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DEVELOPMENT OF TARGET POWER SPECTRAL DENSITY FUNCTIONS COMPATIBLE WITH DESIGN RESPONSE SPECTRA

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ABSTRACT

For seismic analysis of nuclear structures, synthetic acceleration time histories are often required and are generated to envelop design response spectra following the U.S. Nuclear Regulatory Commission, Standard Review Plan (SRP) Section 3.7.1. It has been recognized that without an additional check of the power spectral density (PSD) functions, spectral matching alone may not ensure that synthetic acceleration time histories have adequate power over the frequency range of interest. The SRP Section 3.7.1 Appendix A provides a target PSD function for the Regulatory Guide 1.60 horizontal spectral shape. For other spectral shapes, additional guidance on developing the target PSD functions compatible with the design spectra is desired. This paper presents a general procedure for the development of target PSD functions for any practical design response spectral shapes, which has been incorporated into the recent SRP 3.7.1, Revision 4.

INTRODUCTION

For seismic analysis of nuclear structures, synthetic acceleration time histories are often required and are generated to envelop design response spectra. The U.S. Nuclear Regulatory Commission (NRC) NUREG-0800 [1], Standard Review Plan (SRP) Section 3.7.1, "Seismic Design Parameters," provides two approaches for developing spectramatching acceleration time histories: Approach 1 involves matching response spectra at several damping levels and

Approach 2 provides criteria for matching only 5% damped response spectra. It has been recognized that without an additional check of the power spectral density (PSD) functions, spectral matching alone may not ensure that synthetic acceleration time histories have adequate power over the frequency range of interest, even though the spectral matching criteria are met. NUREG/CR-5347, "Recommendations for Resolution of Public Comments on USI A-40, 'Seismic Design Criteria,'" recommended using target PSDs as a secondary check to ensure adequate power in the synthetic time histories and provided a specific target PSD function for design spectra based on the NRC Regulatory Guide (RG) 1.60, "Design Response Spectra for Seismic Design of Nuclear Power Plants," horizontal spectral shape [2]. SRP 3.7.1 Appendix A describes the use of this target PSD for RG 1.60 design response spectra. For spectral shapes other than the RG 1.60 horizontal design spectra, which account for most design spectra used for new nuclear reactor designs, more guidance is desired on developing the target PSD functions compatible with the design spectra. The compatibility between a design response spectrum and the corresponding target PSD function describes in the sense of expectation (on average) how well the response spectrum of a synthetic time history generated from the target PSD function converges to the design spectrum.

The key issue in the development of such guidance is how to generate a PSD function compatible with a given response spectrum. There are many references in the literature dealing with or involving the generation of a PSD function compatible with a response spectrum [e.g., 3-7]. Most of the procedures described in these references utilize the product of a peak

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factor and the standard deviation of the response of an oscillator to estimate the mean maximum response (i.e., the mean response spectrum) [8-10]. Because different peak factors can lead to somewhat different estimates of the mean maximum responses [5], it would be difficult to identify one peak factor applicable to different response spectral shapes. In addition, Pozzi and Der Kiureghian found that not all response spectral shapes are admissible; for a response spectrum to be admissible, it has to decay sufficiently fast at the tail (toward larger frequencies) but cannot decay too fast in the same time[4]. The same authors also noted that a given response spectrum may not be admissible for practical reasons. For example, some design spectra were developed by fitting simple functions to the response spectra of recorded ground motions and these functions, often piece-wise linear functions, may not necessarily have admissible shapes at the tail.

The recent design certification (DC) applications and the combined license (COL) applications have shown very different response spectral shapes. In light of these differing response spectral shapes and the difficulty in choosing a right peak factor, it is desired to have a procedure that does not require any particular peak factor and has minimal restriction on response spectral shape. In addition, because the PSD check is stated in the SRP 3.7.1 as a secondary check, through a reduction factor of 0.8 applied on the target PSD function in SRP 3.7.1 Appendix A, the compatibility between a target PSD function and the design response spectrum may not need to be at an excessively high degree.

This paper presents the results of a study that improves the fundamental approach in NUREG/CR-5347 by providing a general procedure for the development of target PSD for any practical design response spectra. It also presents various aspects associated with the target PSD development and the basis for the guidance described in Appendix B to SRP 3.7.1, Revision 4, which is applicable to the development of PSD for spectra other than RG 1.60 response spectra. Some of the technical areas addressed in this paper include processing the acceleration database in NUREG/CR-6728 [11], “Technical Basis for Revision of Regulatory Guidance on Design Ground Motions: Hazard- and Risk-consistent Ground Motion Spectra Guidelines,” issues related to multiple consistent design spectra, the development of a suitable factor for implementing the PSD criteria as a secondary check, the role of seed selection, and the frequency limits for the PSD check. The procedure introduced in this paper can be applied in any seismic analysis of a structure or component that requires the development of synthetic acceleration time histories from given design response spectra.

SRP 3.7.1, REV. 4, APPENDIX B PROCEDURE

SRP 3.7.1, Rev. 4, Appendix B was developed to provide guidance on the minimum PSD for response spectral shapes other than RG 1.60 horizontal response spectrum (RS). The initial intent of this development was to use the bin average

response spectra and bin average PSD functions derived from the NUREG/CR-6728 database, which categorizes the acceleration time histories into distance and magnitude bins. However, during processing the acceleration time histories, we found that a bin average PSD function normally is not compatible with the bin average response spectrum. Taking bin CEUS (Central and Easter U.S.) SOIL M75D100.200 as an example, the average (expected) RS based on the bin average PSD function (dashed line in red as shown in Figure 1) is generally lower than the bin average RS described by the other three curves.

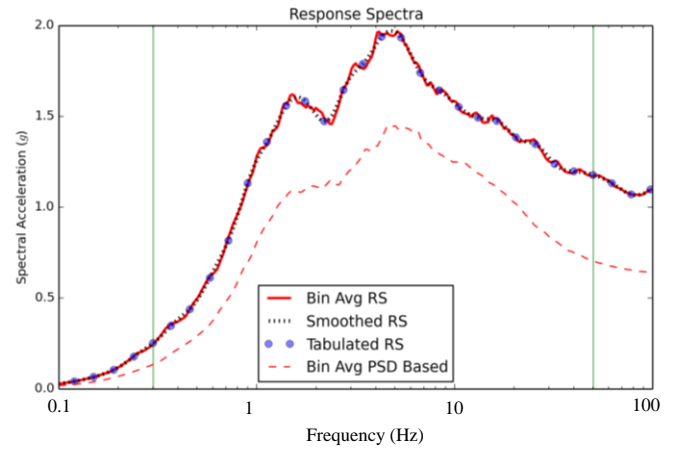


FIGURE 1 RS FOR CEUS SOIL M75D100.200

Therefore, an iterative frequency-by-frequency scaling approach was developed, expanding the fundamental approach in NUREG/CR-5347 to ensure the RS-PSD compatibility by averaging the response spectra of many synthetic acceleration time histories generated from the PSD function. To be consistent with SRP 3.7.1 Appendix A, the PSD function in this approach is defined as the one-sided PSD for an acceleration time history $a(t)$, related to its Fourier amplitude $|F(\omega)|$ by the following equation:

$$S_o(\omega) = \frac{2|F(\omega)|^2}{2\pi T_D} \quad (1)$$

in which T_D is the strong motion duration over which $F(\omega)$ is evaluated and ω represents the circular frequency. The duration T_D represents the duration of near maximum and nearly stationary power of an acceleration time history record as recommended in Appendix B of NUREG/CR-5347, and can often be estimated adequately as the duration corresponding to a 5%-to-75% rise of the cumulative Arias energy of the record. At any frequency, the average one-sided PSD is computed over a frequency window width of $\pm 20\%$ of the subject frequency.

Furthermore, in light of the existence of different normalization factors for discrete Fourier transform and its inverse, the Fourier amplitude $|F(\omega)|$ in Equation (1) (also for SRP 3.7.1 Appendix A) is defined herein at each circular frequency ω_n as:

$$|F(\omega_n)| = \Delta t \left| \sum_{j=0}^{N-1} a(t_j) e^{-2\pi i \left(\frac{nj}{N}\right)} \right| \quad (2)$$

where, $a(t_j)$ is the strong motion portion of the acceleration time history with N data points (after proper tapering at both ends), $t_j = j \Delta t$, $j = 0, 1, \dots, N-1$, and $n = 0, 1, \dots, N/2$. For $N/2 < n \leq N-1$, ω_n represents the negative frequencies and does not appear in the one-sided PSD calculation.

The iterative frequency-by-frequency scaling approach is described in SRP 3.7.1 Appendix B in terms of the NUREG/CR-6728 bin representative design response spectra, but can be applied to any other practical design spectra. A NUREG/CR-6728 bin representative design response spectrum is defined as a NUREG/CR-6728 design response spectrum (using Equation 3 or 4 in SRP 3.7.1 Appendix B) with the moment magnitude M and fault distance R equal to the midpoint bin values. For example, for bin M6-7 D010-050, the midpoint bin values are $M=6.5$ and $R=30$ km.

The iterative frequency-by-frequency scaling approach was found to be able to produce a target PSD compatible with a typical design RS (RS_{design}) with 10 iterations. The procedure consists of the following steps:

- (1) Determine a proper initial PSD function. The initial PSD does not need to be very close to the target PSD, which is to be obtained through this iterative procedure; a proper initial PSD only speeds up the convergence process. The bin average PSDs for the NUREG/CR-6728 time history database were used in developing the target PSDs for the NUREG/CR-6728 bin representative design response spectra (shown in Tables 1 & 2 of SRP 3.7.1, Rev. 4, Appendix B).
- (2) Generate M number of synthetic time histories from the current PSD function, where M took a value of 10, 20, ... 100 for iteration 1, 2, ... 10, respectively. To generate synthetic time histories from a PSD function, (a) Fourier spectra were constructed by using random phase angles and Fourier magnitudes computed following Equation 1, and (b) time histories were then generated through inverse FFT. The synthetic time histories were assumed to have 4096 data points and a time step of 0.005 s. Since the synthetic time histories are stationary at this point (the envelope function is applied in step (3) below), the entire duration of 20.48 s was used as the strong motion duration to generate the Fourier spectrum using Equation 1. The PSD function was linearly interpolated in the log-log scale to fill all frequency

points for the Fourier coefficients. This method produces the same acceleration time histories as the method in Appendix B of NUREG/CR-5347.

- (3) Apply a trapezoidal envelope function to the synthetic time histories (rise time = 1.4 s, strong motion duration = 10.24 s, and decay time = 7.0 s), which is Function B in Appendix B of NUREG/CR-5347.
- (4) Calculate the 5% damped absolute acceleration response spectra for the synthetic time histories and obtain the arithmetic average RS_{avg} .
- (5) Multiply the PSD frequency-by-frequency by $(RS_{\text{design}}/RS_{\text{avg}})^2$, and use this adjusted PSD in the next iteration.

Convergence to RS_{design} can be quickly achieved in the dominant frequency range of interest to structural response. However, in some cases, at very low and/or very high frequencies, successive iterations could lead to increase or decrease of the PSD values without noticeable improvement to the RS match. This behavior may be due to the inadmissibility of some design spectra; for example, the NUREG/CR-6728 bin representative design response spectra were developed by statistically fitting to the bin average RS shapes and may not necessarily be physical at these extreme frequencies. Therefore, in those cases, the PSD values at a few very low frequencies (close to 0.1 Hz) or very high frequencies (close to 100 Hz) need to be manually adjusted. The tabulated target PSD's in Tables 1 and 2 of SRP 3.7.1 Rev. 4 Appendix B are values after the manual adjustment to some bins.

The converged target PSD's may be smoothed using cubic splines at the frequency points as shown in the first column of Tables 1 and 2 of SRP 3.7.1 Rev. 4 Appendix B to improve the quality of the PSD function for the purpose of representing the mean PSD function.

Figure 2 through Figure 4 show three typical cases: (1) that does not require manual adjustment, (2) that shows very large PSD values at very low frequencies, which requires manually adjustment, and (3) that has been manually adjusted, respectively. These figures were generated for the NUREG/CR-6728 design spectra. Each figure has two plots: the one on the top shows various RS and the one at the bottom shows various PSD functions. Each PSD plot includes four curves: (1) bin average PSD, (2) iterated target PSD, (3) smoothed PSD, and (4) tabulated PSD. Each RS plot also includes four curves: (1) bin average RS, (2) bin representative RS, (3) bin average PSD based RS, and (4) tabulated target PSD based RS. Similar to the case of a soil site shown in Figure 1, it can be seen that the bin average PSD based RS and bin average RS generally do not agree well for rock sites, indicating that the bin average PSDs are not compatible with the bin average RS. The critical message in these figures is that the RS generated from the tabulated target PSDs closely match the bin representative NUREG/CR-6728 design RS, demonstrating their compatibility. For tabulated

target PSDs that require minor manual adjustments at a few very low frequencies and/or very high frequencies, the adjustments do not have noticeable effect on the level of agreement between the tabulated PSD based RS and the bin representative RS.

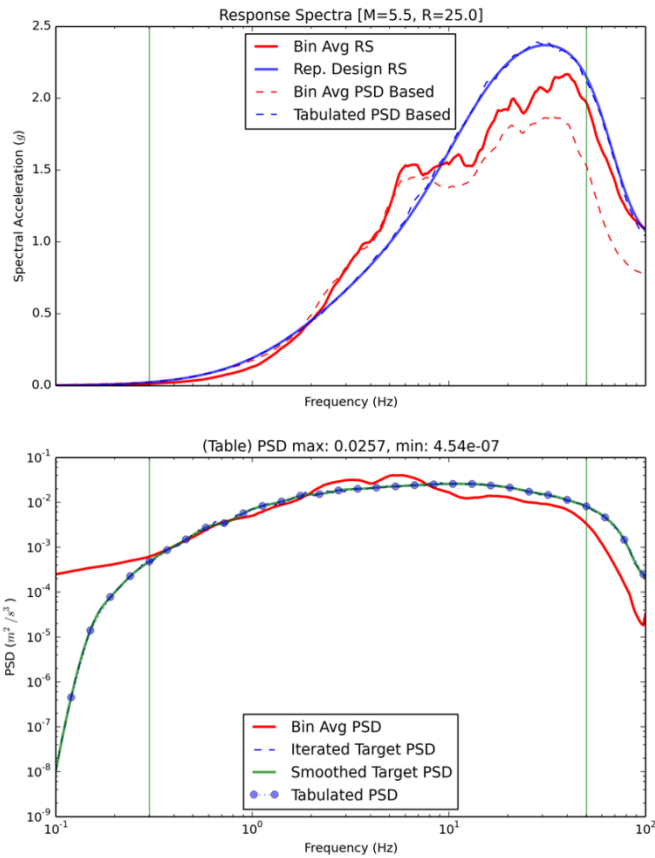


FIGURE 2 RS AND PSD FOR CEUS ROCK M55D000.050

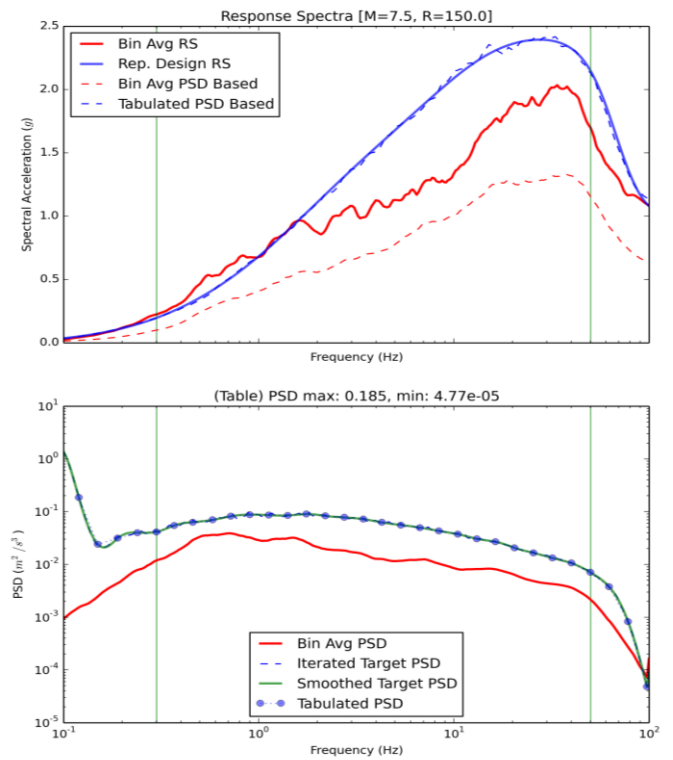


FIGURE 3 RS AND PSD FOR CEUS ROCK M75D100.200

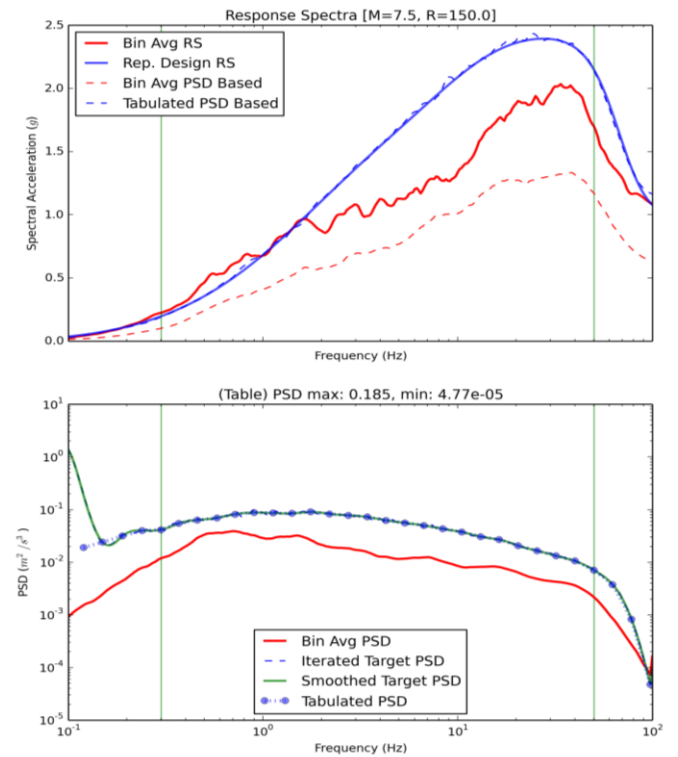


FIGURE 4 RS AND PSD FOR CEUS ROCK M75D100.200 (MANUALLY ADJUSTED)

ISSUES RELATED TO THE APPLICATION OF THE APPENDIX B PROCEDURE

PSD Check as a Secondary Check

SRP 3.7.1 Appendix A states that the minimum PSD of the synthetic acceleration time history should be at least 80% of the target PSD. This is set “so as to be sufficiently high to prevent a deficiency of power over any broad frequency band, but not so high that it introduces additional conservatism over that already embodied in the RG 1.60 response spectrum.” Although a synthetic acceleration time history following the SRP 3.7.1 guidance envelops the design response spectrum, its PSD function usually still shows fluctuations around the target PSD. Setting the minimum PSD check at 80% of the target PSD ensures that substantial valleys can be detected in some spectrally matched time histories, which occur only occasionally due to the conservatism in the spectral matching process. The same philosophy is used for the minimum PSD guidance in Appendix B.

In SRP 3.7.1 Appendix B, a reduction factor of 70% is applied upon the target PSD for PSD check. Using the iterative procedure described in this paper, the average response spectrum of the synthetic acceleration time histories generated from the target PSD is very close to the given design response spectrum, as shown in Figure 2 through Figure 4. This is different from the criterion used in the development of the target PSD for SRP 3.7.1 Appendix A, as described in NUREG/CR-5347, in that the response spectrum of an acceleration time history generated from the target PSD is generally lower than the RG 1.60 response spectrum. To achieve a PSD check consistent with SRP 3.7.1 Appendix A, the computed PSD from the synthetic time history is expected to be above 70% of the target PSD developed based on SRP 3.7.1 Appendix B, as opposed to the 80% factor used in SRP 3.7.1 Appendix A.

To derive the factor of 70%, the iterative procedure described in SRP 3.7.1 Appendix B was applied to the RG 1.60 response spectrum and a target PSD was determined accordingly. The frequency-by-frequency ratios of the target PSD defined by Equation 2 of SRP 3.7.1 Appendix A over the target PSD developed based on the SRP 3.7.1 Appendix B procedure were calculated for the frequency range of 0.3 Hz to 24 Hz and the geometric mean of these ratios was found to be 0.89. Therefore, the adjusted factor for use with target PSDs in SRP 3.7.1 Appendix B can be determined as $0.89 \times 80\% \approx 70\%$.

Frequency Range for PSD Check

SRP 3.7.1 Appendix A states that the PSD check should be performed for frequencies in the range of 0.3 Hz to 24 Hz, because power below 0.3 Hz generally has no influence on stiff nuclear plant structures and the power above 24 Hz for the target PSD is so low as to be inconsequential. As a comparison, the zero period acceleration (ZPA) frequency is 33 Hz for RG 1.60 spectra.

For SRP 3.7.1 Appendix B, the lower bound frequency for PSD check is the same as that in Appendix A, while the upper bound frequency (cutoff frequency) should be consistent with the design response spectrum. The reason for not setting a fixed upper bound frequency lies in the vastly different frequency contents at higher frequencies among existing design response spectra, especially between those in the Western U.S. (WUS) and Central and Eastern U.S. (CEUS).

A sensitivity study was performed to examine whether upper frequency limits can be determined through a cumulative power level equal to what the 24 Hz frequency limit implies in SRP 3.7.1 Appendix A. A plateau in the cumulative power with respect to frequency could indicate that the power beyond a certain frequency (e.g., 24 Hz in SRP 3.7.1 Appendix A) is very small and that frequency could be used to develop an upper frequency limit for PSD check. Using the target PSD in SRP 3.7.1 Appendix A, the level of cumulative target PSD that corresponds to the upper bound frequency 24 Hz was determined to be 0.9955 of the maximum cumulative target PSD. However, for the target PSDs presented in Tables 1 and 2 of Appendix B, this approach led to cutoff frequencies in the range of 9 Hz to 64 Hz, which is too wide to be useful for any practical applications. In addition, a small difference in these cumulative measures (e.g., 0.99 versus 0.9955) can lead to large differences in cutoff frequency estimates because the cumulative curves are very flat at higher frequencies.

Most importantly, these estimated cutoff frequencies can lead to significant incompatibility between the truncated PSD (by removing power above the cutoff frequency) and the RS. For each bin shown in Tables 1 and 2 of Appendix B, a sensitivity study was performed by progressively removing the PSD value at the highest frequency from the target PSD and generating the corresponding response spectra through averaging the response spectra of 100 time histories generated from the truncated target PSD curve. Figure 5 and Figure 6 show two representative comparisons of the resultant response spectra (dashed lines) and the NUREG/CR-6728 bin representative design response spectra (solid blue lines). The vertical lines in these figures indicate the frequencies at which the PSD curves were truncated from higher frequencies. These figures can be used for the determination of upper bound (cutoff) frequencies for PSD check, together with other considerations such as the ZPA frequency of the design response spectra and the dynamic characteristics of the soil-structure-equipment system.

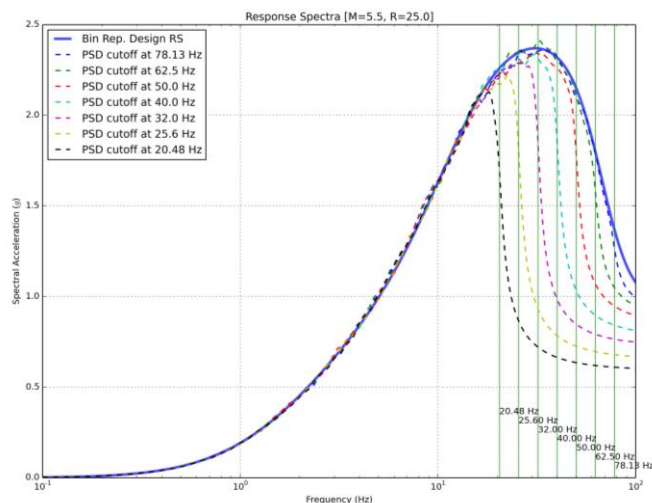


FIGURE 5 SENSITIVITY OF TARGET PSD CUTOFF FREQUENCY FOR CEUS ROCK M55D000.050

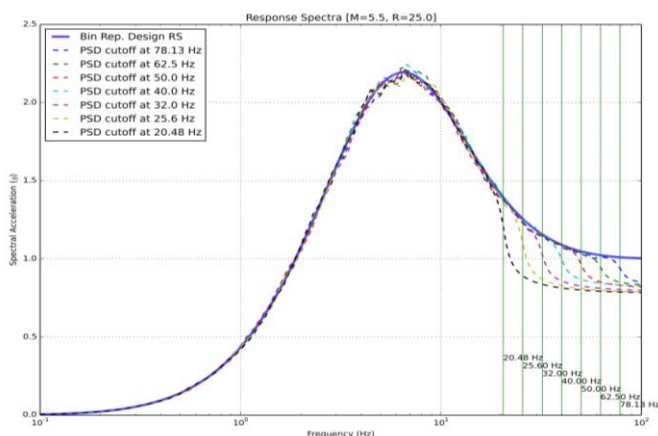


FIGURE 6 SENSITIVITY OF TARGET PSD CUTOFF FREQUENCY FOR WUS ROCK M55D000.050

Development of Target PSD for Multiple Consistent Response Spectra

The target PSD functions presented in Tables 1 and 2 of SRP 3.7.1 Appendix B were developed for the NUREG/CR-6728 bin representative design acceleration response spectra, which are based on a damping ratio of 5%. On the other hand, the target PSD for SRP 3.7.1 Appendix A was developed based on 2% damped pseudo relative velocity response spectra. It is noted that the development of target PSD following the SRP 3.7.1 Appendix B procedure should not be sensitive to the selection of a particular damping value because the calculation of PSD is independent of damping, which is confirmed by the study described below.

To demonstrate that the tabulated target PSD values in SRP 3.7.1 Appendix B are not sensitive to damping ratios, two

representative bins were selected from the NUREG/CR-6728 database in this study. Response spectra consistent with the bin representative RS were generated for damping values 2% and 10% using 1,000 synthetic acceleration time histories generated from the relevant target PSDs in Appendix B. Then target PSD functions were developed based on these 2% damped RS and 10% damped RS, and then compared to the tabulated target PSDs in Appendix B, which were computed from the 5% damped bin representative RS.

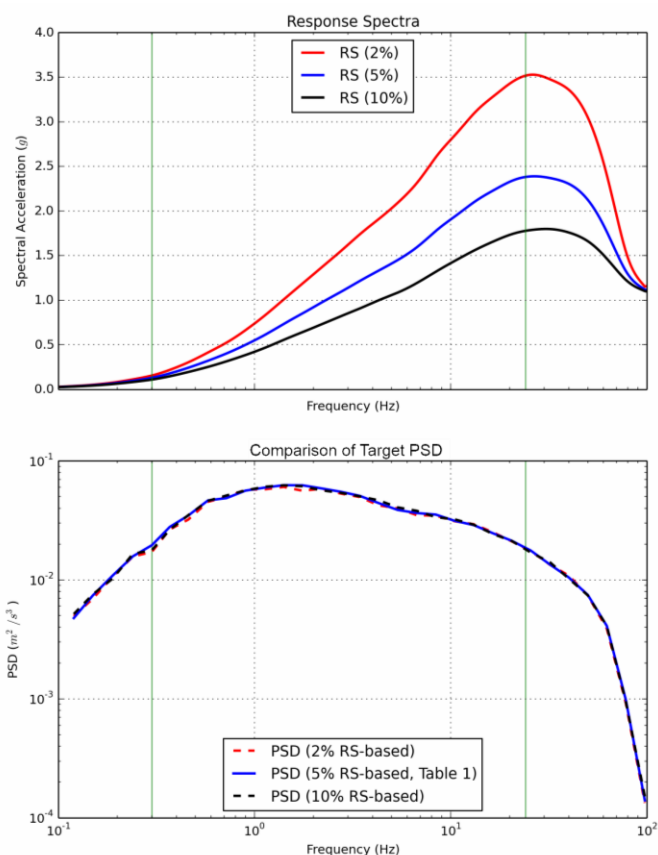


FIGURE 7 COMPARISON OF TARGET PSDS GENERATED FROM 2%, 5%, AND 10% DAMPED RS (CEUS ROCK M75D000.010)

Figure 7 shows the consistent 2%, 5%, and 10% damped RS in the top plot and an excellent agreement between the corresponding target PSDs in the bottom plot. Therefore, the development of target PSD is not sensitive to the damping ratio associated with the response spectrum. The minor difference among the target PSDs developed based on differently damped response spectra does not represent any problem in using the target PSD as a secondary check of the synthetic acceleration time histories to detect any potential deficiency of power.

The Role of Seed Selection in PSD Check

As described in SRP 3.7.1, the seed recorded time histories should have a similar response spectral shape to the target response spectra across the frequency range of interest to the analysis and the phasing characteristics of the earthquake records should not change significantly. In addition, seed records can play an important role in achieving a satisfactory PSD check as well, when they do not exhibit sufficient frequency-stationarity in the strong-motion duration. For both SRP 3.7.1 Appendices A and B, the strong motion duration of an acceleration time history is used to calculate the PSD because it represents the duration of “near maximum and nearly stationary power of the acceleration time history.” In general, the PSD estimate is sensitive to how the strong motion duration is selected.

Stationarity can be manifested in both amplitude stationarity and frequency stationarity, the former of which can be fairly represented by a straight line in the Husid plot but the latter cannot as easily be represented. Since a PSD function describes power distribution over frequencies, a frequency non-stationarity in the strong motion usually leads to underestimating the true power that a structure experiences (but for a shorter time) for those frequencies that do not exist for the entire strong motion duration. A power deficiency, shown as large valleys in the PSD function below the 70% target PSD, can have three possible scenarios:

- (1) the affected frequencies do not exist at all in the strong motion,
- (2) the affected frequencies exist but have insufficient power, or
- (3) the affected frequencies have sufficient power but do not exist in the entire strong motion duration.

The first two cases are obviously unfavorable for structural design, but the last case can be unfavorable as well because the waves at different frequency bands may not combine adequately due to lack of sufficient overlap in the strong motion and consequently the structural responses could potentially be underestimated.

For many acceleration time history records, stationarity is well demonstrated in the strong motion portion and thus the use of the strong motion duration is sufficient for the PSD check. However, there are cases where the stationarity cannot be easily identified for a proper determination of the strong motion duration, and therefore, can lead to an unsatisfactory PSD check. In such cases, different techniques to identify the strong motion duration, such as a duration corresponding to the 5%-to-75% rise of the cumulative Arias energy or a nearly linear portion of it, may show power deficiency at different frequency ranges.

An unsatisfactory PSD check often indicates that the strong motion portion of the seed recorded ground motion is not frequency-stationary. In such cases, a different seed may be pursued. Seeds of shorter strong motion durations often show higher level of stationarity (frequency stationarity in particular) and can make the PSD check easier to satisfy.

SUMMARY

This paper presents a procedure to compute a target PSD compatible with a response spectrum. This procedure does not rely on any particular peak factor but instead it calculates the average response spectrum from those of many synthetic acceleration time histories generated from the target PSD. This procedure involves iterative frequency-by-frequency scaling of an initial PSD to reach the converged target PSD. Although the computation involved in this procedure is relatively intense, the current computer hardware allows the calculation to be completed in the scale of minutes.

This paper also discusses several issues related to the development or application of the new Appendix B to the SRP 3.7.1 Rev. 4. A reduction factor of 70% is used for Appendix B in place of the 80% factor in Appendix A; both factors provide similar level of PSD check as a secondary check. The upper bound frequency for a PSD check should be consistent with the response spectral shape. The development of target PSD was found to be insensitive to the choice of damping values for the response spectrum, as long as the response spectra at different damping values are consistent. Finally, the seed records used for the generation of synthetic acceleration time histories have been found to have an important role in PSD check particularly for those frequency-non-stationary records.

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