Predicting Leakage through Composite Landfill Liners

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Abstract: Leakage through composite landfil liners having various characteristics was analyzed using existing analytical and numerical models developed for the study. Three-dimensional numerical models were used to analyze leakage through circular defects and two-dimensional numerical models were used to analyze leakage from defective seams. Leakage rates predicted with the numerical models were compared to leakage rates predicted using existing equations and analytical models currently being used. These comparisons show that existing equations and analytical models all have limitations and no universal equation or method is available for predicting leakage rates. To overcome some of the deficiencie in the existing equations and models, new equations were developed based on results from the numerical models. Recommendations are made for using the new equations, existing equations, and analytical models to predict leakage rates in thick composite liners having a geomembrane overlaying a compacted soil liner and thin composite liners having a geomembrane overlaying a geosynthetic clay liner.

INTRODUCTION

Composite liners consisting of a geomembrane overlaying a compacted soil liner or a geosynthetic clay liner (GCL), are used in a variety of applications including liquid containment and solid waste landfills Although the geomembrane component of a composite liner is nearly impervious to liquid flo , defects in the geomembrane occur, even with carefully controlled manufacture and installation (Laine et al. 1988; Giroud and Bonaparte 1989; Brennecke and Corser 1998; Rollin et al. 1999). Defects can range in size from pinholes having a diameter less than the thickness of the geomembrane to defective seams between geomembrane panels that are several meters long.

Several analytical and experimental studies have focused on predicting leakage rates through composite liners having defects in the geomembrane [e.g., Jayawickrama et al. (1988), Giroud et al. (1989, 1992, 1998), Walton and Sagar (1990), Walton et al. (1997), Rowe (1998), and Touze-Foltz et al. (2000)]. The most commonly used equations were presented by Giroud et al. (1989, 1992) based on the methodology developed in Giroud and Bonaparte (1989). These equations are empirical and were developed using a relatively small database of laboratory results reported by several investigators. Other equations have been developed based on theoretical principles [e.g., Walton and Sagar (1990), Walton et al. (1997), Rowe (1998), and Touze-Foltz et al. (2000)].

Although many equations have been proposed, no study to date has compared these equations or assessed their ability to represent the 3D flo regime in a composite liner. The objectives of this study were to (1) assess the efficac of equations available for predicting leakage rates in composite liners; and (2) make recommendations regarding use of existing equations. The assessment was made using 2D and 3D numerical models.

BACKGROUND

Flow through composite liners is believed to consist of three processes (Fig. 1): (1) flo through the defect in the geomembrane; (2) flo through an interfacial zone between the geomembrane and soil liner; and (3) flo through the soil liner. The rate of leakage and the breadth of flo in the soil liner depend on the ease with which flo can occur in the interfacial zone. All other factors being equal, a greater leakage rate and larger flo area occur when the interfacial zone is more permeable (Foose 1998; Rowe 1998). The interfacial zone is often described using qualitative terms, namely "perfect" contact (no interfacial zone), "excellent" fiel conditions (Giroud and Bonaparte 1989), "good" contact, and "poor" contact (Giroud 1997). However, little data are available for characterizing properties of the interfacial zone.

Leakage Rates for Perfect Contact

Walton and Sagar (1990) identifie Forchheimer's equation (Giroud et al. 1994) as an analytical solution for calculating leakage through a small circular defect Q_c in an infinitel thick composite liner having perfect contact between the geomembrane and soil liner. Forchheimer's equation is

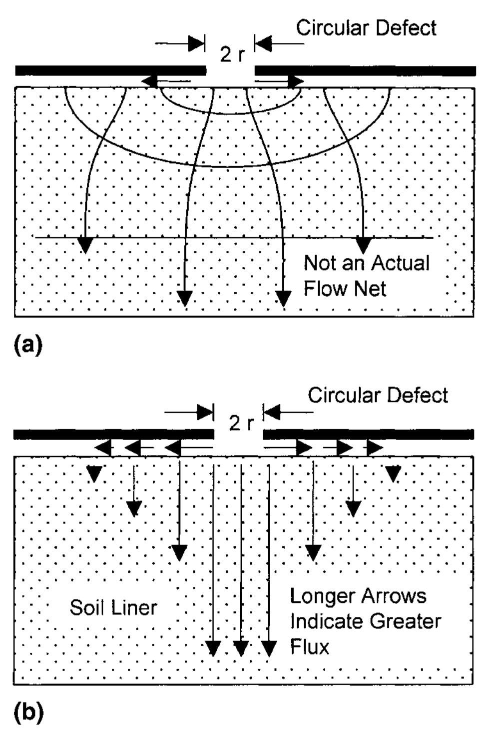

FIG. 1. (a) 3D Flow through Defects in Composite Liners; (b) Conceptualization of Flow through Defects in Composite Liners Used by Rowe (1998)

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$$Q_c = 4K_s h_t r \tag{1}$$

where K_s = saturated hydraulic conductivity of the soil liner; h_t = total head drop across the composite liner; and r = radius of the defect. Laboratory tests by Walton et al. (1997) and numerical evaluations by Walton and Sagar (1990); Walton et al. (1997), and Foose (1997) show that (1) provides accurate predictions of leakage rates provided the radius of defect is <1/75 of the thickness of the soil liner and the flaw are widely spaced.

The distribution of total head near a defect in a composite liner having a thick soil liner and perfect contact between the geomembrane and soil liner can also be obtained from the analogous heat conduction problem described by Carslaw and Jaeger (1959). The solution is (Foose 1997)

$$h_R(R_r, z) = \frac{2h_t}{\pi} \sin^{-1}\left\{\frac{2r}{[(R_r - r)^2 + z^2]^{1/2} + [(R_r + r)^2 + z^2]^{1/2}}\right\} \tag{2}$$

where h_R = total head at radial distance R_r from the axis of the defect; and z = vertical distance from the base of the defect.

An analysis conducted by Foose (1997) using (1) shows that the error in leakage rate is <1% when the radius of defect is <1/75 of the thickness of the soil liner and the flaw are widely spaced. The error in predicted leakage rate is <10% provided that the radius of defect is <1/8 the thickness of the soil liner. Hence, for composite liners having a 6 mm thick GCL and perfect contact, (1) can be used for defects is having a radius <0.75 mm with the error in predicted leakage rate remaining <10%. Giroud and Bonaparte (1989) recommended that defects having a radius of 5.6 mm (area = 1 cm^2) be used to size components of liner systems and defects having a radius of 1 mm (area = 3.1 mm^2) be used to evaluate the performance of liner systems. Consequently, leakage rates for GCL composite liners can be in error by as much as 40% when (1) is used to analyze the size of defects recommended by Giroud and Bonaparte (1989).

Walton et al. (1997) developed graphical and empirical solutions for leakage rate through a composite liner having perfect contact and circular defects in the geomembrane having a radius as large as 1/10 the thickness of the soil liner. For a 6 mm-thick GCL, the graphical and empirical solutions developed by Walton et al. (1997) are valid for defects having a radius <0.6 mm, but not for defects having the sizes recommended by Giroud and Bonaparte (1989). The graphical solution for defective seams or tears with perfect contact proposed by Walton and Sagar (1990) also is limited to defects having a width <1/10 the thickness of the soil liner.

Walton and Seitz (1992) described an equation for predicting flui flo through fractures in concrete vaults in intimate contact with soil, a situation analogous to leakage through composite liners having defective seams or tears and perfect contact. The solution is based on a Neumann boundary condition (specifie flux representing the defect (Yates 1988). The leakage rate per unit length of defective seam Q_l can be simplifie to

$$Q_l = K_s h_t \varepsilon \tag{3}$$

where ε is

$$\varepsilon = -\left\{\left[\sum_{n=1}^{\infty}\left(\frac{(8L_s)(p_{1j} + p_{1j}^2 p_{2j})}{w[\pi(2n-1)]^2[p_{1j}(1 + p_{2j}^2) + p_{2j}(1 + p_{1j}^2)]\cosh\left(\dfrac{w\pi(2n-1)}{4L_s}\right)}\right)\right] - \frac{L_s}{w}\right\}^{-1} \tag{4}$$

The variables p_{1j} and p_{2j} are define as

$$p_{1j} = \tanh\left[\frac{w\pi(2n-1)}{4L_s}\left(\frac{2R_o}{w} - 1\right)\right] \tag{5a}$$

$$p_{2j} = \tanh\left(\frac{w\pi(2n-1)}{4L_s}\right) \tag{5b}$$

where w = width of defect; R_o = distance between defects; and L_s = thickness of the soil liner.

Harr (1962) presented an analytical solution for a seepage problem analogous to flo through a composite liner having a defective seam or tear, perfect contact, and a Dirichlet boundary (specifie head) at the defect. The equation from Harr (1962) is

$$Q_l = \frac{2K_s h_t}{\Phi} \tag{6}$$

The dimensionless form factor Φ is define as

$$\Phi = \frac{K'}{K} \tag{7}$$

where K' and K = complete elliptic integrals of firs kind with a modulus of m and complementary modulus of m', respectively. The modulus is

$$m = \lambda \, \mathrm{sn}\left(\frac{w}{R_o}\Lambda, \lambda\right) \tag{8}$$

where Λ = complete elliptic integral of firs kind of modulus λ. The ratio between the elliptic integral of firs kind of modulus λ and the complete elliptical integral of firs kind of complementary modulus λ' is define as

$$\frac{\Lambda}{\Lambda'} = \frac{R_o}{2L_s} \tag{9}$$

where Λ' = complete elliptical integral of firs kind of complementary modulus λ'.

A table of elliptic integrals, an approximate function, or a numerical solution is used to determine λ and Λ from the ratio Λ/Λ'. The parameters λ and Λ are substituted in (8) to compute m. The dimensionless form factor Φ (which is K'/K) is determined from m using a table of elliptic integrals, an approximate function, or a numerical solution.

Leakage Rates for Imperfect Contact

Imperfect contact has often been used as a hypothesis to explain why measured leakage rates in the laboratory and fiel are often greater than those predicted using equations for perfect contact [e.g., Giroud and Bonaparte (1989), and Rowe (1998)]. Imperfect contact between the geomembrane and soil liner may exist due to the soil particles, rutting and undulations occurring during construction of the liner, and wrinkles in the geomembrane (Giroud and Bonaparte 1989; Rowe 1998).

The most commonly used equations for calculating the leakage rate in composite liners when imperfect contact exists were presented by Giroud et al. (1989, 1992). Giroud (1997) adapted these empirical equations to apply to a variety of cases. These equations are referred to herein as Giroud's (1997) equations. Giroud's (1997) equation for leakage through circular defects is

$$Q_{c,G} = \beta_c \left[1 + 0.1\left(\frac{h_w}{L_s}\right)^{0.95}\right] a^{0.1} h_w^{0.9} K_s^{0.74} \tag{10}$$

where h_w = depth of leachate above the geomembrane; and a = area of defect. The units for h_w, L_s, a, and K_s are m, m, m^2, and m/s, respectively. Giroud's (1997) equation for long defects (tears or defective seams) is

$$Q_{l,G} = \beta_l \left[1 + 0.2\left(\frac{h_w}{L_s}\right)^{0.95}\right] w^{0.1} h_w^{0.45} K_s^{0.87} \tag{11}$$

where w = width of defect in m.

The coefficient β_c and β_l in (10) and (11) are 0.21 and 0.52, respectively, for good contact, and 1.15 and 1.22, respectively, for poor contact. Good and poor contact are descriptive of fiel conditions. The limitations of (10) and (11) and definition of good and poor contact are discussed in Giroud (1997).

For circular defects in composite liners having good contact, the radius of wetting R_c is (Giroud et al. 1992)

$$R_c = 0.26 a^{0.05} h_w^{0.45} K_s^{-0.13} \tag{12}$$

For long defects in composite liners having good contact, the width of wetting W_l is (Giroud et al. 1992)

$$W_l = 0.26 w^{0.1} h_w^{0.45} K_s^{-0.13} \tag{13}$$

The coefficien 0.26 used in (12) and (13) for good contact is increased to 0.61 for poor contact.

Rowe (1998) developed an analytical equation for leakage through circular defects in composite liners based on the approach formulated by Jayawickrama et al. (1988). Multidimensional flo in the soil liner [Fig. 1(a)] is conceptualized as radial flo along the interface between the soil and geomembrane and then vertical flo in the soil liner [Fig. 1(b)]. This solution, herein referred to as Rowe's (1998) solution, is the only available analytical method in which the transmissivity of the interface between the geomembrane and soil liner is explicitly included and provides a direct method to evaluate how the interface between the geomembrane and soil liner affects the leakage rate. However, the assumptions regarding flo in the interface and soil liner have not been verifie numerically or experimentally.

For a composite liner having a total head of zero at the base of the liner, Rowe's (1998) solution is

$$Q_{c,R} = \pi K_s \left(r^2 \frac{h_t}{L_s} + 2\frac{h_t}{L_s}\Delta_1 + 2\frac{h_t}{L_s}\Delta_2 - \frac{2h_w}{L_s}\Delta_2\right) \tag{14}$$

where Δ_1 and Δ_2 = expressions involving Bessel functions (see Appendix), the radius of the defect r, and the wetted radius R_c. Transmissivity T of the interface is embedded in the method to obtain R as shown in the Appendix.

Rowe (1998) proposed another equation for calculating leakage through circular defects occurring at the crest of a wave or "wrinkle" in the geomembrane in which flo is controlled by the soil liner and the lateral spreading of flo is not restricted. Rowe's (1998) wrinkle equation is

$$Q_{w,R} = 2SK_s \left[\sqrt{\frac{B}{2} + \left(\frac{L_s T}{K_s}\right)} \frac{h_t}{L_s}\right] \tag{15}$$

where S = length of the wrinkle; B = width of the wrinkle; T = transmissivity of the interface; and other terms are as define previously. Eq. (15) is also based on the assumption that flo in the soil liner is 1D. Rowe (1998) and Touze-Foltz et al. (2000) also presented solutions for predicting leakage rates through circular defects that occur in wrinkles or waves in which resistance to flo through the defect is considered, as well as the effects of flo from defects in closely spaced wrinkles.

The absolute maximum leakage rate through circular defects $Q_{c,m}$ can be computed using Bernoulli's equation for free flo through an orific having sharp edges (Giroud and Bonaparte 1989)

$$Q_{c,\max} = 0.6a\sqrt{2gh_w} \tag{16}$$

where g = gravitational constant.

Interface Properties

Jayawickrama et al. (1988) applied Newton's viscosity law for flo between two smooth parallel plates to back-calculate the interface thickness between a geomembrane and soil liner using flo rates measured in a laboratory test of a composite liner. The calculated interface thickness ranged from 0.02 mm for a soil liner having a hydraulic conductivity of 1×10^{-7} cm/s to 0.15 mm for a soil liner having a hydraulic conductivity of 1×10^{-4} cm/s. The model used to back-calculate the thickness is based on the assumption that flo in the soil liner is unidirectional, as shown in Fig. 1(b).

Rowe (1998) used his model to back-calculate the transmissivity of the interface between the geomembrane and compacted soil liner necessary to yield leakage rates through defects comparable to those obtained from Giroud's (1997) equations. For a 60 cm thick composite liner having good contact, Rowe (1998) found that the transmissivity of the interface is approximately 1.6×10^{-8} m^2/s. For a 60 cm-thick composite liner having poor contact, Rowe (1998) found that transmissivity of the interface is approximately 1×10^{-7} m^2/s.

An important limitation of these results for interface properties is that they have not been measured directly. Values for transmissivity of the interface have been back-calculated using analytical models that are based on the assumption of unidirectional flo in the soil liner. This assumption has not been verifie experimentally, analytically, or numerically. No direct measurements of an interfacial gap or interfacial flo have been made in the laboratory or in the fiel for leakage through defects in composite liners having a compacted soil liner. Inferences regarding properties of the interface in composite liners have been made based solely on observed flo rates [e.g., Jayawickrama et al. (1988), Giroud and Bonaparte (1989), and Rowe (1998)]. Nevertheless, these are the best estimates of interface conditions that can be made presently.

Harpur et al. (1993) analyzed the interface between a geomembrane and a GCL where the bentonite was encased between two geotextiles. A GCL with the bentonite glued to a geomembrane was also tested, although an interface did not exist for this GCL. A radial transmissivity device was used in which flo passed through a circular defect in the geomembrane and along the interface between the GCL and geomembrane. Flow was collected along the perimeter of the specimen and did not pass through the soil liner as would be the case for flo in a composite liner. For the GCL with bentonite encased in geotextiles, the range for transmissivity of the interface was calculated to be 6×10^{-12} m^2/s to 2×10^{-10} m^2/s.

LEAKAGE MODEL

The equations available for calculating leakage rates for composite liners have been obtained empirically from a limited number of laboratory tests or are analytical solutions based on assumptions such as perfect contact, thick soil liners, or unidirectional flo in the soil liner. The best method to assess the validity of these equations would be to use data from carefully controlled experiments. The experience of Brown et al. (1987) as well as attempts made by the writers have demonstrated that laboratory experiments of this type are often plagued by

difficultie caused by the boundaries of the testing device and consolidation water from the soil liner. These difficultie confound the observed flo rates and the resulting errors are as large or larger than the leakage rates attempting to be measured. Walton et al. (1997) was able to overcome some of these difficultie by using sand as a soil liner for the purpose of verifying (1). However, the surface characteristics of sands are very different from those for clay liners.

Field data could also be used for comparison, but they are plagued by inadequate knowledge of the size, shape, and number of defects in the geomembrane, unknown hydraulic conductivity of the soil liner or GCL, unknown depth of leachate on the liner, and confounding effects such as consolidation water. In lieu of experimental or fiel data, the next best approach is to use an established and verifie multidimensional numerical model, based on minimal assumptions, that can be used to represent conditions likely to exist in a composite liner as accurately as possible. This latter approach was used in this study.

3D and 2D numerical models were developed to analyze leakage through defects in composite liners having perfect and imperfect contact between the geomembrane and soil liner. Results obtained with these models were then compared to the leakage rates computed using the equations by Harr (1962), Walton and Seitz (1992), Giroud (1997), and Rowe (1998).

The finite-di ference flo model MODFLOW (McDonald and Harbaugh 1988) was used to develop a model for leakage through composite liners. MODFLOW was used because it is a well recognized and verifie 3D model and there are several preprocessors available to build models. MODFLOW solves the governing equation for 3D flo of water through porous media

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h_t}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h_t}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h_t}{\partial z}\right) - W = S_s\frac{\partial h_t}{\partial t} \tag{17}$$

where K_{xx}, K_{yy}, and K_{zz} = hydraulic conductivity along the x-, y-, and z-axes parallel to the major axes of hydraulic conductivity; W = flo rate of sinks and sources; S_s = specifi storage; and t = time. For this study, (17) was solved for the steady-state condition using the strongly implicit solver in MODFLOW. Mass balance errors for all simulations were <1% (Foose 1997).

Leakage through circular defects was modeled as a radially symmetric system using only one quadrant. The area around a circular defect was modeled as a block of soil with the axes of the defect lying on the edge of the cube (Fig. 2). A layer of no-flo cells was used to simulate the geomembrane and constant head cells were used to simulate a defect in the geomembrane. For the conditions modeled, head loss that occurs in the layer above the geomembrane or through the defect was found to be negligible because the soil liner has much lower hydraulic conductivity than the layer generally placed above the geomembrane.

The defect was modeled using rectangular cells. Circular defects were approximated using a fin mesh of rectangular cells. The bottom boundary was modeled as a freely draining boundary having a constant head of zero. The soil liner was assumed to be saturated, homogeneous, and isotropic.

Composite liners having defective seams and tears in the geomembrane were modeled in two dimensions with the dimension in the y-direction having unit length. To determine the total quantity of flo from a defective seam, results from a circular defect can be added to the solution for the defective seam to account for flo from the ends (Foose 1997).

The interface between the geomembrane and soil liner was modeled as a thin layer (up to 0.030 mm-thick) having a transmissivity and thickness representative of the interface contact ranging from perfect contact (no interface layer) to varying degrees of imperfect contact. Thickness of the interface t_i was calculated using Newton's viscosity law for flo between two smooth parallel plates

$$t_i = \left(\frac{12\eta T}{\rho g}\right)^{1/3} \tag{18}$$

where η = kinematic viscosity of water; T = transmissivity of the interface; and ρ = density of water (Giroud and Bonaparte 1989). The transmissivity is

$$T = K_i t_i \tag{19}$$

where K_i = hydraulic conductivity of the interface. The transmissivity in terms of the hydraulic conductivity of the interface can be solved for by cubing both sides of (18), substituting (19) into the resulting equation, solving for t, multiplying both sides of the resulting equation by K_i, and substituting in (19), which leads to

$$T = K_i^{3/2}\sqrt{\frac{12\eta}{\rho g}} \tag{20}$$

Near the defect the grid spacing was 0.001 mm in all directions. The grid was expanded by a factor of 1.1 for distances within 1 mm of the center of the defect, 1.2 for distances 1–5 mm from the center of the defect, and 1.4 at distances >5 mm from the center of the defect. The largest model had 518,400 finite-di ference cells. An exceptionally small grid spacing was required near the defect because of the

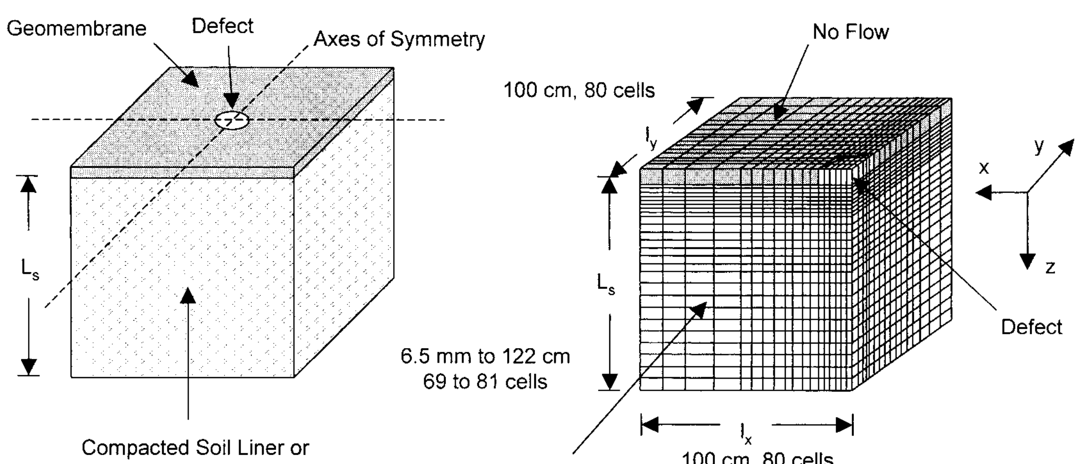

FIG. 2. Conceptual Model for Flow through Defects in Composite Liners

high rate of change of the gradient near the defect (Foose et al. 1998).

ANALYSIS OF LEAKAGE RATE PREDICTIONS

Two composite liners were analyzed to assess leakage through defects in composite liners: (1) a 6.5 mm thick GCL overlain with a geomembrane (a popular alternative liner system); and (2) a 61 cm thick compacted clay liner overlain with a geomembrane (the liner prescribed in Subtitle D of the Resource Conservation and Recovery Act). The hydraulic conductivity of the soil liner in the Subtitle D liner was assumed to be 1×10^{-7} cm/s, which is the commonly regulatory standard for compacted soil liners. The hydraulic conductivity of the GCL was assumed to be 1×10^{-9} cm/s, which is representative of values reported in the literature for water [e.g., Ruhl and Daniel (1997)]. However, the hydraulic conductivity of the GCL was varied up to 2×10^{-6} cm/s corresponding to the scenario in which the hydraulic conductivity of the GCL is increased as a result of chemical interactions with leachate [e.g., Petrov and Rowe (1997) and Quaranta et al. (1997)]. The depth of leachate was assumed to be 30 cm, which is the common regulatory standard.

Leakage through Circular Defects—Perfect Contact

Leakage rates for the two liners analyzed with perfect contact between the geomembrane and soil liner are shown in Fig. 3, along with leakage rates predicted using Forchheimer's equation for circular defects of various sizes [(1)]. For the Subtitle D liner, results from the 3D finite-di ference model replicate results from (1), which serves to verify the numerical model. This also indicates that the discretization error resulting from approximating a circular defect using small rectangular cells is small. As shown in Fig. 3(b), total heads in the Subtitle D liner predicted using the numerical model and total heads determined from the analytical equation [(2)] also compare favorably.

The leakage rate from the GCL composite liner ($K_s = 1 \times 10^{-9}$ cm/s) is about two orders of magnitude less than that for the Subtitle D liner. Eq. 1 tends to underpredict leakage from the GCL composite liner because (1) is based on the assumption of a semiinfinit permeable medium, which does not correspond to a thin GCL. For small defects (radius ~1 mm), (1) underestimates the leakage rate by 11%. For larger defects (radius ~6 mm), (1) underestimates the leakage rate by 44%.

Leakage through Defective Seams or Tears—Perfect Contact

Leakage rates per unit length of defect for a GCL composite liner and a Subtitle D liner are shown in Fig. 4. For both liners, the geomembrane and soil liner are in perfect contact. The leakage rates shown in Fig. 4 do not include flo from the ends of the defect. For a leachate depth of 30 cm, the leakage

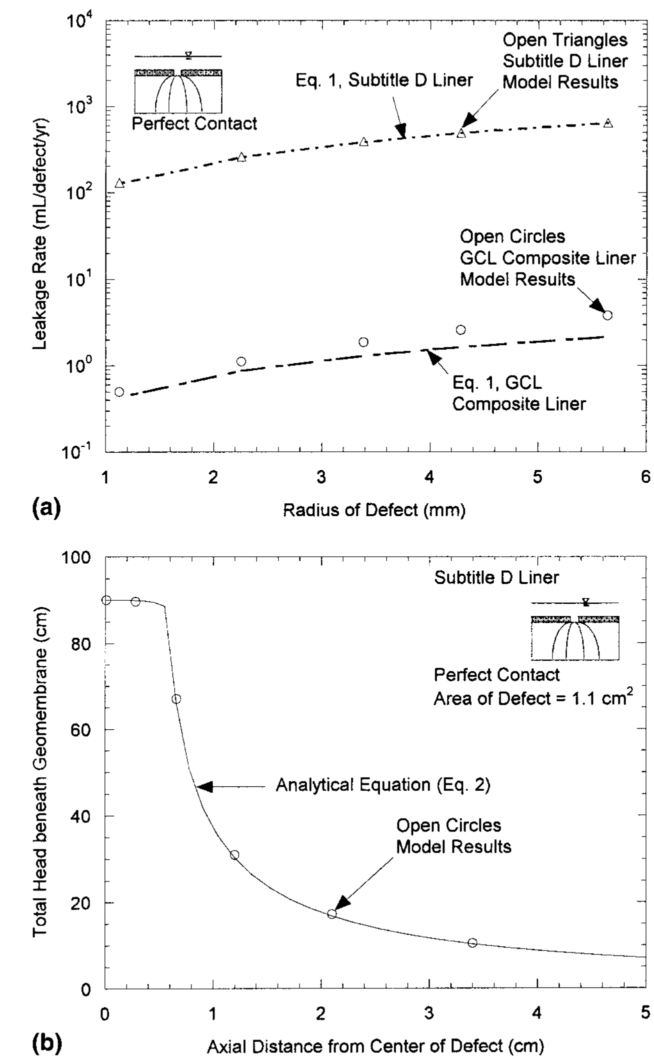

FIG. 3. (a) Leakage Rates through Circular Defects in Composite Liners Having Perfect Contact; (b) Total Head versus Axial Distance from Center of Defect for Subtitle D Liner

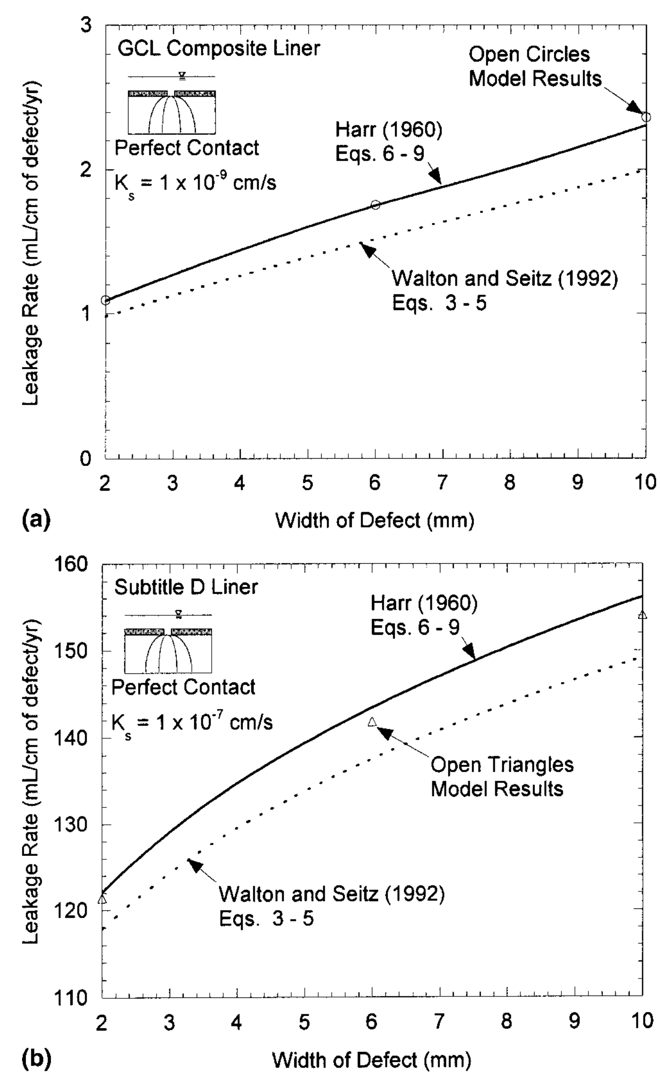

FIG. 4. Leakage Rates through Defective Seams in Composite Liners Having Perfect Contact: (a) GCL Composite Liner; (b) Subtitle D Liner

rate for the GCL composite liner having a hydraulic conductivity of 1×10^{-9} cm/s is two orders of magnitude less than that for the Subtitle D liner. The ratio of leakage rates equals the ratio of the hydraulic conductivity of the soil liner and the GCL.

Also shown in Fig. 4 are leakage rates predicted using the solution by Walton and Seitz (1992) [(3)–(5)] and the analytical solution from Harr (1962) [(6)–(9)]. Leakage rates from the analytical solution by Walton and Seitz (1992), which uses a specifie flu boundary condition for the defect, are lower than those from the numerical model for both liners. The numerical model is expected to be more realistic because a specifie head boundary condition for the defect is likely to be representative of fiel conditions.

Leakage rates from the solution in Harr (1962) are close (within 2%) to those obtained with the numerical model. Asymptotic expansion series were used to compute values of λ and Λ from the ratio Λ/Λ' because values for the modulus m that were used were near the asymptotes of tabulated values (Abramowitz and Stegun 1972). The discrepancy in the calculated leakage rates is due to truncation errors in the asymptotic expansion series used in computing the elliptic integrals.

Leakage rates through defective seams can be much greater than that through a circular defect in the geomembrane panel. For a defect 1 m long and 2 mm wide in a Subtitle D liner, the leakage rate is approximately 12,000 mL/year. In contrast, the leakage rate for a single circular defect having a diameter of 2 mm is 115 mL/year. Thus, leakage from a small portion of defective seam or tear can far outweigh leakage from numerous widely spaced small defects. Given that most leaks occur in fiel seams (Darilek et al. 1989; Rollin et al. 1999), construction quality-assurance programs should focus on the quality of the seams and estimates of leakage rates made during design should consider the likelihood of seam failures.

Leakage through Circular Defects—Imperfect Contact

Leakage rates for composite liners with imperfect contact and circular defects having an area of 1 cm^2 were analyzed with the numerical model. For the GCL composite liner, transmissivity of the interface between the geomembrane and GCL was varied from perfect contact to 2×10^{-10} m^2/s, based on measurements made by Harpur et al. (1993). For the Subtitle D liner, the transmissivity of the interface was varied from perfect contact to 2×10^{-12} m^2/s.

GCL Composite Liner

Leakage rates for a GCL composite liner having a hydraulic conductivity of 2×10^{-8} cm/s are shown in Fig. 5 along with leakage rates computed using Rowe's (1998) solution for circular defects [(14)] and Giroud's (1997) equation for circular defects [(10)]. The shaded box near the bottom of the graph depicts the range of interface transmissivities reported by Harpur et al. (1993) for the geotextile-encased GCL. The leakage rate increases as the transmissivity of the interface increases and is particularly sensitive for transmissivities $>10^{-12}$ m^2/s. Rowe's (1998) solution underestimates the leakage range for transmissivities $<2 \times 10^{-12}$ m^2/s, with the maximum error being about a factor of 2 in the limiting case of perfect contact. For transmissivities $>2 \times 10^{-12}$ m^2/s, leakage rates predicted using Rowe's (1998) solution are essentially the same as those obtained from the numerical model.

Rowe's (1998) solution underestimates the leakage rate at transmissivities $<2 \times 10^{-12}$ m^2/s because flo in the GCL is multidimensional rather than 1D, as assumed by Rowe (1998). This effect is shown in Fig. 6 where streamlines obtained from the numerical model are shown for a GCL composite liner

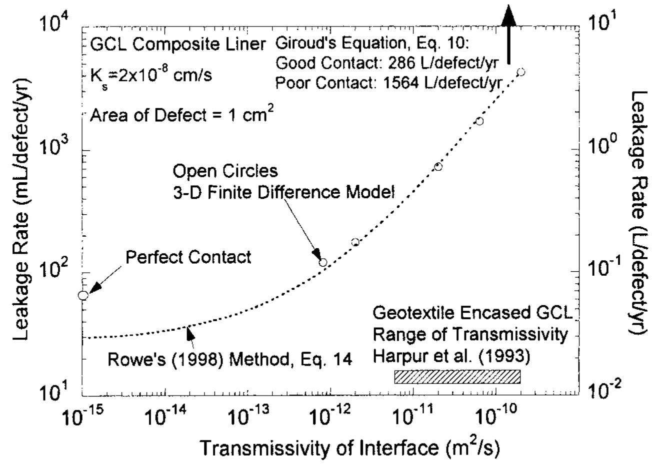

FIG. 5. Leakage Rate through Circular Defect in GCL Composite Liner Having Imperfect Contact

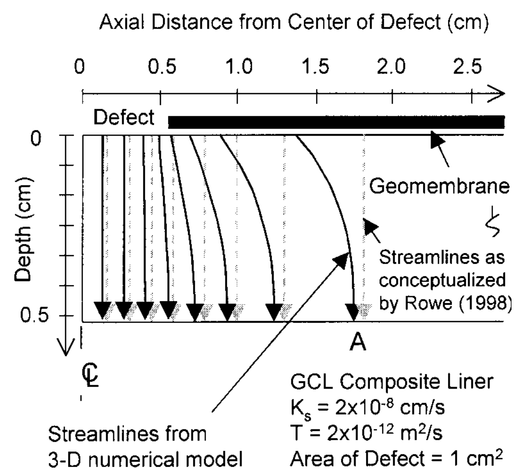

FIG. 6. Streamlines for Flow through Circular Defect in GCL Composite Liner Having Imperfect Contact

having a hydraulic conductivity of 2×10^{-8} cm/s and an interface transmissivity of 2×10^{-12} m^2/s along with streamlines as conceptualized by Rowe (1998).

Rowe's (1998) solution underpredicts leakage rates for transmissivities $<2 \times 10^{-12}$ m^2/s because the gradient along a given streamline is underestimated. Consider the streamlines that exit at location A shown in Fig. 6. The streamline for flui particles that exit at location A following the path assumed by Rowe's (1998) solution is longer than the streamline for 3D flo . Since the drop in total head along the streamlines must be the same, the gradients along streamlines obtained from Rowe's (1998) solution will be smaller than those based on 3D flo . Thus, Rowe's (1998) solution will tend to underpredict the leakage rate. For greater interface transmissivities, more lateral spreading of flo occurs in the interface, streamlines in the soil liner tend to become more undirectional, and leakage rates predicted using Rowe's (1998) solution approach those from the numerical model.

Giroud's (1997) equation [(10)] predicts a leakage rate of 286 L/defect/year for good contact and a leakage rate of 1,564 L/defect/year for poor contact (Fig. 5). The leakage rates obtained using Giroud's (1997) equation are substantially greater than those from the numerical model and Rowe's (1998) solution for the values of transmissivity of the interface measured by Harpur et al. (1993). One possible reason for this is that the values of transmissivity measured in the laboratory may not be representative of good and poor contact, define by Giroud (1997), which are for fiel conditions.

The area of the liner conducting flo beneath the defect in a GCL composite liner was also analyzed. For a GCL composite liner having a hydraulic conductivity of 2×10^{-8} cm/s and a transmissivity of the interface of 2×10^{-11} m^2/s, 98% of the flo was conducted within 10 cm of the center of the defect. For the same properties, the wetted radius predicted using Rowe's (1998) solution is 9 cm, which is comparable to the wetted radius obtained from the numerical model. The wetted radius predicted using Giroud's (1997) equation for good contact [(12)] is 1.7 m, which is much greater than that obtained from the numerical model or Rowe's (1998) solution, and is consistent with the higher leakage rates predicted with Giroud's (1997) equation [(10)].

A comparison of leakage rates predicted using Rowe's (1998) solution and results from the numerical model is shown in Fig. 7 for various hydraulic conductivities of the GCL. The abscissa is the ratio of hydraulic conductivity of the interface and hydraulic conductivity of the GCL (K_i/K_s). Leakage rates from Rowe's (1998) solution $Q_{c,R}$ and the numerical model $Q_{c,m}$ differ by <10% provided that K_iK_s is $>10^4$. Therefore, flo in the GCL can be approximated as unidirectional when the hydraulic contrast between the interface and soil liner is $>10^4$.

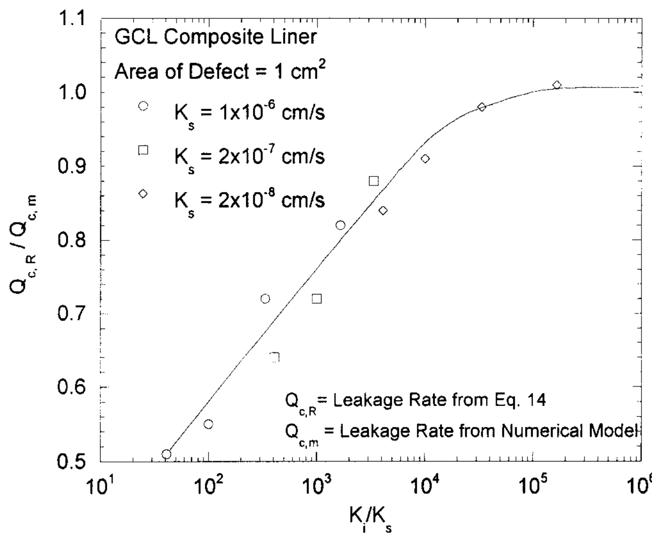

FIG. 7. Ratio of Leakage Rates Predicted Using Rowe (1998) to Leakage Rates from Numerical Model for Circular Defects in GCL Composite Liner

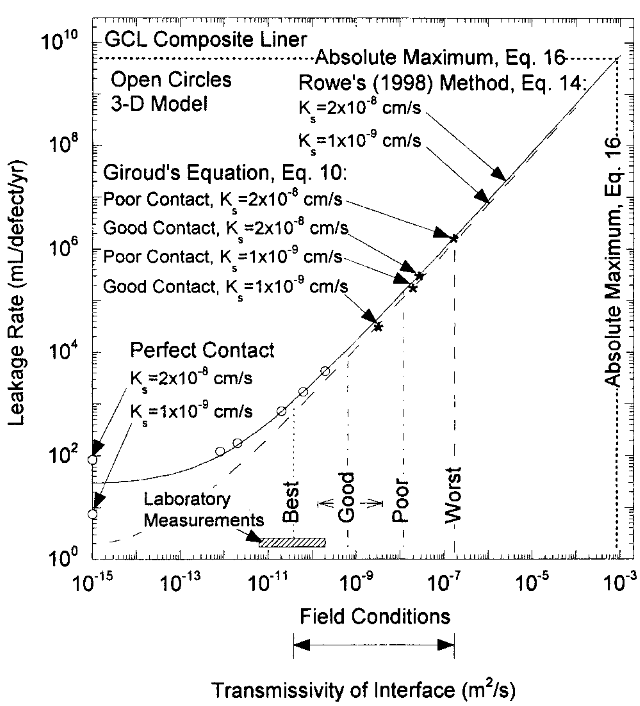

FIG. 8. Leakage Rate through Circular Defect in GCL Composite Liner for Field Conditions

The transmissivity of the interface for a GCL composite liner for fiel conditions can be estimated using the approach that Giroud and Bonaparte (1989) used for composite liners having a compacted soil liner and leakage rates computed using Rowe (1998). The "best" fiel case is assumed to correspond to the middle of the range of transmissivities of the interface measured in the laboratory by Harpur et al. (1993). Following the approach used by Giroud and Bonaparte (1989), the transmissivity for the "worst" fiel condition is assumed to be halfway between the best fiel case and the transmissivity corresponding to the absolute maximum leakage rate [(16)]. The range of transmissivity between the best and worst fiel conditions is arbitrarily divided into thirds (Fig. 8) and the values of transmissivity corresponding to good and poor conditions for a GCL composite liner are 6×10^{-10} m^2/s and 1×10^{-8} m^2/s, respectively. Based on the range of laboratory measurements of the transmissivity of the interface between a GCL and geomembrane reported by Harpur et al. (1993), the transmissivity corresponding to good fiel conditions may range between 1×10^{-10} and 4×10^{-9} m^2/s (Fig. 8).

Subtitle D Liner

Leakage through a Subtitle D liner having imperfect contact between the geomembrane and soil liner was also investigated. Rowe (1998) found that the transmissivity of the interface is 1.6×10^{-8} m^2/s for good contact conditions, as define by Giroud (1997). Convergence problems with the 3D numerical model for the Subtitle D liner prevented simulations with transmissivities $>2 \times 10^{-12}$ m^2/s. Results of simulations for the two transmissivities evaluated are shown in Table 1. For interface transmissivities $<2 \times 10^{-12}$ m^2/s, Rowe's (1998) solution underpredicts the leakage rate. This occurs because for these cases, flo is 3D rather than undirectional, as conceptualized by Rowe (1998).

Provided that the Subtitle D liner behaves similarly to the GCL composite liner, flo in the soil liner of the Subtitle D liner can be approximated as unidirectional when $K_i/K_s > 10^4$, which corresponds to a transmissivity of 3.5×10^{-11} m^2/s when $K_s = 1 \times 10^{-7}$ cm/s. Therefore, for transmissivities representative of good contact (1.6×10^{-8} m^2/s), as back-calculated by Rowe (1998), Rowe's (1998) solution and Giroud's (1997) equation should provide reasonable estimates of the leakage rate for the Subtitle D liner. Similar results were found from a more complete analysis of flo through defective seams in a Subtitle D liner, and are discussed in a subsequent section.

TABLE 1. Comparison of Leakage Rates Predicted Using Numerical Model and Rowe's (1998) Solution for Subtitle D Liner

Interface transmissivity (m^2/s)	Leakage rate numerical model $Q_{c,m}$ (mL/year)	Leakage rate Rowe's (1998) solution $Q_{c,R}$ (mL/year)	Ratio $Q_{c,m}/Q_{c,R}$
Perfect contact	648	—	—
2×10^{-14}	657	9	73
2×10^{-12}	894	99	9

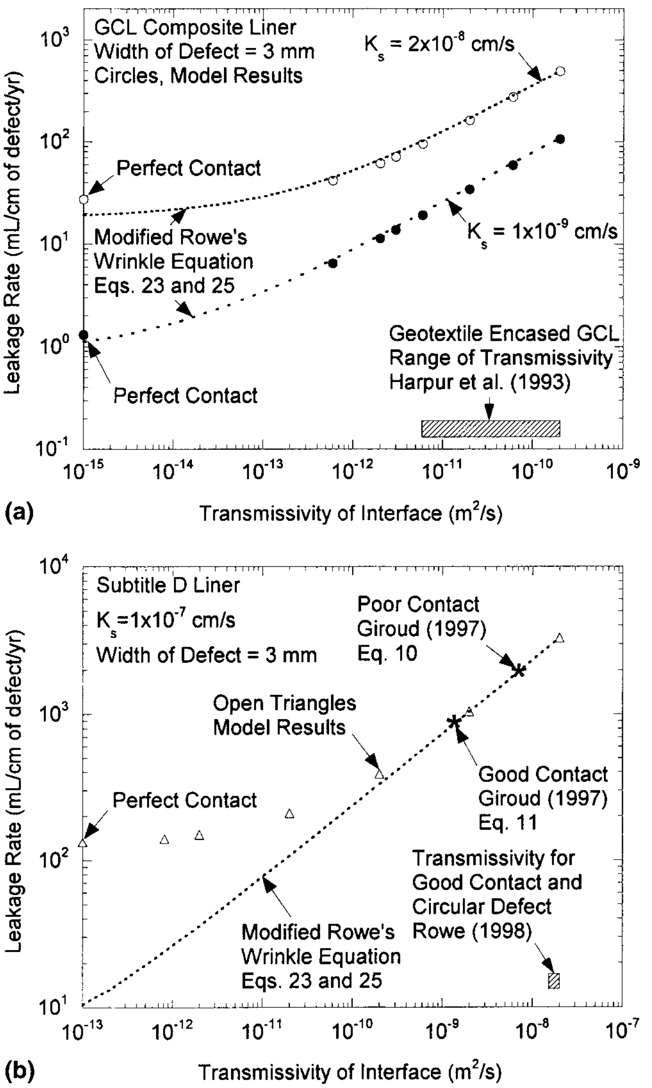

FIG. 9. Leakage Rates through Defective Seams in Composite Liners Having Imperfect Contact: (a) GCL Composite Liner; (b) Subtitle D Liner

Leakage through Long Defects—Imperfect Contact

GCL Composite Liner

Analyses for leakage from long defects employed the same properties of the interface as those for circular defects. Leakage rates for a 3 mm wide defective seam or tear in the geomembrane of a GCL composite liner are shown in Fig. 9(a) in terms of leakage rate per unit length of defect. If perfect contact exists between the geomembrane and GCL, an increase in the hydraulic conductivity of the GCL by a factor of 20 results in an increase in the leakage rate by a factor of 20. For cases in which imperfect contact exists and the transmissivity of the interface between the geomembrane and GCL is $\geq 3 \times 10^{-12}$ m^2/s, a 20-fold increase in hydraulic conductivity results in the leakage rate increasing by a factor of 5. Leakage rate is more sensitive to hydraulic conductivity of the GCL when the interface is less transmissive because the contrast between the hydraulic conductivity of the interface and the GCL is greater. As a result, lateral flo along the interface becomes more significant Similar results were found for circular defects.

Leakage through defective seams can also be analyzed using a modifie version of Rowe's (1998) solution for analyzing leakage from a circular defect in a composite liner located at the crest of a wrinkle in the geomembrane [(15)], herein referred to as the modifie Rowe's (1998) wrinkle equation. The modificatio consists of using the width of the defective seam or tear w in place of the width of the wrinkle B and redefinin S to be equal to the length of the defect. As shown in Fig. 9(a), the modifie Rowe's (1998) wrinkle equation yields leakage rates that are nearly the same as those from the numerical model for all interface transmissivities.

Giroud's (1997) equation for defective seams was also used to calculate the leakage rate through a 3 mm wide defective seam or tear in a GCL composite liner. For a GCL having a hydraulic conductivity of 1×10^{-9} cm/s and good contact, the leakage rate predicted using Giroud's (1997) equation is 124 mL/year/cm, which is slightly greater than the upper bound of 107 mL/year/cm from the numerical model for the GCL having a hydraulic conductivity of 1×10^{-9} cm/s. For a GCL having a hydraulic conductivity of 2×10^{-8} cm/s and good contact, Giroud's (1997) equation yields 1,679 mL/year/cm, which is more than three times greater than the upper bound obtained from the numerical model for a GGL having a hydraulic conductivity of 2×10^{-8} cm/s and interface transmissivity of 2×10^{-10} m^2/s. This occurs because the transmissivity of the interface representative of good contact is $>2 \times 10^{-10}$ m^2/s.

Subtitle D Liner

Leakage rates through a 3 mm wide defect in a Subtitle D liner are shown in Fig. 9(b) in terms of leakage per unit length of defect as a function of transmissivity. The leakage rates were obtained from the numerical model, Giroud's (1997) equation for long defects, and the modifie Rowe's (1998) wrinkle equation. The leakage rates predicted with Giroud's (1997) equation are plotted along the trend for results from the numerical model to identify interface transmissivities that correspond to the predicted leakage rates.

For good contact, the numerical model and Giroud's (1997) equation coincide at a transmissivity of approximately 1×10^{-9} m^2/s. In comparison, Rowe (1998) found that the transmissivity for good contact was 1.6×10^{-8} m^2/s based on leakage rates from Giroud's (1997) equation for circular defects. A similar result for the transmissivity for poor contact is also shown in Fig. 9(b). The numerical model for long defects suggests that the transmissivity for poor contact should be 7×10^{-9} m^2/s, whereas Rowe's (1998) analysis of circular defects yielded a transmissivity of 1×10^{-7} m^2/s. The transmissivities representative of good and poor contact should not depend on the geometry of the defect. This inconsistency may be due to approximations used by Giroud et al. (1992) to extrapolate the empirical equations for circular defects to long defects.

Leakage rates predicted using the modifie version of Rowe's (1998) wrinkle equation are similar to results from the numerical model for cases in which the transmissivity of the interface is $>2 \times 10^{-10}$ m^2/s. This transmissivity correspond to a ratio of $K_i/K_s = 3 \times 10^4$, which is similar to the ratio of K_i/K_s yielding good agreement for the GCL composite liner having circular defects (Fig. 5). The leakage rates compare favorably for transmissivities $>2 \times 10^{-10}$ m^2/s because flo in the soil liner of the Subtitle D liner can be approximated as unidirectional for transmissivities $>2 \times 10^{-10}$ m^2/s.

RECOMMENDED EQUATIONS FOR PREDICTING LEAKAGE RATES

Perfect Contact

Comparison of leakage rates from the numerical model and those from the analytical equation for perfect contact [(1)] shows that the analytical equation can be used to accurately predict leakage rates through thick composite liners. For thin composite lines employing GCLs, (1) tends to underpredict

leakage rates. Analysis of results from the numerical simulations showed that an equation similar to (1) can be used to predict leakage rates for GCL composite liners. This equation is

$$Q_c = F_c K_s h_t r \tag{21}$$

where F_c = nondimensional flo factor for circular defects. The product of F_c and r is analogous to the inverse of the form factor, which represents the geometric characteristics of the flo net for a composite liner, in the method of fragments described by Harr (1962).

An expression for F_c was back-calculated from results of the numerical model, as shown in Fig. 10(a) in terms of r/L_s. The flo factor is a linear function of r/L_s

$$F_c = 4 + 3.35 \frac{r}{L_s} \tag{22}$$

The intercept is $F_c = 4$ for $r/L_s = 0$, which corresponds to the semiinfinit composite liner described in (1).

A similar equation was obtained for the leakage rate per unit length Q_l for long defects

$$Q_l = F_l K_s h_t \tag{23}$$

where F_l = nondimensional flo factor for long defects. An expression for F_l was back-calculated from results of the numerical model shown in Fig. 10(b).

$$F_l = \frac{1}{0.52 - 0.76 \log\left(\frac{w}{L_s}\right)} \tag{24}$$

Eqs. (21)–(24) are easier to use than previously published equations for calculating leakage through defects in composite liners of finit thickness having perfect contact because the solution does not include infinit series, elliptic integrals, or elliptic functions. These equations can also be used for GCL composite liners whereas previously developed equations can only be used for thicker liners [e.g., Walton and Sagar (1990) and Walton et al. (1997)].

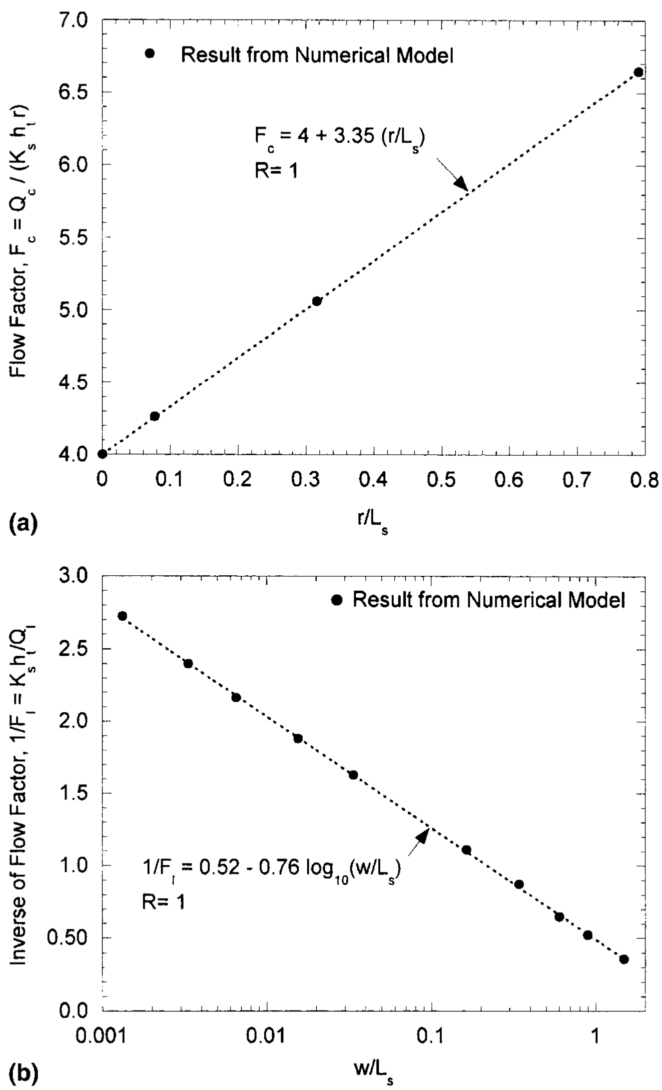

FIG. 10. Nondimensional Flow Factors for Circular Defects in Composite Liners Having Perfect Contact: (a) Circular Defects; (b) Long Defects

Imperfect Contact

For thick composite liners having circular defects and a more permeable interface ($K_i/K_s > 3 \times 10^4$, which corresponds to $T = 1.8 \times 10^{-10}$ m^2/s for $K_s = 1 \times 10^{-7}$ cm/s), Rowe's (1998) solution is recommended. Alternatively, Giroud's (1997) equation can be used for circular defects in thick composite liners, e.g., Subtitle D liner or thicker. Rowe's (1998) solution is also recommended for composite liners employing a GCL having a more permeable interface ($K_i/K_s > 10^4$). The transmissivity for good contact for a composite liner having a GCL should be approximately 6×10^{-10} m^2/s. A drawback of using Rowe's (1998) solution is its complexity. However, the solution can be programmed in a commercial spreadsheet and then used repeatedly thereafter. A spreadsheet of the solution can be obtained from the writers.

For composite liners having defective seams or tears and a more permeable interface ($K_i/K_s > 10^4$ for composite liners employing a GCL and $K_i/K_s > 3 \times 10^4$ for thicker composite liners) a modifie version of Rowe's (1998) wrinkle equation can be used to compute F_l

$$F_l = \frac{2}{L_s}\left[\frac{w}{2} + \sqrt{\frac{L_s T}{K_s}}\right] \tag{25}$$

The nondimensional flo factor F_l is then substituted into (23) to compute the leakage rate.

Rowe's (1998) solution for circular defects and (23) and (25) for defective seams or tears tend to underestimate leakage rates for less permeable interfaces, i.e., when $K_i/K_s < 10^4$ for

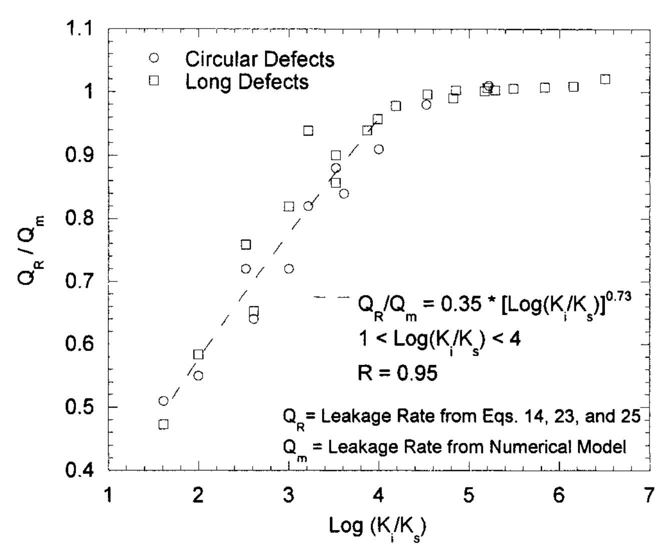

FIG. 11. Ratio of Leakage Rates Predicted Using Rowe (1998) to Leakage Rates from Numerical Model for Defects in GCL Composite Liner

composite liners employing a GCL and $K_i/K_s < 3 \times 10^4$ for thicker composite liners. As shown in Fig. 11, the discrepancy is consistent for circular defects and defective seams or tears in the GCL composite liner. Using the results shown in Fig. 11, the leakage rate for a GCL composite liner using Rowe's (1998) solution or (23) and (25) Q_R can be adjusted using the following relationship:

$$Q_a = 2.85 Q_R \left[\log\left(\frac{K_i}{K_s}\right)\right]^{-0.73} \tag{26}$$

where Q_a = leakage rate in a GCL composite liner adjusted for the discrepancy resulting from approximating the streamlines as 1D; and Q_R = leakage rate predicted using Rowe's (1998) solution or (23) and (25). Eq. (26) is limited to values of K_i/K_s between 1 and 4. The error in predicted leakage rate associated with this adjustment is <10%. For thick composite liners (e.g., Subtitle D liner or thicker) having a less permeable interface (i.e., when $K_i/K_s < 3 \times 10^4$), a numerical model can be used to estimate the leakage rate.

A summary of the recommended methods for computing leakage rates in composite liners is listed in Table 2. In Table 2, thick composite liners are liners in which a geomembrane overlays a compacted soil liner having a thickness ≥61 cm. GCL composite liners consist of a geomembrane overlaying a GCL. Table 2 lists methods for circular or long defects (tears or failed seams) for various interface conditions that range from perfect contact to cases in which the interface has a hydraulic conductivity much greater than the hydraulic conductivity of the underlying soil liner.

SUMMARY AND CONCLUSIONS

The results and analyses presented in this paper provide a fundamental analysis of leakage through defects in composite liners. 3D and 2D numerical models were used to simulate flo from circular and long defects in composite liners having either imperfect or perfect contact between the geomembrane and soil liner. Results obtained with the models were used to assess the efficac of various equations used for predicting leakage rates. Numerical models were used as the basis of comparison instead of experimental data because of the diffi culties inherent in measuring the small leakage rates from composite liners and uncertainties in existing fiel data.

TABLE 2. Recommended Equations and Methods for Calculating Leakage Rates

	Interface condition $[\log(K_i/K_s)]$[a]	Recommended equations and methods
Circular defects		
Perfect contact	1	Eqs. (21) and (22)
Thick composite liner[b]	>4.5	Rowe (1998) or Giroud (1997)
Thick composite liner	<4.5	Numerical model
GCL composite liner	>4	Rowe (1998)
GCL composite liner	<4	Rowe (1998), adjust $Q_{c,R}$ using Eq. (26)
Long defects		
Perfect contact	1	Eqs. (23) and (24)
Thick composite liner	>4.5	Eqs. (23) and (25)
Thick composite liner	<4.5	Numerical model
GCL composite liner	>4	Eqs. (23) and (25)
GCL composite liner	<4	Eqs. (23) and (25), adjust Q_l using Eq. (26)

[a]K_i = hydraulic conductivity of interface; K_s = hydraulic conductivity of soil liner or GCL.
[b]Thick composite liner has soil liner thickness ≥61 cm.

Comparison of leakage rates from the numerical models and those from existing analytical and empirical models shows that all of the existing models and equations have limitations on their applicability. Therefore, no universal equation or model exists for predicting leakage rates in composite liners. Results from the comparison show which models are appropriate and under what conditions. Based on the comparison, recommendations have been made regarding the appropriate methods for predicting leakage rates in composite liners. Analytical equations and equations based on results from the numerical models are presented for calculating leakage rates in composite liners having perfect contact. For composite liners having imperfect contact, the appropriate method for calculating leakage rates should be selected based on the thickness of the soil liner and the ratio between the hydraulic conductivity of the interface and the hydraulic conductivity of the soil liner. A summary of the recommendations is listed in Table 2.

APPENDIX. ROWE'S (1998) SOLUTION

The term Δ_1 is define as

$$\Delta_1 = -[R\lambda_1(r, R)K_1(\alpha R) + R\lambda_2(r, R)I_1(\alpha R)] /\alpha + r\lambda_1(r, R)K_1(\alpha r)/\alpha + r\lambda_2(r, R)I_1(\alpha r)/\alpha \tag{27a}$$

and Δ_2 is define as

$$\Delta_2 = [-R\lambda_1(R, r)K_1(\alpha R) - R\lambda_2(R, r)I_1(\alpha R)] /\alpha + [r\lambda_1(R, r)K_1(\alpha r) + r\lambda_2(R, r)I_1(\alpha r)]/\alpha \tag{27b}$$

where K_1 and I_1 are modifie Bessel functions of order 1. The term $\lambda_1(X, Y)$ is define as

$$\lambda_1(X, Y) = \frac{I_o(\alpha Y)}{K_o(\alpha X)I_o(\alpha Y) - K_o(\alpha Y)I_o(\alpha X)} \tag{28a}$$

and $\lambda_2(X, Y)$ is define as

$$\lambda_2(X, Y) = \frac{K_o(\alpha Y)}{K_o(\alpha X)I_o(\alpha Y) - K_o(\alpha Y)I_o(\alpha X)} \tag{28b}$$

where K_o and I_o are modifie Bessel functions of zero order. The wetted radius R is computed by solving the following equation:

$$(-h_w - L_s)\Lambda_1 - (L_s)\Lambda_2 = 0 \tag{29}$$

where

$$\Lambda_1 = \frac{-\alpha K_1(\alpha R)I_o(\alpha R) - \alpha K_o(\alpha R)I_1(\alpha R)}{K_o(\alpha r)I_o(\alpha R) - K_o(\alpha R)I_o(\alpha r)} \tag{30a}$$

$$\Lambda_2 = \frac{-\alpha K_1(\alpha R)I_o(\alpha r) - \alpha K_o(\alpha r)I_1(\alpha R)}{K_o(\alpha R)I_o(\alpha r) - K_o(\alpha r)I_o(\alpha R)} \tag{30b}$$

and α is define by

$$\alpha^2 = \frac{K_s}{L_s T} \tag{31}$$

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NOTATION

The following symbols are used in this paper:

a = area of defect;
B = width of wrinkle;
F_c = nondimensional flo factor for circular defects;
F_l = nondimensional flo factor for long defects;
g = gravitational constant;
h_R = total head at radial distance R_r;
h_t = total head drop across composite liner;
h_w = depth of leachate above geomembrane;
K_i = saturated hydraulic conductivity of interface;
K_s = saturated hydraulic conductivity of soil liner;
K_{xx}, K_{yy}, K_{zz} = hydraulic conductivity along x-, y-, and z-axes, respectively;
L_s = thickness of soil liner;
Q_a = adjusted leakage rate for low transmissivity interface;
Q_c = leakage through circular defect in composite liner;
$Q_{c,G}$ = leakage rate through circular defect using Giroud’s (1997) equation;
$Q_{c,m}$ = leakage rate through circular defect from numerical model;
$Q_{c,\max}$ = absolute maximum leakage rate through circular defect;
$Q_{c,R}$ = leakage rate through circular defect using Rowe’s (1998) equation;
Q_l = leakage rate per unit length of long defect;
$Q_{l,G}$ = leakage rate through long defect using Giroud’s (1997) equation;
Q_m = leakage rate from numerical model;
Q_R = leakage rate predicted using Rowe’s (1998) solution or Eqs. (23) and (25);
$Q_{w,R}$ = leakage rate through a circular defect Rowe’s (1998) “wrinkle” equation;
R_c = radius of wetting for circular defects;
R_o = distance between long defects;
R_r = radial distance from axis of defect;
r = radius of defect;
S = length of wrinkle;
S_s = specifi storage;
T = transmissivity;
t_i = thickness of interface;
W = flo rate of sinks and sources;
W_l = width of wetting for long defects;
w = width of defect;
z = vertical distance from defect;
β_c = contact factor for circular defects;
β_l = contact factor for long defects;
Φ = dimensionless form factor;
η = kinematic viscosity of water;
K = complete elliptical integral of firs kind with modulus m';
K' = complete elliptical integral of firs kind with modulus m;
Λ = complete elliptical integral of firs kind of modulus λ;
Λ' = complete elliptical integral of firs kind of complementary modulus λ'; and
ρ = density of water.