

**Theoretical Analysis
of Pressure Vessels**

Wanda Berent, David Richard, Mark Kirk
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Outline



- Introduction and Motivation
- Analysis Methods
- Results
- Conclusion

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RG 1.161 and Appendix K Overview



- Regulatory Guide 1.161 and ASME Section XI, Nonmandatory Appendix K provide procedures for calculating adequate Charpy upper shelf energy to protect against ductile fracture of the vessel.
- The applied J -Integral is estimated from stress intensity factor equations.

$$K_{Ip}^{Axial} = (SF) p_a [1 + (R_t/t)] (\pi a)^{0.5} F_1$$

$$F_1 = 0.982 + 1.006(a/t)^2$$

$$K_{II} = [(CR)/1000]^{2.5} F_3$$

$$F_3 = 0.69 + 3.217(a/t) - 7.435(a/t)^2 + 3.532(a/t)^3$$

$$a_c = a + \frac{1}{6\pi} \left(\frac{K_{Ip} + K_{II}}{\sigma_y} \right)^2$$

Re-evaluate using a_c ,
giving K_{Ip}' and K_{II}'

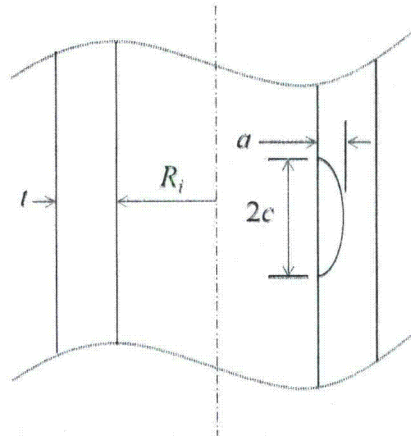
$$J = 1000 \frac{(K_{Ip}' + K_{II}')^2}{E'}$$

RG 1.161 and Appendix K

Overview



- Flaw geometry, $a/c = 1/3$



RG 1.161 and Appendix K

Motivation



- These stress intensity factor equations were derived for $R/t = 10$, which applies to pressurized water reactor vessels.
- Boiling water reactor vessels typically have $R/t = 20$.
- The thinner vessels are expected to be less susceptible to steep thermal gradients.
- This work further explores the validity of using the derived equations for the $R/t = 20$ case, through the use of independent methods of calculating stress intensity factor.

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Temperature Profiles

The Heat Equation



- Parabolic partial differential equation: $\rho c_p \frac{\partial T}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left(kr \frac{\partial T}{\partial r} \right)$
- Boundary conditions:
 - $q = h_c (T_f - T_s)$ for $r = R_i$ and all τ
 - $q = 0$ for $r = R_o$ and all τ
 - $T_f = T_0 - (CR)\tau$
- Initial conditions: $T = T_0$ for all r and $\tau = 0$

The Heat Equation

Solving Strategies for Temperature Profiles



- Finite element analysis
- Crank-Nicolson method
- Analytical solutions
- Resource: Kreyszig, Advanced Engineering Mathematics
- This work
 - Commercial engineering coding software with a built-in PDE solver
 - Skeel and Berzins, "A Method for the Spatial Discretization of Parabolic Equations in One Space Variable," SIAM J. Sci. and Stat. Comput.

Calculation of Stresses

Pressure and Thermal Hoop Stresses



- Pressure stresses in a thick-walled cylinder:

$$\sigma_{\theta\theta} = \frac{pR_i^2}{R_o^2 - R_i^2} \left[1 + \left(\frac{R_o}{r} \right)^2 \right]$$

- Thermal stresses in a cylinder:

$$\sigma_{\theta\theta} = \frac{\alpha E}{1 - \nu} \frac{1}{r^2} \left(\frac{r^2 + R_i^2}{R_o^2 - R_i^2} \int_{R_i}^{R_o} T r dr + \int_{R_i}^r T r dr - T r^2 \right)$$

Calculation of Stress Intensity Factor

Three Independent Methods



- American Petroleum Institute Standard 579-1:

$$\sigma(x) = \sigma_0 + \sigma_1 \left(\frac{x}{t} \right) + \sigma_2 \left(\frac{x}{t} \right)^2 + \sigma_3 \left(\frac{x}{t} \right)^3 + \sigma_4 \left(\frac{x}{t} \right)^4$$

$$K_I = \left[\sigma_0 G_0 + \sigma_1 G_1 \left(\frac{a}{t} \right) + \sigma_2 G_2 \left(\frac{a}{t} \right)^2 \right] \sqrt{\frac{\pi a}{Q}} + \dots$$

$$\left[\sigma_3 G_3 \left(\frac{a}{t} \right)^3 + \sigma_4 G_4 \left(\frac{a}{t} \right)^4 \right] \sqrt{\frac{\pi a}{Q}}$$

$$Q = 1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \quad \text{for } a/c \leq 1.0 \quad Q = 1 + 1.464 \left(\frac{c}{a} \right)^{1.65} \quad \text{for } a/c > 1.0$$

Calculation of Stress Intensity Factor

Three Independent Methods



- Fracture Analysis of Vessels – Oak Ridge (FAVOR) method:
 - Similar to the API-579-1 method, except that a 3rd order polynomial is used for the stress distribution.
- Newman and Raju for internal pressure:

$$K_I = \frac{pR_i}{t} \sqrt{\pi \frac{a}{Q}} F \left(\frac{a}{c}, \frac{a}{t}, \frac{R}{t}, \phi \right)$$

Calculation of Stress Intensity Factor

Application of the Methods



- API-579-1 and FAVOR methods:
 - Are based upon the principle of elastic superposition.
 - May be applied for multiple loading conditions, if:
 - All conditions are mode I and
 - The polynomial used to describe the stress fits the actual stress profile well.
- The Newman and Raju equation was derived only for internal pressure and should be used only for that case.

Inputs

Geometry, Material Properties, etc.



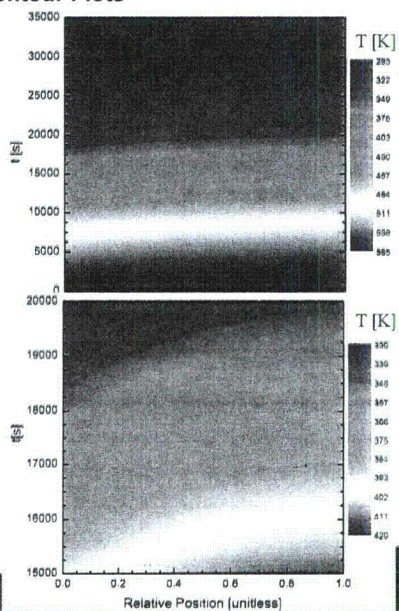
Description	Value
Geometry	
t	0.16 m [0.51 ft]
R/t	20.5
a/t	0.1 to 0.8
a/c	1/3
Material Properties	
E	2.0×10^6 MPa [4.2×10^6 kip/ft ²]
ν	0.30
α	1.30×10^{-6} m/m/K [7.20×10^{-6} ft/ft/R]
k	38 W/(m·K) [22 BTU/(hr·ft·R)]
ρ	7770 kg/m ³ [485 lb _m /ft ³]
c_p	536 J/(kg·K) [0.13 BTU/(lb _m ·R)]
Thermal Loading Conditions	
h_c	5673 W/(m ² ·K) [1000 BTU/(hr·ft ² ·R)]
C/R	0.02 K/s [100 R/hr]
T_∞	561 K [1010 R]
Mechanical Loading Conditions	
p	19.0 MPa [396 kip/ft ²]

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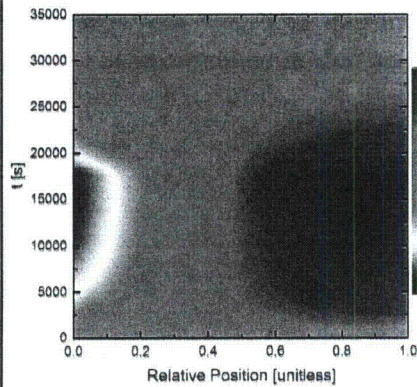
Temperature Distribution Contour Plots



- Small gradients at the beginning and end of the cooldown.
- Highest gradient observed between 15 000 s and 20 000 s.

Thermal Stress Distribution

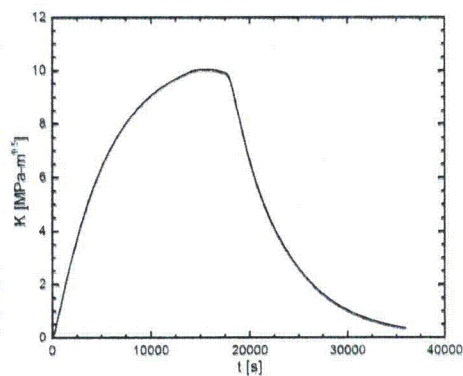
Hoop Stress Contour



- Vanishing thermal stresses early and late in the transient.
- Highest magnitude stresses observed between 5 000 and 20 000 s.
- Tension at ID, compression at OD.

Stress Intensity Factor: Thermal

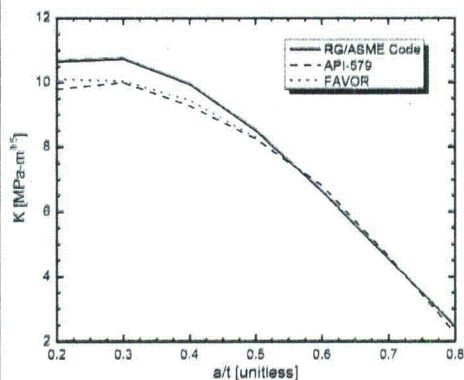
Time History



- API-579-1 method with $a/c = 1/3$, $a/t = 1/4$.
- Highest stress intensity factor observed at 14 000 s.

Stress Intensity Factor: Thermal

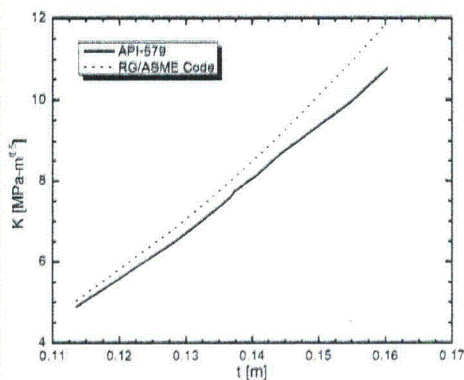
Comparison of Different Methods



- Maximum K in the transient.
- RG/ASME Code equations bound API-579-1 and FAVOR methods for $a/t < 0.55$.
- Subsequent agreement.

Stress Intensity Factor: Thermal

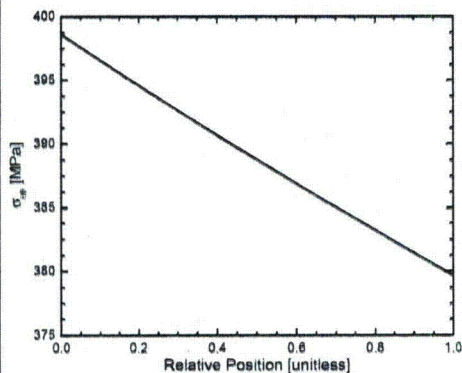
Effect of Vessel Thickness



- RG/ASME Code equation conservatively bounds the API-579-1 method.

Pressure Stress Distribution

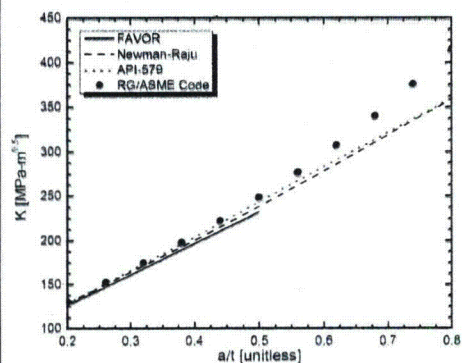
Hoop Stress



- Approximately linear distribution
- ID: ~400 MPa, OD: ~380 MPa

Stress Intensity Factor: Pressure

Comparison of Different Methods



- Agreement up to $a/t = 0.5$.
- RG/ASME Code equation becomes increasingly conservative for $a/t > 0.5$.

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Conclusion



- The RG/ASME Code equations for estimating stress intensity factor under thermal and internal pressure loading conditions agree with or conservatively bound independent methods for $R/t = 20$.
- Reference: Conference Proceeding PVP2012-78229.