

# ***Effect of Weld Residual Stress Fitting on Stress Intensity Factor for Circumferential Surface Cracks in Pipe***

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# Acknowledgments

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***The views expressed herein are those of the author and do not represent an official position of the US Nuclear Regulatory Commission.***

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# Background

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- ***Recent studies have shown that the crack growth of PWSCC is mainly driven by the weld residual stress (WRS) within the dissimilar metal weld***
- ***The existing stress intensity factor (K) solutions for surface cracks in pipe typically require a polynomial stress distribution through the pipe wall thickness. (e.g., in API RP 579, the through thickness stress distribution can be represented as a 4th order polynomial fit)***
- ***However, if the through thickness stress distribution contains steep gradients or sharp fluctuation (as in typical WRS distributions), it may not be feasible to accurately represent the stress distribution with a 4th order polynomial fit***
- ***Furthermore, the uncertainty in the accuracy of calculated stress intensity factors is questionable***

## Background (cont'd)

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- ***Universal Weight Function Method (UWFM) which does not require a polynomial fit to the actual stress distribution has been proposed for implementation into the ASME Code Section XI – [See PVP2012-78236]***
- ***In this method, piece-wise cubic variation of stress between discrete locations where stresses are known are used to calculate the stress intensity factor***
- ***Solutions available for the deepest point and the surface point of a semi-elliptical surface crack in a cylinder or a flat plate***

# Universal Weight Function Method

## ■ UWFM for circumferential surface crack

$$K_I = \int_0^a \sigma(x) m(x, a) dx$$

### ◆ Deepest point

$$m(x, a) = \frac{2}{(2\pi(a-x))^{1/2}} \left[ 1 + M_1 \left( 1 - \frac{x}{a} \right)^{1/2} + M_2 \left( 1 - \frac{x}{a} \right) + M_3 \left( 1 - \frac{x}{a} \right)^{3/2} \right]$$

$$M_1 = \frac{2\pi}{\sqrt{2Q}} (3G_1 - G_0) - \frac{24}{5}$$

$$M_2 = 3$$

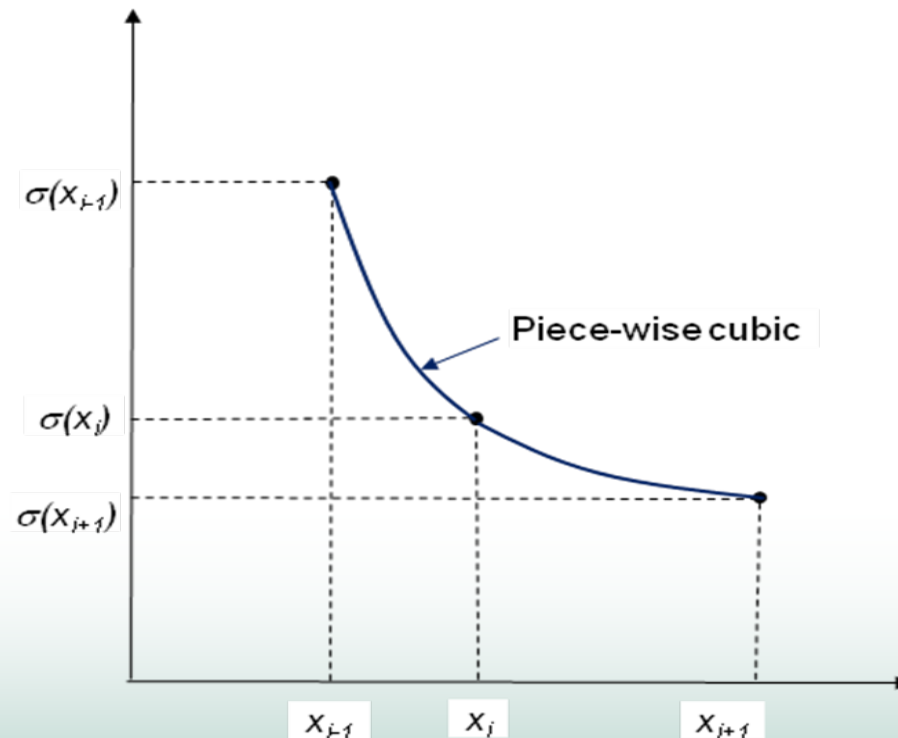
$$M_3 = \frac{6\pi}{\sqrt{2Q}} (G_0 - 2G_1) + \frac{8}{5}$$

→ **API 579 (2000) solutions**

# Universal Weight Function Method (cont'd)

## ■ Piece-wise cubic stress interpolation

$$\sigma_i(x) = d_{0i} + d_{1i}x + d_{2i}x^2 + d_{3i}x^3 \quad (i = 1, \dots, n)$$



# Polynomial Stress Distribution Method

- **Polynomial stress distribution through the wall thickness - e.g. 4th order polynomial stress distribution represented as:**

$$\sigma(x) = \sigma_o + \sigma_1 \left( \frac{x}{t} \right) + \sigma_2 \left( \frac{x}{t} \right)^2 + \sigma_3 \left( \frac{x}{t} \right)^3 + \sigma_4 \left( \frac{x}{t} \right)^4$$

## ❖ **$G_0, G_1$ - API 579 (2000) solutions**

$$K_I = \sum_{i=0}^4 \sigma_i G_i \left( \frac{a}{t} \right)^i \sqrt{\frac{\pi a}{Q}}$$

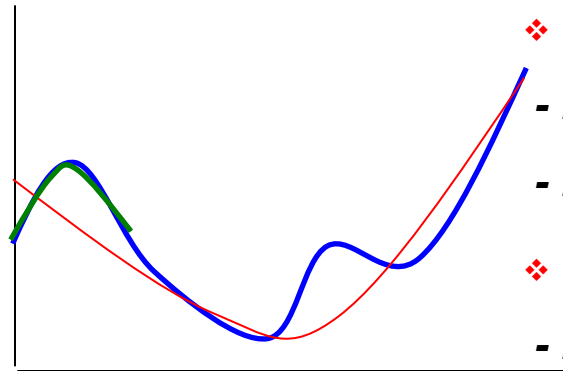
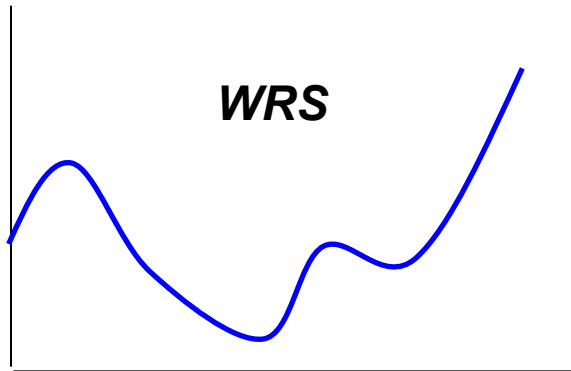
$$G_2 = \frac{\sqrt{2Q}}{\pi} \left( \frac{16}{15} + \frac{1}{3} M_1 + \frac{16}{105} M_2 + \frac{1}{12} M_3 \right)$$

$$G_3 = \frac{\sqrt{2Q}}{\pi} \left( \frac{32}{35} + \frac{1}{4} M_1 + \frac{32}{315} M_2 + \frac{1}{20} M_3 \right)$$

$$G_4 = \frac{\sqrt{2Q}}{\pi} \left( \frac{256}{315} + \frac{1}{5} M_1 + \frac{256}{3465} M_2 + \frac{1}{30} M_3 \right)$$



# Workscope

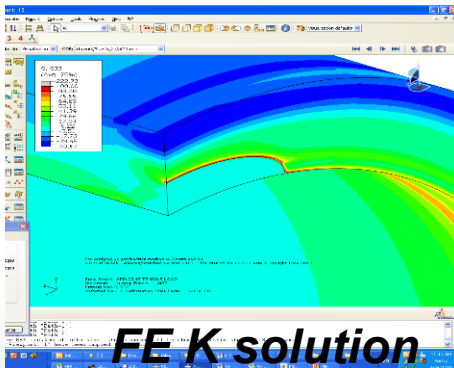


## ❖ Polynomial fit method

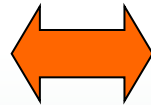
- Entire polynomial fit
- Partial polynomial fit

## ❖ UWFM solution

- Piece-wise cubic fit

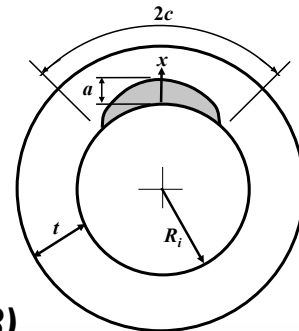


Compare



## ❖ K solutions for various semi-elliptical crack sizes:

- $a/t=0.2, 0.4, 0.6, 0.8$  (fix  $c/a=5, R_i/t=3$ )
- K values at deepest and surface points



**Investigate the potential sensitivity of stress intensity factors to the 4th order polynomial fitting artifacts**

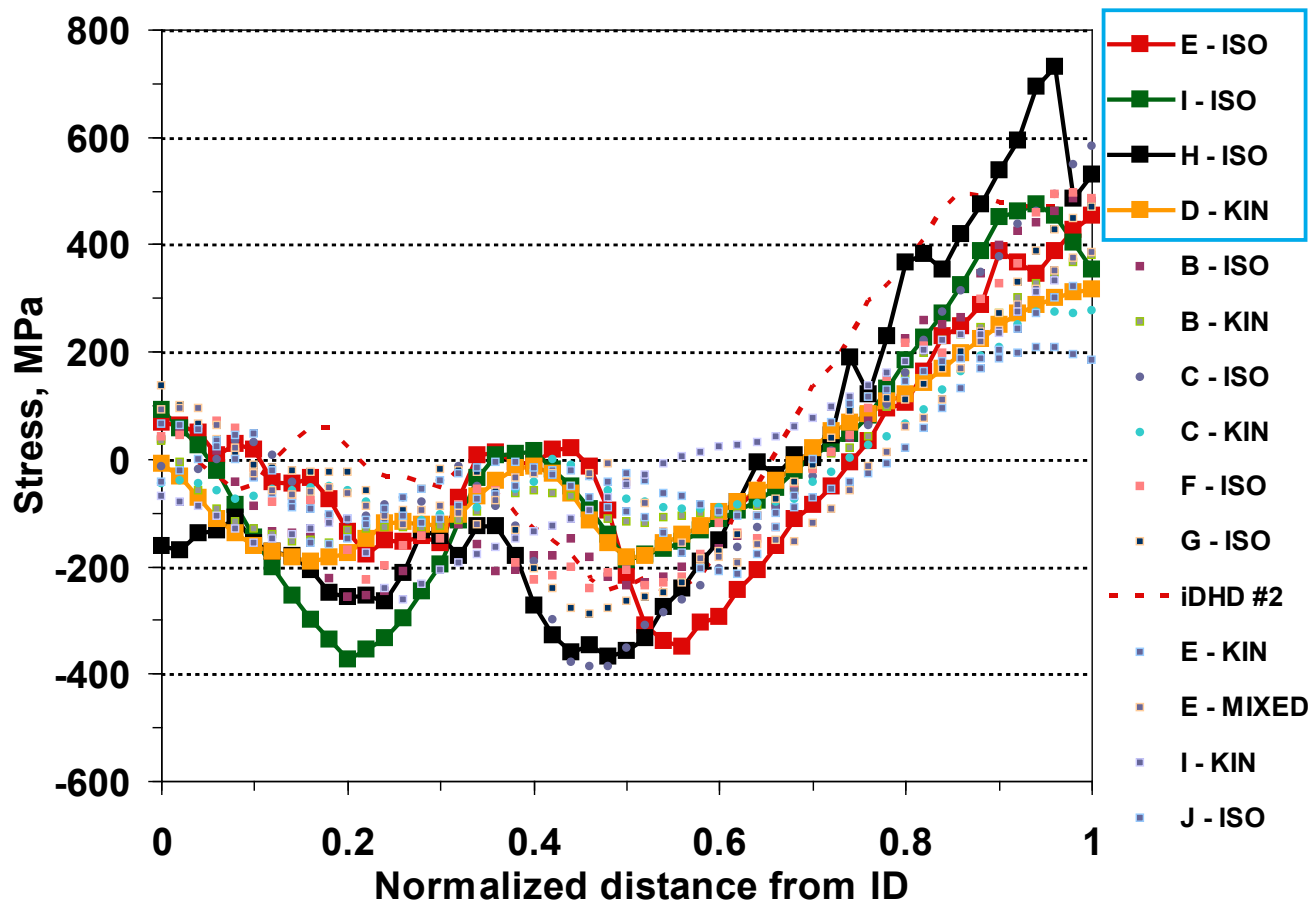
## Select Welding Residual Stress

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- ***Selected from the Phase II of the NRC/EPRI Welding Residual Stress Validation Program***
- ***Cases that could not be well represented by 3<sup>rd</sup> (or 4<sup>th</sup>) order polynomial fits***
- ***Cases that had multiple ‘peaks and valleys’***
- ***Cases that bounded the entire data set***

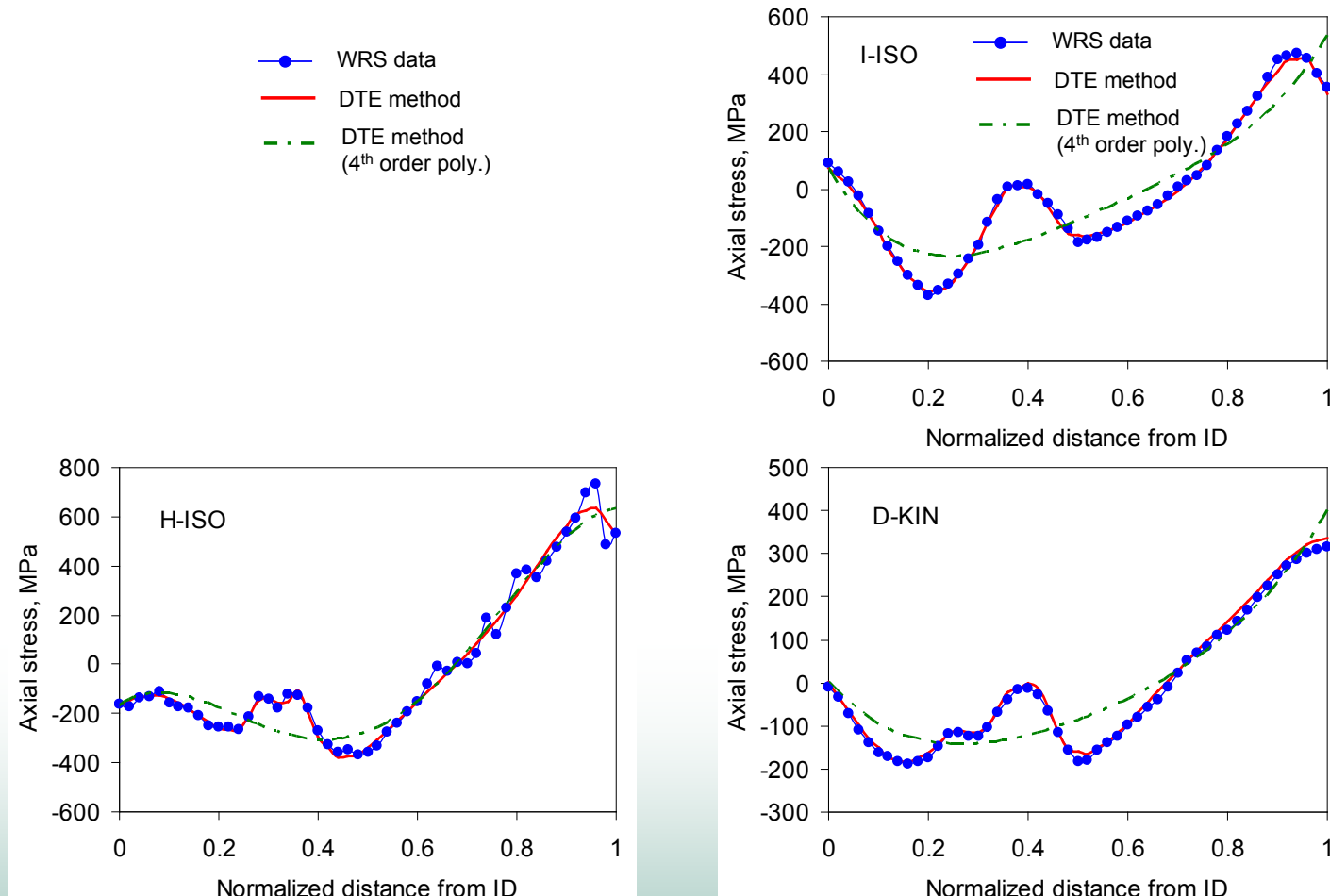
## Selected Welding Residual Stress (cont'd)

- Selected from the Phase II of the NRC/EPRI Welding Residual Stress Validation Program



# Selected Welding Residual Stress (cont'd)

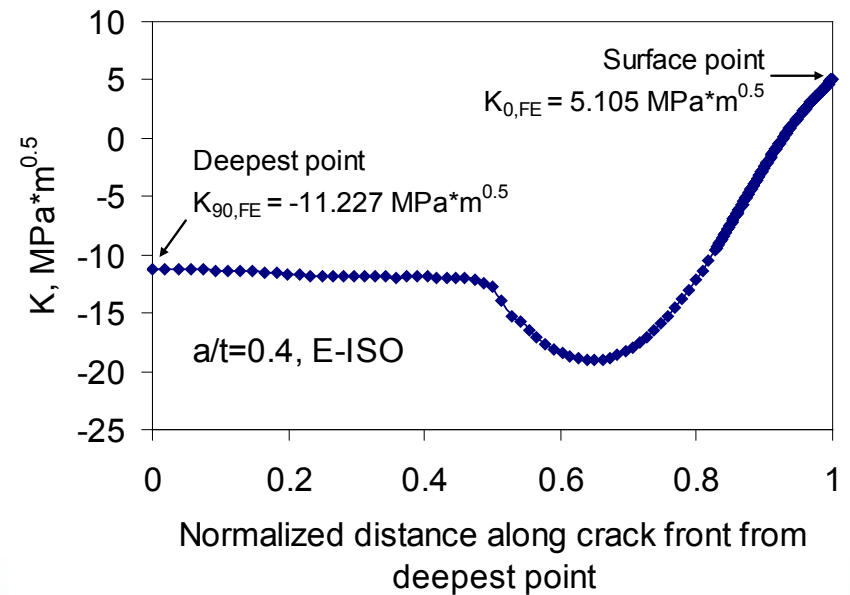
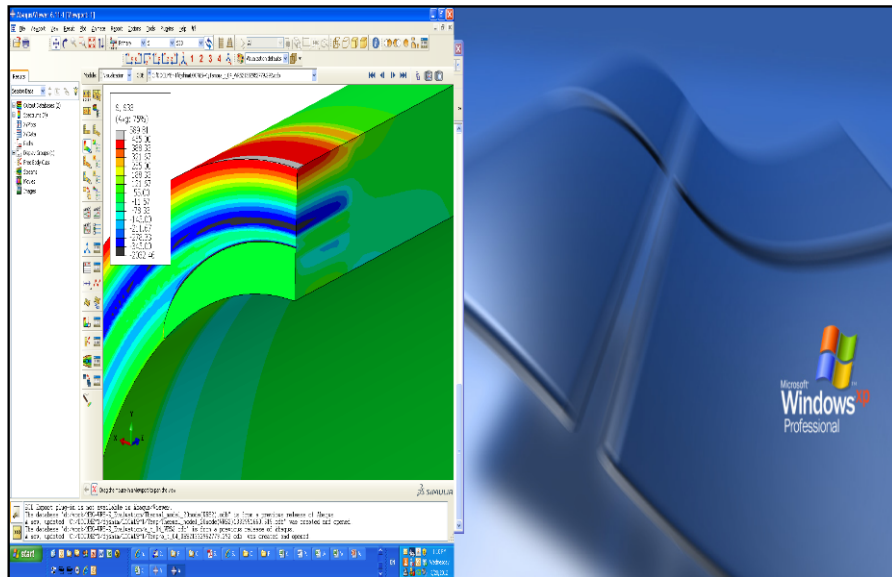
## ■ 4<sup>th</sup> order polynomial representation



**DTE (Differential Thermal Expansion) Method : A calibrated thermal gradient is applied through the pipe thickness to generate the WRS profile**

# Finite Element Based $K$ Solutions

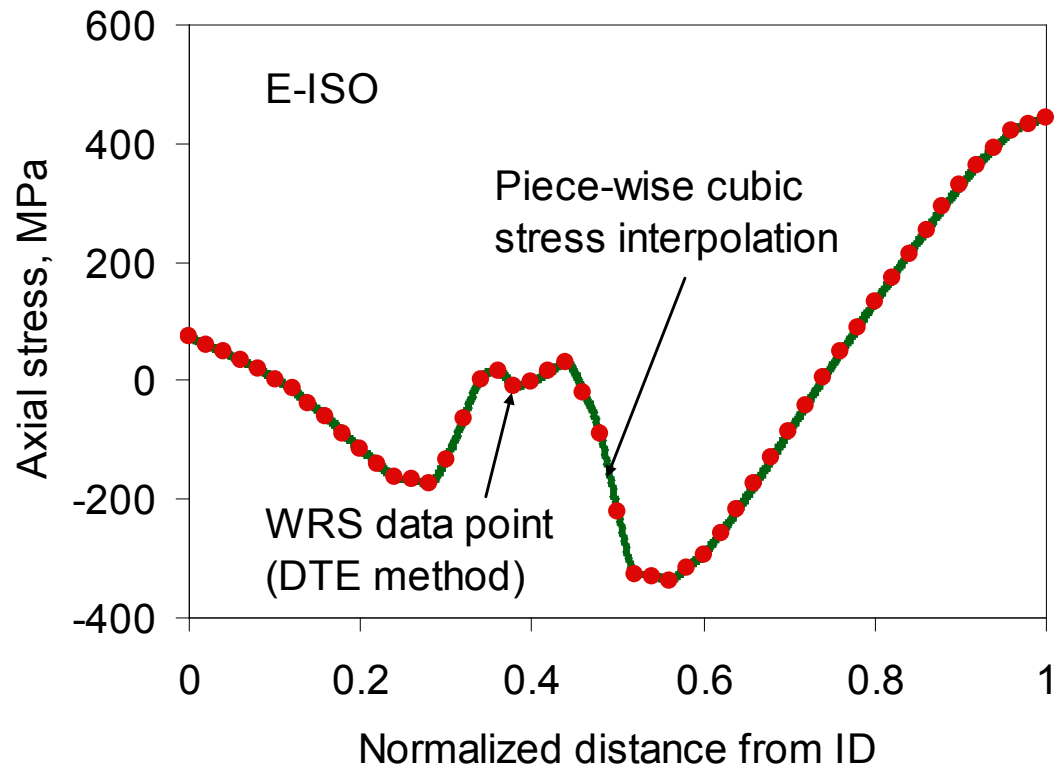
## ■ FE based $K$ solution



# Piece-wise Cubic Stress Fit

- **Example of piece-wise cubic stress fit for UWFM**

- ◆ **51 data points through the thickness**



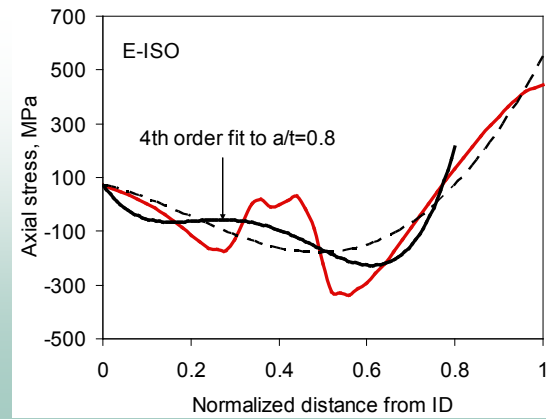
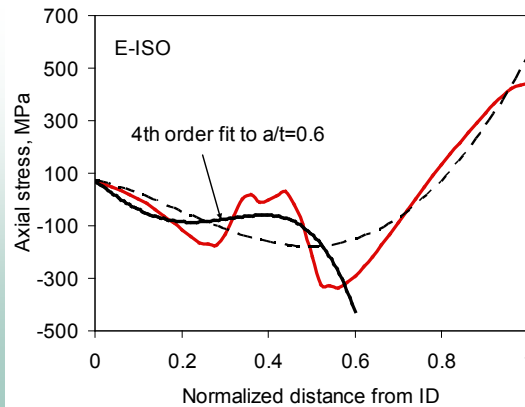
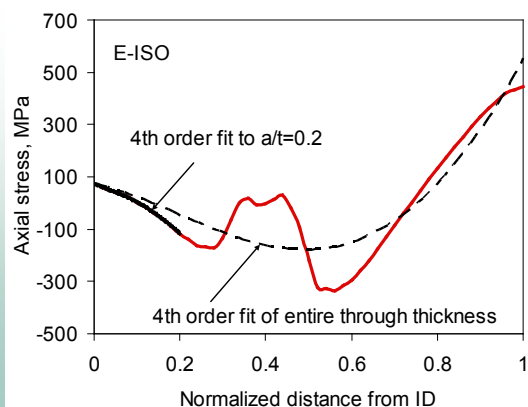
# Polynomial Fits

## ■ Examples of 4<sup>th</sup> order polynomial fit

### ◆ Using entire thickness data

- WRS data
- DTE method
- - - DTE method (4<sup>th</sup> order poly.)

### ◆ Using data up to crack-tip



# Comparison of Results

## ■ Deepest point

WRS	a/t	$\frac{K_{90,UWFM}}{K_{90,FE}}$	$\frac{K_{90,Poly}}{K_{90,FE}}$	$\frac{K_{90,Poly-tip}}{K_{90,FE}}$
E-ISO	0.2	1.015	-0.025	1.021
	0.4	1.059	1.678	0.984
	0.6	0.992	0.797	1.041
	0.8	1.026	0.980	1.013
I-ISO	0.2	1.014	0.810	1.013
	0.4	1.012	1.313	0.987
	0.6	1.014	0.934	1.062
	0.8	1.019	0.915	1.022
H-ISO	0.2	0.999	0.780	0.996
	0.4	1.019	1.149	0.995
	0.6	1.020	0.973	1.026
	0.8	1.004	0.920	0.943
D-KIN	0.2	1.022	0.730	1.021
	0.4	0.991	1.240	0.987
	0.6	1.001	0.836	1.021
	0.8	1.016	0.921	0.924
Average		1.014	0.934	1.003
Standard deviation		0.016	0.353	0.034

## ■ Surface point

WRS	a/t	$\frac{K_{0,UWFM}}{K_{0,FE}}$	$\frac{K_{0,Poly}}{K_{0,FE}}$	$\frac{K_{0,Poly-tip}}{K_{0,FE}}$
E-ISO	0.2	0.989	1.088	1.037
	0.4	0.962	1.234	1.238
	0.6	0.903	1.343	0.232
	0.8	1.156	-0.291	6.553
I-ISO	0.2	1.092	0.394	1.149
	0.4	1.040	2.012	0.872
	0.6	1.043	1.369	1.806
	0.8	1.005	1.217	1.514
H-ISO	0.2	0.968	1.006	1.021
	0.4	0.959	0.958	0.950
	0.6	0.945	0.961	1.125
	0.8	0.945	0.975	1.024
D-KIN	0.2	1.010	0.775	1.110
	0.4	1.012	0.787	1.178
	0.6	0.982	0.848	1.253
	0.8	0.959	0.874	1.057
Average		0.998	0.972	1.445
Standard deviation		0.062	0.486	1.400



# Application to Crack Growth Analyses

## ■ Input for crack growth calculations

### Geometry Used

Ri: 142 mm

t: 47.33 mm

Ri/t: 3

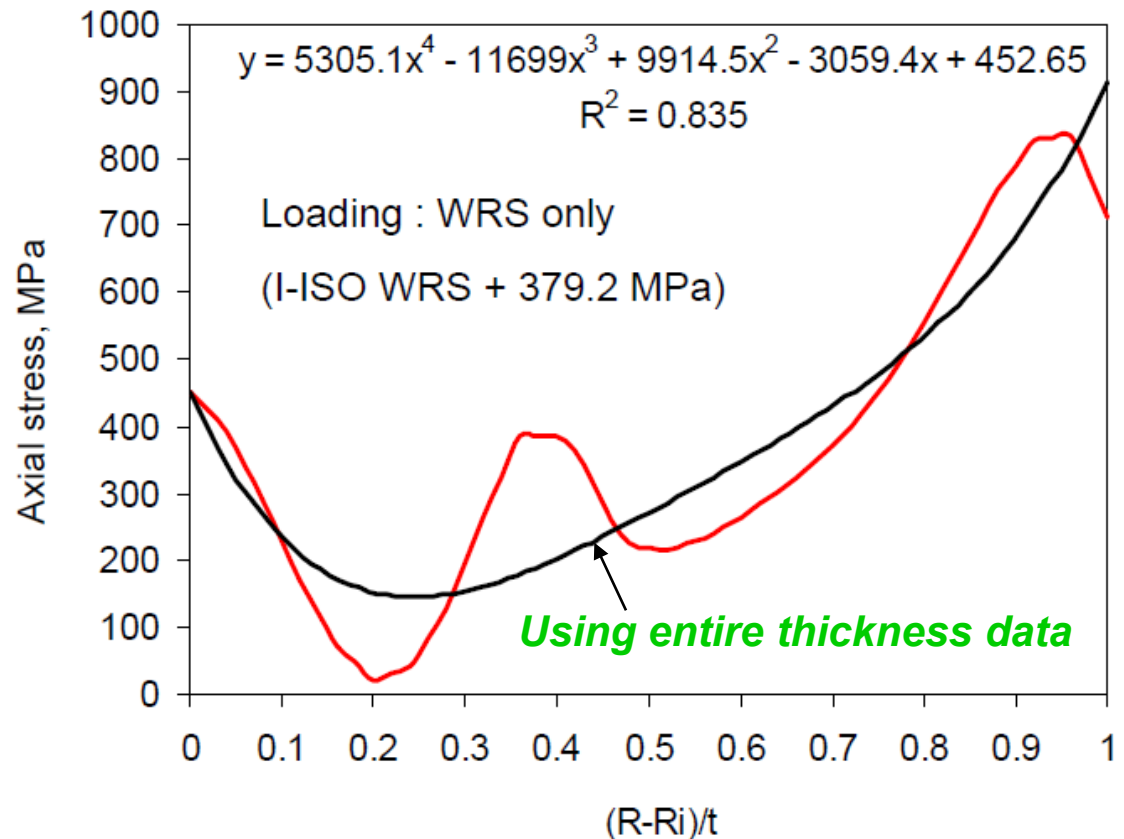
### Initial Crack Size

c/a: 20 (c=94.66 mm)

a/t: 0.1 (a=4.733 mm)

### Crack Growth

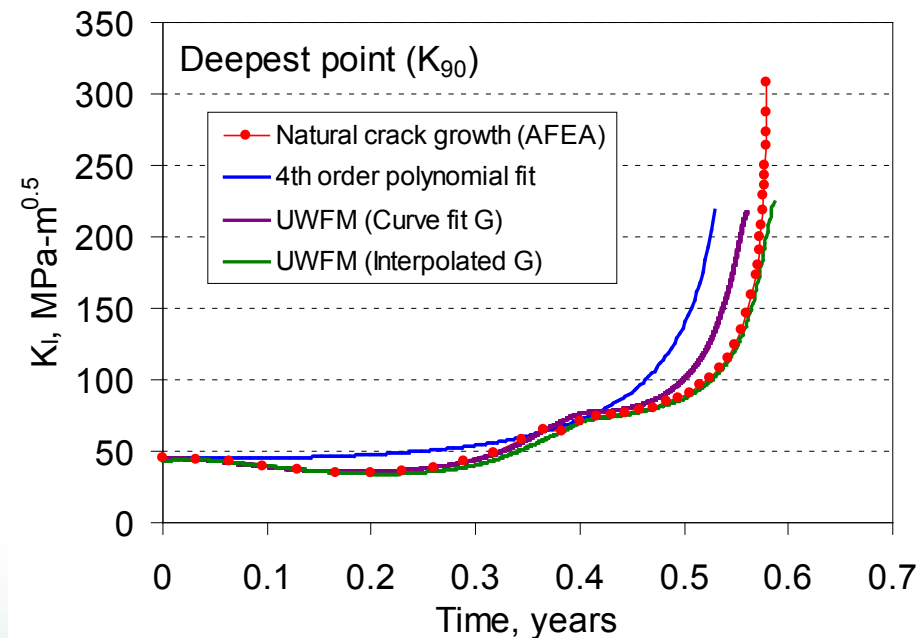
Alloy182 CGR at 644F (340C)



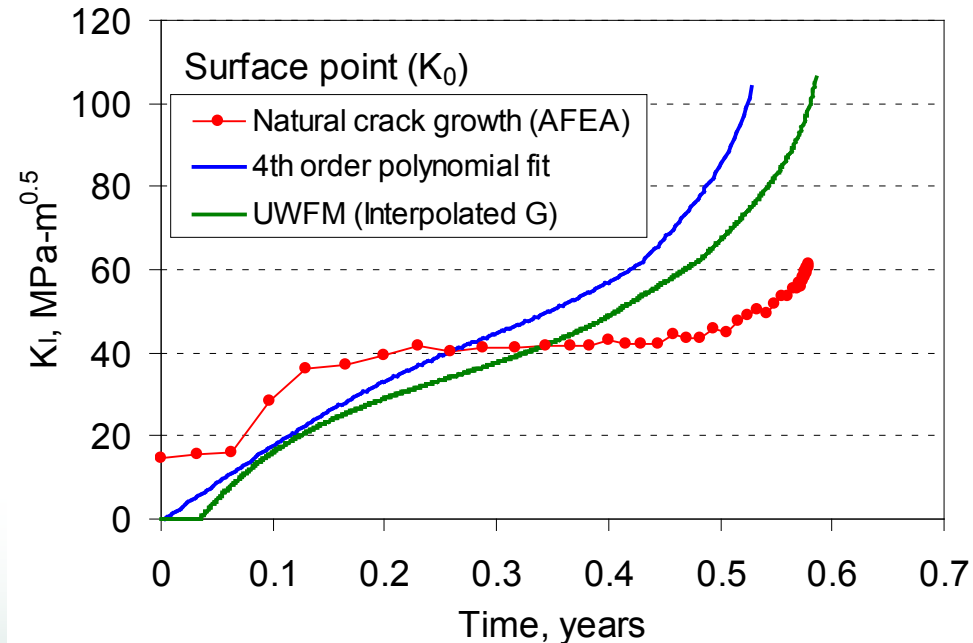
# Application to Crack Growth Analyses (cont'd)

## ■ Crack growth results – $K$ values

### ◆ Deepest point



### ◆ Surface point

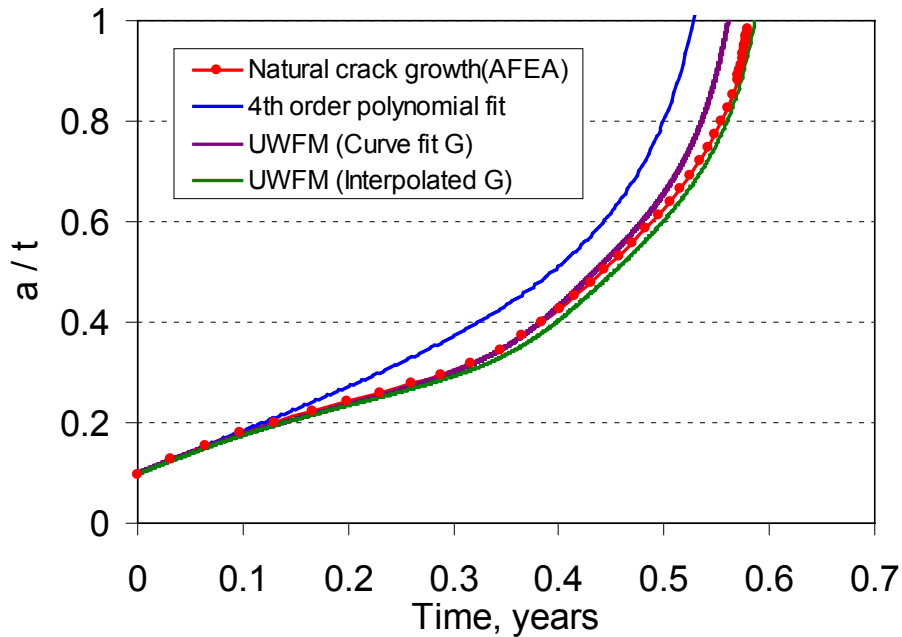


**Note : 4<sup>th</sup> order polynomial fit using entire thickness data**

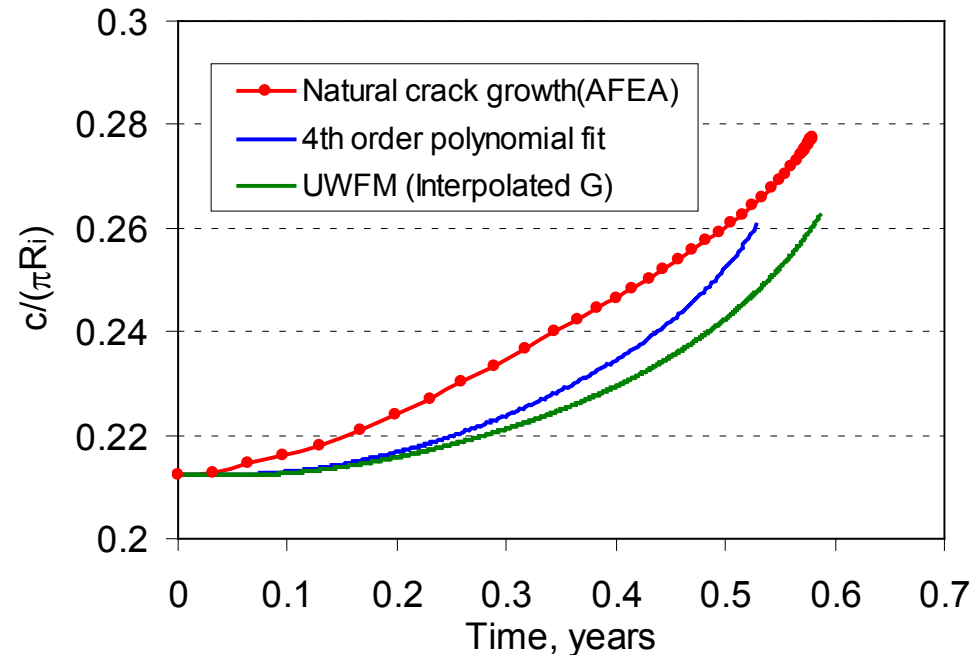
# Application to Crack Growth Analyses (cont'd)

## ■ Crack growth results – Crack shapes

### ◆ Crack depth



### ◆ Crack length



**Note : 4<sup>th</sup> order polynomial fit using entire thickness data**

## Concluding Remarks

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- ***The UWFM solutions showed good agreement (within 1.5% difference on average) with the FE results – for both deepest and surface points***
- ***The results of the polynomial stress distribution method showed larger difference compared to the UWFM results. For the deepest point, the difference was reduced when the polynomial fit was conducted using the stress data up to the crack-tip. However, this trend was not shown for the surface point.***
- ***The results of this study demonstrate the potential sensitivity of stress intensity factors to polynomial stress fitting artifacts***

## Concluding Remarks (cont'd)

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- *The piece-wise WRS representations used in the UWFM is not sensitive to these fitting artifacts*
- *Crack growth analyses results demonstrate that the polynomial stress fitting artifacts can affect the crack growth calculation results*