



Characterizing Uncertainties

Workshop 2

Learning Objectives

- Practice quantifying aleatory uncertainty
- Practice identifying and characterizing sources of epistemic uncertainty
- Practice using probability to characterize epistemic uncertainty

Important:

- a) The workshop problems can be performed as group exercises.
- b) The purpose is to exercise the modeling thought process, not to get the “right answer.”
- c) Workshop 4 will address Bayesian quantification of uncertainties

Problem #1 – Aleatory Uncertainties

- A. If the average U.S. NPP CDF is $5 \times 10^{-5}/\text{yr}$, what is the probability of 0 core damage events in the U.S. in 20 years? In 50 years?
- B. What is the probability of at least 1 core damage event in 20 years?
- C. Same as B, but with an average CDF of $1 \times 10^{-4}/\text{yr}$.

Problem #2 – Aleatory and Epistemic Uncertainties

This problem deals with the annual maximum for daily snowfall (in inches), an aleatory variable, whose characteristics are unknown.

A. Self-rating

- i. How many years have you lived in the Boston area?
- ii. Do you pay attention to snowfall data?

Problem #2 (cont.)

- B. Define X_n as the maximum daily snowfall for Boston in year n (in inches). Thus, for example, X_{2017} is the maximum daily snowfall observed in 2017.
- For the period 1982-2017, what are your best guesses for the mean, minimum, and maximum values of X_n (i.e., $E[X_n]$, $\min[X_n]$, and $\max[X_n]$)?
 - Pick three plausible values for $E[X_n]$ and assign probabilities to these values. What is the mean value of your distribution for $E[X_n]$? How does this compare with your best guess?
 - Adjust your best guesses, your probability distribution, or both as you see fit (but keep track of the original values).

Boston	Best Guess				Distribution			
	Min[X_n]	$E[X_n]$	Max[X_n]		i=1	i=2	i=3	Mean
Original				$E[X_n]_i$				
				$P\{E[X_n]_i\}$				
Revised				$E[X_n]_i$				
				$P\{E[X_n]_i\}$				

Problem #2 (cont.)

- C. Define Y_n as the maximum daily snowfall for Worcester in year n .
- How do you think Y_n compares with X_n ? What are the reasons for your answer? (If you are using a mental model, what is that model and what are some key assumptions?) How confident are you in that model?
 - For the period 1982-1994 and 2003-2017 (there are no data for the middle years), what is are your best guesses for $\min[Z_n]$, $E[Z_n]$, and $\max[Z_n]$ where $Z_n = Y_n - X_n$?

Worcester - Boston	Best Guess		
	Min[Z_n]	E[Z_n]	Max[Z_n]
Original			

Problem #2 (cont.)

- D. Assume X_n and Z_n are independent and are lognormally distributed (general properties given below).
- i. What are the uncertainties you need to consider in developing distributions for X_{2019} and Y_{2019} ?
 - ii. How would you compute the probability that Y_{2019} will be greater than 24”?

Lognormal Distribution Characteristics

Characteristic	Formula
pdf	$\frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}$
Mean	$e^{\mu + \frac{1}{2}\sigma^2}$
Variance	$(E[X])^2 e^{\sigma^2 - 1}$
5 th Percentile	$e^{\mu - 1.6448\sigma}$
50 th Percentile (median)	e^{μ}
95 th percentile	$e^{\mu + 1.6448\sigma}$
Most Likely (mode)	$e^{\mu - \sigma^2}$
Range Factor (95 th /50 th)	$e^{1.6448\sigma}$

Extra Problem – Epistemic and Aleatory Uncertainties

John (a real person) drives a 2005 Subaru Outback to work.

- A. In the following table, mark the box that best represents your personal knowledge of automobile reliability.

Know very little	Somewhat knowledgeable	Know a lot

- B. In the following table, (i) mark the value that is your best guess as to how many times John has started the car over its lifetime; and (ii) provide your probability for each value.

	Total Number of Starts				
	2,500	5,000	10,000	20,000	40,000
Best Guess (i)					
Probability (ii)					

Extra Problem (cont.)

- C. In the following table, (i) mark the value that best represents your guess as to how many times the car has failed to start; and (ii) provide your probability for each value.

	Total Number of Failures				
	0	2	4	8	16
Best Guess (i)					
Probability (ii)					

- D. Using the results of (B) and (C), what is (i) your best guess for the probability of failing to start, and (ii) your average value? Are you comfortable with these results?
- E. What is your probability that John's car fails to start tomorrow?

Extra Problem (cont.)

F. Class exercise.

- i. What is the class distribution of failure rates based on best guesses (i.e., the answers to D.i)?
- ii. What is the class average value based on the answer to D.i?
- iii. What is the class distribution of failure rates based on average values (i.e., the answers to D.ii)?
- iv. What is the class average value based on the answer to D.ii?
- v. How do the above compare with the actual best estimate (to be supplied)?