

## Outline



- Introduction and Motivation
- Analysis Methods
- Results
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## RG 1.161 and Appendix K

### Overview



- Regulatory Guide 1.161 and ASME Section XI, Nonmandatory Appendix K provide procedures for calculating adequate Charpy upper shelf energy to protect against ductile fracture of the vessel.
- The applied  $J$ -Integral is estimated from stress intensity factor equations.

$$K_{I_p}^{ASME} = (SF) P_a [1 + (R_t/t)] (\pi a)^{0.5} F_1$$

$$F_1 = 0.982 + 1.006(a/t)^2$$

$$K_{II} = [(t/R)/1000]^{2.5} F_1$$

$$F_1 = 0.69 + 3.217(a/t) + 7.435(a/t)^2 + 3.532(a/t)^3$$

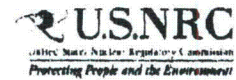
$$a_c = a + \frac{1}{6\pi} \left( \frac{K_{I_p} + K_{II}}{\sigma_y} \right)^2$$

Re-evaluate using  $a_c$ ,  
giving  $K_{I_p}'$  and  $K_{II}'$

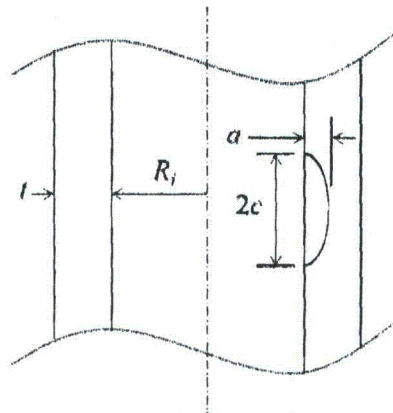
$$J = 1000 \frac{(K_{I_p}' + K_{II}')^2}{E'}$$

## RG 1.161 and Appendix K

### Overview



- Flaw geometry,  $a/c = 1/3$



## RG 1.161 and Appendix K

### Motivation



- These stress intensity factor equations were derived for  $R/t = 10$ , which applies to pressurized water reactor vessels.
- Boiling water reactor vessels typically have  $R/t = 20$ .
- The thinner vessels are expected to be less susceptible to steep thermal gradients.
- This work further explores the validity of using the derived equations for the  $R/t = 20$  case, through the use of independent methods of calculating stress intensity factor.

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## Temperature Profiles

### The Heat Equation



- Parabolic partial differential equation:  $\rho c_p \frac{\partial T}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right)$
- Boundary conditions:
 
$$q = h_c (T_f - T_s) \text{ for } r = R_i \text{ and all } \tau$$

$$q = 0 \text{ for } r = R_o \text{ and all } \tau$$

$$T_f = T_o - (CR)\tau$$
- Initial conditions:  $T = T_o \text{ for all } r \text{ and } \tau = 0$



## The Heat Equation

### Solving Strategies for Temperature Profiles



- Finite element analysis
- Crank-Nicolson method
- Analytical solutions
- Resource: Kreyszig, Advanced Engineering Mathematics
- This work
  - Commercial engineering coding software with a built-in PDE solver
  - Skeel and Berzins, "A Method for the Spatial Discretization of Parabolic Equations in One Space Variable," SIAM J. Sci. and Stat. Comput.

## Calculation of Stresses

### Pressure and Thermal Hoop Stresses



- Pressure stresses in a thick-walled cylinder:

$$\sigma_{\theta\theta} = \frac{pR_o^2}{R_o^2 - R_i^2} \left[ 1 + \left( \frac{R_o}{r} \right)^2 \right]$$

- Thermal stresses in a cylinder:

$$\sigma_{\theta\theta} = \frac{\alpha E}{1-\nu} \frac{1}{r^2} \left( \frac{r_o^2 + R_i^2}{R_o^2 - R_i^2} \int_{R_i}^{R_o} T r dr + \int_{R_i}^r T r dr - T r^2 \right)$$

## Calculation of Stress Intensity Factor

### Three Independent Methods



- American Petroleum Institute Standard 579-1:

$$\sigma(x) = \sigma_0 + \sigma_1 \left( \frac{x}{t} \right) + \sigma_2 \left( \frac{x}{t} \right)^2 + \sigma_3 \left( \frac{x}{t} \right)^3 + \sigma_4 \left( \frac{x}{t} \right)^4$$

$$K_I = \left[ \sigma_0 G_0 + \sigma_1 G_1 \left( \frac{a}{t} \right) + \sigma_2 G_2 \left( \frac{a}{t} \right)^2 \right] \sqrt{\frac{\pi a}{Q}} + \dots$$

$$\left[ \sigma_3 G_3 \left( \frac{a}{t} \right)^3 + \sigma_4 G_4 \left( \frac{a}{t} \right)^4 \right] \sqrt{\frac{\pi a}{Q}}$$

$$Q = 1 + 1.464 \left( \frac{a}{c} \right)^{1.65} \quad \text{for } a/c \leq 1.0 \quad Q = 1 + 1.464 \left( \frac{c}{a} \right)^{1.65} \quad \text{for } a/c > 1.0$$

## Calculation of Stress Intensity Factor

### Three Independent Methods



- Fracture Analysis of Vessels – Oak Ridge (FAVOR) method:
  - Similar to the API-579-1 method, except that a 3<sup>rd</sup> order polynomial is used for the stress distribution.
- Newman and Raju for internal pressure:

$$K_I = \frac{pR_i}{t} \sqrt{\pi \frac{a}{Q}} F \left( \frac{a}{c}, \frac{a}{t}, \frac{R_i}{t}, \phi \right)$$

## Calculation of Stress Intensity Factor

### Application of the Methods



- API-579-1 and FAVOR methods:
  - Are based upon the principle of elastic superposition.
  - May be applied for multiple loading conditions, if:
    - All conditions are mode I and
    - The polynomial used to describe the stress fits the actual stress profile well.
- The Newman and Raju equation was derived only for internal pressure and should be used only for that case.

## Inputs

Geometry, Material Properties, etc.



Description	Value
<b>Geometry</b>	
$t$	0.18 m [0.61 ft]
$R/t$	20.8
$a/t$	0.1 to 0.8
$a/x$	1/3
<b>Material Properties</b>	
$E$	$2.0 \times 10^6$ MPa [ $4.2 \times 10^6$ kip/in <sup>2</sup> ]
$\nu$	0.30
$\alpha$	$1.30 \times 10^{-6}$ m/m/K [ $7.20 \times 10^{-6}$ in/in/R]
$k$	38 W/(m·K) [22 BTU/(hr·ft·R)]
$\rho$	7770 kg/m <sup>3</sup> [488 lb <sub>m</sub> /ft <sup>3</sup> ]
$c_p$	536 J/(kg·K) [0.13 BTU/(lb <sub>m</sub> ·R)]
<b>Thermal Loading Conditions</b>	
$h_o$	5673 W/(m <sup>2</sup> ·K) [1000 BTU/(hr·ft <sup>2</sup> ·R)]
$C^*k$	0.02 K/s [100 R/hr]
$T_o$	581 K [1010 R]
<b>Mechanical Loading Conditions</b>	
$p$	18.0 MPa [386 kip/in <sup>2</sup> ]

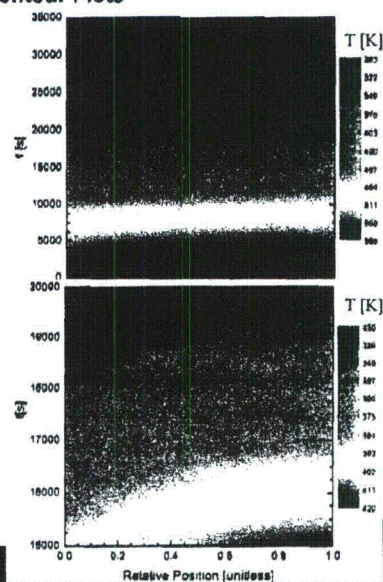


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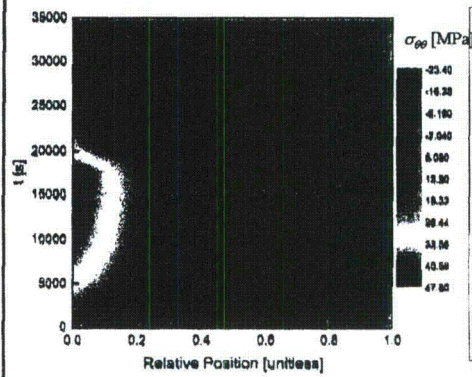
## Temperature Distribution Contour Plots



- Small gradients at the beginning and end of the cooldown.
- Highest gradient observed between 15 000 s and 20 000 s.

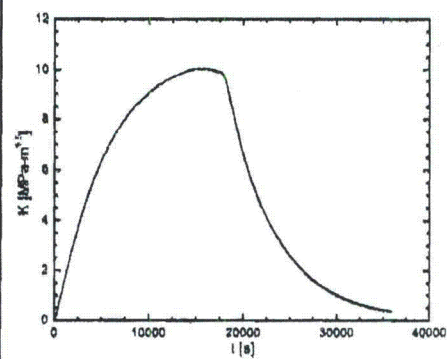


## Thermal Stress Distribution Hoop Stress Contour



- Vanishing thermal stresses early and late in the transient.
- Highest magnitude stresses observed between 5 000 and 20 000 s.
- Tension at ID, compression at OD.

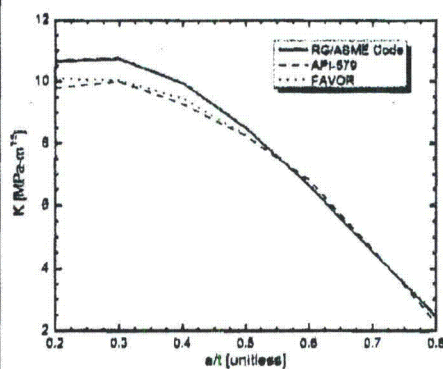
## Stress Intensity Factor: Thermal Time History



- API-579-1 method with  $a/c = 1/3$ ,  $a/t = 1/4$ .
- Highest stress intensity factor observed at 14 000 s.

## Stress Intensity Factor: Thermal

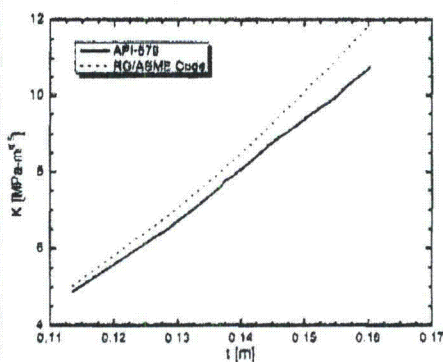
### Comparison of Different Methods



- Maximum  $K$  in the transient.
- RG/ASME Code equations bound API-579-1 and FAVOR methods for  $a/t < 0.55$ .
- Subsequent agreement.

## Stress Intensity Factor: Thermal

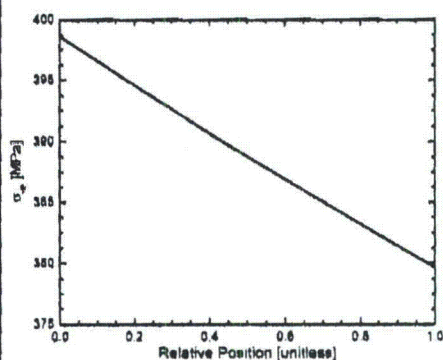
### Effect of Vessel Thickness



- RG/ASME Code equation conservatively bounds the API-579-1 method.

## Pressure Stress Distribution

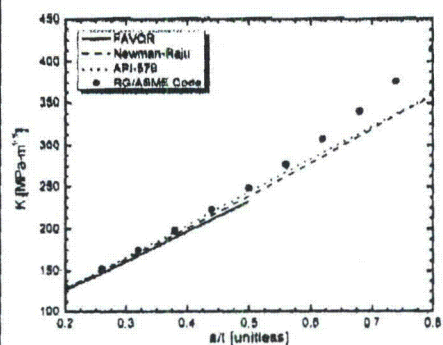
### Hoop Stress



- Approximately linear distribution
- ID: ~400 MPa, OD: ~380 MPa

## Stress Intensity Factor: Pressure

### Comparison of Different Methods



- Agreement up to  $a/t = 0.5$ .
- RG/ASME Code equation becomes increasingly conservative for  $a/t > 0.5$ .



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## Conclusion



- The RG/ASME Code equations for estimating stress intensity factor under thermal and internal pressure loading conditions agree with or conservatively bound independent methods for  $R/t = 20$ .
- Reference: Conference Proceeding PVP2012-78229.