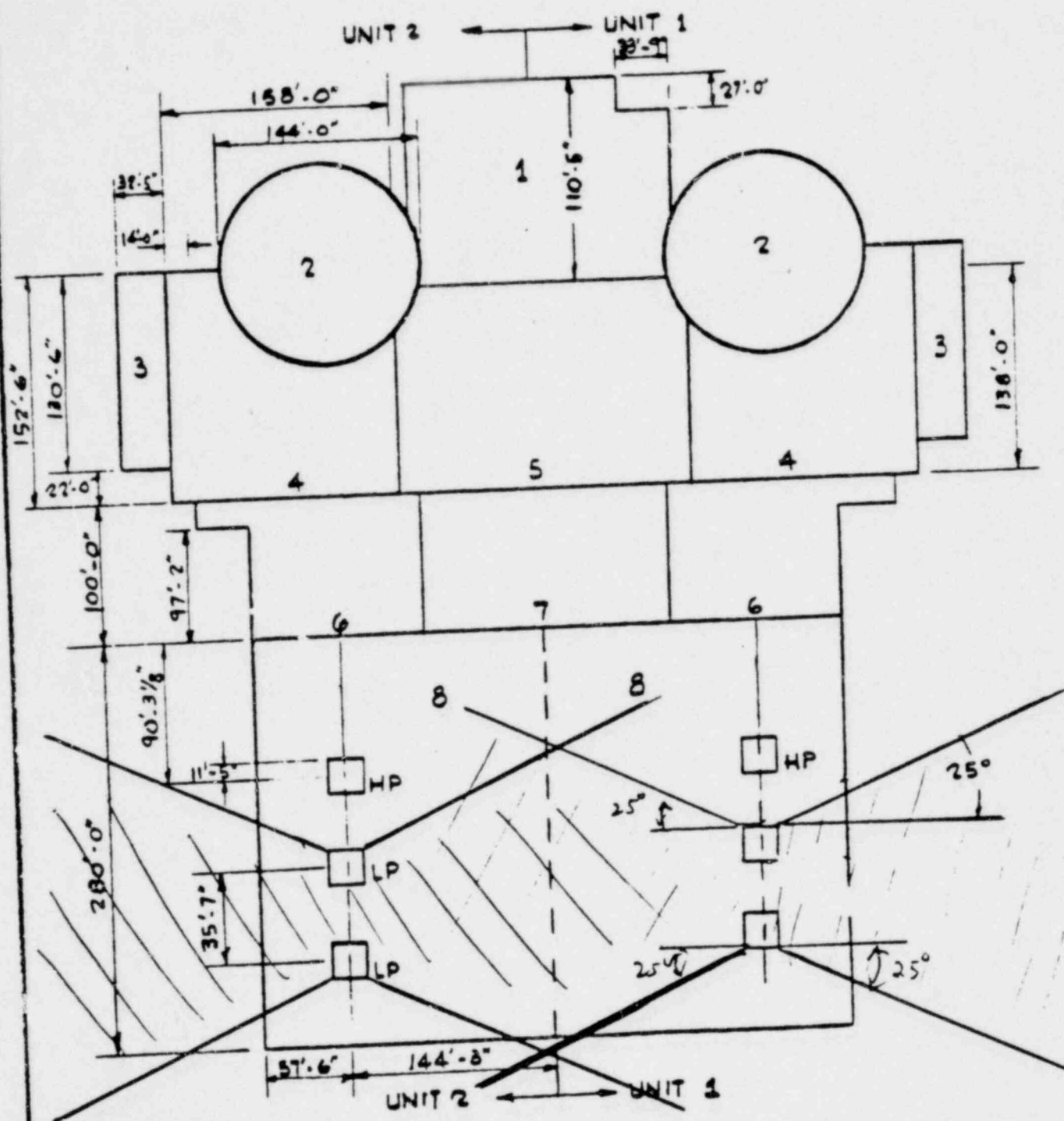
Summation: 3.6×10^{-4}

CPS-3/F-10A
Table 3.5-8 10

LP TURBINE DISK MISSILE CHARACTERISTICS

	LP DISK NUMBER				
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
Postulated Burst Speed, ^{ev} r/min	3312	3312	3312	3268	3312
Net energy of missile leaving outer casing (10 ⁶ ft. lbf)	20.23	9.89	16.07	17.47	18.81
Weight of missile leaving outer casing (lb)	5848	5775	4349	5500	7535
Translational velocity of missile leaving outer casing assuming all energy is translational (ft/s)	472	332	488	452	401
Minimum projected area of missile (ft ²) ^{ec}	1.83	1.85	1.51	1.89	2.70
Maximum projected area of missile (ft ²)	14.1	14.5	15.2	15.5	15.5

Source: References 5 and 8



1. FUEL BLDG.
2. REACTOR BLDG.
3. DIESEL BLDG.
4. SAFEGUARDS BLDG.
5. AUXILIARY BLDG.
6. SWITCHGEAR BLDG.
7. ELECTRICAL & CONTROL BLDG.
8. TURBINE BLDG.

COMANCHE PEAK S.E.S.
FINAL SAFETY ANALYSIS REPORT
UNITS 1 and 2

TURBINE MISSILE STRIKE ZONE
PLAN VIEW

FIGURE 3.5-3

APPENDIX 3.5A

3.5A

THEORY OF MISSILE ENERGY [8]

3.5A.1 ^{Appendix} Introduction

This ~~section~~ ^{Appendix} presents the theory and mathematical models used to determine the initial missile energy at the moment of disk burst and dissipation of this energy as the missile crashes through the inner and outer casings of the turbine. The net energy determines the missile velocity leaving the outer casing.

3.5A.2 General Approach and Assumptions

The approach of this analysis employs theory and assumptions which tend to yield maximum net energy of the missile leaving the outer casing of the turbine.

~~This general approach is considered appropriate for safety analysis since any actual turbine failures could only produce missiles less hazardous than those calculated in this report.~~

The same basic theory is applicable to all disks and burst speeds considered in this report. ~~In developing this theory, reference was made to various reports dealing with turbine rotor failure as well as other publications indicated in the list of references given at the end of this report.~~

3.5A.3

3.3 Sector Angle of Highest Energy Missiles

The translational energy of a disk missile E_{MT} at the instant of failure is:

$$E_{MT} = \frac{1}{2} m_M V_{MT}^2 \quad (1)$$

where:

m_M = mass of the missile

V_{MT} = translational speed of missile

In terms of the missile sector angle φ , the energy equation can be written as follows:

$$E_{MT} = \frac{1}{2} A_M \gamma_0 \varphi \rho \left[\frac{I_M}{S} \frac{\sin \varphi/2}{\varphi/2} \omega_M \right]^2 \quad (2)$$

where:

A_M = area of cross-section of missile

γ_0 = distance from turbine axis to the center of gravity of cross-section of missile

I_M = moment of inertia of cross-section of missile with respect to turbine axis

S = static moment of cross-section of missile with respect to turbine axis

ρ = density of the missile material

ω_M = angular velocity of the rotor at the moment of bursting

After combining the constants into a single constant K , equation (2) is reduced to:

$$E_{MT} = K \frac{\sin^2 \varphi/2}{\varphi/2} \quad (3)$$

which can be differentiated as follows:

$$\frac{dE_{MT}}{d\varphi} = K \frac{\sin \varphi/2 \cos \varphi/2 \varphi - \sin^2 \varphi/2}{\varphi^2} \quad (4)$$

The translational energy is a maximum at $\frac{dE_{MT}}{d\varphi} = 0$ which occurs when:

$$\sin \varphi/2 [(\cos \varphi/2) \varphi - \sin \varphi/2] = 0 \quad (5)$$

or

$$\tan \varphi/2 = \varphi \quad (6)$$

which results in the numerical value:

$\varphi = 133.6^\circ$ for the missile with maximum initial translational energy.

Further calculations are based on a missile sector having an angle of 133.6° .

3.5A.4 Initial Missile Energy and Trajectory

The unrestrained trajectory of the missile up to collision with the turbine inner casing is shown in Fig. ~~4.07~~ ^{3.5-4}. The corner points follow cycloid paths. The momentary centers of rotation of the missile lie along a straight line through the turbine axis parallel to the path of the center of gravity of the missile.

The total energy of the missile E_{mo} at the moment of rotor burst is:

$$E_{mo} = \frac{1}{2} I_{pmo} \omega_{mo}^2 \quad (7)$$

where:

I_{pmo} = polar moment of inertia of the missile at the moment of burst
referred to turbine axis

ω_{mo} = angular velocity of the missile at the moment of burst

This consists of translational energy:

$$E_{mto} = \frac{1}{2} m_{mo} y_s^2 \omega_{mo}^2 \quad (8)$$

where:

m_{mo} = mass of the missile at the moment of burst

y_s = distance from the turbine axis to the missile center of gravity
(including blades) at the moment of burst

and rotational energy:

$$E_{mro} = E_{mo} - E_{mto} \quad (9)$$

or

$$E_{mro} = \frac{1}{2} I_{pmcg} \omega_{mo}^2 \quad (10)$$

where:

I_{pmcg} = polar moment of inertia of the missile referred to its center of gravity at the moment of burst

3.5A.5

Dissipation of Missile Energy within the Turbine

The net missile energy as it leaves the turbine outer casing is determined from the following energy balance:

$$E_m = E_{mo} - \sum \Delta E_{m,i} \quad (11)$$

The individual portions of energy dissipation ΔE_{mi} are described in the following sub-sections.

It is assumed that the total energy of the missile leaving the turbine casing is in the form of translational energy. This assumption is conservative because a part of the energy is likely to be rotational which does not contribute significantly to the capability of the missile penetrating the reactor containment or other parts of the power plant.

3.5A.5.1

Blade Deformation

In accordance with observed turbine rotor failure patterns, it is assumed that half of the turbine blades on a disk are deformed by being crushed and stressed up to their yield stress. However, this occurs over only one third of their length because due to the susceptibility of the upper blade part to buckling, substantial crushing only occurs in the lower blade part.

The remaining half of the blades are assumed to suffer considerable bending deformation prior to destruction. It is conservatively assumed that a permanent 90° bending occurs in the lower blade region.

3.5A.5.1.1

Blade Crushing

Only the lower third of the blade vane, which cannot buckle, is considered as crushable. The energy dissipation is:

$$\Delta E_{mi} = A_B \sigma_{ys} \frac{l_b}{3} \frac{m}{2} \quad (12)$$

where:

A_b = cross-sectional area of the lower third portion of blade vane

σ_{ys} = yield stress of the blade material

l_b = blade vane length

m = number of blades on the disk missile

3.5A.5.1.2

~~3.5.1.2~~ Blade Bending

The bending moment for full plastic bending is $M_{bb} = 1.5 Z_b \sigma_{ys}$

Therefore, the blade bending energy dissipation is:

$$\Delta E_{m2} = \frac{m}{2} \int M_{bb} d\psi_b, \text{ or } \Delta E_{m2} = 1.5 Z_b \sigma_{ys} \frac{\pi}{2} \frac{m}{2} \quad (13)$$

where:

M_{bb} = bending moment of blade

Z_b = section modulus of the blade vane at the root in the circumferential direction

ψ_b = bending angle of blade

3.5A.2

~~3.5.2~~ Break-off of Blade Vanes

~~Based on the literature References (8) to (12).~~ Break-off of the blade vanes is assumed to occur in two phases in which 50% of the blade vanes (Blade Group No. 1) break in collision of the disk with the inner casing. The other 50% of the blading is assumed to break off either by collision with other portions of the rotor or during penetration of the disk missile through the inner casing.

$$\Delta E_{m3} = \frac{1}{2} 0.5 m_b (r_m \omega_m)^2 + \frac{1}{2} 0.5 I_{pb} \omega_m^2 \text{ for Blade Group 1} \quad (14)$$

where:

m_b = mass of the blades associated with the missile

I_{ps} = polar moment of inertia of the blades associated with the missile referred to their center of gravity

r_{m1} = distance from the center of mass of 50% of the blades to the momentary point of rotation at the onset of Phase 1

$$\Delta E_{m1} = \frac{1}{2} 0.5 m_b (r_{m2} \omega_m)^2 + \frac{1}{2} 0.5 I_{ps} \omega_m^2 \quad \text{for Blade Group 2} \quad (15)$$

where:

r_{m2} = distance from the center of mass of the remaining 50% of the blades to the momentary point of rotation at the onset of Phase 2.

3.5A.3

Friction between Missile and Inner Casing

The frictional energy at the area of contact between the missile and the inner casing (see Figure ^{3.5A-1} ~~NA-1.08~~) is determined in accordance with the following equations 16 to 21. Due to the very "rough" surface of the missile, a very high coefficient of friction (probably greater than 0.5) is to be expected. However, in order to use conservative assumptions, only half of this value is used for the calculations, i.e. $\mu = 0.25$. Furthermore, the calculations are done using average constant values for ω_m and the distance " r ", whereby the reduction of ω_m due to friction is taken to be 10% as an average. This assumption can be checked by subsequent evaluation of the energy components.

In accordance with the theory of impulse and momentum, the unknown integral value $\int_{t_s}^{t_e} F_n dt$ for the impulse is proportional to the difference between the initial and final velocities of the missile normal to the casing wall during the friction process. If $V_{nt} \cos \alpha$ is used for the initial velocity, a conservative energy dissipation value results.

The value of the final velocity is not yet known in this phase of the calculation procedure. For this reason, the calculations proceed on the basis of an estimated value which can be improved subsequently by means of trial and error until the required accuracy is achieved. For the calculations in this ~~report~~ ^{section} the error was reduced to 10^{-6} .

The procedure is described by the following equations:

$$\Delta E_{MS} = \int_{t_s}^{t_e} \mu F_n d(\omega_n t) \quad (16)$$

For the first approximation the following values are inserted:

μ = constant, and

$\tilde{\omega}_n$ = constant (average during the period t_s to t_e)

yielding the following:

$$\Delta E_{MS} = a \mu \tilde{\omega}_n \int_{t_s}^{t_e} F_n dt \quad (17)$$

where:

a = distance between point of friction and momentary center of rotation,
at right angles to the frictional force

μ = coefficient of friction

$\tilde{\omega}_m$ = average angular velocity during the frictional phase

$\int_{t_1}^{t_2} F_n dt$ = change of momentum of the missile during the frictional phase

In accordance with impulse-momentum theory:

$$\int_{t_1}^{t_2} F_n dt = (m_{m0} - m_B)(v_{mT} \cos \alpha - v'_m) \quad (18)$$

$(m_{m0} - m_B)$ = missile mass without blades

α = angle between the direction of translation of the missile and a line which is perpendicular to the tangent at the point of contact of the missile and inner casing

v'_m = velocity of missile normal to the inner casing wall after penetrating the wall

The velocity of the missile after leaving the inner casing is maximum when the total energy of the missile is translational:

$$E_{mB} = \frac{1}{2} (m_{m0} - m_B) (v'_m)^2, \text{ or} \quad (19)$$

$$v'_m = \sqrt{\frac{2E_{mB}}{m_{m0} - m_B}} \quad (20)$$

where:

E_{mB} = energy of the missile after leaving the inner casing

Using greatest final momentum of the missile results in the smallest energy dissipation due to friction, and thus to a calculation which is on the conservative side.

The equation for the frictional energy is therefore:

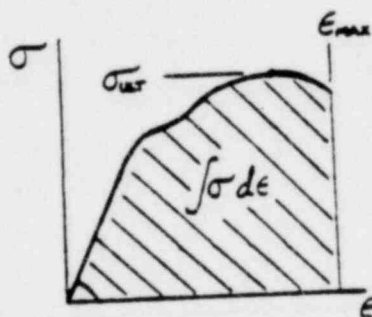
$$\Delta E_{MS} = a \mu \tilde{\omega}_m [v_{MT} \cos \alpha (m_{m0} - m_s) - \sqrt{2E_{MS}} (m_{m0} - m_s)] \quad (21)$$

3.5A.4

3.5.4 Deformation of the Inner Casing up to Breakage

It is assumed the missiles will deform the inner casing until it breaks from tension and bending stresses.

The energy dissipation due to deformation by tension of the casing up to the point of breaking is determined by the area under the stress-strain curve shown in the following diagram:

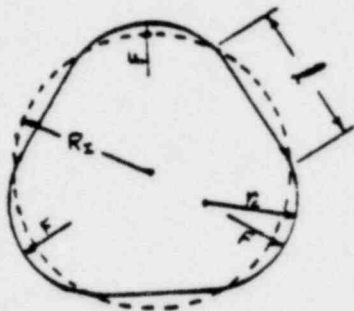


The evaluation of the stress-strain diagram results in the following average values for steels used to fabricate the turbine inner casing:

$$\int \sigma d\epsilon = 0.85 \sigma_{max} \epsilon_{max} \quad (22)$$

Bending deformation of the inner casing is modeled to result from three equal

disk pieces (missiles) as illustrated in the following diagram:



With an assumed deformation radius

r_c of 30% of the casing radius

~~(see reference (12))~~ ^{SEFC} the total
tensile-stressed length $3l$ will be

70% of the casing circumference.

F = initial force of missile

Therefore, the following deformation energies for each disk missile by tension and bending are obtained:

By tension:

$$\Delta E_{M6} = \frac{(0.7) (0.85 \sigma_{UTZ} E_{MAXZ}) V_Z}{3}$$

$$\Delta E_{M6} = 0.2 \sigma_{UTZ} E_{MAXZ} V_Z$$

(23)

By bending with same assumption as in 3.5/1.2:

$$\Delta E_{M7} = \int M_{bZ} d\psi_Z \approx 1.5 \sigma_{a2Z} Z_Z \int d\psi_Z$$

$$\Delta E_{M7} = 1.5 \sigma_{a2Z} Z_Z \left[\frac{2r_c \pi}{3} \left(\frac{1}{r_c} - \frac{1}{R_i} \right) + l \frac{1}{R_i} \right]$$

Since $l = \frac{2}{3} \pi (R_i - r_c)$ and $r_c = 0.3 R_i$:

$$\Delta E_{M7} = 1.4 \pi \sigma_{a2Z} Z_Z$$

(24)

where:

M_{bz} = bending moment of the inner casing wall

ψ_z = bending angle of the inner casing wall

$\sigma_{0.2}$ = yield stress (0.2% offset) for inner casing wall material

Z_i = section modulus of the inner casing wall

r_c = radius of inner casing wall at point of impact at the moment of breaking

R_i = radius of inner casing before impact

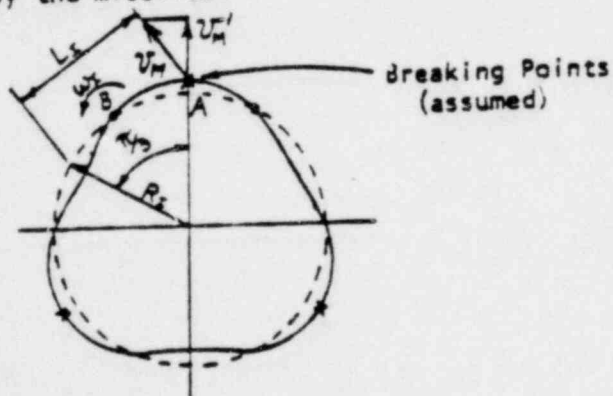
l = length of tensile-stressed portion of inner casing wall at the moment of breaking

V_i = total volume of inner casing around the missiles

3.5A.5

Kinetic Energy of the Inner Casing Fragments

For the inner casing breakage, the shape shown in the following sketch is conservatively assumed because it results in the smallest energy dissipation by the missiles:



Assumptions:

Velocity v_M at the point A of the casing fragment at breakage has been applied conservatively as v_M' the exit velocity of the rotor missile.

Half of the casing fragment is approximated by a flat plate, with a length of L_I

The entire energy of the casing fragment is assumed to be rotation of the fragment around its center of gravity, point B with angular velocity ω_I

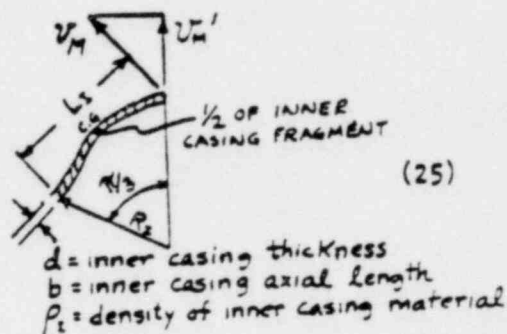
Three disk missiles produce three casing fragments, and therefore, the energy dissipation for each missile is:

$$\Delta E_{M8} = 2 \frac{1}{2} I_{PI} \left(\frac{V_M}{L_I/2} \right)^2$$

$$I_{PI} = \frac{1}{12} \rho_i b d L_I (d^2 + L_I^2) \approx \frac{1}{12} \rho_i b d L_I^3$$

$$\Delta E_{M8} = \frac{1}{3} \overbrace{\rho_i b d L_I}^{V_i/6} V_M^2$$

$$\Delta E_{M8} = \frac{1}{18} \rho_i V_i V_M^2$$



where:

I_{PI} = polar moment of inertia of half of the inner casing fragment with respect to its center of gravity.

3.5A.6

Penetration of the Outer Casing

Calculation in accordance with the Stanford formula in Reference (6):

$$\Delta E_{M9} = D_M \sigma_{UT0} [0.344 T^2 + 8.06 \times 10^{-3} WT] \quad (26)$$

where D_m is the equivalent diameter of the missile:

$$D_m = 4 \frac{A_{min}}{U}$$

A_{min} = minimum projected area of missile

U = perimeter of minimum projected area of missile

T = thickness of outer casing

W = distance between supports of outer casing wall

3.5A.7

3.5.7 Kinetic Energy of the Outer Casing Fragment

Calculation in accordance with Recht and Ipson per Reference (5):

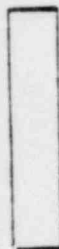
$$\Delta E_{M10} = E_{M9} \left[1 - \left(\frac{m_{M10}}{m_{M10} - m_p} \right)^2 \right] \quad (27)$$

where:

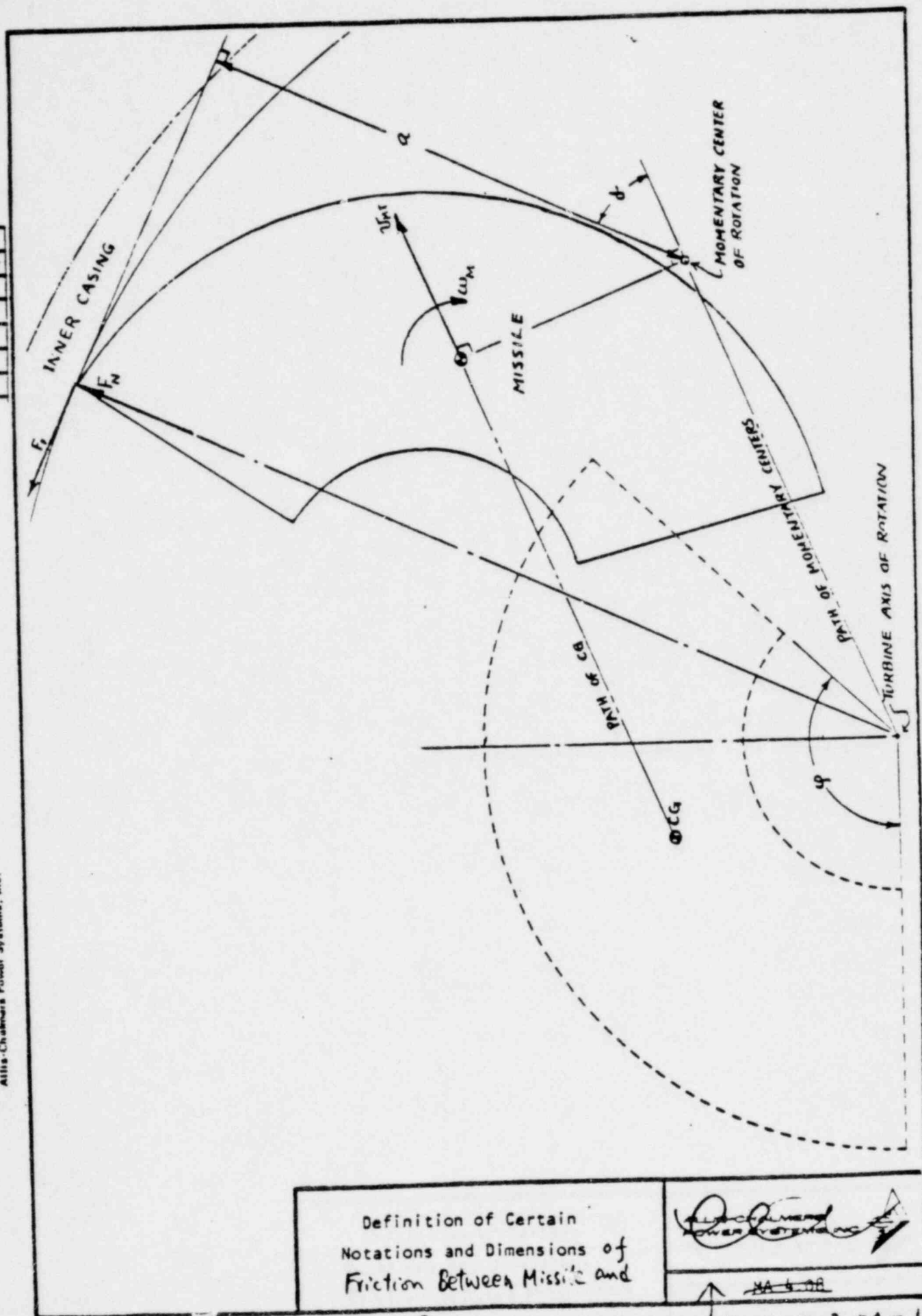
m_p = mass of the outer casing fragment

E_{M9} = missile energy after penetration of the inner casing minus energy loss in penetration of the outer casing, i.e. $E_{M9} = E_{M8} - \Delta E_{M8}$

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Definition of Certain
Notations and Dimensions of
Friction Between Missile and

Inner Casing

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NA 4-28

FIGURE 3.5A-1

APPENDIX 3.5B

Turbine Missile Strike Probability Distributions

3.5B.1 Introduction

A generated turbine missile will fly away from the turbine with a given speed and direction. The direction is defined by the angle from the horizontal plane and angle of horizontal component from the turbine axis. Figure 3.5B.1 shows the relationship between the disk plane and missile exit direction.

3.5B.2 Distribution of Exit Velocity and Directions

With a given exit velocity and direction, the turbine missile trajectory is determined by gravity. If the initial velocity and direction are random variables, we need to describe the distributions of the initial velocities and directions in order to estimate the strike probabilities. There are three random variables we used in our analysis:

the random variable v representing the initial velocity,

the random variable θ_1 representing the angle of the vertical component (i.e., the projection on disc plane) from the horizontal plane,

and the random variable θ_2 representing the angle from the disk plane (see Figure 3.5B-1).

We assumed that:

- (1) The initial velocity v is uniformly distributed between v_1 and v_2 , where $v_1 < v_2$. We have used $(v_1, v_2) = (385, 415)$ ft/sec.
- (2) The angle θ_1 is uniformly distributed between 0° and $\pi/2$.
- (3) The angle θ_2 is uniformly distributed between 0° and Δ° . where we used $\Delta = 25^\circ$ in our analysis

The Technique for Determining Turbine Missile Strike Probability Distribution Arbitrarily Oriented Target

Consider the coordinate system shown in Fig. 3.5B-1. The origin is the center of the disk which ruptures, the x-axis corresponds to the turbine axis, and the disc rotates in the y-z plane. The missile will leave the origin with an initial speed v_0 and initial direction defined by θ_0 and ϕ_0 (see Fig. 3.5B-1). For a given point (x_0, y_0, z_0) we want to find the combinations (v_0, ϕ_0, θ_0) which will result in the trajectory of the missile passing through the point.

Since the only force acting on the missile is that of gravity (we ignore air resistance) the trajectory will lie in the plane $\theta_0 = \theta_0$, where

$$\theta_0 = \tan^{-1}(x_0/y_0) \quad (1)$$

and the original problem reduces to a two-dimensional problem. If ρ is the distance from the origin along the intersection of the x-y plane and the $\theta_0 = \theta_0$ plane then the problem is as shown in Fig. 3.5B-2 where

$$\rho_0 = \sqrt{x_0^2 + y_0^2} \quad (2)$$

The equation of the trajectory in the ρ -z plane is

$$z = \rho \tan \phi - \frac{g\rho^2}{2v^2 \cos^2 \phi} \quad (3)$$

where g is the acceleration of gravity. For a given (ρ_0, z_0) we can solve this for v as a function of ϕ getting

$$v = \rho_0 \sqrt{\frac{g}{2 \cos^2 \phi (\rho_0 \tan \phi - z_0)}} \quad (4)$$

and from this it follows that ϕ is restricted to the range

$$\tan^{-1}(z_0/\rho_0) < \phi < 90^\circ \quad (5)$$

Using a differential argument we get that the range in v , dv , is given by

$$dv = \sqrt{\frac{a}{2\cos^2\phi}} \left[\frac{p_0(\sin \xi + \cos \xi \tan \phi) - 2z_0 \cos \xi}{2(p_0 \tan \phi - z_0)^{3/2}} \right] \Delta_1 \quad (6)$$

In Fig. 3.5B-2

consider an area centered at (x_0, y_0, z_0) , or equivalently at (p_0, z_0, θ_0) , whose orientation is defined by its normal vector $\vec{N} = (\cos \alpha, \cos \beta, \cos \gamma)$, where α , β , and γ are the direction cosines of \vec{N} .

the $p-z$ plane, L_1 , is the line which corresponds to that in Fig. 3.5B-2.

It can be shown that the slope of this line in the $p-z$ plane is given by

$$\tan \xi = - \frac{(\sin \theta \cos \alpha + \cos \theta \cos \beta)}{\cos \gamma} \quad (7)$$

Equation (6)

With this angle, ξ , can be used to calculate the band of allowed values

in the ϕ - v plane, ~~as shown in Fig. 2.5.~~

Let L_2 be the line in the target plane which is perpendicular to L_1 . The extent of the target in this direction must be covered by varying θ_3 through an angle $\Delta\theta_3$. If $\vec{n} = (\cos \theta_3, -\sin \theta_3, 0)$ is the normal to the ρ - z plane then L_2 is a scalar multiple of $\vec{N} \times (\vec{N} \times \vec{n})$, where

$$\begin{aligned} \vec{N} \times (\vec{N} \times \vec{n}) = & [-\sin \theta_3 \cos \alpha \cos \beta - \cos \theta_3 (\cos^2 \beta + \cos^2 \gamma)] \vec{i} \\ & + [\sin \theta_3 (\cos^2 \alpha + \cos^2 \gamma) + \cos \theta_3 \cos \alpha \cos \beta] \vec{j} \\ & + [\cos \gamma (\cos \theta_3 \cos \alpha - \sin \theta_3 \cos \beta)] \vec{k} \end{aligned} \quad (8)$$

and \vec{i} , \vec{j} , and \vec{k} are the unit vectors in the x , y , and z directions, respectively. We define \vec{L} to be the unit vector in this direction

$$\vec{L} = \frac{\vec{N} \times (\vec{N} \times \vec{n})}{||\vec{N} \times (\vec{N} \times \vec{n})||} \quad (9)$$

and we want to find the projection of \vec{L} onto \vec{n} .

$$\vec{L} \cdot \vec{n} = \frac{-2 \sin \theta_3 \cos \theta_3 \cos \alpha \cos \beta - \sin^2 \theta_3 \cos^2 \alpha - \cos^2 \theta_3 \cos^2 \beta - \cos^2 \gamma}{||\vec{N} \times (\vec{N} \times \vec{n})||} \quad (10)$$

If the dimension of the target area along L_2 is Δ_2 then $\Delta\theta_3$ must satisfy

$$\rho_0 \Delta\theta_3 = |\vec{L} \cdot \vec{n}| \Delta_2 \quad (11)$$

or

$$\Delta\theta_3 = \frac{|\vec{L} \cdot \vec{n}| \Delta_2}{\rho_0} \quad (12)$$

3.5B.3.2 Strike Probability Calculation Method

The probability of a missile having the initial velocity and direction within a specified range R can formally be written as

$$\text{Prob}[(\theta_1, \theta_2, v) \in R] = \iiint_{(\theta_1, \theta_2, v) \in R} f(\theta_1, \theta_2, v) dv d\theta_2 d\theta_1, \quad (13)$$

where the probability density $f(\theta_1, \theta_2, v)$ has the form

$$f(\theta_1, \theta_2, v) = \begin{cases} \frac{1}{\frac{\pi}{2} \Delta(v_2 - v_1)}, & \text{if } v_1 < v < v_2, 0 < \theta_1 < \frac{\pi}{2}, 0 < \theta_2 < \pi, \\ 0, & \text{otherwise,} \end{cases} \quad (14)$$

since the random variables v , θ_1 and θ_2 were assumed to be uniformly distributed between v_1 and v_2 , 0 and $\pi/2$, and 0 and Δ , respectively. The distributions and Eq. (5-18) are expressed in terms of θ_1 , θ_2 and v , which we will call the distribution space hereafter, but the target, the missile trajectory, and its speed are all expressed in the exit dynamics space. The exit dynamics space consists of the exit velocity v , the angle ϕ from the horizontal plane, and the angle θ_3 of the horizontal component from the disk plane. The relationship between the distribution space and the exit dynamics space is given in Figure 3.5B-1. In order to be consistent, Eq. (5-18) is rewritten using the coordinates of the exit dynamics space: (13)

$$\text{Prob}[(\theta_1, \theta_2, v) \in R] = \text{Prob}[(\phi, \theta_3, v) \in R']$$

$$= \iiint_{(\phi, \theta_3, v) \in R'} g(\phi, \theta_3, v) |J| dv d\theta_3 d\phi, \quad (15)$$

where $g(\varphi, \theta_3, v) = f[\theta_1(\varphi, \theta_3), \theta_2(\varphi, \theta_3), v]$

R' in the exit dynamics space corresponds to R in the distribution space, and J is the Jacobian of transformation

$$J = \frac{\partial(\theta_1, \theta_2, v)}{\partial(\varphi, \theta_3, v)} = \frac{\partial(\theta_1, \theta_2)}{\partial(\varphi, \theta_3)}.$$

The expressions for J , $\theta_1(\varphi, \theta_3)$ and $\theta_2(\varphi, \theta_3)$ can be obtained from the relationships between (θ_1, θ_2) and (φ, θ_3) :

((6))

$$\begin{aligned}\tan \theta_1 &= \tan \varphi \sec \theta_3, \\ \sin \theta_2 &= \cos \varphi \sin \theta_3.\end{aligned}$$

The Jacobian of transformation has the form:

((17))

$$|J| = \frac{(1 + \sin^2 \varphi \tan^2 \theta_3)}{|\cos \varphi| (1 + \tan^2 \varphi \sec^2 \theta_3) \sqrt{1 - \cos^2 \varphi \sin^2 \theta_3}}$$

The probability of hitting the above target is given by the integral of the form $\int_{\theta_1, \theta_2} f(\theta_1, \theta_2, v) d\theta_1 d\theta_2 dv$ with the proper limits of integration, which are determined by the location of target, its size, and its orientation. Now to evaluate the integral of (15), we replace the inner double integral of (15)

$$\iint_{(\theta_1, \theta_2)} g(\varphi, \theta_3, v) |J| dv d\theta_3 \quad \text{with} \quad g(\varphi, \theta_3, v) |J| \Delta v \Delta \theta_3,$$

where
 Δv
 is from Eq. (6), and

is from Eq. (12) with $(\vec{L} \cdot \vec{n})$ given by Eq. (10). The angle ξ is given by Eq. (7):

and the target area being a unit area means that $\Delta_1 \Delta_2 = 1$.

Now, the multiple integral of Eq. (15) can be replaced by the following simple integral

$$\int_{\varnothing} g(\varnothing, \theta, v(\varnothing)) |J| \Delta v \Delta \theta d\varnothing,$$

where

$$v(\varnothing) = p_0 \sqrt{\frac{a}{2 \cos^2 \varnothing (p_0 \tan \varnothing - z_0)}}$$

and \varnothing is restricted to the range

$$\tan^{-1}\left(\frac{z_0}{p_0}\right) < \varnothing < \frac{\pi}{2}.$$

Because of the nature of the physical problem, we are interested only in those trajectories which result in the missile hitting the target from above, and this leads to another restriction on \varnothing

$$\varnothing > \tan^{-1}\left(\frac{z_0}{p_0} - \tan \xi\right).$$

The fact that the random variable θ_2 is uniformly distributed between 0° and Δ° , i.e., $0 \leq \theta_2 \leq \Delta$, places another restriction on \varnothing , which depends on the θ_3 value,

(22)

~~(5-30)~~

$$h(\theta_3) < \varphi < \frac{\pi}{2},$$

where

$$h(\theta_3) = \begin{cases} 0, & \text{if } 0 \leq \theta \leq \Delta, \\ \sin^{-1} \left(\sqrt{\cos^2 \Delta - \frac{\sin^2 \Delta}{\tan^2 \theta_3}} \right), & \text{if } \Delta < \theta_3 \leq \frac{\pi}{2}. \end{cases}$$

The limits of integration for the integral of \bar{C}_φ , (18) can now be obtained from the fact that the random variable v is uniformly distributed between v_1 and v_2 , i.e., $v_1 \leq v \leq v_2$, and the restrictions in E_φ , (20), (21) and (22) on the allowed φ values. If we define

$$\varphi_{\min} = \max \left\{ \tan^{-1} \left(\frac{z_0}{p_0} \right), \tan^{-1} \left(\frac{z_0}{p_0} - \tan \xi \right), h(\theta_3) \right\},$$

(23)

~~(5-31)~~

then the integral of (18) can be rewritten as

$$\int_{\varphi_{l_1}}^{\varphi_{u_1}} + \int_{\varphi_{l_2}}^{\varphi_{u_2}} \{g[\varphi, \theta, v(\varphi)] |J| \Delta v \Delta \theta\} d\varphi$$

(24)

~~(5-32)~~

with an additional restriction $\varphi \geq \varphi_{\min}$. In the integral of E_φ , (24), φ_{l_1} and φ_{u_1} are obtained by solving Eq. (19) for φ with $v = v_2$ and φ_{l_2} and φ_{u_2} are obtained by solving Eq. (19) for φ with $v = v_1$. In general, there are two values of φ satisfying Eq. (19) for each v . φ_{l_1} is the smaller of the two φ values corresponding to v_2 and φ_{u_1} is the smaller of the two φ values corresponding to v_1 . For a given value of v , one φ value satisfying Eq. (19) is given by

$$\varphi = \frac{1}{2} (\xi + \sin^{-1} A),$$

~~(5-33)~~

(25)

where

$$A = \left(\frac{g p_0^2}{v^2} + z_0 \right) / \sqrt{p_0^2 + z_0^2},$$

and the other φ value is given by

$$\varphi = \frac{1}{2} (\pi + \xi - \sin^{-1} A).$$

(26)

~~(5-34)~~

If $|A| > 1$ for some v , then there exists no solution for φ , and in particular if $|A| \geq 1$ for some v_1 , we set $\varphi_{u_1} = \varphi_{l_2} = \frac{\pi}{4}$.

In summary, the probability of hitting a target plane of small area $\Delta_1 \times \Delta_2$, whose center is located at (p_0, θ_0, Z_0) with its orientation given by its normal vector $\vec{N} = (\cos \alpha, \cos \beta, \cos \gamma)$, is given by the integral of Eq. (24) with the limits of integration ϕ_1, ϕ_u, ϕ_2 and ϕ_u determined from Eqs. (23), (25) & (26). The integral of Eq. (24) is obtained numerically using Romberg's method of integration. A computer program for evaluating the probability of hitting a target was written in Fortran, and a listing of the program as well as an explanation of the input data is given at the end of this section. Using this computer program and some typical distributions for missile velocities and directions we obtained the probabilities of hitting various targets of unit size, and the ^{sample problem and} results are reported in ^{3.5 B. 4.} tables 1 through 4.

~~probability of hitting~~
~~adjusting the integral~~

If the area of a target is large, one cannot use the differential approximation form of Eqs. (6) and (12) for the entire area. Therefore, the probability of hitting the target can be obtained by

first dividing the target area into small targets, for each small target evaluating the integral of Eq. (24) in order to obtain the probability of hitting a small target, then finally by summing up all the probabilities of hitting small targets.

3.5B.4 Sample Problem

The computer program ~~did~~ calculates the strike probability of hitting a small target (1.0 ft^2). We ~~present~~ ^{present below} one example in Dr. S.H. Bush's paper - "Probability of Damage to Nuclear Components Due to Turbine Failure" to describe the input of the program.

The sample problem is an outer disk hit on the Containment dome with an initial exit velocity of 300 ft/sec .

Input Card No. 1.

Word 1. DEL = the maximum allowed angle in degrees between the disc plane and the missile direction
= 25.

Word 2. V_1 = the minimum allowed initial missile velocity in ft/sec

= 270.

Word 3. V_2 = the maximum allowed initial missile velocity in ft/sec

= 330.

Word 4. IORD = the order of Romberg's numerical integration,
= 4

Input Card No. 2

Word 1. RH = the radial distance (ft) to the target of interest
= 225.

Word 2. Z = the height (ft) of the target from the horizontal plane
= 189.

Word 3. $CA = \cos \alpha = 0$
Word 4. $CB = \cos \beta = 0$
Word 5. $CG = \cos \gamma = 1$ } the unit normal to the target

Input Card No. 3

Word 1. NTH = the number of different angle θ_3 's (deg) for the target of interest, corresponding to card No. 2.

= 7

Word 2. $\text{THETA3}(1) = -15.$

Word 3. $\text{THETA3}(2) = -10.$

Word 4. $\text{THETA3}(3) = -5.$

Word 5. $\text{THETA3}(4) = 0$

Word 6. $\text{THETA3}(5) = 5.$

Word 7. $\text{THETA3}(6) = 10.$

Word 8. $\text{THETA3}(7) = 15.$

The result is P_2 is 5.22×10^{-8} per square ft.
The effective target area for HCM on the containment dome is 12000 ft². So that we ^{obtain} ~~get~~ 6.26×10^{-4} for the strike probability, which is very close to Dr. Bush's calculation result of 6.1×10^{-4} .

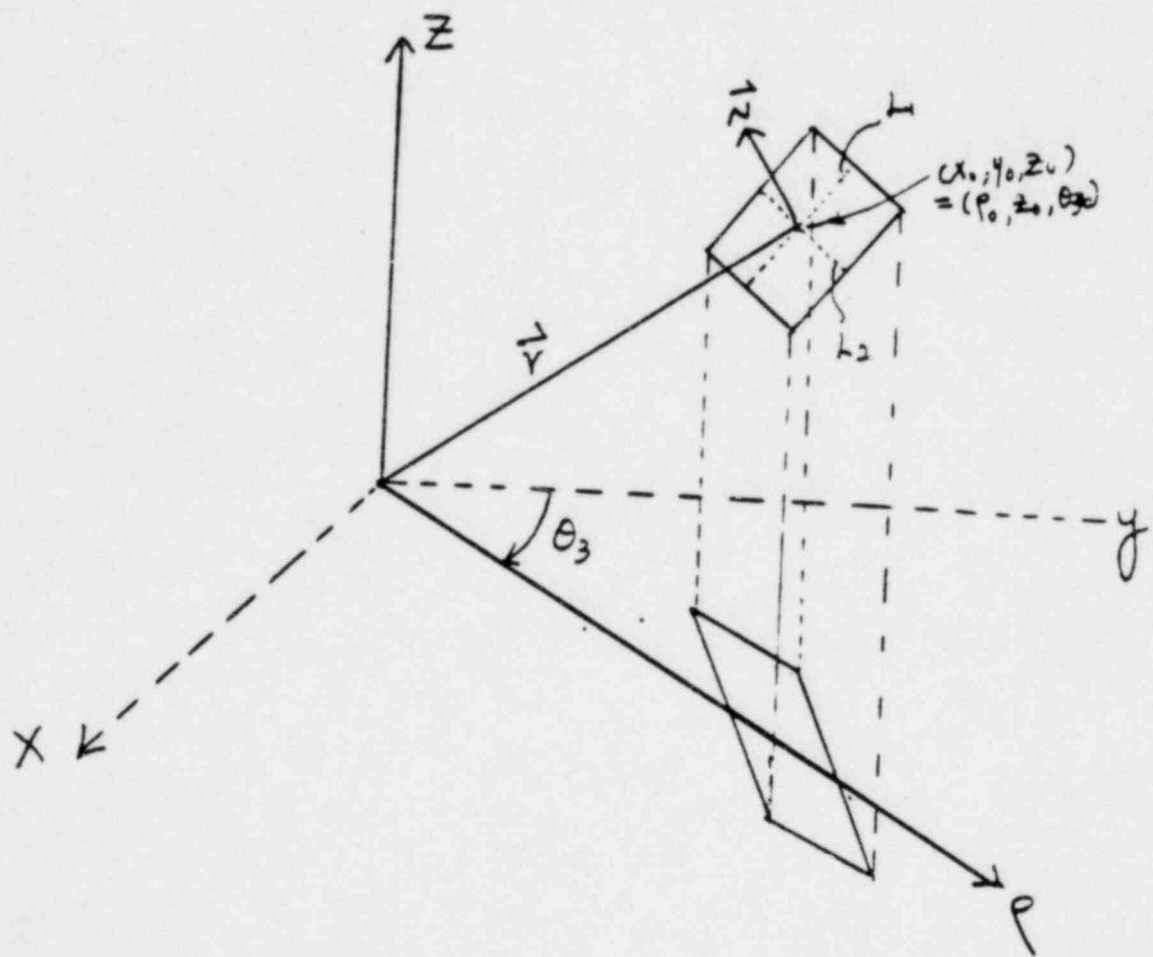


Figure 3.5B - 2 The Relationship Between an Arbitrarily Oriented target Area and Exit Direction