Probabilistic Modeling for NPP PRA

Lecture 3-1

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Schedule

	Wednesday 1/16	Thursday 1/17	Friday 1/18	Tuesday 1/22	Wednesday 1/23
Module	***1: Introduction***	***3: Characterizing Uncertainty***	***5: Basic Events***	***7: Learning from Operational Events***	***9: The PRA Frontier***
9:00-9:45	L1-1: What is RIDM?	L3-1: Probabilistic modeling for NPP PRA	L5-1: Evidence and estimation	L7-1: Retrospective PRA	L9-1: Challenges for NPP PRA
9:45-10:00	**Break**	**Break**	**Break**	**Break**	**Break**
10:00-11:00	L1-2: RIDM in the nuclear industry	L3-2: Uncertainty and uncertainties	L5-2: Human Reliability Analysis (HRA)	L7-2: Notable events and lessons for PRA	L9-2: Improved PRA using existing technology
11:00-12:00	W1: Risk-informed thinking	W2: Characterizing uncertainties	W4: Bayesian estimation	W6: Retrospective Analysis	L9-3: The frontier: grand challenges and advanced methods
12:00-1:30	**Lunch**	**Lunch**	**Lunch**	**Lunch**	**Lunch**
Module	***2: PRA Overview***	***4: Accident Sequence Modeling***	***6: Special Technical Topics***	***8: Applications and Challenges***	***10: Recap***
1:30-2:15	L2-1: NPP PRA and RIDM: early history	L4-1: Initiating events	L6-1: Dependent failures	L8-1: Risk-informed regulatory applications L8-2: PRA and RIDM infrastructure	L10-1: Summary and closing remarks
2:15-2:30	**Break**	**Break**	**Break**	**Break**	
2:30-3:30	L2-2: NPP PRA models and results	L4-2: Modeling plant and system response	L6-2: Spatial hazards and dependencies	L8-3: Risk-informed fire protection	Discussion: course feedback
3:30-4:30	L2-3: PRA and RIDM: point-counterpoint	W3: Plant systems modeling	L6-3: Other operational modes L6-4: Level 2/3 PRA: beyond core damage	L8-4: Risk communication	Open Discussion
4:30-4:45	**Break**	**Break**	**Break**	**Break**	
4:45-5:30	Open Discussion	W3: Plant systems modeling (cont.)	W5: External Hazards modeling	Open Discussion	
5:30-6:00		Open Discussion	Open Discussion		

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Key Topics

- Characteristics of basic stochastic models used in NPP PRA: Poisson and Bernoulli processes
- Distribution functions and expected values: concepts and notation
- Combinations of random variables
  - Useful results
  - Underlying theory

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Resources

- A. Papoulis, Probability, *Random Variables, and Stochastic Processes*, McGraw-Hill, New York, 1965.
- R.L. Winkler and W.L Hays, *Statistics: Probability, Inference and Decision, Second Edition*, Holt, Rinehart and Winston, New York, 1975.
- G. Apostolakis, “The concept of probability in safety assessments of technological systems,” *Science*, **250**, 1359–1364, 1990.

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

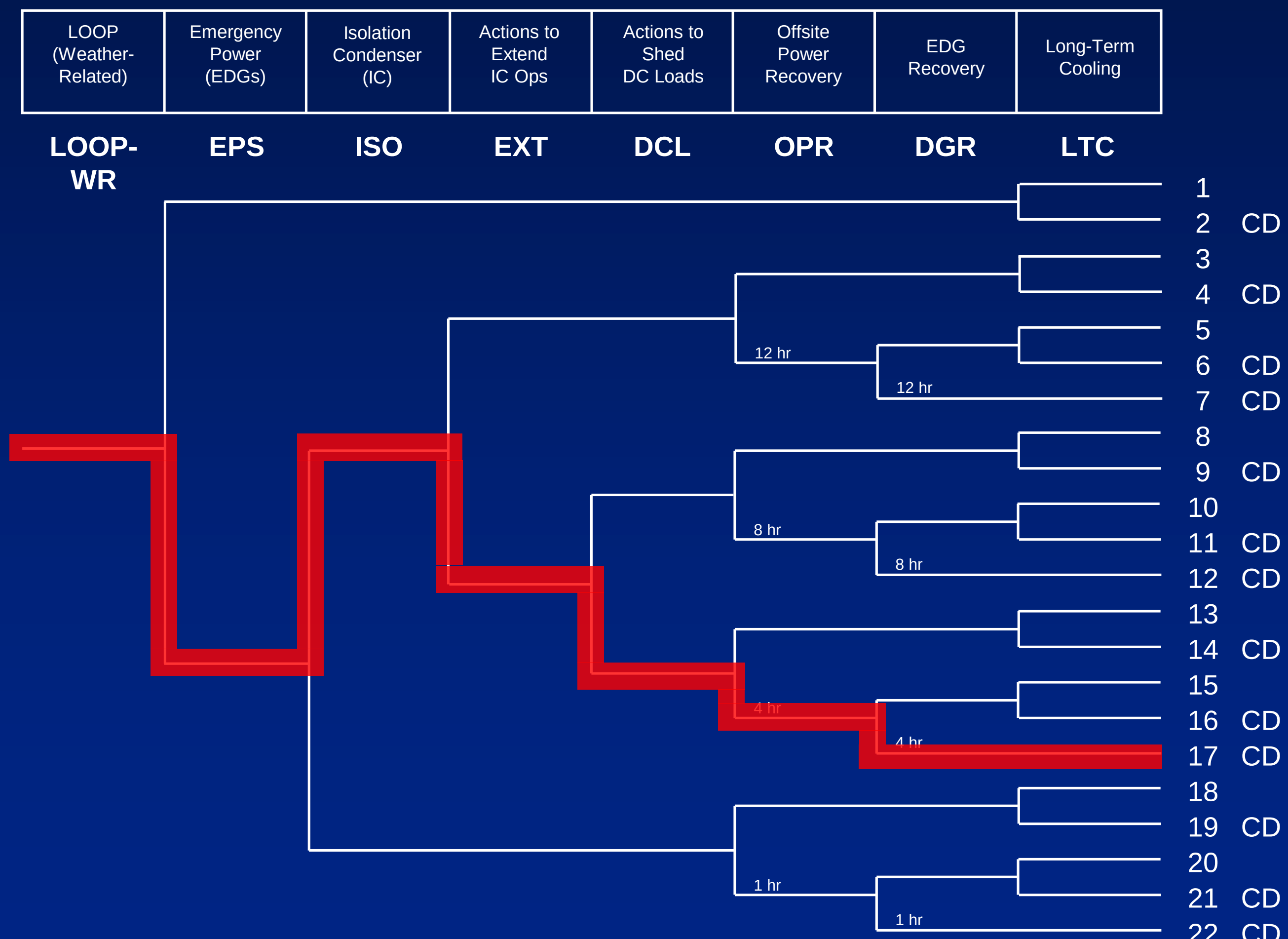
Other References

- N.D. Singpurwalla, *Reliability and Risk: A Bayesian Perspective*, Wiley, Chichester, 2006.
- A.E. Green and A.J. Bourne, *Reliability Technology*, Wiley-Interscience, London, 1972.

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

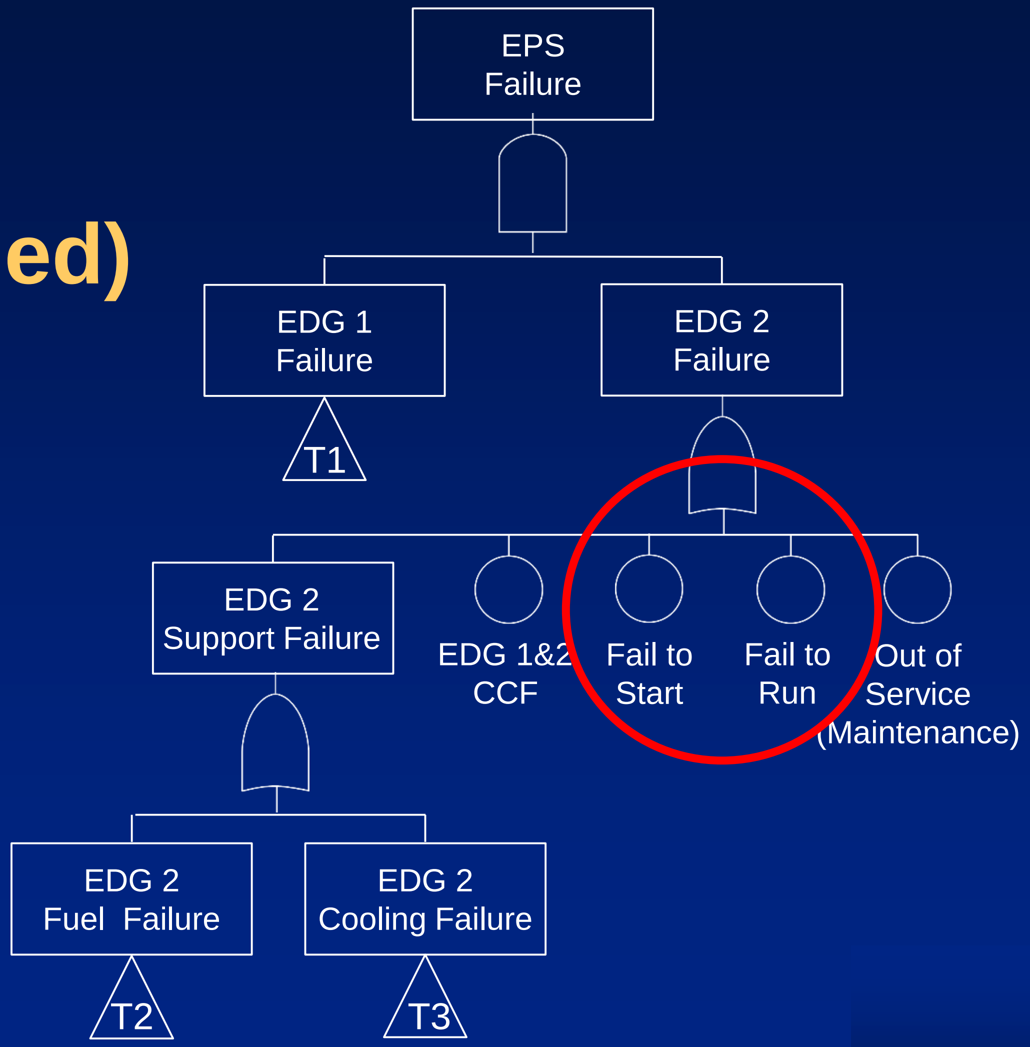
$$\text{Risk} \equiv \{s_i, C_i, p_i\}$$

Sequences



U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

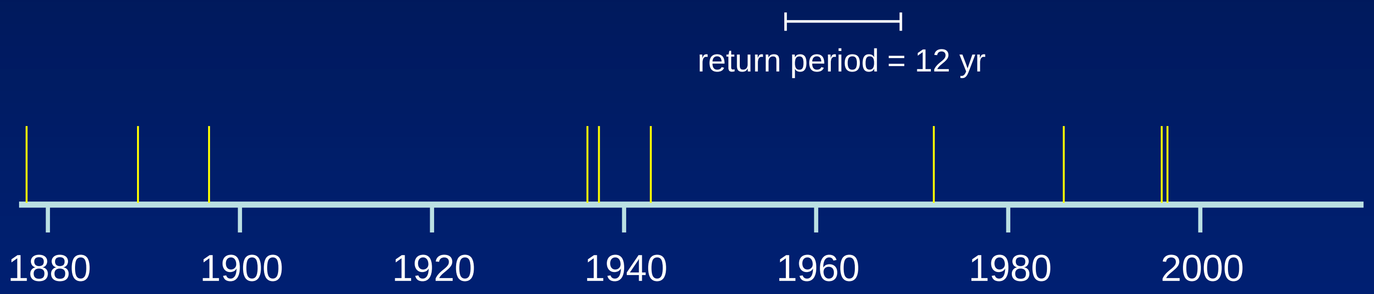
NPP Emergency Power System Example (simplified)



Modeling “Random Processes”

Ran·dom (*adj.*): “occurring without definite aim, purpose, or reason”

- Example: flooding events at Harpers Ferry, WV



- For long-term planning purposes, it’s useful to treat the flood generating process as a random process. (Of course, as a major storm approaches, behavior is much less random.)
- Even for processes involving definite aim/purpose/reason (e.g., operator actions), important contextual features can be viewed as random, resulting in an overall random process.

Two Fundamental Stochastic Process Models in NPP PRA

- Poisson
  - Used for events occurring over time
  - Example PRA uses
    - Initiating events
    - Failures during operation
- Bernoulli ("coin flip")
  - Used for events occurring on demand
  - Example PRA uses
    - Failures to start
    - Failures to change position

- Sto·chas·tic (*adj.*): "pertaining to process involving a randomly determined sequence of observations"
- Other stochastic processes of interest to NPP PRA
  - Infant mortality and aging processes ("bathtub curve")
  - Extreme values
  - Gaussian (sums of large numbers of random variables)

Poisson Process - Assumptions

- For non-overlapping time intervals, the (random) number of events in each interval are independent ("independent increments"). Example: N_1 is independent of N_2.

- The probability of events in an interval depends only on the length of the interval ("stationary process")

$$P\{N = n \; in \; (t_1, t_2)\} = P\{N = n \; in \; |t_2 - t_1|\}$$

- The probability of an event in an increment Δt is proportional to Δt

$$P\{N = 1 \; in \; \Delta t\} = \lambda \Delta t + o(\Delta t) \; where \; \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$$

- The probability of more than one event in Δt goes to zero as Δt goes to zero

$$P\{N > 1 \; in \; \Delta t\} = o(\Delta t)$$

Proportionality constant "frequency"

Notation

- Random variables are denoted with capital letters

  N = no. events

  T = occurrence time

- Specific values are denoted by lower case letters

Poisson Process - Distributions

Notation

function type

value

$$p_N(n/\underline{C})$$

random variable

condition

- **Poisson probability distribution for <u>number of events</u> in a fixed time interval**:
  - Probability mass function

$$\boxed{p_N(n|t,\lambda)} \equiv P\{N = n \text{ in } (0,t)|\lambda\} = \boxed{\frac{(\lambda t)^n e^{-\lambda t}}{n!}}$$

  - Cumulative distribution function

$$P_N(n|t,\lambda) \equiv P\{N \leq n \text{ in } (0,t)|\lambda\} = \sum_{i=0}^{n} p_N(i|t,\lambda) = \sum_{i=0}^{n} \frac{(\lambda t)^i e^{-\lambda t}}{i!}$$

  - Complementary cumulative distribution function

$$\bar{P}_N(n|t,\lambda) \equiv P\{N > n \text{ in } (0,t)|\lambda\} = \sum_{i=n+1}^{\infty} p_N(i|t,\lambda) = \sum_{i=n+1}^{\infty} \frac{(\lambda t)^i e^{-\lambda t}}{i!}$$

Poisson Process - Distributions

Notation

Expectation operator

$E[N/\underline{C}]$

random variable

condition

- **Poisson probability distribution for <u>number of events</u> in a fixed time interval**:
  - Mean (aka "average," "expected value") and variance

$$\boxed{E[N|t,\lambda]} \equiv \sum_{i=0}^{\infty} i \cdot p_N(i|t,\lambda) = \boxed{\lambda t}$$

$$Var[N|t,\lambda] \equiv E[(N - E[N|t,\lambda])^2] = \sum_{i=0}^{\infty} (i - E[N|t,\lambda])^2 \cdot p_N(i|t,\lambda) = \lambda t$$

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Poisson Process - Distributions

- **Exponential probability distribution for <u>event occurrence time</u>:**
  - Probability density function

$$\boxed{f_T(t|\lambda)} \equiv \lim_{\Delta t \to 0} \frac{P\{t \le T < t + \Delta t\}}{\Delta t} = \boxed{\lambda e^{-\lambda t}}$$

  - Cumulative distribution function

$$\boxed{F_T(t|\lambda)} \equiv P\{T \le t|\lambda\} = \int_0^t f_T(t'|\lambda)dt' = \boxed{1 - e^{-\lambda t}}$$

  - Complementary cumulative distribution function

$$\bar{F}_T(t|\lambda) \equiv P\{T > t|\lambda\} = \int_t^\infty f_T(t'|\lambda)dt' = e^{-\lambda t}$$

Poisson Process - Distributions

- **Exponential probability distribution for event occurrence time:**
  - Mean (aka "average," "expected value") and variance

$$\boxed{E[T|\lambda]} \equiv \int_0^{\infty} t' \cdot f_T(t'|\lambda)dt' = \boxed{\frac{1}{\lambda}}$$

$$Var[T|\lambda] \equiv E[(T - E[T|\lambda])^2] = \int_0^{\infty} (T - E[T|\lambda])^2 \cdot f_T(t'|\lambda)dt' = \frac{1}{\lambda^2}$$

  - Percentiles (value of T for which the cumulative probability equals a specified value). Example (95th percentile):

$$0.95 = \int_0^{T_{0.95}} f_T(t'|\lambda)dt' = 1 - e^{-\lambda t} \Longrightarrow T_{0.95} = -\frac{1}{\lambda} ln(1 - 0.95)$$

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Poisson Process – Notes

- The model parameter λ has units of inverse time and is called "frequency." This does not imply regular occurrence.
- The mean value of T (i.e., 1/ λ) is often called the "return period." Again, this does not imply regularity.
- If $\lambda t < 0.1$, $F_T(t|\lambda) \approx \lambda t$ (rare event approximation)
- Poisson process is memoryless – the conditional probability of an event in the interval $(t + \Delta t)$, given the system state at time t, is independent of past history (i.e., how the system arrived at its current state).
- Characteristic time trace: clusters of events with intervening large gaps. (See earlier flooding example.)

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Expected Values – Note on Additivity

- Consider the joint density function for random variables X and Y:

$$f_{X,Y}(x,y) \equiv \lim_{\Delta x \to 0, \Delta y \to 0} \frac{P\{x \leq X < x + \Delta x \ AND \ y \leq Y < y + \Delta y\}}{\Delta x \Delta y}$$

- The expected value of X + Y is the sum of the expected values for X and Y, regardless of the uncertainty in X and Y, and regardless of the dependence between X and Y.

$$E[X+Y] \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x+y) f_{X,Y}(x,y)dxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot f_{X,Y}(x,y)dxdy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \cdot f_{X,Y}(x,y)dxdy$$

$$= E[X] + E[Y]$$

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Knowledge Check

- Probability mass/density functions

$$\sum_{i=0}^{\infty} p_N(i|t,\lambda) = ? \qquad \int_0^{\infty} f_T(t'|\lambda)dt' = ?$$

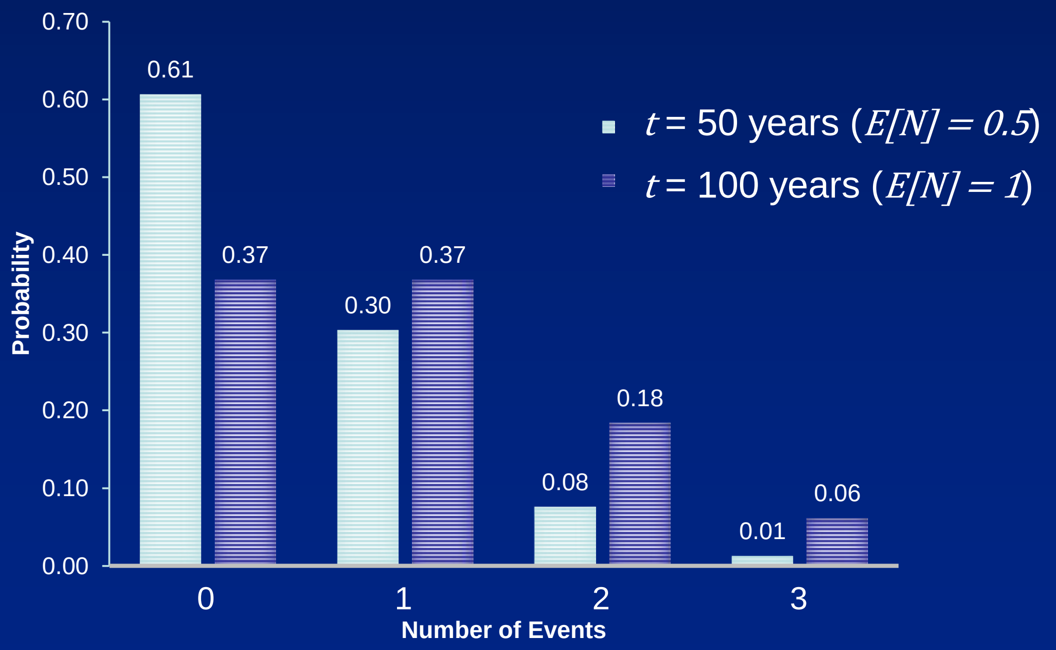
- Cumulative distribution functions

$$P_N(\infty|t,\lambda) = ? \qquad F_T(\infty|\lambda) = ?$$

- Mean (“Expected”) values
  - If $\lambda = 10^{-9}$/yr, average occurrence time for event = ?
  - Mean value of a coin flip?

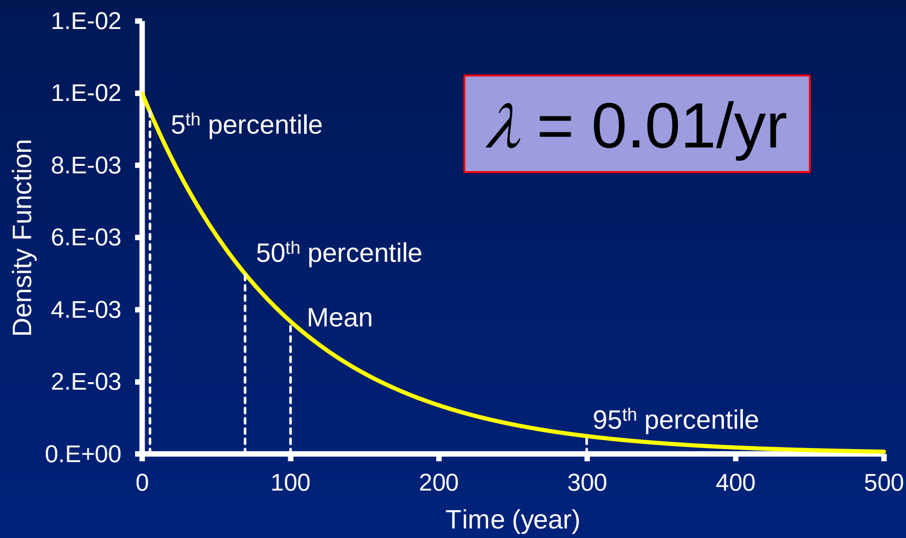
Poisson Process – Example (cont.)

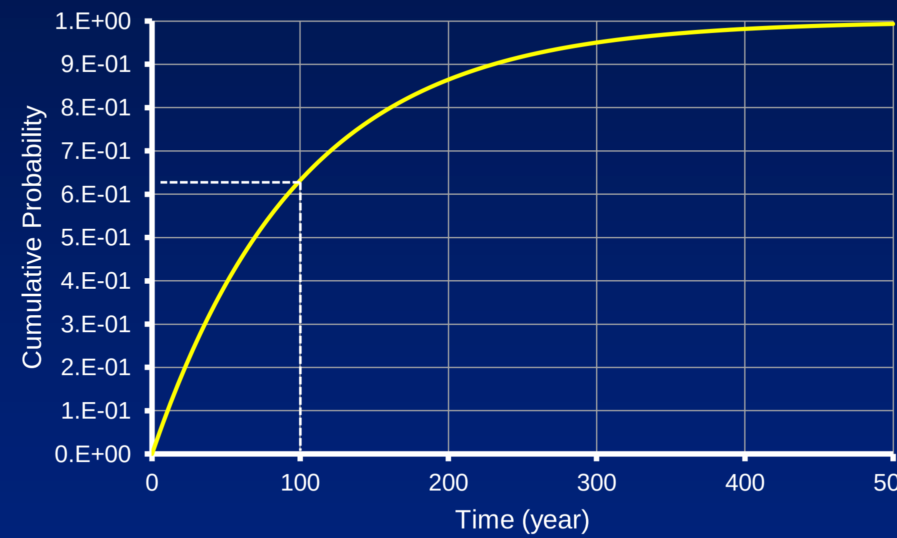
$\lambda = 0.01/\text{yr}$



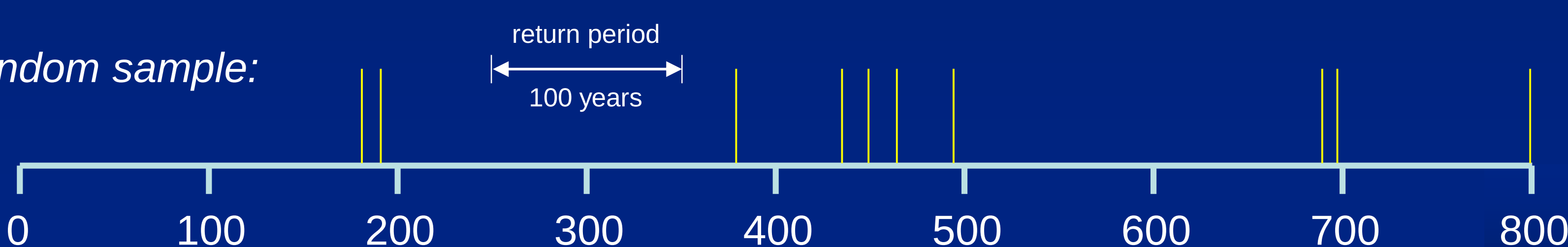
U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Poisson Process - Example





A random sample:



U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Bernoulli Process

- Independent, identical trials => memoryless "coin flip" process
- Binomial probability distribution

$$\boxed{p_N(n|m,\phi)} \equiv P\{N = n \; in \; m \; trials|\phi\} = \boxed{\binom{m}{n}\phi^n(1-\phi)^{m-n}}$$

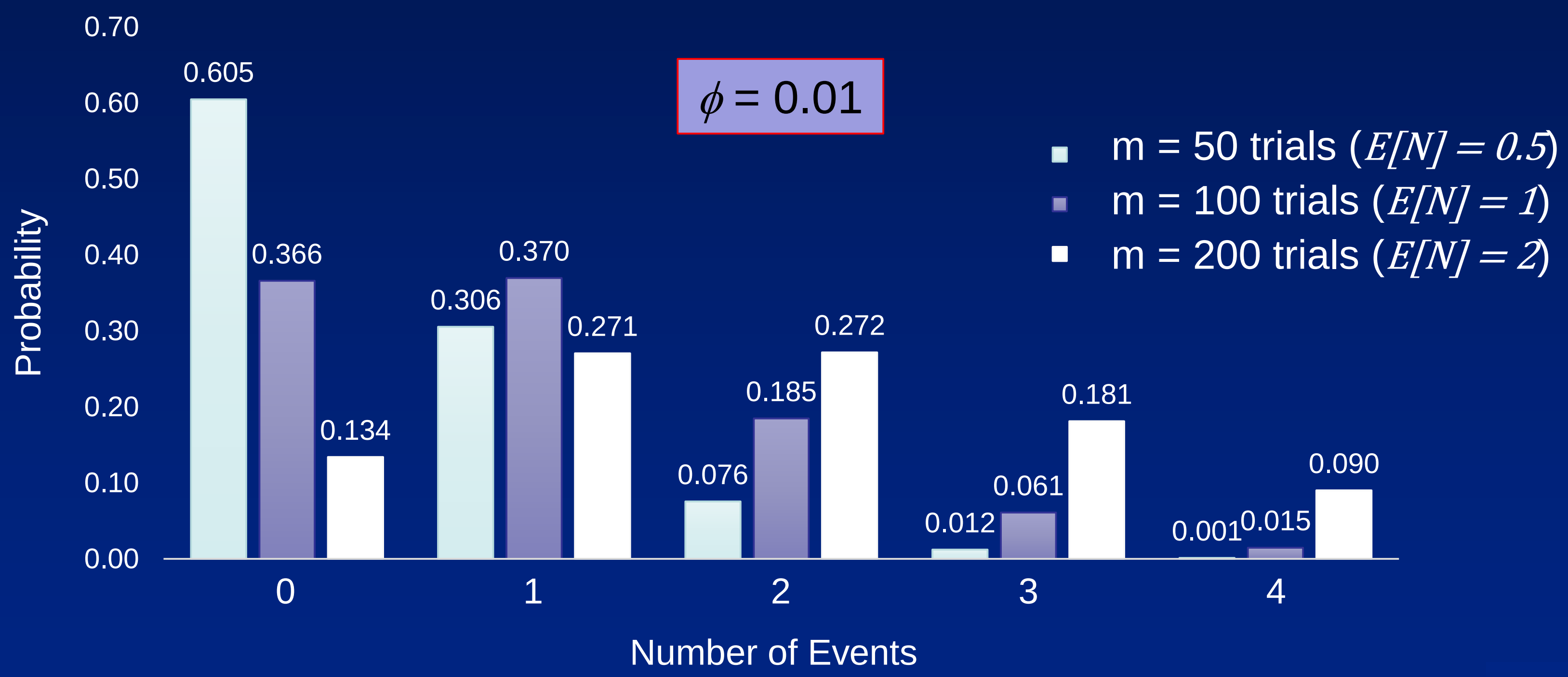
where $\binom{m}{n} \equiv \dfrac{m!}{n!\,(m-n)!}$

- Moments

$$\boxed{E[N|m,\phi]} \equiv \sum_{i=0}^{\infty} i \cdot p_N(i|m,\phi) = \boxed{m\phi}$$

$$Var[N|m,\phi] = \sum_{i=0}^{\infty}(i - E[N|m,\phi])^2 \cdot p_N(i|m,\phi) = m\phi(1-\phi)$$

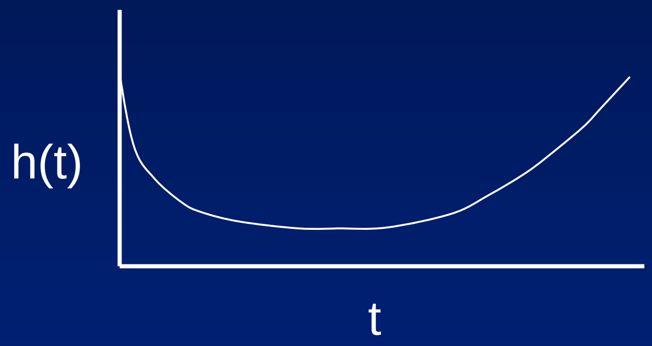
Bernoulli Process Example



U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Non-Stationary Processes

- Examples
  - Extreme weather
  - Passive component ageing
  - Recovery and repair
- Models
  - Parametric
    - Multi-parameter (2+ parameters)
    - empirical and/or derived
  - Simulation



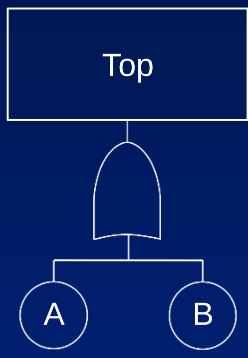
Combining Events – Simple Cases

- OR (U) Gate

$$P_{Top} = P_{A \cup B} = P_A + P_B - P_{A \cap B}$$

$P_{Top} = P_A + P_B$ if A and B are mutually exclusive

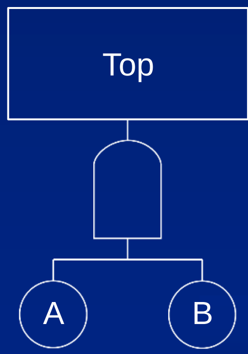
$P_{Top} \approx P_A + P_B$ if A and B are independent and if P_A and P_B are small



- AND (∩) Gate

$$P_{Top} = P_{A \cap B} = P_A \cdot P_{B|A}$$

$= P_A \cdot P_B$ if A and B are independent



Risk concern: situations where $P_{B|A} \gg P_B$

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

More Complex Situations: Functions of Random Variables

- PRA models can involve combinations of random variables. Expected values might behave intuitively, but full distributions might not.
- Example: an operator action requires the performance of two tasks in sequence. The time to perform each task is exponentially distributed with rate λ_i (i = 1, 2).
  - Mean time to perform overall action

$$E[T] = E[T_1] + E[T_2] = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

  - Probability density function of time to perform overall action

$$f_T(t|\lambda_1, \lambda_2) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{\lambda_2 t}\right)$$

- Probability calculus can be used to develop distributions for many situations

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Results for Three Situations of Interest

- Event tree sequences. The occurrence of a sequence, which involves a Poisson-distributed initiating event (frequency λ) followed by a string of subsequent Bernoulli events (probability ϕ_i), is a Poisson process with frequency $\lambda\phi_1\phi_2\ldots$
- Event tree end states. The occurrence of an end state (e.g., core damage) that can be reached by one or more event tree sequences is a Poisson process with frequency equal to the sum of the sequence frequencies.
- Time-reliability. If T_a is the (random) time available to perform required actions, T_n is the (random) time needed to perform these actions, and T_a and T_n are independent, the probability of failure is

$$P\{T_n > T_a\} = \int_0^{\infty} f_{T_n}(t) F_{T_a}(t)\,dt$$

Note: this is an example of a general stress-strength model where failure occurs when the "stress" exceeds the "strength."

U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Time-Reliability Derivation Sketch

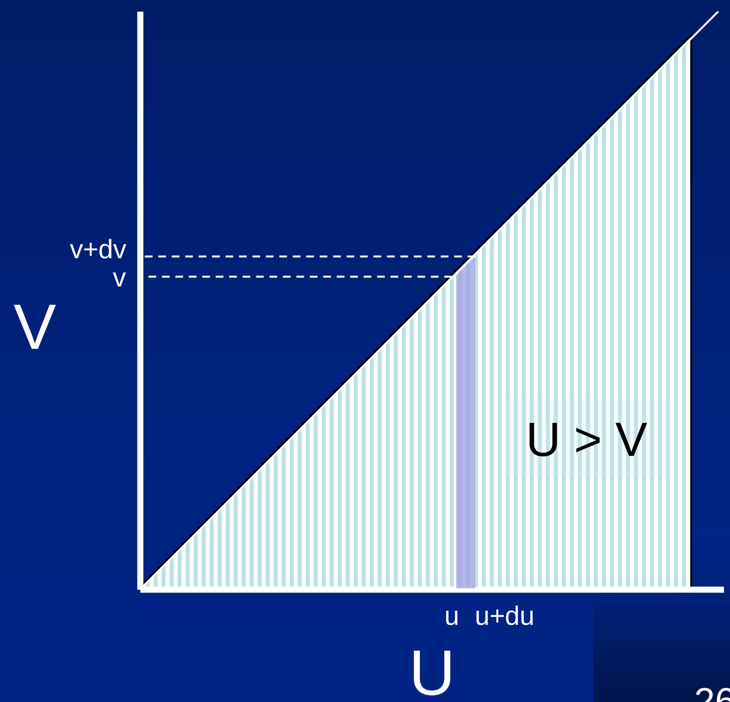
1) For notational simplicity, use U to represent T_n and V to represent T_a. Recall $f_X(\bullet)$ is the probability density function for random variable X and $F_X(\bullet)$ is the cumulative distribution function; $f_{X,Y}(\bullet, \bullet)$ is the joint density function for X and Y.
2) The probability of failure is the probability that U and V are in the shaded area.

$$P\{U > V\} = \int_0^\infty \int_0^u f_{U,V}(u', v')\, dv' du'$$

3) If U and V are independent,

$$f_{U,V}(u,v) = f_U(u) f_V(v)$$

$$\therefore P\{U > V\} = \int_0^\infty f_U(u') F_V(u') du'$$



U.S.NRC
United States Nuclear Regulatory Commission
Protecting People and the Environment

Closing Comments

- Mathematical details given in this lecture provide
  - Basis for parameter estimation procedure (Lecture 5-1)
  - Conditions where standard results break down (large λ, non-stationary processes)
  - Partial basis for confidence in PRA foundation
  - Basis for approaches to concerns (e.g., adding mean values when uncertainties are large)
- Additional details are provided in the background slides and are the subject of numerous texts on probability, statistics, stochastic processes, and reliability engineering

Probability math is essential the consistent application of logic throughout the analysis (If X then Y, given C), but is far from the entirety of NPP PRA.