Bayesian Estimation

Workshop 4

Learning Objectives

- Additional practice constructing an informative prior distribution (see also Workshop 2)
- Practice using conjugate likelihood-prior pairs
- Exposure to solution techniques for non-conjugate pairs

Important:

a) The workshop problems can be performed as group exercises.

b) The purpose is to exercise the modeling thought process, not to get the “right answer.”

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Weather-Related Loss of Power in the Boston Area

Weather-related losses of offsite power can be important in NPP PRA. This workshop uses real data and simple models for grid losses in the Boston area to provide practice with the mechanics of Bayesian estimation.

Self-rating

i. How many years have you lived in the Boston area?
ii. Do you have direct experience with weather-related losses of the power grid?

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Problem #1 – Frequency of Weather-Related Loss of Grid (WRLOG)

A. Is the Poisson model a good model for the occurrence of WRLOG? Why or why not?

B. Assume WRLOG can be adequately characterized by a Poisson distribution with frequency λ_{WR}.

   i. What is your "best guess" for λ_{WR}?

   ii. What are reasonable upper and lower "bounds" for λ_{WR}?

   iii. Assuming that your state of knowledge can be roughly characterized by an exponential distribution,* develop appropriate values for the parameter β where

$$\pi(\lambda|\beta) = \beta e^{-\beta\lambda} \qquad \Pi(\lambda|\beta) = 1 - e^{-\beta\lambda} \qquad E[\lambda] = \frac{1}{\beta} \quad SD[\lambda] = \frac{1}{\beta}$$

*The exponential is likely an unsatisfactory form for realistic analysis but more complicated forms require numerical analysis.

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Problem #1 (cont.)

C. Consider the given data for WRLOG in Eastern Massachusetts. The data are from 1/1/2001 through 6/30/2014.*
   i. The data do not include the August 14, 2003 Northeast Blackout which affected numerous U.S. and Canadian plants, but did not extend to the Boston area. For the purpose of a PRA, should it?
   ii. What are the mean and standard deviation for the posterior distribution for λ_{WR}? (Note that the posterior distribution will be a gamma distribution.)

Date	Years Since Last Event
2/8/2013	0.76
10/29/2012	1.00
10/29/2011	0.17
8/28/2011	0.63
1/12/2011	2.08
12/12/2008	0.51
6/10/2008	0.60
11/3/2007	0.55
4/14/2007	0.70
8/2/2006	1.95
8/20/2004	3.64**

**Since 1/1/2001

*Jordan Wirfs-Brock, “Data: Explore 15 Years of Power Outages,” Inside Energy, August 18, 2014. Available from http://insideenergy.org/2014/08/18/data-explore-15-years-of-power-outages/.

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Problem #2 – Duration of Weather-Related Loss of Grid (WRLOG)

A. Is the Poisson model a good model for the duration of WRLOG? Why or why not?

B. Assume WRLOG duration can be characterized by a lognormal distribution with parameters μ_{WR} and σ_{WR}.

   i. How would you develop a prior distribution for μ_{WR} and σ_{WR}?

   ii. Given the data for WRLOG recovery times, how would you update this prior distribution?

Date	Duration (hr)
2/8/2013	79
10/29/2012	28
10/29/2011	216
8/28/2011	0
1/12/2011	8
12/12/2008	239
6/10/2008	10
11/3/2007	12
4/14/2007	2
8/2/2006	6
8/20/2004	6

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Conjugate Likelihood-Prior Pairs

- Binomial Likelihood, Beta Prior
  - Likelihood Function

$$L(n, m|\phi) = \binom{m}{n} \phi^n (1-\phi)^{m-n}$$

  - Prior Distribution

$$\pi_0(\phi|a, b) = \frac{\Gamma(b+a)}{\Gamma(b)\Gamma(a)} \phi^{a-1}(1-\phi)^{b-1} \qquad E_0[\phi] = \frac{a}{a+b}$$

  - Posterior Distribution

$$\pi_1(\phi|a', b') = \frac{\Gamma(b'+a')}{\Gamma(b')\Gamma(a')} \phi^{a'-1}(1-\phi)^{b'-1} \qquad E_1[\phi] = \frac{a'}{a'+b'}$$

$$a' = a + n \qquad b' = b + (m-n)$$

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Conjugate Likelihood-Prior Pairs

- Poisson Likelihood, Gamma Prior
  - Likelihood Function

$$L(n,t|\lambda) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

  - Prior Distribution

$$\pi_0(\lambda|\alpha,\beta)\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda} \qquad E_0[\lambda] = \frac{\alpha}{\beta} \qquad SD_0[\lambda] = \frac{\sqrt{\alpha}}{\beta}$$

  - Posterior Distribution

$$\pi_1(\lambda|\alpha',\beta')\frac{\beta'^{\alpha'}}{\Gamma(\alpha')}\lambda^{\alpha'-1}e^{-\beta'\lambda} \qquad E_1[\lambda] = \frac{\alpha'}{\beta'} \qquad SD_1[\lambda] = \frac{\sqrt{\alpha'}}{\beta'}$$

Lognormal Distribution Characteristics

Characteristic	Formula
pdf	$\frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{1}{2}\left(\frac{ln x-\mu}{\sigma}\right)^2}$
Mean	$e^{\mu+\frac{1}{2}\sigma^2}$
Variance	$(E[X])^2 e^{\sigma^2-1}$
5th Percentile	$e^{\mu-1.6448\sigma}$
50th Percentile (median)	e^{μ}
95th percentile	$e^{\mu+1.6448\sigma}$
Most Likely (mode)	$e^{\mu-\sigma^2}$
Range Factor (95th/50th)	$e^{1.6448\sigma}$