5100. Reinforced Concrete

5110
- 5110 - Introduction: Materials & Design Methods

5120 & 5130
- 5120 - Moment Design of Beams
- 5130 - Shear Design of Beams

5140 & 5150
- 5140 - Footing Design
- 5150 - Column Design

5160 & 5170
- 5160 - Development & Splices of Reinforcement
- 5170 - Strut and Tie Model

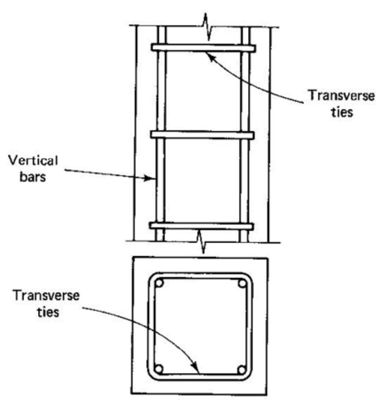
5180
- 5180 - Two-way Slabs

5150 - Column Design
Columns (ACI 2.1)

- Member with a ratio of height-to-least lateral dimension exceeding 3 used primarily to support axial compressive load
- Shorter members may be unreinforced and treated as pedestal footings
- Concrete compression members may be classified into three categories:
  - Pedestals or short compression blocks
  - Short reinforced concrete columns
  - Long or slender reinforced concrete columns

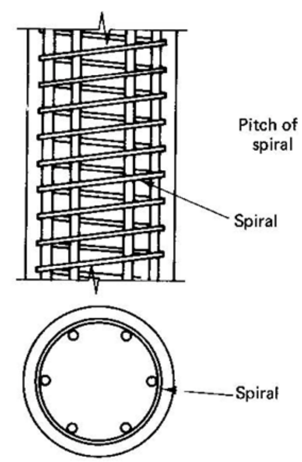
Design of Concrete Structures
Columns - Types

- Tied columns
  - Usually square or rectangular, but can also be made into octagonal, round, L-shape, etc.
  - Reinforcing bars supported by separate lateral ties.



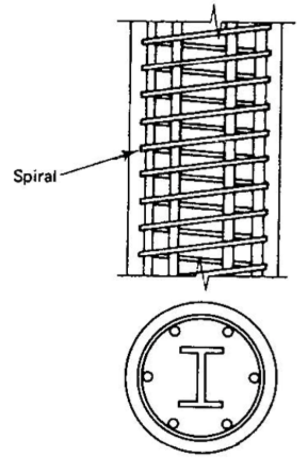
Design of Concrete Structures
Columns - Types

- Spiral columns
- Usually circular but can also be made into square, octagonal, or other shapes. For such columns, circular arrangements of the bars are still used.
- Longitudinal reinforcing bars are arranged in a circle and wrapped by a spiral.
- More expensive compared to tied columns



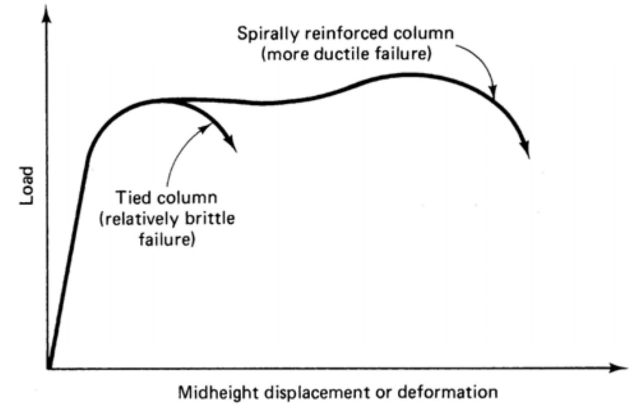
Design of Concrete Structures
Columns - Types

- Composite columns
  - Steel encased concrete core
  - Structural Steel shapes encased in concrete and further reinforced by longitudinal & lateral reinforcement



Design of Concrete Structures
Columns - Types

- Comparison of load-deformation behavior of tied and spirally bound columns



Design of Concrete Structures
Column Design

- Same Basis/Assumptions as for Flexural Design
- Forces and Moments from Elastic Analysis
- Need to Consider:
  - Effect of Axial Load/Variable Moment of Inertia on Member Stiffness
  - Effect of Deflections on Moments and Forces
  - Effect of Duration of Load (Creep)

Design of Concrete Structures
Column Design – Concentrically Loaded Short Column

- Maximum ultimate load P_0 on a concentrically loaded short column given by:
- $P_o = 0.85 f_c' (A_g - A_{st}) + f_y A_{st}$

OR

- $P_o = A_g [0.85 f_c' (1-\rho) + f_y \rho]$
  - Where, $\rho = A_{st}/A_g$

Design of Concrete Structures
Column Design – Concentrically Loaded Short Column

- Concentrically loaded columns are uncommon or non existent and eccentricity may result from

(a) end conditions

(b) inaccuracy of manufacture

(c) variations of material even if load is concentric

Design of Concrete Structures
Column Design – Concentrically Loaded Short Column

- In 1963 & 1971 codes specified minimum design eccentricities so as to reduce the axial load design strength of a section in pure compression to account for accidental eccentricities. (not less than 1 inch or 0.05h for spirally reinforced members and 0.1h for tied columns - (h = overall thickness of member)

Design of Concrete Structures
Column Design – Concentrically Loaded Short Column

- In codes since 1977 the reduction is done directly by limiting the design axial load strength of a section in pure compression to 85% (spiral columns) or 80% (tied columns) of the nominal strength

For Spiral Columns:

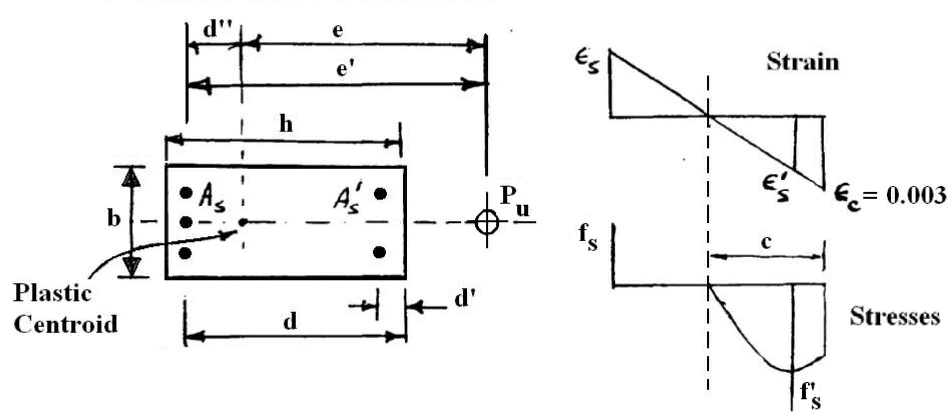
- $P_{n(max)} = 0.85P_o = 0.85[0.85\, f_c'\ (A_g - A_{st}) + f_y A_{st}]$

For Tied Columns:

- $P_{n(max)} = 0.80P_o = 0.80[0.85\, f_c'\ (A_g - A_{st}) + f_y A_{st}]$

Design of Concrete Structures
Column Design – Concentrically Loaded Rectangular Column

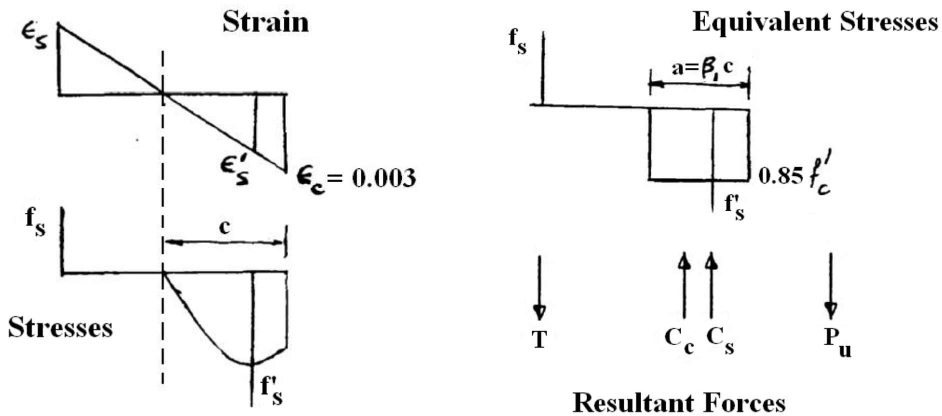
- **Column Section at Ultimate Load**



Design of Concrete Structures

Column Design – Concentrically Loaded Rectangular Column

- **Column Section at Ultimate Load (cont'd)**



Design of Concrete Structures

Column Design – Concentrically Loaded Short Column

- Failure may occur in tension or compression (failure is dependent on the axial load level)
- At ultimate load in eccentrically loaded columns, the compression steel usually reaches its yield strength; hence, it is usual to assume that the compression steel is yielding.

Design of Concrete Structures

Column Design – Concentrically Loaded Short Column

- Assuming $f_s' = f_y$
  - Equilibrium:
    - $P_u = \phi[0.85 f_c' ab (d - 0.5a) + A_s' f_y (d-d')]$
  - Taking moments about tension steel:
    - $P_u e' = \phi[0.85 f_c' ab (d - 0.5a) + A_s' f_y (d-d')]$
- e' = eccentricity of ultimate load P_u from centroid of tension steel
- e = eccentricity of P_u from plastic centroid

Design of Concrete Structures

Column Design – Concentrically Loaded Short Column

- Plastic Centroid
  - centroid of resistance of section
    - Assume uniform strain & maximum stresses ($0.85 f_c'$ for all the concrete, f_y for all the steel)
  - point of application of external load that produces an axially loaded condition at failure
- for symmetrically reinforced members, plastic centroid $\equiv$ centroid of cross-section

Design of Concrete Structures
Column Design – Concentrically Loaded Short Column

- Plastic Centroid (cont'd)
  - To locate plastic centroid, take moments about centroid of A_s
- $0.85f_c'bh\,(d - 0.5h) + A_s'f_y\,(d-d') = P_o d'' = [0.85f_c'bh + (A_s + A_s')f_y]d''$

$$d'' = \frac{0.85f_c'bh\,(d - 0.5h) + A_s'f_y\,(d-d')}{0.85f_c'bh + (A_s + A_s')\,f_y}$$

  - For the eccentrically loaded columns, take moments about plastic centroid
- $P_u\,e = \phi[0.85f_c'ab\,(d-d'' - 0.5a) + A_s'f_y\,(d - d' - d'') + A_s f_s\,d'']$

Design of Concrete Structures
Column Design – Concentrically Loaded Short Column

- Plastic Centroid (cont'd)
  - For balanced condition: steel reaches its yield strength when the extreme fiber concrete reaches a compressive strain of 0.003 at the same time
- From similar triangles

$$\frac{0.003}{c_b} = \frac{f_y/E_s}{d - c_b} \qquad \text{or} \qquad c_b = \frac{0.003\,E_s d}{f_y + 0.003E_s}$$

c_b = c at balanced condition

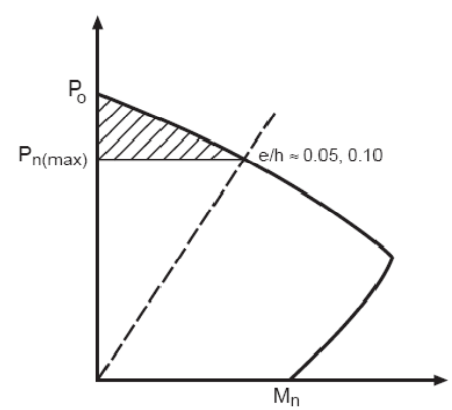
- $a_b = \beta_1\,c_b = \dfrac{0.003\,E_s\,\beta_1 d}{f_y + 0.003E_s}$

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Recall
  - For Spirally Reinforced Members,
    - $P_{n(max)} = 0.85P_o = 0.85[0.85\,f_c'\,(A_g - A_{st}) + f_y\,A_{st}]$
  - For Tied Reinforced Members,
    - $P_{n(max)} = 0.80P_o = 0.80[0.85\,f_c'\,(A_g - A_{st}) + f_y\,A_{st}]$

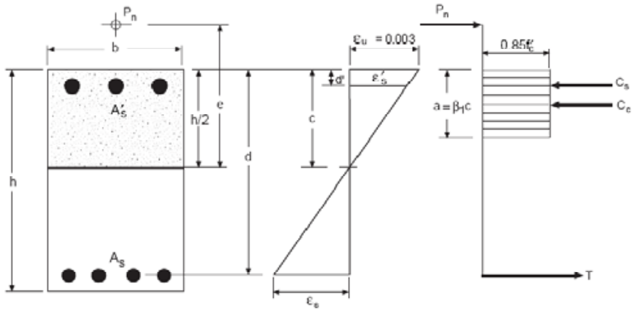
Design of Concrete Structures
Column Design – Columns in Pure Compression

- Maximum Axial Strength
  - Axial Force vs. Moment



Design of Concrete Structures
Column Design – Columns in Pure Compression

- Nominal Strength for combined Flexure & Axial Load

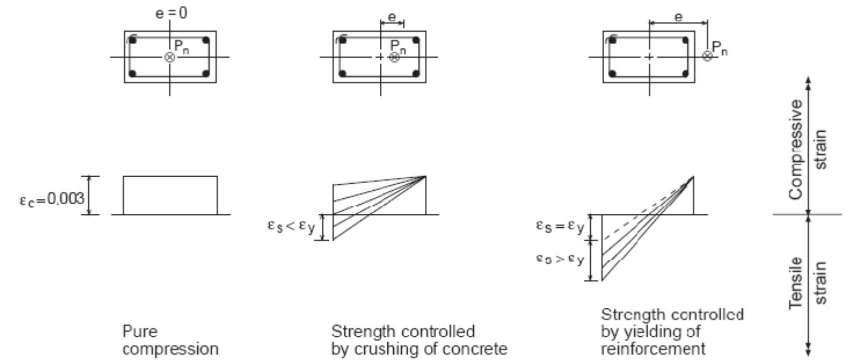


$\varepsilon_s \leq \varepsilon_y$ (Compression Controls)
$\varepsilon_s = \varepsilon_y$ (Balanced Condition)
$0.005 > \varepsilon_s > \varepsilon_y$ (Transition)
$\varepsilon_s \geq 0.005\varepsilon_y$ (Tension Controls)

Strain & Equivalent Stress Distribution

Design of Concrete Structures
Column Design – Columns in Pure Compression

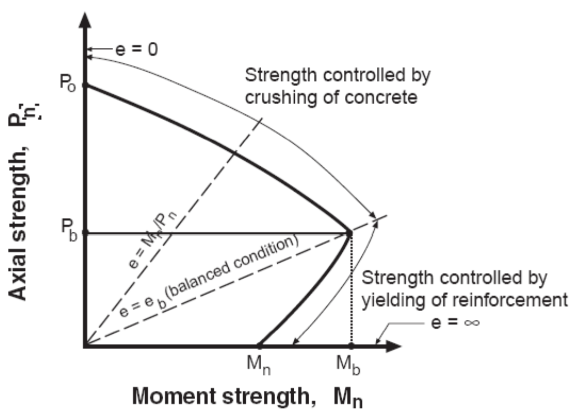
- Nominal Strength for combined Flexure & Axial Load



Strain Variation for Full Range of Load – Moment Interaction

Design of Concrete Structures
Column Design – Columns in Pure Compression

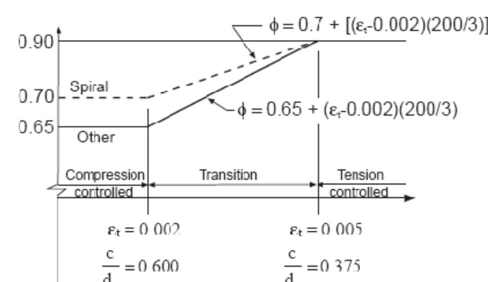
- Nominal Strength for combined Flexure & Axial Load



Axial Load and Moment Interaction Diagram

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Nominal Strength for combined Flexure & Axial Load



$$\text{Interpolation on } c/d_t\text{: Spiral } \phi = 0.70 + 0.20\left[\left(\frac{1}{c/d_t}\right) - \left(\frac{5}{3}\right)\right]$$

$$\text{Other } \phi = 0.65 + 0.25\left[\left(\frac{1}{c/d_t}\right) - \left(\frac{5}{3}\right)\right]$$

ϕ vs Net Tensile ε_t and c/dt for Grade 60 Steel

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Tension-Controlled Sections and Transition (ACI 10.3.4)

$$c_t = 0.375 d_t$$

$$a_t = \beta_1 c_t = 0.375 \beta_1 d_t$$

$$C_t = 0.85 f'_c b a_t = 0.319 \beta_1 f'_c b d_t$$

$$T = A_s f_y = C_t$$

$$A_s = 0.319 \beta_1 f'_c b d_t / f_y$$

$$\rho_t = A_s / (b d_t) = 0.319 \beta_1 f'_c / f_y$$

$$\omega_t = \frac{\rho_t f_y}{f'_c} = 0.319 \beta_1$$

$$M_{nt} = \omega_t (1 - 0.59 \omega_t) f'_c b d_t^2$$

$$R_{nt} = \frac{M_{nt}}{b d_t^2} = \omega_t (1 - 0.59 \omega_t) f'_c$$

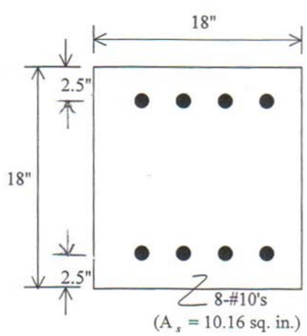
Design of Concrete Structures
Column Design – Columns in Pure Compression

- Tension-Controlled Sections and Transition (ACI 10.3.4)
  - Recall, design parameters for Tension Controlled Sections

			$f'_c = 3000$ $\beta_1 = 0.85$	$f'_c = 4000$ $\beta_1 = 0.85$	$f'_c = 5000$ $\beta_1 = 0.80$	$f'_c = 6000$ $\beta_1 = 0.75$	$f'_c = 8000$ $\beta_1 = 0.65$	$f'_c = 10,000$ $\beta_1 = 0.65$
R_{nt}			683	911	1084	1233	1455	1819
ϕR_{nt}			615	820	975	1109	1310	1637
ω_t			0.2709	0.2709	0.2550	0.2391	0.2072	0.2072
ρ_t	Grade	40	0.02032	0.02709	0.03187	0.03586	0.04144	0.05180
		60	0.01355	0.01806	0.02125	0.02391	0.02762	0.03453
		75	0.01084	0.01445	0.01700	0.01912	0.02210	0.02762

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Interaction Diagram Calculations



$f_c' = 3{,}750$ psi

$f_y = 50{,}000$ psi

$E_s = 2.8 \times 10^7$ psi

- Point 1: Zero Moment

$\varepsilon_c = .003$ in/in

$\varepsilon_y = \dfrac{5.0 \times 10^4}{2.9 \times 10^7} = .001724$ in/in

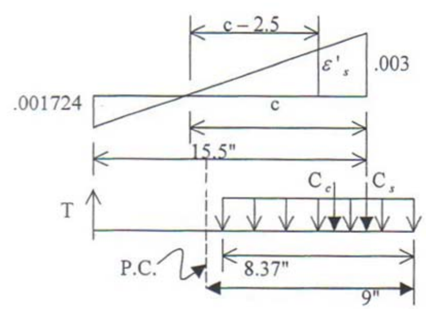
$P_u = .85\, f_c'\, A_c + A_s\, f_y$

$= .85\,(3.75)(\,324 - 10.16) + 10.16(50) = 1506^k$

$\boxed{P = 1506^k \quad M = 0}$

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Point 2: Balance Point



$$c = \frac{15.5(.003)}{.00472} = 9.85"$$

$$a = .85c = 8.37"$$

$$T = A_s f_y = 5.08(50) = 254^k$$

$$0.85\, f'_c = 3.188 \text{ ksi}$$

$$C_c = .85\, f'_c\, ab = 3.188(8.37)(18) = 480.2^k$$

$$C_s = A'_s f_y$$

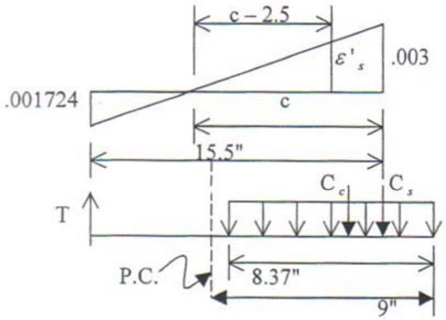
$$= 254^k$$

$\varepsilon'_s = .003\,(7.35/9.85) = .00224$ in/in

$\therefore$ Comp Steel has Yielded

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Point 2: Balance Point (cont'd)



$P = C_s + C_c – T$

$P = 254 + 480.2 - 254 = 480.2^k$

Σ Moments @ Plastic Centroid

$a/2 = 4.19$ in.

$M = (9 - 4.19)\,C_c + 6.5\,C_s + 6.5\,T$

$= 4.82\,(480.2) + 2\,(6.5)\,254$

$= 467.8^{1-k}$

$P = 480.2^k = P_b$

$M = 467.8^{1-k} = M_b$

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Point 3: Zero Axial Load (see analysis of doubly reinforced beams)

$\rho = \rho' = \dfrac{4(1.27)}{18(15.5)} = .0182 < \rho_b$ ∴ Comp. Steel Has Not Yielded

$< (\rho - \rho')_{min}$

$.85\,f'_c = 3.188$ ksi

$$\{3.18(18)(.85c)\} + 4(1.27)\left[29{,}000(.003)\left(\frac{c-2.5}{c}\right) - 3.18\right] = \{50(4)(1.27)\}$$

$c = 3.314$ in.

$a = 2.817$ in.

Design of Concrete Structures
Column Design – Columns in Pure Compression

- Point 3: Zero Axial Load (cont'd)

$C_c = 161.62^k = 0.85\,f'_c\,ab$

$T = 254^k$

$P = C_s + C_c - T = 0^k$

$C_s = 92.38^k$, from:

$C_s = A'_s(f'_s - 0.85\,f'_c)$

$f'_s = 87\left(\dfrac{c-d'}{c}\right) \le f_y = 21.37$ ksi

$C_s = 4(1.27)(18.19) = 92.38^k$

M = 6.5(254) + 6.5(92.38k) + (9-1.41)(161.62)

= 290^{1-k} (∑ Moments @ Plastic Centroid)

$P = 0^k$

$M = 290^{1-k}$

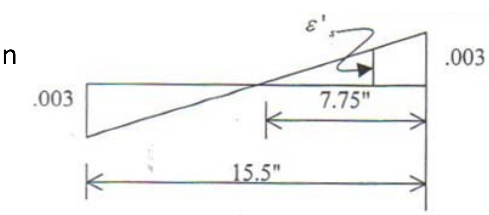
Design of Concrete Structures
Column Design – Columns in Pure Compression

- Point 4: $\varepsilon_c = .003$ in/in and $\varepsilon_s = .003$ in/in

$\varepsilon_s' = (5.25/7.75)(.003) = .00203$ in/in

∴ Comp Steel has Yielded

$a = .85(7.75) = 6.58$



$T = C_s = 254^k$ $C_c = 3.188\,(6.58)\,(18) = 377^k$

$P = 377^k$

$M = (9-3.29)\,377 + 2(6.5)254 = 454^{1-k}$

$P = 377^k$

$P = 454^{1-k}$

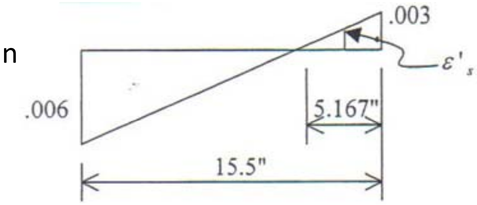
Design of Concrete Structures
Column Design – Columns in Pure Compression

- <u>Point 5: ε_u = .003 in/in and ε_s = .006 in/in</u>

ε_s' = (2.667/5.167)(.003) = .00155 in/in

∴ Comp Steel has Not Yielded

a = .85(5.167) = 4.38”



C_c = 3.188 (4.38) (18) = 251^k C_s = (.00155/.001724)(254) = 228^k

T = 254^k P = $C_s + C_c - T$ = 226^k

M = (6.5)228 + (6.5)254 + (9-2.19)251 = $454^{1\text{-}k}$

P = 226^k
M = $404^{1\text{-}k}$

Design of Concrete Structures
Column Design – Columns in Pure Compression

- <u>Point 6: ε_c = .003 in/in and ε_s = 0 in/in</u>

C = 15.5 in.

a = .85(15.5) = 13.16 in.

C_c = 3.188 (4.38) (18) = 754^k C_s = 254^k

T = 0^k P = $C_s + C_c - T$ = 1008^k

M = (6.5)254 + (9-6.58)754 = $289.5^{1\text{-}k}$

P = 1008^k
M = $289.5^{1\text{-}k}$

Design of Concrete Structures
Column Design – Columns in Pure Compression

- <u>Point 7: ε_c = .003 in/in and ε_s = .0005 in/in</u>

C = (.003/.0035)(15.5) = 13.29 in.

a = .85(13.29) = 11.29 in.

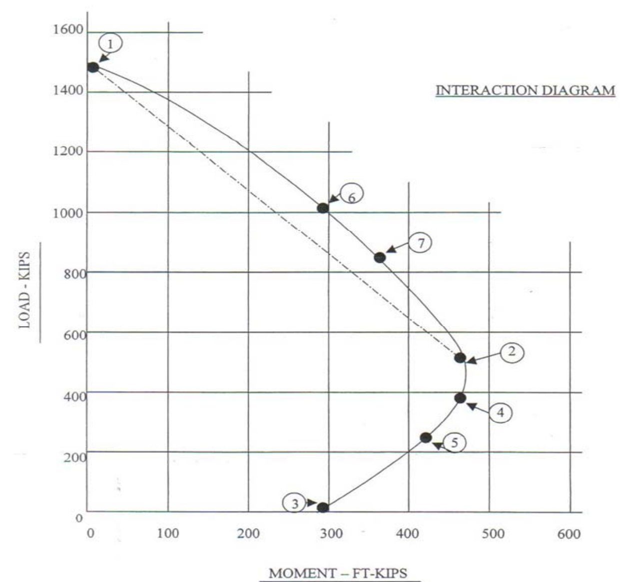
C_c = 3.188 (11.29) (18) = 646^k C_s = 254^k

T = (.0005/.001724)(254) = 76.1^k P = $C_s + C_c - T$ = 824^k

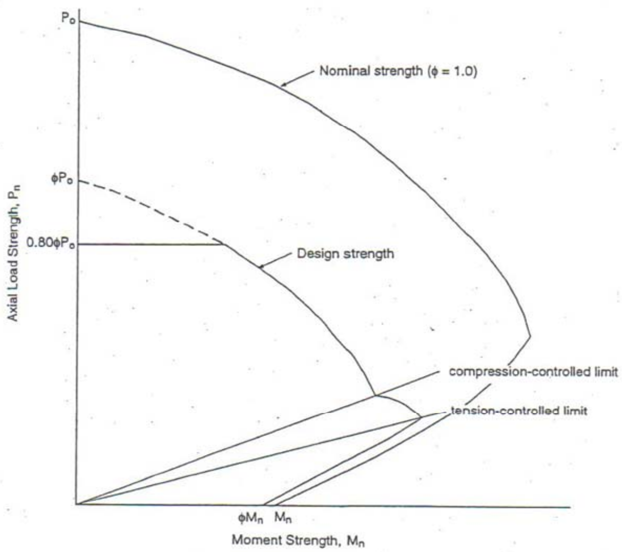
M = (6.5)254 + 6.5(76.1) + (9-5.65)646 = $364^{1\text{-}k}$

P = 824^k
M = $364^{1\text{-}k}$

Design of Concrete Structures
Column Design – Columns in Pure Compression



Design of Concrete Structures
Column Design – Columns in Pure Compression



Design Load-Moment Strength Diagram (tied column)

Design of Concrete Structures
Column Design – Design for Reinforcement

- Select reinforcement for a 10in x 25in (A_g = 250in²) tied reinforced column section using <u>Interaction Diagrams</u>.
  – Neglect Slenderness
  – Required Strength: f_c' = 5,000 psi
    - P_u = 290^k and M_u (about strong axis) = 315^k f_y = 60,000 psi
  – Compute $K_n = P_u / (\phi f_c' A_g)$ = 290/(.65*5*250) = .357
  – Compute $R_n = P_u e / (\phi f_c' A_g h)$ = 315*12/(.65*5*250*25)= .186
  – Estimate γ $\gamma = (h – 5)/h$ = 25-5/25 = .8

Design of Concrete Structures
Column Design – Design for Reinforcement (cont'd)

- For a rectangular section

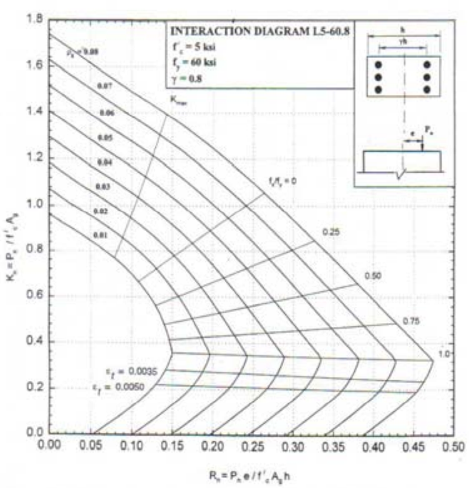
with bars along two faces,

  - f_c' = 5 ksi
  - f_y = 60,000 ksi
  - γ = .8

  – Find $\rho_g \approx .019$

- $A_{st} = \rho_g A_g$

  = .019(250) = 4.75 in²

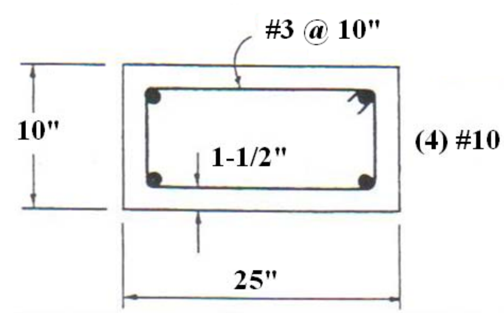


- Select (4) #10 bars (A_s = 5.08 in²)

Design of Concrete Structures
Column Design – Design for Reinforcement (cont'd)

- Select Lateral Ties
- Use #3 ties with #10 longitudinal bars.

Spacing not greater than:
16 (1.27) = 20.3"
48 (0.375) = 18"
Column Size = 10"



Design of Concrete Structures
Column Design – Slender Columns

- For design use ACI 10.10 – 10.14
  - ACI 10. 10 – Slenderness Effects in Compression
  - ACI 10.11 – Magnified Moments
  - ACI 10.12 – Magnified Moments – Non-Sway Frames
  - ACI 10.13 – Magnified Moments – Sway Frames

Design of Concrete Structures
Column Design – Slender Columns

I) Braced vs. unbraced (Non-Sway vs. Sway)

- Most columns are neither completely braced or completely unbraced.

1. A column is assumed to be non-sway (braced) if the increase in column end moments due to second-order effects does not exceed 5% of the first-order end moments (ACI 10.11.4.1)

Design of Concrete Structures
Column Design – Slender Columns

I) Braced vs. unbraced (cont'd)

2. Also a story within a structure is assumed non-sway if

$$Q = \frac{\sum P_u \Delta_o}{V_u l_c} \leq 0.05$$ (ACI 10.11.4.2)

- P_u = total vertical load
- V_u = story shear
- Δ_o = first-order relative deflection between the top and bottom of that story due to V_u
- l_c = length of compression member in a frame, measured from center to center of the joints in the frame.

Design of Concrete Structures
Column Design – Slender Columns

II) Braced (Non-Sway) Frames

(a) Slenderness

- Slenderness effects neglected when,

$$kl_u/r \leq 34 - 12(M_1/M_2)$$ (ACI 10-7)

- M_1/M_2 is not taken less than -0.5
- M_1 = smaller factored end moment (positive if member is in single curvature, negative if bent in double curvature)
- M_2 = larger factored end moment (always positive)

Design of Concrete Structures
Column Design – Slender Columns

II) Braced (Non-Sway) Frames (cont'd)

(a) Slenderness (cont'd)

- Slenderness effects neglected when,

$$kl_u/r \le 34 - 12(M_1/M_2) \qquad \text{(ACI 10-7)}$$

- k = effective length factor (ACI 10.12.1)
  - shall be taken as 1.0 unless analysis shows that a lower value is justified (equations in ACI R10.12.1 or the Jackson and Moreland Alignment Charts may be used)
- l_u = unsupported length (ACI 10.11.3.1 and 10.11.3.2)

Design of Concrete Structures
Column Design – Slender Columns

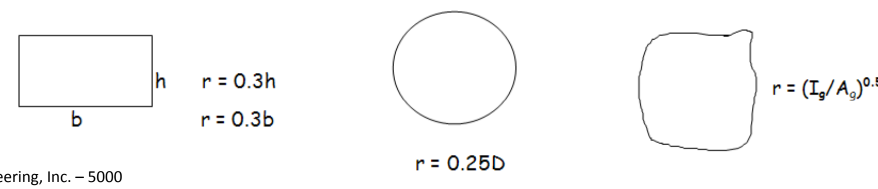
II) Braced (Non-Sway) Frames (cont'd)

(a) Slenderness (cont'd)

- Slenderness effects neglected when,

$$kl_u/r \le 34 - 12(M_1/M_2) \qquad \text{(ACI 10-7)}$$

- r = radius of gyration = $(I_g/A_g)^{0.5}$ and may be taken as
  - 0.3 times the dimension in the direction of analysis for a rectangular section and 0.25 times the diameter of a circular section (ACI 10.11.2)



Design of Concrete Structures
Column Design – Slender Columns

II) Braced (Non-Sway) Frames (cont'd)

(b) Design

- Braced columns shall be designed for factored axial load P_u and amplified moment M_c

$$M_c = \delta_{ns} M_2 \qquad \text{(ACI 10-8)}$$

- δ_{ns} is a moment magnification factor that reflects effects of member curvature between ends of member

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \ge 1.0 \qquad \text{(ACI 10-9)}$$

5100. Reinforced Concrete

5110	• 5110 - Introduction: Materials & Design Methods
5120 & 5130	• 5120 - Moment Design of Beams • 5130 - Shear Design of Beams
5140 & 5150	• 5140 - Footing Design • 5150 - Column Design
5160 & 5170	• 5160 - Development & Splices of Reinforcement • 5170 - Strut and Tie Model
5180	• 5180 - Two-way Slabs

5160 - Development & Splices of Reinforcement

- ***Deformed Bars and Wires In Tension***
  - The Code requires that the distance from the point of peak stress to the near end of a bar be equal to or exceed the development length, ℓ_d
  - ℓ_d is the minimum embedment length required to anchor a bar that is stressed to the yield point

Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- ***Deformed Bars and Wires In Tension (cont'd)***

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \quad \text{(ACI 12-1)}$$

- ℓ_d = development length, in.
- d_b = nominal diameter of bar or wire, in.
- f_c' = specified compressive strength of concrete, psi
- f_y = specified yield strength of nonprestressed bar or wire, psi
- ψ_t = **reinforcement location factor**
  = 1.3 for horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast below the bar being developed or spliced
  = 1.0 for other reinforcement
- λ = **lightweight aggregate concrete factor**
  = 1.3 when lightweight aggregate concrete is used, or
  = $6.7(f_c')^{0.5} / f_{ct}$, but not less than 1.0, when f_{ct} is specified
  = 1.0 for normal weight concrete

Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- ***Deformed Bars and Wires In Tension (cont'd)***

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \quad \text{(ACI 12-1)}$$

- ψ_s = **reinforcement size factor**
  = 1.0 for #7 and larger bars
  = 0.8 for #6 and smaller bars and deformed wires
- ψ_e = **coating factor**
  = 1.5 for epoxy-coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$
  = 1.0 for uncoated reinforcement
- The product of ψ_t and ψ_e need not be taken greater than 1.7
- c_b = **spacing or cover dimension, in.**
  = the smaller of (1) distance from center of bar or wire being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars or wires being developed

Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- ***Deformed Bars and Wires In Tension (cont'd)***

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \quad \text{(ACI 12-1)}$$

- K_{tr} = **transverse reinforcement index**
  = $\dfrac{A_{tr} f_{yt}}{1500sn}$

Where,

- A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in.^{2}
- f_{yt} = specified yield strength of transverse reinforcement, psi
- s = maximum spacing of transverse reinforcement within l_d, center-to-center, in.
- n = number of bars or wires being developed along the plane of splitting

Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- ***Deformed Bars and Wires In Tension (cont'd)***

- $[(c_b + K_{tr})/d_b] \le 2.5$ (ACI 12.2.3)

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\lambda \sqrt{f_c'}} \frac{\Psi_t \Psi_e \Psi_s}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \quad \text{(ACI 12-1)}$$

- As a design simplification, it is conservative to assume $K_{tr} = 0$, even if transverse reinforcement is present. If a clear cover of $2d_b$ and a clear spacing between bars being developed of $4d_b$ is provided, variable "c_b" would equal $2.5d_b$. For the preceeding conditions, even if $K_{tr} = 0$, the term $[(c_b + K_{tr})/d_b]$ would equal 2.5

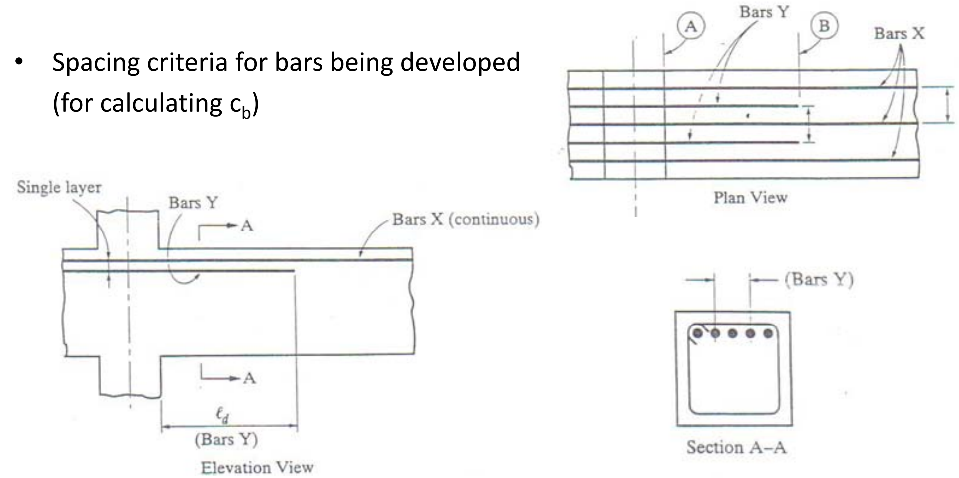
- The term $[(c_b + K_{tr})/d_b]$ in the denominator of Eq. (12-1) accounts for the effects of small cover, close bar spacing, and confinement provided by transverse reinforcement.

Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- ***Deformed Bars and Wires In Tension (cont'd)***

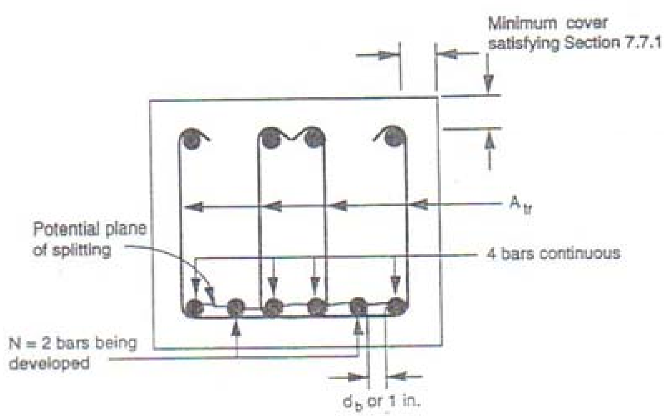
- Spacing criteria for bars being developed (for calculating c_b)



Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- ***Deformed Bars and Wires In Tension (cont'd)***



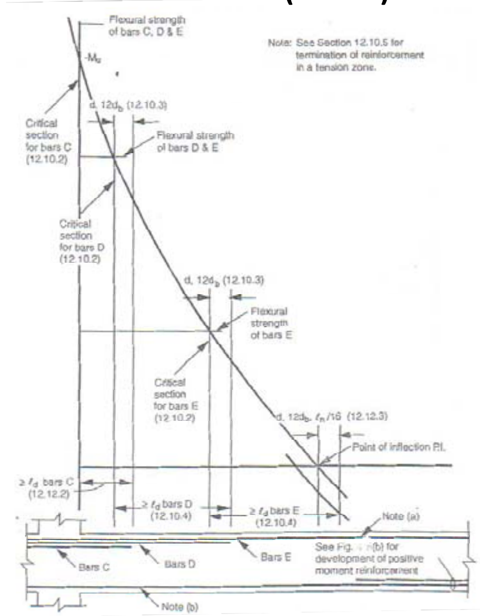
Design of Concrete Structures

Development and Splices of Reinforcement (cont'd)

- Note (a): Portion of total negative reinforcement (A_s^-) must be continuous (or spliced with a Class A splice or a mechanical or welded splice satisfying 12.14.3) along full length of perimeter beams (7.13.2.2)

- (a) Negative moment reinforcement

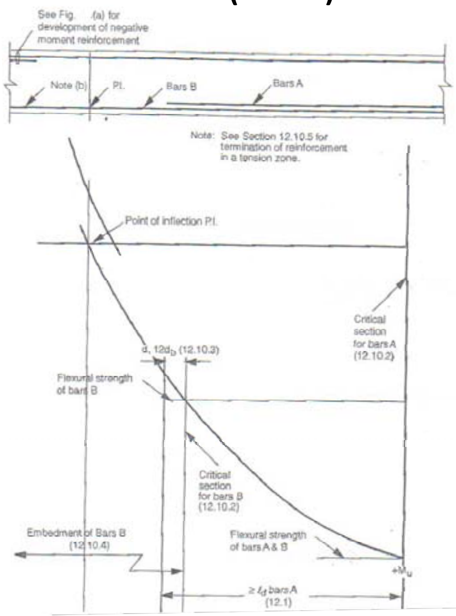
- **Figure** – Development of positive and negative moment reinforcement



Design of Concrete Structures

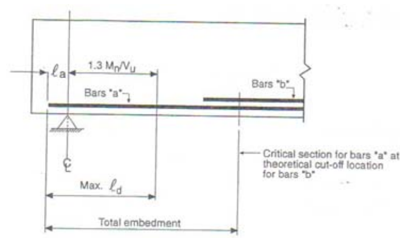
Development and Splices of Reinforcement (cont’d)

- Note (b): Portion of total positive reinforcement (A_s^-) must be continuous (or spliced with a Class A splice or a mechanical or welded splice satisfying 12.14.3) along full length of perimeter beams and of beams without closed stirrups (7.13.2.2) & (7.13.2.4)
- (b) positive moment reinforcement
- **Figure** – Development of positive moment reinforcement

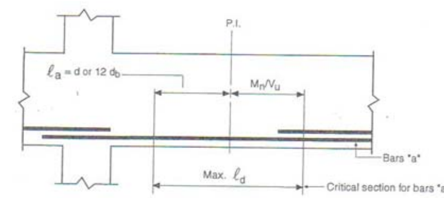


Design of Concrete Structures

Development and Splices of Reinforcement (cont’d)



- Development Length Requirements at Simple Support (straight bars)
- Concept for Determining Maximum size of Bars “a” at Point of Inflection (12.11.3)



5100. Reinforced Concrete

- 5110
  - 5110 - Introduction: Materials & Design Methods
- 5120 & 5130
  - 5120 - Moment Design of Beams
  - 5130 - Shear Design of Beams
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  - 5150 - Column Design
- 5160 & 5170
  - 5160 - Development & Splices of Reinforcement
  - 5170 - Strut and Tie Model
- 5180
  - 5180 - Two-way Slabs

5170 - Strut and Tie Model

- **Introduction**
- The beam theory of reinforced concrete is based on linear strain distribution which results in plane sections remaining plane
  - This assumption is not valid for design problems such as deeps beam or in areas where there are discontinuities of load or member cross section

Design of Concrete Structures
Strut – And – Tie Design

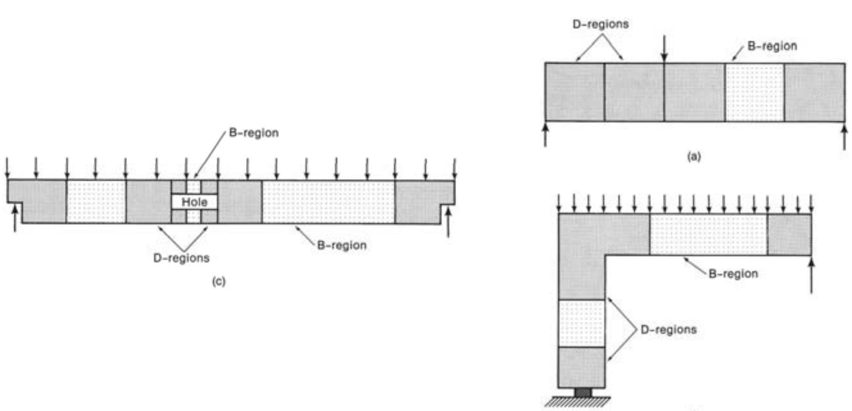
- St. Venant's principle states that strains induced by discontinuities in load or in member cross section vary in approximately linear fashion at distances greater than or equal to the greatest cross-sectional dimension h from the point of load application. Therefore, St. Venant's principle does not apply at locations closer than the distance h to discontinuities in applied load or geometry
- Thus reinforced concrete members may be divided into zones where beam theory is valid (B regions) and zones of discontinuities (or disturbances) where beam theory is not valid (D regions)

Design of Concrete Structures
Strut – And – Tie Design

- The strut-and-tie model was introduced to facilitate design of D regions but may also be used in B regions. The strut-and-tie model represents the D regions with a truss system, consisting of compression struts and tension ties connected at nodes
- The strut-and-tie model evolved in the 1980's and was first introduced through Appendix A in ACI 318-02. The use of strut-and-tie models in ACI 318-08 is permitted to be used in the design of structural concrete through ACI 8.3.4

Design of Concrete Structures
Strut – And – Tie Design

- Examples of *B-regions and D-regions* are shown below

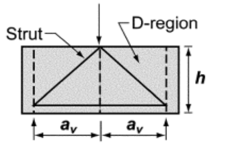


Design of Concrete Structures
Strut – And – Tie Design

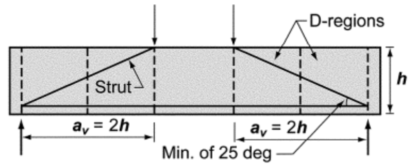
- **Definitions**
  - ***Deep Beams*** – According to ACI 10.7 deep beams may be designed by either taking into account the nonlinear distribution of strain or by using the strut-and-tie method of Appendix A
- Deep beams are members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports and have either
  a) Clear spans, l_n, equal to or less than four times the overall member depth;
  b) Regions with concentrated loads within twice the member depth from the face of the support

Design of Concrete Structures
Strut – And – Tie Design

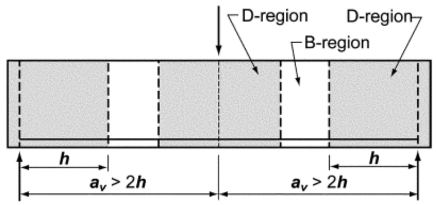
- Examples of *D-regions in deep and slender beams* are shown below



(a) Shear span, $a_v < 2h$, deep beam



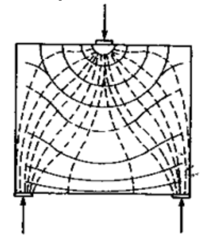
(b) Shear span, $a_v = 2h$, limit for a deep beam



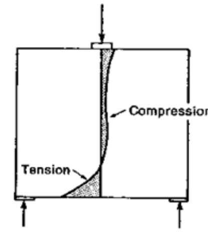
(c) Shear span, $a_v > 2h$, slender beam

Design of Concrete Structures
Strut – And – Tie Design – Deep Beam Examples

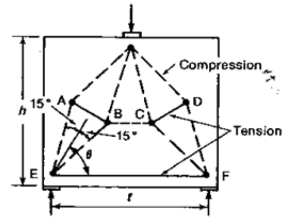
- Deep Beam Under Concentrated Load



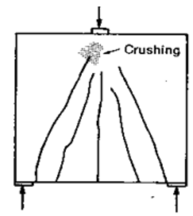
(a) Stress trajectories.



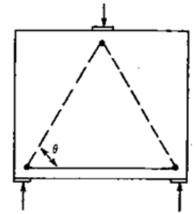
(b) Distribution of theoretical horizontal elastic stresses at midspan.



(c) Truss model.



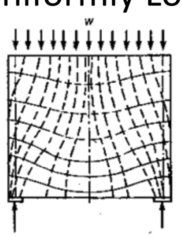
(d) Crack pattern in test.



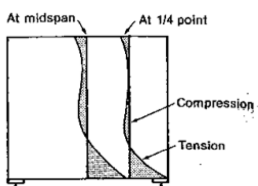
(e) Simplified truss.

Design of Concrete Structures
Strut – And – Tie Design – Deep Beam Examples

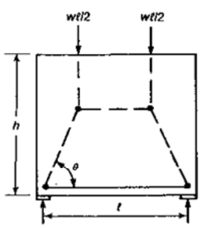
- Uniformly Loaded Deep Beam



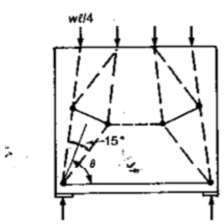
(a) Stress trajectories.



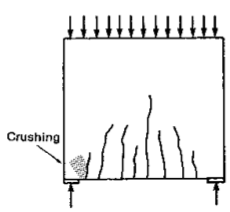
(b) Distribution of theoretical horizontal elastic stresses.



(c) Truss model.
$\theta = 68°$ if $\ell/h \le 1$
$= 54°$ if $\ell/h = 2$



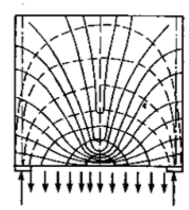
(d) Refined truss model.



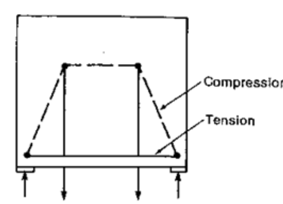
(e) Crack pattern.

Design of Concrete Structures
Strut – And – Tie Design – Deep Beam Examples

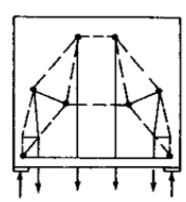
- Deep Beam with bottom edge loading



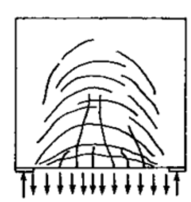
(a) Stress trajectories.



(b) Truss model.



(c) Refined truss model.



(d) Crack pattern.

Design of Concrete Structures
Strut – And – Tie Design

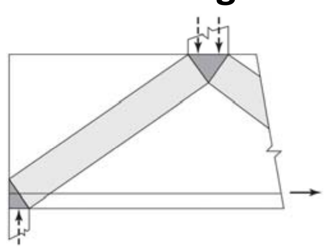
- **Definitions (cont'd)**
  - ***Struts*** – In the strut-and-tie model the struts represent the compression elements that result from the resultants of compression fields. The compression struts which may be bottle shaped are typically idealized as prismatic struts. If the effective compression strength f_{ce} differs at the two ends of a strut due either to different nodal zone strengths at the two ends, or to different bearing lengths, the strut may be idealized as a uniformly tapered compression-member

Design of Concrete Structures
Strut – And – Tie Design

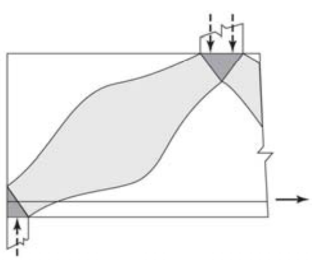
- **Definitions (cont'd)**
  - ***Ties*** – Ties consist of reinforcing and/or prestressing steel and the surrounding concrete that is concentric with the axis of the ties. Although the concrete is neglected in design, it does reduce the elongation of the tie (tension stiffening), especially under service loads
  - ***Nodes*** – A node is the point in a joint in a strut-and-tie model where the axes of the struts, ties, and concentrated forces acting on the joint intersect. Nodes are classified based on the forces acting at the joint as C-C-C. C-C-T, C-T-T or T-T-T

Design of Concrete Structures
Strut – And – Tie Design

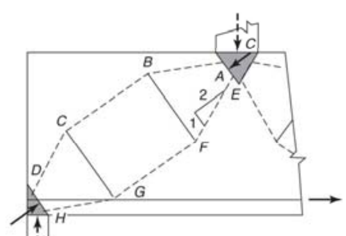
- **Shapes of Struts**



(a) Idealized prismatic strut.



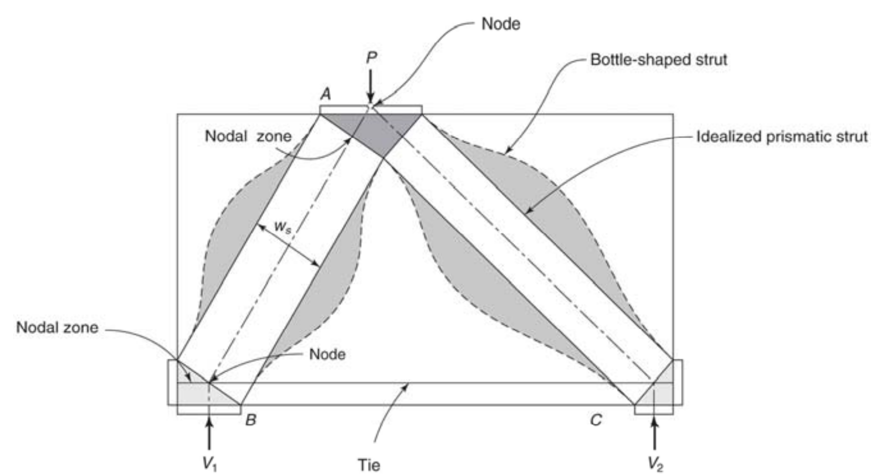
(b) Bottle-shaped strut.



(c) Strut-and-tie model of a bottle-shaped strut.

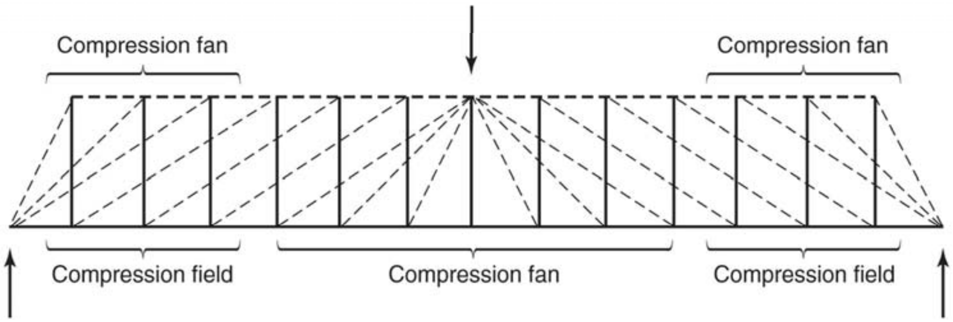
Design of Concrete Structures
Strut – And – Tie Design

- **Shapes of Struts, Ties, Nodes**



Design of Concrete Structures
Strut – And – Tie Design

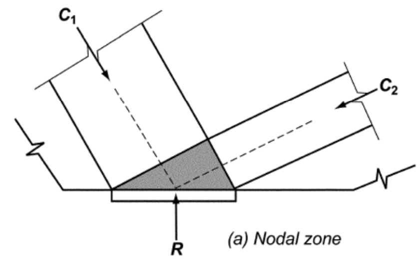
- **Compression Fans and Compression Fields**



- Strut-and-tie models may include struts as part of compression fans or compression fields.

Design of Concrete Structures
Strut – And – Tie Design

- **Definitions (cont'd)**
  - ***Nodal Zones*** - A nodal zone is the volume of concrete around a node that is assumed to transfer strut-and-tie forces through the node. Early strut-and-tie models used hydrostatic nodal zones but these have been recently replaced by extended nodal zones



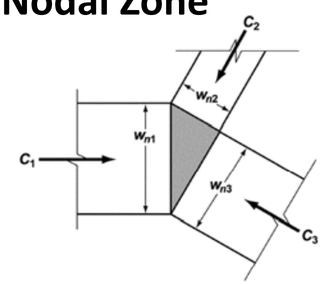
(a) Nodal zone

Design of Concrete Structures
Strut – And – Tie Design

- **Definitions (cont'd)**
  - ***Hydrostatic Nodal Zones*** - are considered to be in a state of hydrostatic compression. Both tensile and compression forces place nodes in compression because tensile forces are treated as if they pass through the node and apply a compressive force on the anchorage face
  - The dimension of one side of the nodal zone is often determined based on the contact area of the load such as bearing plate, column vase, or beam support. The dimensions of the remaining sides are obtained by requiring that the same level of stress be maintained within the node

Design of Concrete Structures
Strut – And – Tie Design

- **Hydrostatic Nodal Zone**



$$\mathbf{C_1 : C_2 : C_3 = w_{n1} : w_{n2} : w_{n3}}$$

- Hydrostatic compression implies that the sides of the nodal zones (W_{n1}, W_{n2}, W_{n3}) are proportional to the strut forces (C_1, C_2, C_3) [This is not a true hydrostatic state since the out-of-plane stresses are not required to be also equal]

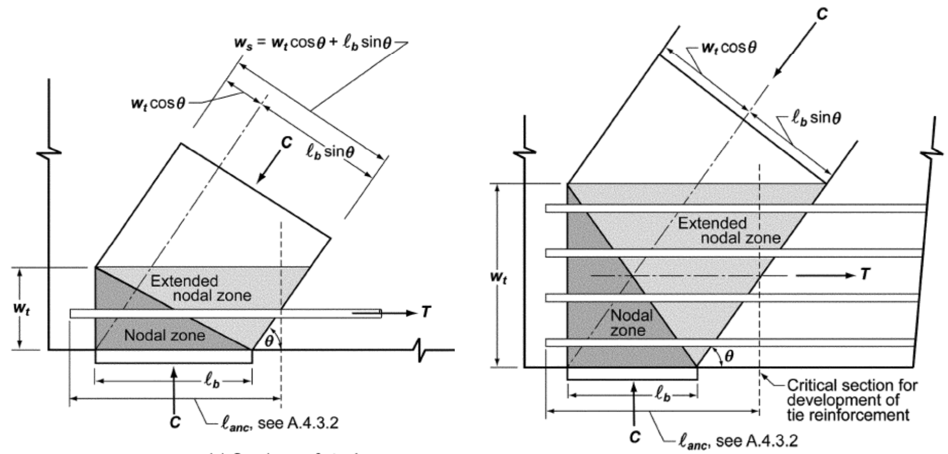
Design of Concrete Structures
Strut – And – Tie Design

- **Definitions (cont'd)**

- ***Extended Nodal Zone*** - The length of hydrostatic zone is often not adequate to allow for anchorage of the reinforcement (Nilson, et. al.). Thus the extended nodal zone bounded by the intersection of the nodal zone and the associated strut is used (Fig. 9). It increases the length within which the tensile force from the tie can be transferred to the concrete. Ties may be developed outside the nodal and extended nodal zones if needed, as shown in Fig. 9. Nodal zones may be subdivided as shown in Fig. 10 to simplify calculations. The reaction R may be divided into R_1, which equilibrates the vertical component of C_1 and R_2, which equilibrates the vertical component of the force C_2 as shown in Fig. 10.

Design of Concrete Structures
Strut – And – Tie Design

- **Extended Nodal Zones**

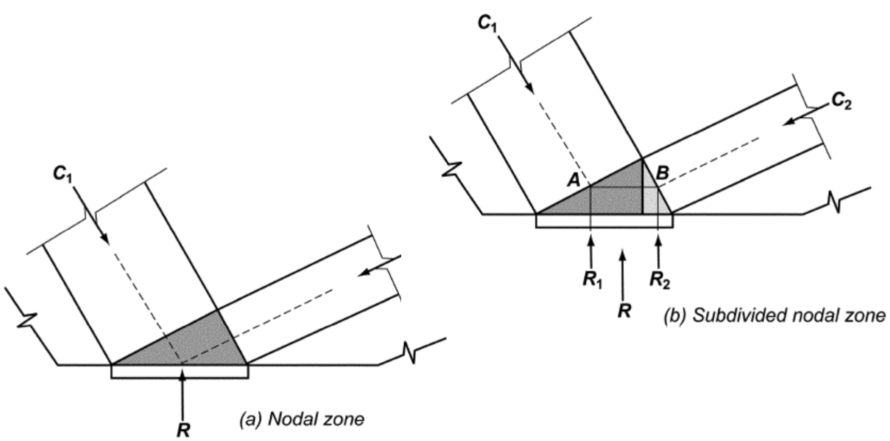


(a) One layer of steel

(b) Distributed steel

Design of Concrete Structures
Strut – And – Tie Design

- **Subdivision of Nodal Zones**



(a) Nodal zone

(b) Subdivided nodal zone

Design of Concrete Structures
Strut – And – Tie Model Design Procedure

- **The steps involved in the strut-and-tie method of design may be summarized as follows:**

  1. Identify the D-regions
  2. Compute the force resultants on each D-region boundary
  3. Select a truss model to transfer the resultant forces, across the D-region such that the axes of the struts and ties are oriented to approximately follow the axes of the compression and tension stress fields

Design of Concrete Structures
Strut – And – Tie Model Design Procedure

- **Steps (cont'd)**

4. Determine the forces in members of the truss model
5. Using the forces calculated in Step 4, and the effective concrete strengths (see ACI A.3.2 and A.5.2), determine the effective width of the struts and nodal zones
6. Design the ties (see ACI A.4.1) and detail the reinforcement for proper anchorage in the nodal zones

Design of Concrete Structures
Strut – And – Tie Model Design Procedure

- ACI provisions for strut-and-ties design of concrete elements requires that,

$$\phi F_n \geq F_u$$

where

- ϕ = strength reduction factor (= 0.75, see ACI 9.3.2.6)
- F_n = nominal capacity of strut, tie, or nodal zone
- F_u = factored force acting in strut, tie, bearing area, or nodal zone

Design of Concrete Structures
Strut – And – Tie Model Design Procedure

- In addition to satisfying strength limit states using the strut-and-ties model, serviceability requirements need to be considered. Traditional elastic analysis can be used for deflection checks. Crack control provisions given in ACI 10.6.4
- The limit on nominal shear strength of deep beams given in ACI 11.8.3 needs to be checked before embarking on detailed design of deep beams

Design of Concrete Structures
Strut – And – Tie Model Design Procedure

- The strut-and-tie model for a given structure is not unique. However, some rules need to be followed. For example the angle , between the axes of any strut and any tie entering a single node shall not be taken as less than 25 degrees (ACI A.2.5). This mitigates cracking problems and avoids incompatibilities due to shortening of the struts and lengthening of the ties in almost the same direction

Design of Concrete Structures
Strut – And – Tie Design – Strength of Struts

- The nominal compressive strength of a strut without longitudinal reinforcement shall be taken as,

$$F_{ns} = f_{ce} A_{cs}$$

at the weaker end of the compression member

  - A_{cs} = cross-sectional area at one end of the strut
  - f_{ce} = the smaller of the effective compressive strength of the concrete in the strut (ACI A.3.2) or in the nodal zone (ACI A.5.2)

Design of Concrete Structures
Strut – And – Tie Design – Strength of Struts (cont'd)

- In struts, $f_{ce} = 0.85\beta_s f_c'$ (ACI A-3)

where $\beta_s = 1.0$ for strut with uniform cross section;
$= 0.75$ for bottle-shaped struts with reinforcement satisfying ACI A.3.3;
$= 0.60\lambda$ for bottle-shaped struts without reinforcement satisfying ACI A.3.3 (λ is given in ACI 8.6.1);
$= 0.40$ for struts in tension members, or the tension flanges of members; and
$= 0.60\lambda$ for all other cases

Design of Concrete Structures
Strut – And – Tie Design – Strength of Struts (cont'd)

- To allow for = 0.75, for concrete strength not exceeding 6000 psi, the reinforcement ratio needed to cross the strut is:

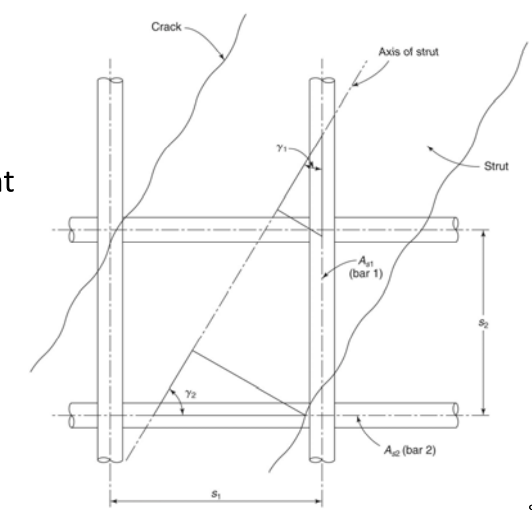
$$\sum \frac{A_{si}}{b_s s_i} \sin\alpha_i \geq 0.003 \qquad \text{(ACI Eq. A-4)}$$

- where A_{si} is the total area of reinforcement at spacing s_i in the i^{th} layer of reinforcement crossing a strut at an angle $_i$ to the axis of the strut and b_s is the width of the strut

Design of Concrete Structures
Strut – And – Tie Design – Strength of Struts (cont'd)

- Reinforcement crossing a strut (ACI RA.3.3)

- Strength of struts may be increased by providing compression reinforcement
  - Parallel to axis of strut
  - must be properly anchored
  - enclose in ties or spirals (ACI 7.10)



Design of Concrete Structures
Strut – And – Tie Design – Strength of Struts (cont'd)

- The nominal strength of such longitudinally reinforced strut is,

$$F_{ns} = f_{ce}A_{cs} + f_s'A_s'$$

- The stress f_s' in the reinforcement in a strut at nominal strength can be obtained from the strains in the strut when the strut crushes. For Grade 40 or 60 reinforcement, f_s' can be taken as f_y (ACI RA.3.5)

Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties

- The nominal strength of ties F_{nt} is taken as the sum of the strengths of the reinforcing steel and prestressing steel within the tie.

$$F_{nt} = A_{st}f_y + A_{ps}(f_{pe} + \Delta f_p)$$

where

- A_{st} = area of reinforcing steel;
- f_y = yield strength of reinforcing steel;
- A_{ps} = area of prestressing steel, if any;
- f_{pe} = effective stress in prestressing steel; and
- f_p = increase in prestressing steel stress due to factored load

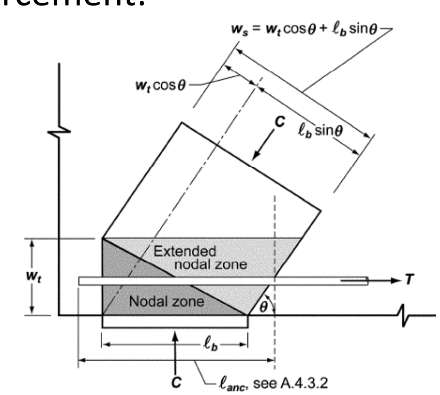
Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties (cont'd)

- The sum $f_{pe} + \Delta f_p$ must be less than or equal to the yield stress of the prestressing reinforcement f_{py}. For nonprestressed members, $A_{ps} = 0$. The value of Δf_p may be found by analysis or a value of 60 ksi maybe used for bonded tendons and 10 ksi may be used for unbonded tendons

Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties (cont'd)

- The effective tie width assumed in design, w_t, can vary between the following limits, depending on the distribution of the tie reinforcement:

  a) If the bars in the tie are in one layer, the effective tie width can be taken as the diameter of the bars in the tie plus twice the cover to the surface of the bars;



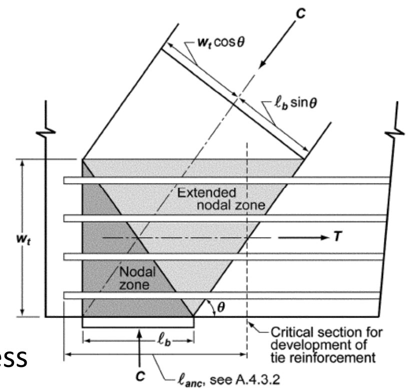
(a) One layer of steel

Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties (cont'd)

- A practical upper limit of the tie width can be taken as the width corresponding to the width in a hydrostatic nodal zone, calculated as:

  $W_{t,max} = F_{nt}/(f_{ce}b_s)$

- Where f_{ce} is computed for the nodal zone in accordance with ACI A.5.2
- If the tie width exceeds the value from (a), the tie reinforcement should be distributed approximately uniformly over the width and thickness of the tie



(b) Distributed steel

Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties (cont'd)

- ACI RA.4.3 states that:
  - Anchorage of ties often requires special attention in nodal zones of corbels or in nodal zones adjacent to exterior supports of deep beams. The reinforcement in a tie should be anchored before it leaves the extended nodal zone at the point defined by the intersection of the centroid of the bars in the tie and the extensions of the outlines of either the strut or the bearing area. This length is ℓ_{anc}

Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties (cont'd)

- ACI RA.4.3 (cont'd) states that:
  - Some of the anchorage may be achieved by extending the reinforcement though the nodal zone and developing it beyond the nodal zone. If the tie is anchored using 90 degree hooks, the hooks should be confined within the reinforcement extending into the beam from the supporting member to avoid cracking along the outside of the hooks in the support region.

Design of Concrete Structures
Strut – And – Tie Design – Strength of Ties (cont'd)

- Figure shows two ties anchored at a nodal zone. Development is required where the centroid of the tie crosses the outline of the extended nodal zone
- The development length of the tie reinforcement can be reduced through hooks, mechanical devices, additional confinement, or by splicing it with several layers of smaller bars

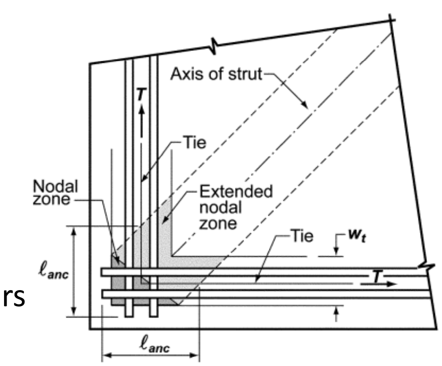


Fig. RA.4.3—Extended nodal zone anchoring two ties.

Design of Concrete Structures
Strut – And – Tie Design – Strength of Nodal Zones

- The nominal compression strength of a nodal zone, F_{nn}, shall be

$$F_{nn} = f_{ce}A_{nz} \qquad \text{(ACI Eq. A-7)}$$

- where f_{ce} is the effective compressive strength of the concrete in the nodal zone and A_{nz} is the smaller of (a) and (b):

  a) the area of the face of the nodal zone on which F_u acts, taken perpendicular to the line of action F_u;

  b) the area of a section through the nodal zone taken perpendicular to the line of action of the resultant force on the section

Design of Concrete Structures
Strut – And – Tie Design – Strength of Nodal Zones (cont'd)

- Unless confining reinforcement is provided within the nodal zone and its effect is supported by tests and analysis, the calculated effective compressive stress, f_{ce}, on a face of a nodal zone due to the strut-and-tie forces shall not exceed the value given by:

$$f_{ce} = 0.85\beta_n f_c'$$

- where the value of β_n is a factor that reflects the degree of disruption in nodal zones

Design of Concrete Structures
Strut – And – Tie Design – Strength of Nodal Zones (cont'd)

- The sign of forces acting on the node influences the capacity at the nodal zones as reflected by the β_n value. The presence of tensile stresses due to ties decreases the nodal zone concrete strength
  - β_n = 1.0 in nodal zones bounded by struts or bearing area (e.g., C-C-C- nodes)
  - β_n = 0.8 in nodal zones anchoring one tie (e.g., C-C-T nodes)
  - β_n = 0.6 in nodal zones anchoring two or more ties (e.g., C-T-T or T-T-T nodes)

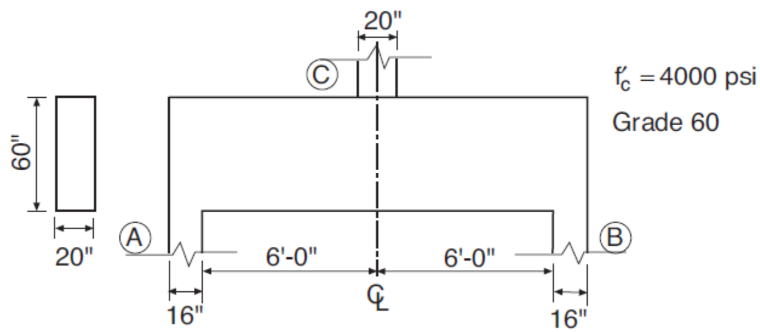
Design of Concrete Structures
Strut – And – Tie Design – Shear Requirements

- **Shear Requirements for Deep Beams**
- ACI 11.8.3 specifies that the nominal shear in a deep beam may not exceed $10(\sqrt{f_c'})b_w d$, where b_w is the width of the web and d is the effective depth. ACI 11.8.4 and 11.8.5 provide minimum steel requirements for horizontal and vertical reinforcement in deep beams. Provisions of ACI 11.8.6 may be used instead of satisfying the requirements in 11.8.4 and 11.8.5

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member

- Determine the required reinforcement for the simply supported transfer girder shown. The single column at midspan subjects the girder to 180 kips dead load and 250 kips live load



Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 1. Calculate factored load and reactions**
  - The transfer girder dead load is conservatively lumped to the column load at midspan
- **Transfer girder dead load is**
  - 5(20/12) [6 + 6 + (32/12)] 0.15 = 18.5 kips
  - P_u = 1.2D + 1.6L = 1.2(18.5+180) + (1.6)250 = 640 kips (ACI Eq. 9-2)
  - $R_A = R_B$ = 640/2 = 320 kips

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 2. Determine if this beam satisfies the definition of a "deep beam"**
  - Overall girder height h = 5 ft
  - Clear span ℓ_n = 12 ft

$$\frac{\ell_n}{h} = \frac{12}{5} = 2.4 < 4$$

- Member is a "deep beam" and will be designed using Appendix A

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

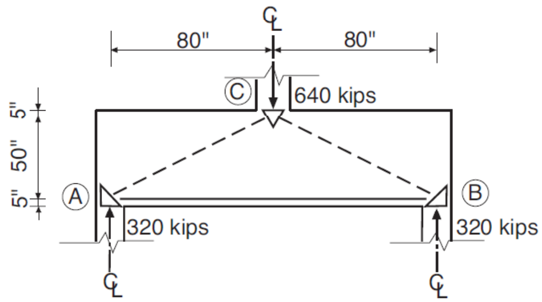
- **Step 3. Check the maximum shear capacity of the cross section**
  - V_u = 320 kips
  - Maximum $\phi V_n = \phi(10\sqrt{f_c'}\, b_w d)$

  = 0.75(10 √[4000] × 20 × 0.9 × 54)/1000 = 512 kips > V_u O.K.

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 4. Establish truss model**
  - Assume that the nodes coincide with the centerline of the columns (supports), and are located 5 in. from the upper or lower edge of the beam as shown. The strut-and-tie model consists of two struts (A-C and B-C), one tie (A-B), and three nodes (A, B, and C). In addition, columns at A and B act as struts representing reactions. The vertical strut at the top of Node C represents the applied load



Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 4. Establish truss model (cont'd)**
  - The length of the diagonal struts = $\sqrt{[50^2 + 80^2]}$ = 94.3 in.
  - The force in the diagonal struts = 320*(94.3/50) = 603 kips
  - The force in the horizontal tie = 320*(80/50) = 512 kips
  - Verify the angle between axis of strut and tie entering Node A
  - The angle between the diagonal struts and the horizontal tie = $\tan^{-1}(50/80) = 32° > 25°$ O.K.

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 5. Calculate the effective concrete strength (f_{ce}) for the struts**
  - Assume reinforcement is provided to resist splitting forces
  - For the "bottle-shaped" Struts A-C & B-C

    $\mathbf{f_{ce} = 0.85\beta_s f_c' = 0.85 \times 0.75 \times 4000 = 2550\ psi}$ (ACI Eq. A-3)
    - where β_s = 0.75 per A.3.2.2(a)
- Note, this effective compressive strength cannot exceed the strength of the nodes at both ends of the strut. See A.3.1
  - The vertical struts at A, B, and C, have uniform cross-sectional area throughout their length

    $\mathbf{f_{ce} = 0.85\beta_s f_c' = 0.85 \times 1.0 \times 4000 = 3400\ psi}$ (ACI Eq. A-3)
    - where β_s = 1.0 per A.3.2.1

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 6. Calculate the effective concrete strength (f_{ce}) for Nodal Zones A, B, & C**
- Nodal Zone C is bounded by three struts
  - So this is a C-C-C nodal zone with β_n = 1.0 *A.5.2.1*

    $\mathbf{f_{ce} = 0.85\beta_n f_c' = 0.85 \times 1.00 \times 4000 = 3400\ psi}$ (ACI Eq. A-8)
  - Nodal Zones A and B are bounded by two struts and a tie.
- For a C-C-T node:
  - β_n = 0.80 *A.5.2.2*

    $\mathbf{f_{ce} = 0.85\beta_n f_c' = 0.85 \times 1.00 \times 4000 = 3400\ psi}$

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 7. Check strength at Node C**
  - Assume that a hydrostatic nodal zone is formed at Node C. This means that the faces of the nodal zone are perpendicular to the axis of the respective struts, and that the stresses are identical on all faces
  - To satisfy the strength criteria for all three struts and the node, the minimum nodal face dimension is determined based on the least strength value of f_{ce} = 2550 psi, thus, governed by the bottle-shaped diagonal struts. The same strength value will be used for Nodes A and B as well

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 7. Check strength at Node C (cont'd)**
- The strength checks for all components of the strut and tie model are based on

$$\phi F_n \geq F_u \quad \text{(ACI Eq. A-1)}$$

  - where ϕ = 0.75 for struts, ties, and nodes 9.3.2.6
- The length of the horizontal face of Nodal Zone C is calculated as

$$\frac{640{,}000}{0.75 \times 2550 \times 20} = 16.7 \text{ in. (less than column width of 20 in.)}$$

- The length of the other faces, perpendicular to the diagonal struts, can be obtained from proportionality: $16.7 \times \frac{603}{640} = 15.7$ in.

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 8. Check truss geometry**
  - The center of the nodal zone is at 4.0 in. from the top of the beam, which is very close to the assumed 5 in.

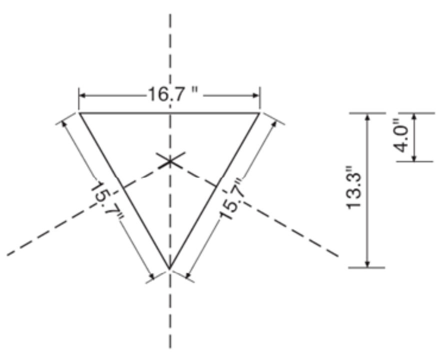


Figure Geometry of Node C

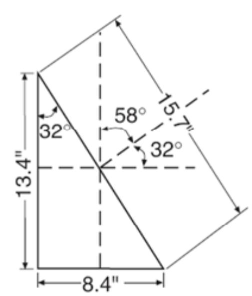


Figure Geometry of Node A

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 8. Check truss geometry (cont'd)**
- The horizontal tie should exert a force on this node to create a stress of 2550 psi
  - Thus size of the vertical face of the nodal zone is

$$\frac{512{,}000}{0.75 \times 2550 \times 20} = 13.4 \text{ in.}$$

- The center of the tie is located 13.4/2 = 6.7 in. from the bottom of the beam. This is reasonably close to the 5 in. originally assumed, so no further iteration is warranted
- Width of node at Support A $\frac{320{,}000}{0.75 \times 2550 \times 20} = 8.4$ in.

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 9. Provide vertical and horizontal reinforcement of diagonal struts**
  - The angle between the vertical ties and the struts is 90º - 32º = 58º

(sin 58º = 0.85)

- Try two overlapping No. 4 ties @ 12 in. O.C.

(to accommodate the longitudinal tie reinforcement)

$$\frac{A_{si}}{b_s s_i}\sin\alpha_i = \frac{4 \times 0.20}{20 \times 12} \times 0.85 = 0.00283$$

- And No. 5 horizontal bars @ 12 in. O.C. on each side face (sin 32º = 0.53)

$$\frac{2 \times 0.31}{20 \times 12} \times 0.53 = 0.00137$$

$$\Sigma\frac{A_{si}}{b_s s_i}\sin\alpha_i = 0.00283 + 0.00137 = 0.0042 > 0.003 \quad \text{O.K.}$$

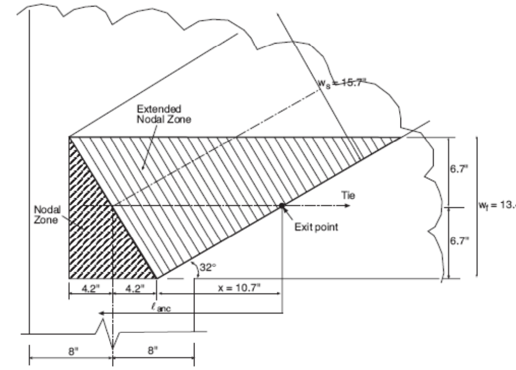
Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 10. Provide horizontal reinforcing steel for the tie**

$$A_{s,req} = \frac{F_u}{\phi f_y} = \frac{512}{0.75 \times 60} = 11.4 \text{ in.}^2$$

Select 16 - No. 8 $A_s = 12.64 \text{ in.}^2$



- These bars must be properly anchored. The anchorage length (ℓ_{anc}) is to be measured from the point where the tie exits the extended nodal zone as shown

Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 10. Provide horizontal reinforcing steel for the tie (cont'd)**
- Distance x = 6.7/tan 32 = 10.7 in.
  - Available space for a straight bar embedment
- 10.7 + 4.2 + 8 - 2.0 (cover) = 20.9 in.
  - This length is inadequate to develop a straight No. 8 bar
- Development length for a No. 8 bar with a standard 90 deg. Hook

$$\ell_{dh} = \left(0.02\psi_e f_y / \lambda\sqrt{f'_c}\right)d_b$$

$$= \left(0.02(1.0)60,000/(1.0)\sqrt{4000}\right)1.0$$

$$= 19.0 < 20.9 \text{ in. O.K.}$$

Note: the 90 degree hooks will be enclosed within the column reinforcement that extends in the transfer girder

Design of Concrete Structures

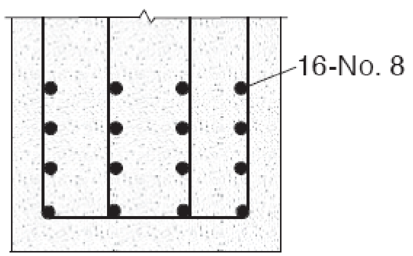
Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Step 10. Provide horizontal reinforcing steel for the tie (cont'd)**
- **Notes:**
  - By providing adequate cover and transverse confinement, the development length of the standard hook could be reduced by the modifiers of ACI 12.5.3
  - Less congested schemes can be devised with reinforcing steel welded to bearing plates, or with the use of prestressing steel
  - The discrepancy in the vertical location of the nodes results in a negligible (about 1.5 percent) difference in the truss forces. Thus, another iteration is not warranted

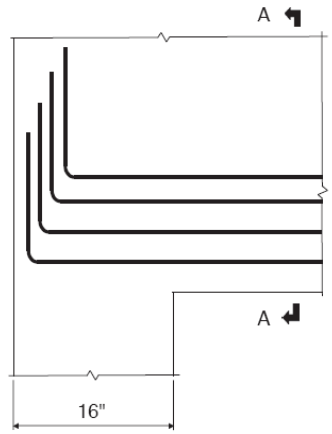
Design of Concrete Structures

Strut – And – Tie Design – Design of Deep Flexural Member (cont'd)

- **Detail of Reinforcement**



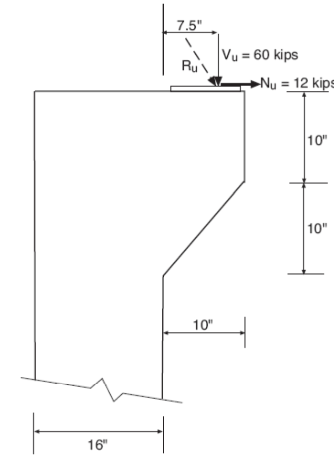
Section A-A



Design of Concrete Structures

Strut – And – Tie Design – Design of Column Corbel

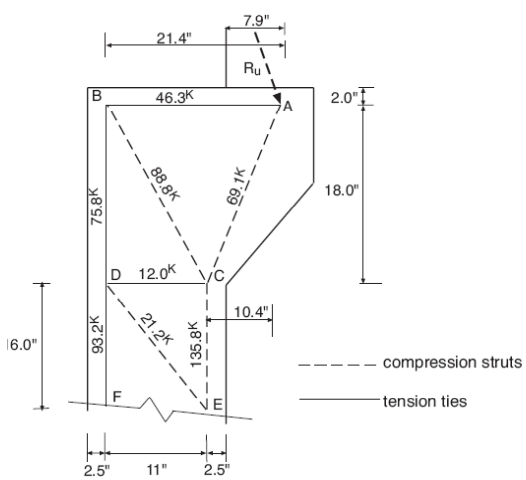
- Design the single corbel of the 16 in. x 16 in. reinforced concrete column for a vertical force V_u = 60 kips and horizontal force N_u = 12 kips
- Assume f_c' = 5000 psi
- Grade 60 reinforcing steel



Design of Concrete Structures

Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Step 1. Establish the geometry of trial truss and calculate force demand in members**



Design of Concrete Structures

Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Step 2. Provide reinforcement for ties**
  - Use ϕ = 0.75 *9.3.2.6*
- The nominal strength of ties is to be taken as:

$$\mathbf{F_{nt} = A_{ts}f_y + A_{tp}\,(f_{se} + \Delta f_p)} \qquad \text{(ACI Eq. A-6)}$$

- where the last term can be ignored as only nonprestressed reinforcement is provided

Tie AB $\quad F_u = 46.3$ kips

$$A_{ts} = \frac{F_u}{\phi f_y} = \frac{46.3}{0.75 \times 60} = 1.03 \text{ in.}^2 \text{ Provide 4-No. 5 framing rebars}$$

Ats = 1.24 in.2

Design of Concrete Structures

Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Step 2. Provide reinforcement for ties (cont'd)**

Tie CD $F_u = 12.0$ kips

$$A_{ts} = \frac{12.0}{0.75 \times 60} = 0.27 \text{ in.}^2 \text{ Provide No. 4 tie (2 legs) } A_{ts} = 0.40 \text{ in.}^2$$

Tie BD & DF $P_u = 93.2$ kips (governs)

$$A_{ts} = \frac{93.2}{0.75 \times 60} = 2.07 \text{ in.}^2$$

- Provide steel in addition of vertical column reinforcement
  - This reinforcement may be added longitudinal bar or a rebar bent at Node A, that is used as Tie AB as well

Design of Concrete Structures

Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Step 3. Calculate strut widths**
- It is assumed that transverse reinforcement will be provided in compliance with A.3.3, so a
  - $\beta_s = 0.75$ can be used in calculating the strut length
- **$f_{ce} = 0.85\beta_s f_c' = 0.85 \times 0.75 \times 5000 = 3187$ kips** (ACI Eq. A-3)
  - $\phi f_{ce} = 0.75 \times 3187 = 2390$ psi

Design of Concrete Structures

Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Step 3. Calculate strut widths (cont'd)**
  - Widths of struts required

Strut AC $P_u = 69.1$ kips

$$w = \frac{69{,}100}{16 \times 2390} = 1.81 \text{ in.}$$

Strut CE

$$w = \frac{135{,}800}{16 \times 2390} = 3.55 \text{ in.}$$

Strut BC

$$w = \frac{88{,}800}{16 \times 2390} = 2.32 \text{ in.}$$

Strut DE

$$w = \frac{21.2}{16 \times 2390} = 0.55 \text{ in.}$$

- The width of the struts will fit within the concrete column with the corbel

Design of Concrete Structures

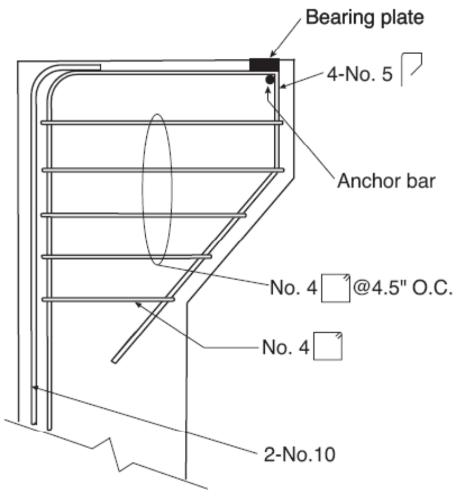
Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Step 3. Calculate strut widths (cont'd)**
  - Provide confinement reinforcement for the struts per A.3.3 in the form of horizontal ties
  - The angle of the diagonal struts to the horizontal hoops is 58 degree. Provide No. 4 hoops at 4.5 in. on center

$$\frac{A_s}{b_s s} \sin\alpha = \frac{2 \times 0.20}{24 \times 4.5} \sin 58^\circ = 0.0031 > 0.003 \quad \text{O.K.}$$

Design of Concrete Structures
Strut – And – Tie Design – Design of Column Corbel (cont'd)

- **Reinforcement Details**



5100. Reinforced Concrete

5110
- 2110 - Introduction: Materials & Design Methods

5120 & 5130
- 5120 - Moment Design of Beams
- 5130 - Shear Design of Beams

5140 & 5150
- 5140 - Footing Design
- 5150 - Column Design

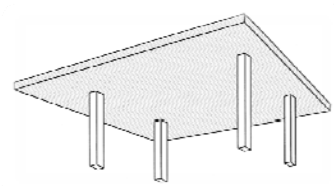
5160 & 5170
- 5160 - Development & Splices of Reinforcement
- 5170 - Strut and Tie Model

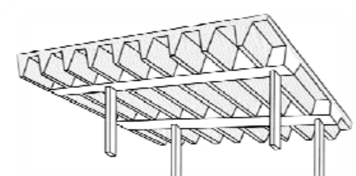
5180
- 5180 - Two-way Slabs

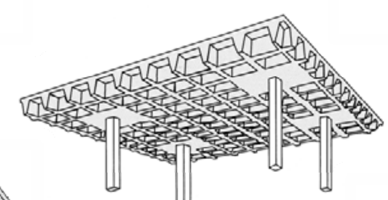
5180 - Two-way Slabs

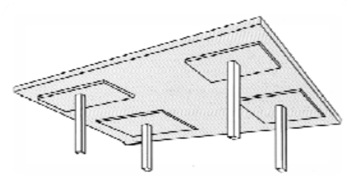
- <u>Two-Way Beam-Supported Slab</u>
  - This basic and common type of floor is one of the original slab systems in reinforced concrete
    - When, long side/short side ≥ 2,
      - one way action may be assumed
- Advantages:
  - Economical for longer spans
- Disadvantages:
  - Greater story height
  - Beams may not allow flexibility of partition location

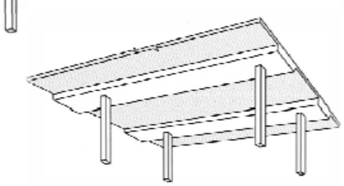
Design of Concrete Structures
Two-Way Slabs - Types


Flat Plate

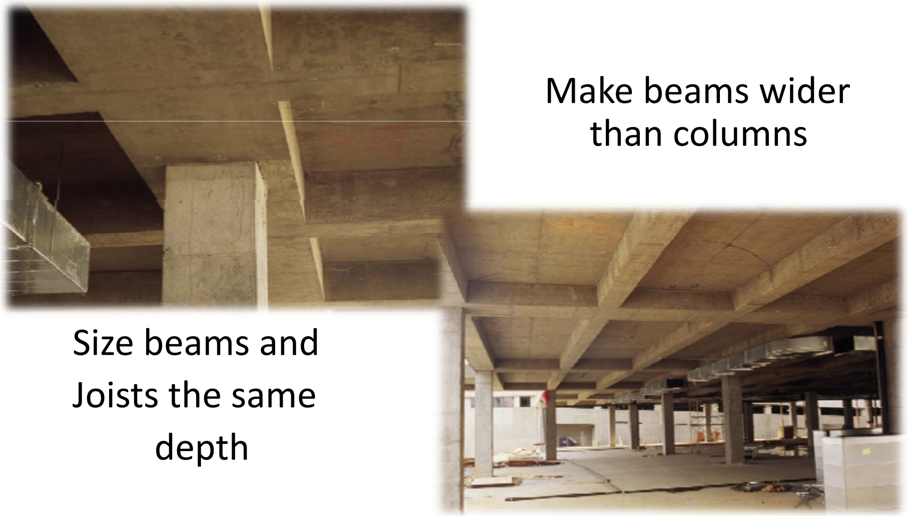

Two-Way Joists


Waffle Slab


Two-Way Slab Plate w. Drop Panel


One-Way Slab and Beam

Design of Concrete Structures
Two-Way Slabs



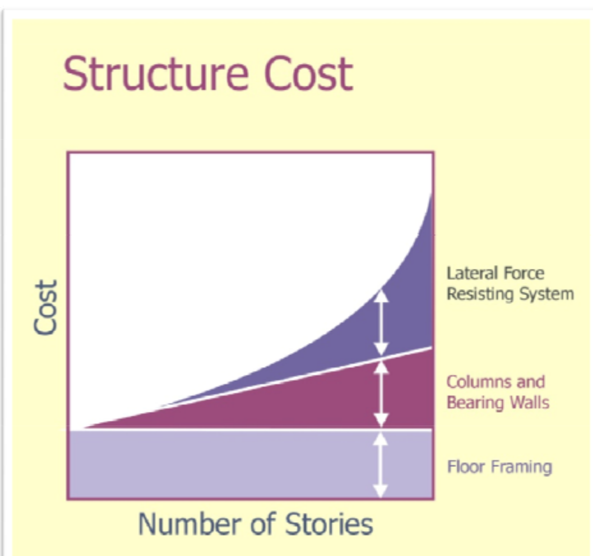
Make beams wider than columns

Size beams and Joists the same depth

Design of Concrete Structures
Two-Way Slabs

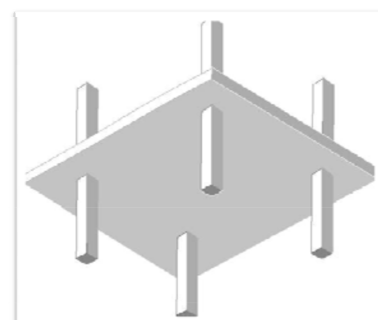
- Floor Framing Systems
  - Select one floor framing system
  - Use shallowest system
  - For most buildings floor framing costs dominate
  - Vertical element costs become more significant in taller buildings or in moderate to strong seismic zones

Design of Concrete Structures
Two-Way Slabs



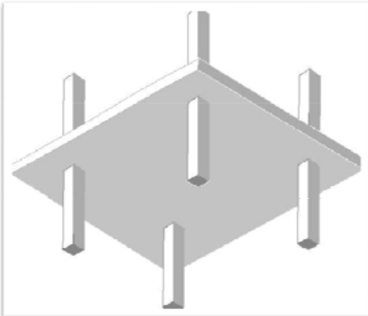
Design of Concrete Structures
Two-Way Slabs

- Flat Plate
- The column line beams that used to be the standard gradually began to disappear giving rise to flat plate systems. The flat plate is currently the most widely used slab system for multi-story construction
- Span Length:
  - Practical range = 15 ft to 30 ft
  - Economical range = 15 ft to 25 ft
- Dimensions:
  - Slab thickness between 5 & 10 in.



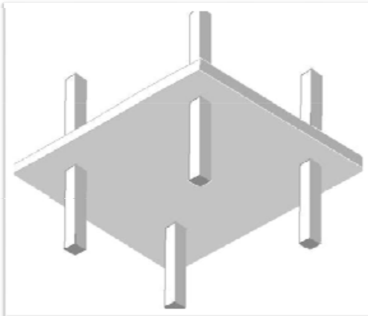
Design of Concrete Structures
Two-Way Slabs

- Flat Plate (Advantages)
  - Most economical short span structural system (formwork costs 50% of floor system cost)
  - Minimizes floor-to-floor height
  - Shortest construction time with least field labor
  - Simplest formwork and reinforcing steel layout
  - Greatest flexibility in layout of columns, partitions



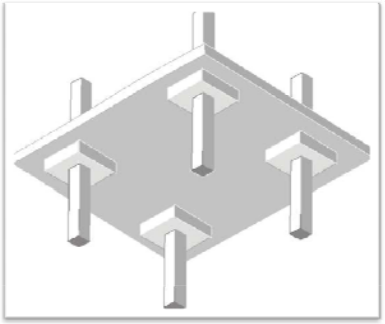
Design of Concrete Structures
Two-Way Slabs

- Flat Plate (Disadvantages)
  - Economical only for short & medium spans and for moderate live loads
  - Note: Shear strength near columns is increased by the use of multiple U stirrups or structural steel usually known as shear head reinforcement



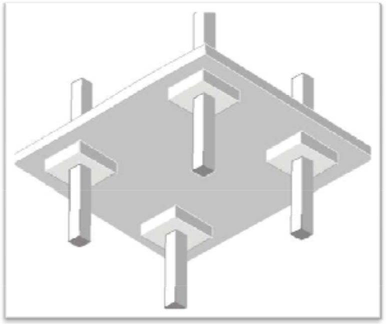
Design of Concrete Structures
Two-Way Slabs

- Two-Way Flat Plate with Drop Panels (AKA Flat Slab)
  - Drop panels and/or column capitals allow for larger shear resistance compared to flat plates
  - Historically predates both the two-way slab on beams system and the flat plate
  - First patented in the U.S. in 1902



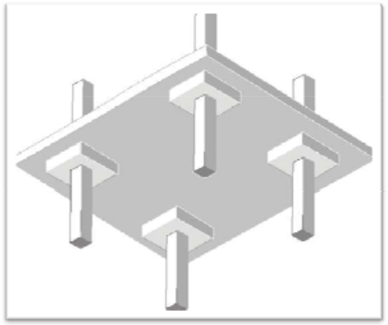
Design of Concrete Structures
Two-Way Slabs

- Two-Way Flat Plate with Drop Panels (cont'd)
- Dimensions:
  - Slab thickness 5 to 10 in.
  - Drop panels 2¼ in. to 8 in.
- Span Length:
  - Practical range = 15 ft to 30 ft
  - Economical range = 18 ft to 30 ft



Design of Concrete Structures
Two-Way Slabs

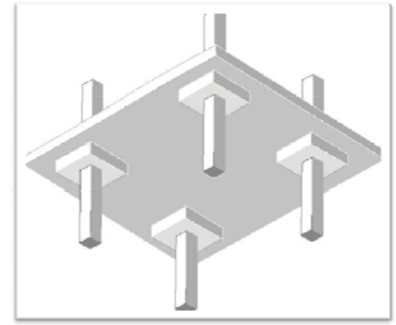
- Two-Way Flat Plate with Drop Panels (Advantages)
  - Very economical system for relatively square bays and multiple bays in each direction
  - Uses smaller columns than Two-Way Flat Plate with longer spans
  - Provides uniform clear space below slab
  - Provides flexibility in layout of columns, partitions



Design of Concrete Structures
Two-Way Slabs

- Two-Way Flat Plate with Drop Panels (Advantages cont'd)

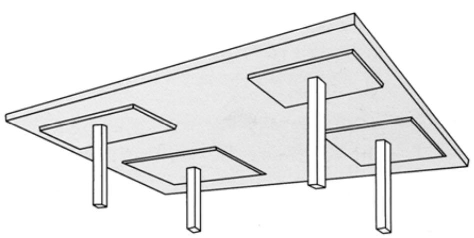
- Simple construction and formwork (formwork costs approximately 51% of floor system cost)
- Finish can be applied directly
- Adequate fire resistance
- Allows lower story heights

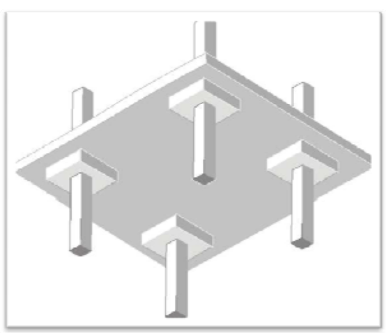


Design of Concrete Structures
Two-Way Slabs

- Two-Way Flat Plate with Drop Panels (Disadvantages)

- Economically viable only for short and medium, heavily loaded spans

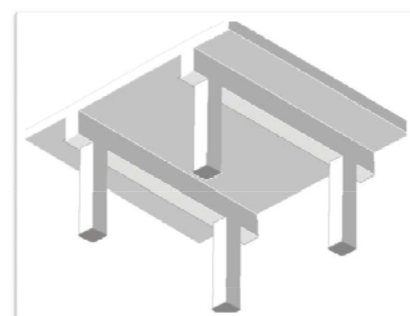




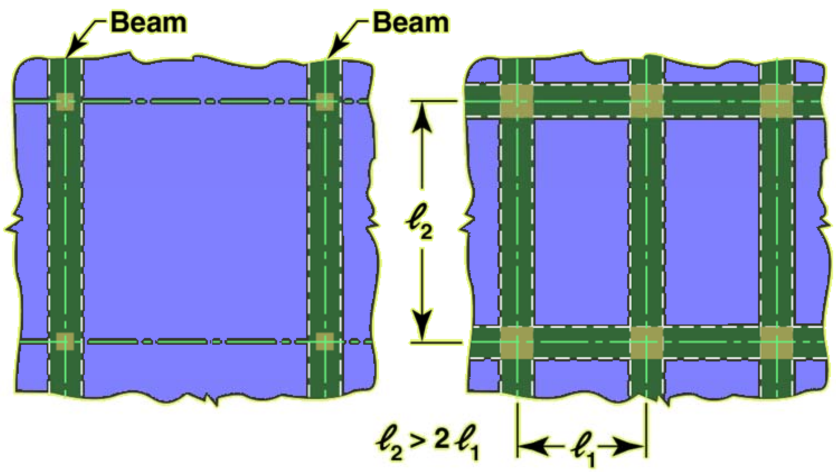
Design of Concrete Structures
Two-Way Slabs

- One-Way Slab and Beam (AKA One-Way Joist)

- Dimensions:
  - Thickness 3 to 5 in. (based on fire or structural requirements)
  - Joists 8 to 20 in. below slab and 5 to 7 in. wide
- Span Length:
  - Practical range = 15 ft to 40 ft
  - Economical range = 25 ft to 40 ft

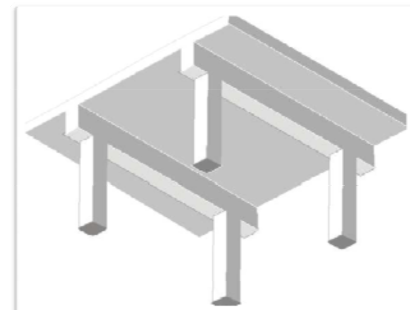


Design of Concrete Structures
Two-Way Slabs



Design of Concrete Structures
Two-Way Slabs

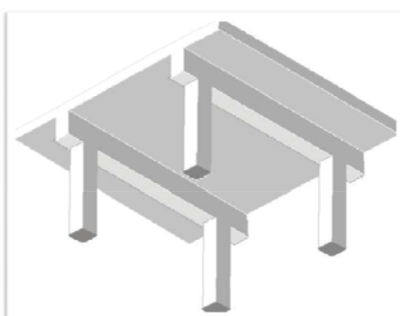
- <u>One-Way Slab and Beam</u> (Advantages)
  - Commonly used for parking structures and elevator and stair areas
  - Good for concentrated and heavy load areas
  - Excellent vibration characteristics
  - Basis for more complex framing systems
  - Popular for use in commercial buildings
  - Adaptable to custom



Design of Concrete Structures
Two-Way Slabs

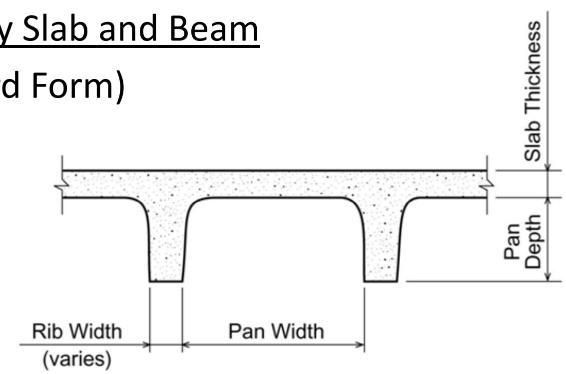
- <u>One-Way Slab and Beam</u> (Disadvantages)

- Higher formwork costs (about 58% of floor system costs)
- Not economical for short spans
- Greater story heights



Design of Concrete Structures
Two-Way Slabs

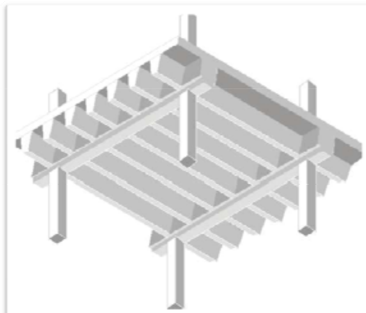
- <u>One-Way Slab and Beam</u>
  (Standard Form)



Pan Width (in.)	Pan Depth (in.)
30	8, 10, 12, 14, 16, 20, 24
53	16, 20, 24
66	14, 16, 20, 24

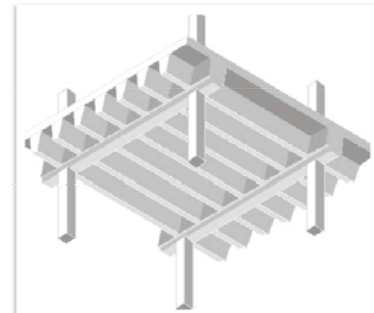
Design of Concrete Structures
Two-Way Slabs

- Two-Way Joists
- Dimensions:
  - Thickness 3 to 5 in. (based on fire or structural requirements)
  - Joists 8 to 24 in. below slab & 6 to 8 in. wide
- Span Length:
  - Practical range = 15 ft to 40 ft
  - Economical range = 35 ft to 40 ft



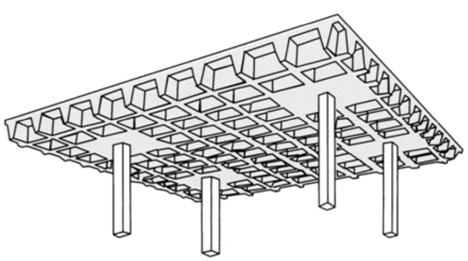
Design of Concrete Structures
Two-Way Slabs

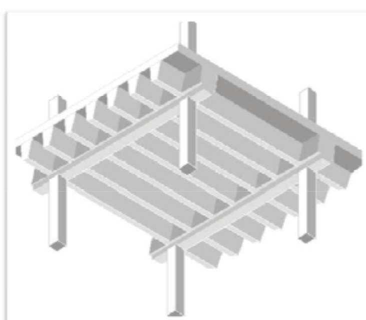
- Two-Way Joists (Advantages)
  - Provides depth for stiffness and increased load bearing capacity
  - Efficient use of concrete and reinforcing materials
  - Standard reusable forms readily removed and re-erected
  - Accommodates floor penetrations and mechanical systems



Design of Concrete Structures
Two-Way Slabs

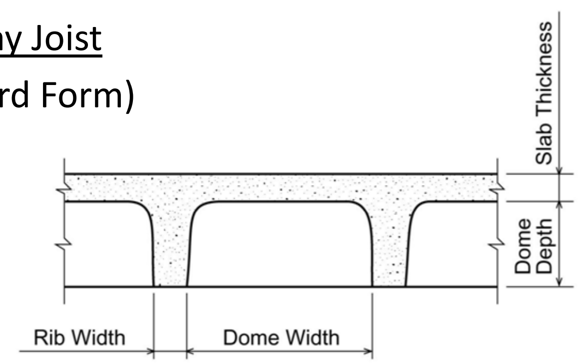
- Two-Way Joists (Disadvantages)
- Not economical for short spans or for light to medium loads
- Higher formwork cost (about 54% of floor system costs)
- Greater story height





Design of Concrete Structures
Two-Way Slabs

- Two-Way Joist (Standard Form)



Dome Width (in.)	Dome Depth (in.)	Rib Width (in.)
30	8, 10, 12, 14, 16, 20, 24	6
41	14, 16, 20, 24	7
52	14, 16, 20, 24	8

Design of Concrete Structures
Two-Way Slabs – Design Introduction

- **Introduction**
  - Design of two-way slabs may include the following steps

1. Choose the layout and type of slab to be used based on architectural, construction and other considerations
2. Perform preliminary design for slab thickness
   - To control deflection (ACI 9.5.3.2, Table 9.5c)
   - To provide adequate shear strength at both interior and exterior columns
   - Check wide beam action (ACI 11.12.1.1)
   - Check two-way action (ACI 11.12.2.1)

Design of Concrete Structures
Two-Way Slabs – Design Introduction

- **Introduction (cont'd)**

3. Choose a design method
   - Direct Design Method uses coefficients to compute positive and negative moments in the various panels in the slab
   - Equivalent Frame Method uses an elastic frame analysis to compute these moments
4. Compute the positive and negative moments in the slab

Design of Concrete Structures
Two-Way Slabs – Design Introduction

- **Introduction (cont'd)**

5. Determine the distribution of the moments across the width of the slab. The lateral distribution of moments within a panel depends on the geometry of the slab and the stiffness of the beams (if any)
   - This procedure is the same in both design methods

Design of Concrete Structures
Two-Way Slabs – Design Introduction

- **Introduction (cont'd)**

6. If there are beams, a portion of the moment must be assigned to the beams
7. Reinforcement is designed for the moments from steps 5 and 6
8. The shear strengths at the columns are checked

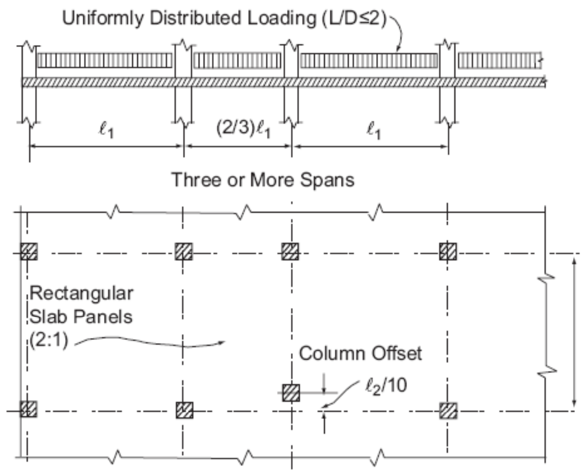
Design of Concrete Structures
Two-Way Slabs – Design Introduction

- **Minimum Two-Way Slab Thickness**

Yield Stress f_y, psi	Without drop panels			With drop panels		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
40,000	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
Minimum Thickness	5 in. (120 mm)			4 in. (100 mm)		
Experience	$\frac{\ell_n}{20}$ to $\frac{\ell_n}{30}$					

Design of Concrete Structures
Two-Way Slabs – Direct Design Method

- Conditions for Analysis by Direct Design Method



Design of Concrete Structures
Two-Way Slabs – Direct Design Method

- **Limitations of the Direct Design Method** **(ACI 13.6.1)**
- The direct design method is subject to the following restrictions:
  1. There shall be a minimum of three continuous spans in each direction
  2. The panels shall be rectangular, with the ratio of the longer to the shorter spans within a panel not greater than 2
  3. The successive span lengths in each direction shall not differ by more than one-third the longer span

Design of Concrete Structures
Two-Way Slabs – Direct Design Method

- **Limitations of the Direct Design Method (cont'd)**

4. Columns may be offset a maximum of 10% of the span in the direction of the offset from either axis between centerlines of successive columns
5. Loads shall be due to gravity only and the live load shall not exceed 2 times the dead load
6. If beams are used on the column lines, the relative stiffness of the beams in the two perpendicular directions, given by the ratio $\alpha_1\ell^2_2/\alpha_2\ell^2_1$, must be between 0.2 and 5.0

Design of Concrete Structures
Two-Way Slabs – Direct Design Method

- **Factored Total Statical Moment M_o**
  - The factored total statical moment M_o for unrestrained uniformly loaded slab-beam with a width ℓ_2, and a clear span ℓ_n and uniform load w_u is given by

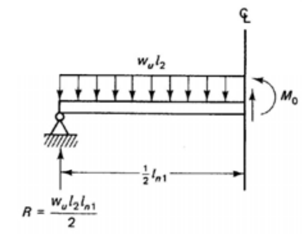
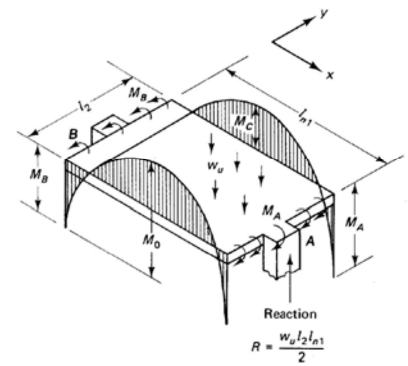
$$M_o = \frac{q_u \ell_2 \ell_n^2}{8}$$

  - ℓ_n measured between faces of supports must not be less than 0.65 ℓ_1
    - ℓ_1 = center-to-center distance between supports

Design of Concrete Structures
Two-Way Slabs

- The ACI code provides coefficients (ACI 13.6.3) for distributing M_o to the positive and negative moment regions depending on the degree of restraint such that

$$M_o = M_c + ½(M_A + M_B)$$

Design of Concrete Structures
Two-Way Slabs

- **Steps in the Direct Design Method**

1. Compute simple beam moments for each span (ACI 13.6.2)
2. Use ACI coefficients (ACI 13.6.3) to distribute M_o to the positive and negative moment regions of the slab-beam
3. Distribute the positive and negative moments across the width of the slab-beam between the column and the middle strips
   - If the column strip contains a beam, the column strip moment is divided between the beam and the balance of the column strip
4. Determine size and distribution of reinforcement in the two orthogonal directions

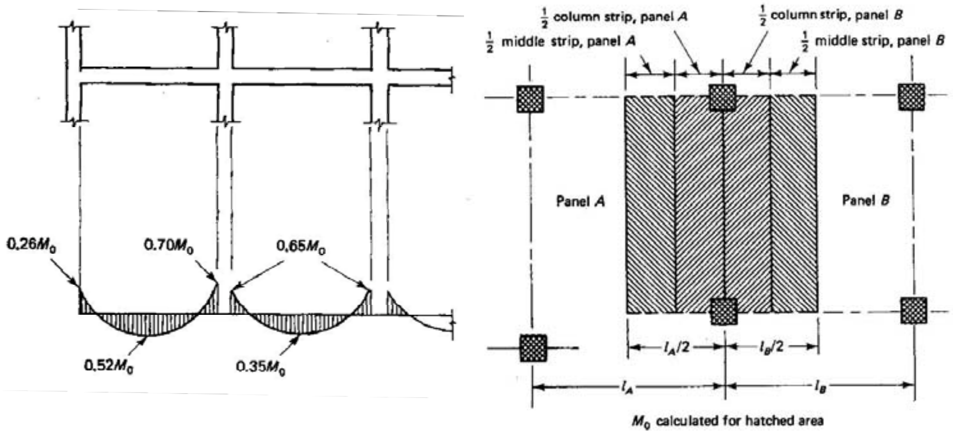
Design of Concrete Structures
Two-Way Slabs

- **Distribution of M_o** **(ACI 13.6.3)**

- Distribution coefficients for a flat plate with no edge beams are shown in the figure on the next page. The code provides coefficients for unrestrained, restrained, and with or without beams between supports

Design of Concrete Structures
Two-Way Slabs

- **Distribution of M_o - For a Plate with no Edge Beams**



Design of Concrete Structures
Two-Way Slabs

- **Transverse Distribution of Moments**
- After distributing M_o to the positive and negative moment regions, the positive and negative moments must be distributed across the column and middle strips. A column strip has a width on each side of the column equal to $0.25\ell_2$ or $0.25\ell_1$, whichever is less. The middle strip is the area bound between two column strips. The moment in each strip is assumed constant unless a beam is present on the column line

Design of Concrete Structures
Two-Way Slabs

- **Transverse Distribution of Moments (cont'd)**
- A beam when present tends to take a larger share of the column strip moment than the adjacent slab. The transverse distribution of moments is a function of ℓ_2/ℓ_1, the relative stiffness of the beam and the slab, and the degree of torsional restraint provided by the edge beam

Design of Concrete Structures
Two-Way Slabs

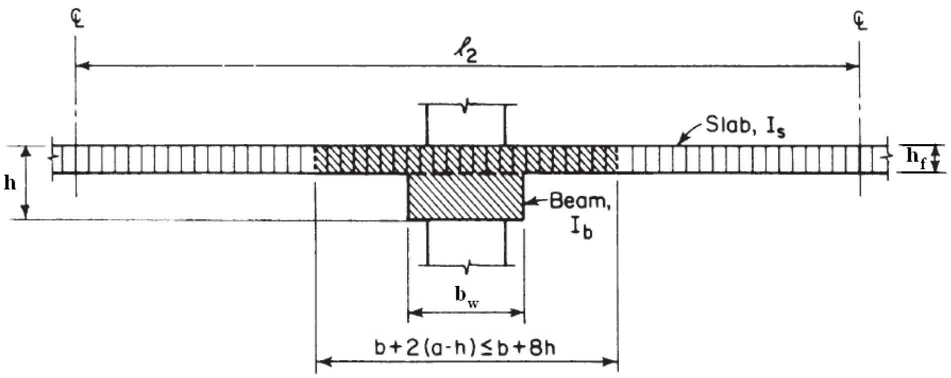
- **Transverse Distribution of Moments (cont'd)**
- The parameter α defines the relative stiffness between the beam and slab,

$$\alpha = \frac{E_{cb}\, I_b}{E_{cs}\, I_s} \qquad \text{(ACI 13.6.4)}$$

  - where E_{cb} and E_{cs} are moduli of elasticity of beam and slab,
  - I_b and I_s are moments of inertia of the effective beam and slab. α_1 and α_2 may be calculated for direction ℓ_1 and ℓ_2. In calculating EI, reinforcements, cracking, column capitals and drop panels may be neglected

Design of Concrete Structures
Two-Way Slabs

- **Transverse Distribution of Moments (cont'd)**
  – Cross Section of a Slab and Effective Beam



Design of Concrete Structures
Two-Way Slabs

- **Transverse Distribution of Moments (cont'd)**
- The relative restraint provided by the torsional resistance of the effective transverse edge beam is represented by the parameter β_t,

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3}$$

- C = cross-sectional constant to define torsional properties calculated by dividing the section into component rectangles, each having smaller dimension x and larger dimension y, and summing the contributions of all the parts

Design of Concrete Structures
Two-Way Slabs

- **Transverse Distribution of Moments (cont'd)**
- Column strip moment, percent of total moment at critical section
  – Positive Factored Moments

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

  – Interior Negative Factored Moments

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Design of Concrete Structures
Two-Way Slabs

- **Transverse Distribution of Moments (cont'd)**
- Column strip moment, percent of total moment at critical section
  – Exterior Negative Moments

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Design of Concrete Structures
Two-Way Slabs

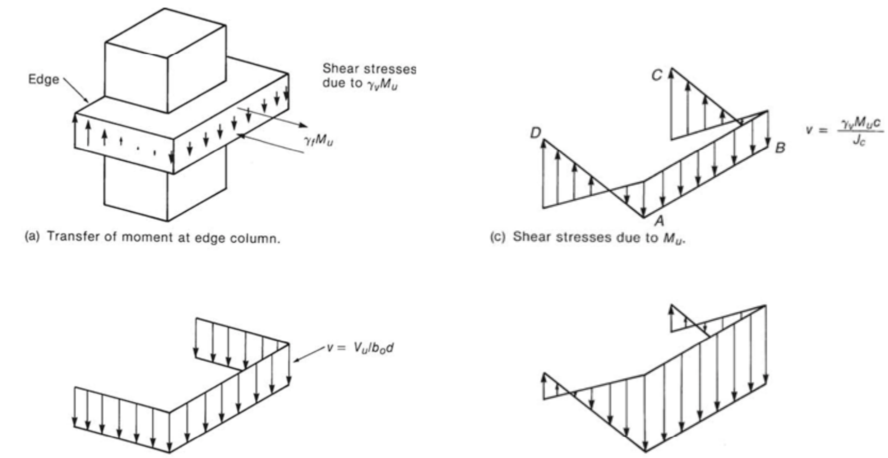
- **Moments in Columns**
  - Columns in two-way construction must be designed to resist the moments found from analysis of the slab-beam system. At interior locations, slab negative moments are found assuming that dead and full live loads act. For the column, a more severe condition will result with partial live loading. For interior columns the Code requires for interior columns to resist,

$$\mathbf{M = 0.07\,[(w_d + 0.5\,w_\ell)\,\ell_2\,\ell_n^2 - w_d'\,\ell_2'\,(\ell_n')^2]} \qquad \text{(ACI 13-4)}$$

  - Where the prime quantities refer to the shorter of the two adjacent spans

Design of Concrete Structures
Two-Way Slabs

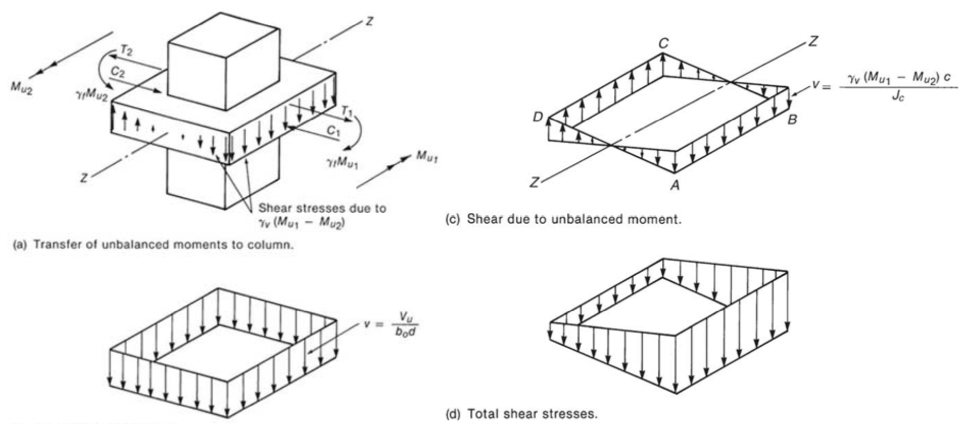
- **Transfer of Moment and Shear**



(a) Transfer of moment at edge column.
(c) Shear stresses due to M_u.
(b) Shear stresses due to V_u.
(d) Total shear stresses.

Design of Concrete Structures
Two-Way Slabs

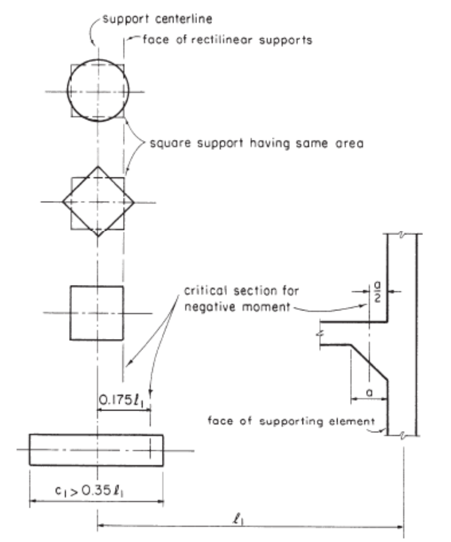
- **Transfer of Moment and Shear (cont'd)**



(a) Transfer of unbalanced moments to column.
(c) Shear due to unbalanced moment.
(b) Shear stresses due to V_u.
(d) Total shear stresses.

Design of Concrete Structures
Two-Way Slabs

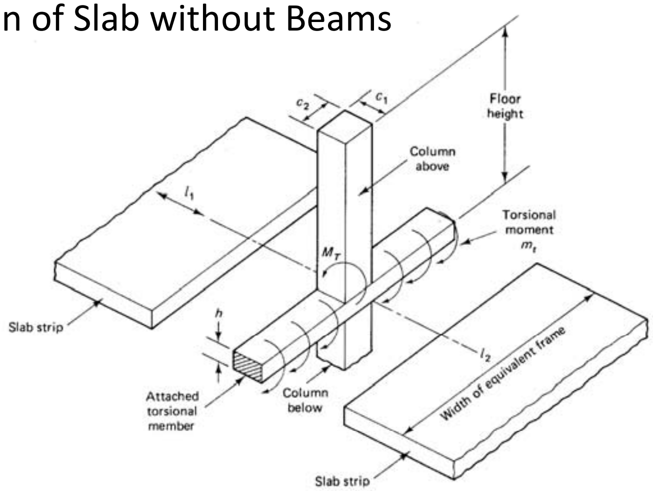
- Critical Sections for Negative Design Moment



(a) Interior & Exterior Column or Wall Supports
(b) Exterior Supports with Brackets or Corbels

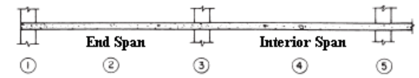
Design of Concrete Structures
Two-Way Slabs

- Transfer of Negative Moment at Exterior Support Section of Slab without Beams



Design of Concrete Structures
Two-Way Slabs – Design Moment Coefficients

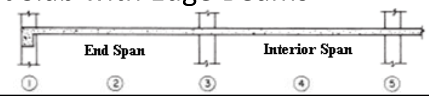
- Flat Plate or Flat Slab Supported Directly on Columns



Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	$0.26M_o$	$0.52M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.26M_o$	$0.31M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.21M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

- Flat Plate or Flat Slab with Edge Beams

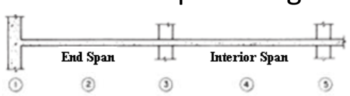


Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	$0.30M_o$	$0.50M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.23M_o$	$0.30M_o$	$0.53M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.07M_o$	$0.20M_o$	$0.17M_o$	$0.14M_o$	$0.16M_o$

Notes: (1) All negative moments are at face of support.
(2) Torsional stiffness of edge beam is such that $\beta_t \geq 2.5$. For values of β_t less than 2.5, exterior negative column strip moment increases to $(0.30 - 0.03\beta_t)M_o$

Design of Concrete Structures
Two-Way Slabs – Design Moment Coefficients

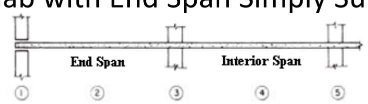
- Flat Plate or Flat Slab with End Span Integral With Wall



Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	$0.65M_o$	$0.35M_o$	$0.65M_o$	$0.35M_o$	$0.65M_o$
Column Strip	$0.49M_o$	$0.21M_o$	$0.49M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	$0.16M_o$	$0.14M_o$	$0.16M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

- Flat Plate or Flat Slab with End Span Simply Supported on Wall

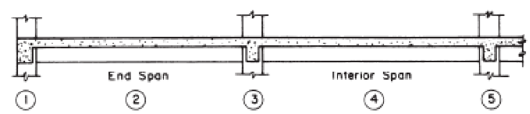


Slab Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
Total Moment	0	$0.63M_o$	$0.75M_o$	$0.35M_o$	$0.65M_o$
Column Strip	0	$0.38M_o$	$0.56M_o$	$0.21M_o$	$0.49M_o$
Middle Strip	0	$0.25M_o$	$0.19M_o$	$0.14M_o$	$0.16M_o$

Note: All negative moments are at face of support.

Design of Concrete Structures
Two-Way Slabs – Design Moment Coefficients

- Two-Way Beam-Supported Slab

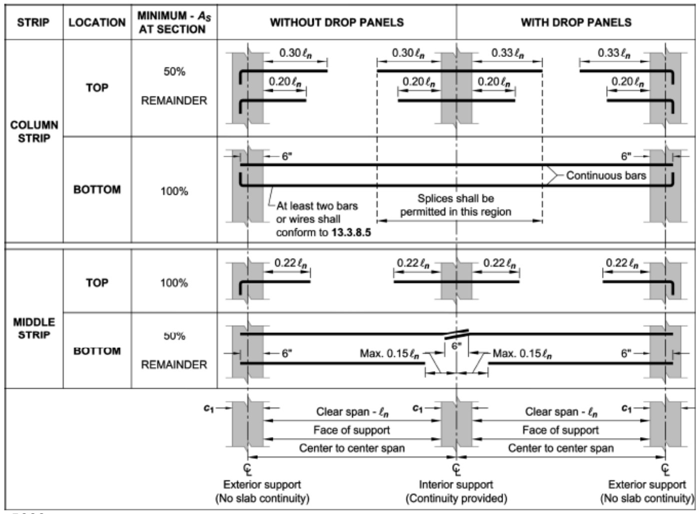


Span Ratio ℓ_2/ℓ_1	Slab and Beam Moments	End Span (1) Exterior Negative	End Span (2) Positive	End Span (3) First Interior Negative	Interior Span (4) Positive	Interior Span (5) Interior Negative
	Total Moment	$0.16M_o$	$0.57M_o$	$0.70M_o$	$0.35M_o$	$0.65M_o$
0.5	Column Strip Beam	$0.12M_o$	$0.43M_o$	$0.54M_o$	$0.27M_o$	$0.50M_o$
	Column Strip Slab	$0.02M_o$	$0.08M_o$	$0.09M_o$	$0.05M_o$	$0.09M_o$
	Middle Strip	$0.02M_o$	$0.06M_o$	$0.07M_o$	$0.03M_o$	$0.06M_o$
1.0	Column Strip Beam	$0.10M_o$	$0.37M_o$	$0.45M_o$	$0.22M_o$	$0.42M_o$
	Column Strip Slab	$0.02M_o$	$0.06M_o$	$0.08M_o$	$0.04M_o$	$0.07M_o$
	Middle Strip	$0.04M_o$	$0.14M_o$	$0.17M_o$	$0.09M_o$	$0.16M_o$
2.0	Column Strip Beam	$0.06M_o$	$0.22M_o$	$0.27M_o$	$0.14M_o$	$0.25M_o$
	Column Strip Slab	$0.01M_o$	$0.04M_o$	$0.05M_o$	$0.02M_o$	$0.04M_o$
	Middle Strip	$0.09M_o$	$0.31M_o$	$0.38M_o$	$0.19M_o$	$0.36M_o$

Notes: (1) All negative moments are at face of support.
(2) Torsional stiffness of edge beam is such that $\beta_t \geq 2.5$
(3) $\alpha_{f1}\ell_2/\ell_1 \geq 1.0$

Design of Concrete Structures
Two-Way Slabs

- Minimum extensions for reinforcement in slabs without beams

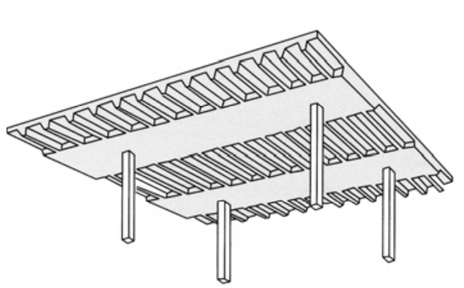


Design of Concrete Structures
Two-Way Slab Systems

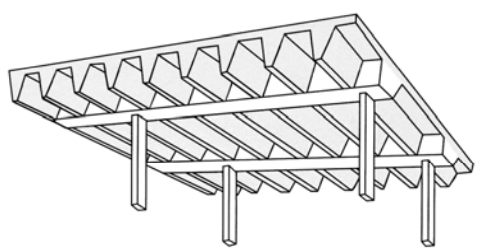
- Design Procedures
  - The ACI code (13.5.1) permits the design of a slab system by any procedure satisfying conditions of equilibrium and geometric compatibility that satisfy code strength requirements (9.2 and 9.3) and all applicable code serviceability requirements including limits on deflections (9.5.3)

Design of Concrete Structures
Two-Way Slab Systems

- <u>Types of Two-Way Slab Systems</u>



Standard



Wide Module

Design of Concrete Structures
Two-Way Slab Systems

- *Analysis for Gravity Loads*
  - The ACI code includes the Direct Design Method (DDM)(13.6) and the Equivalent Frame Method (EFM)(13.7) for slab systems between supports and supporting columns or walls laid out on a basically orthogonal grid
  - Both methods apply to two-way slabs with beams as well as to flat slabs and flat plates

Design of Concrete Structures
Two-Way Slab Systems

- *Analysis for Lateral Loads*
  - The Code (13.5.1.2) states that for lateral loads, analysis of frames shall take into account effects of cracking and reinforcement on stiffness of frame members
    - Cracking of slabs should be considered so that drift caused by wind or earthquake is not grossly underestimated (R13.5.1.2)
  - Any methods that that satisfy compatibility and equilibrium and that are in reasonable agreement with test data can be used. Acceptable methods include plate bending finite element models, effective width models, and equivalent frame methods

Design of Concrete Structures
Two-Way Slab Systems

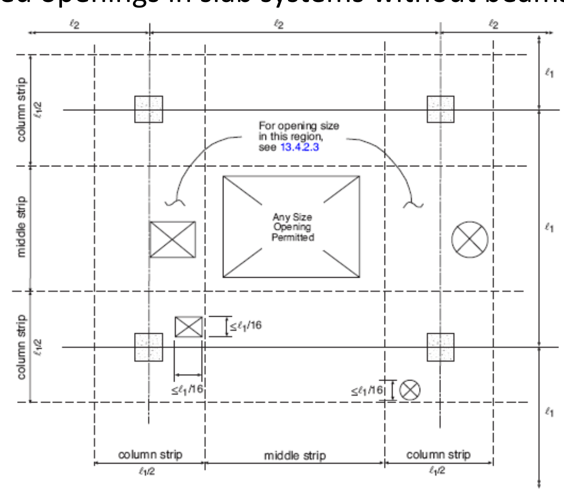
- *Analysis for Lateral Loads* (cont'd)
  - The stiffness of slab members is affected not only by cracking, but also by other parameters such as ℓ_2/ℓ_1, c_1/ℓ_1, c_2/c_1, and concentration of reinforcement in the slab near the column supports

Design of Concrete Structures
Two-Way Slab Systems

- Openings in Slab Systems
- The ACI Code (13.4) permits openings of any size provided that analysis shows that code design strength (9.2 & 9.3) and serviceability requirements are satisfied
- This requirement is waived for slabs without beams when the provisions of section 13.4.2.1 through 13.4.2.4 are satisfied

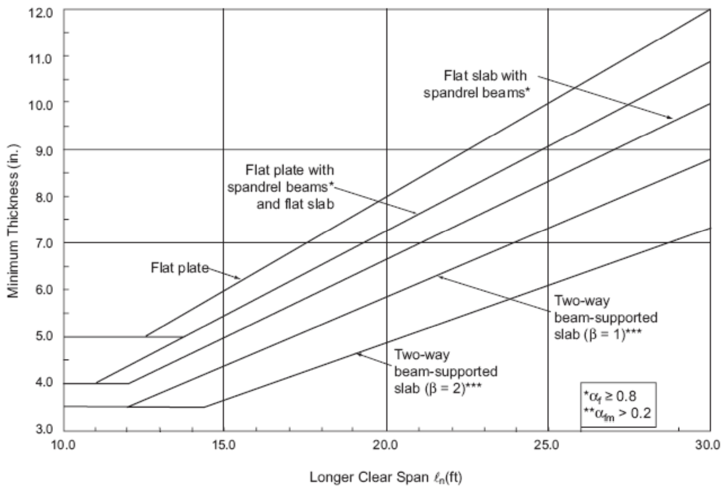
Design of Concrete Structures
Two-Way Slab Systems

- Openings in Slab Systems
  - Permitted openings in slab systems without beams



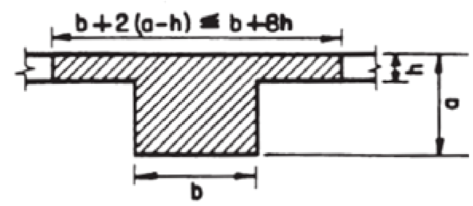
Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Minimum Slab thickness

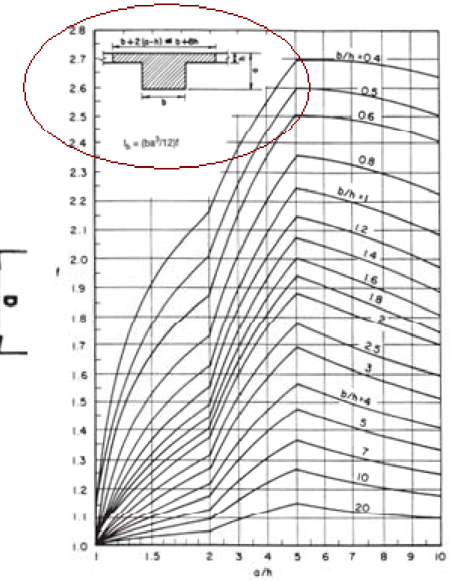


Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Beam Stiffness
  - Interior beams

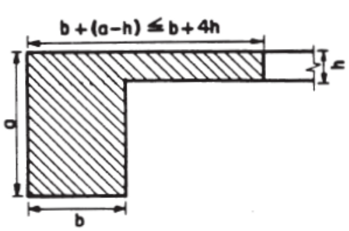


$I_b = (ba^3/12)f$

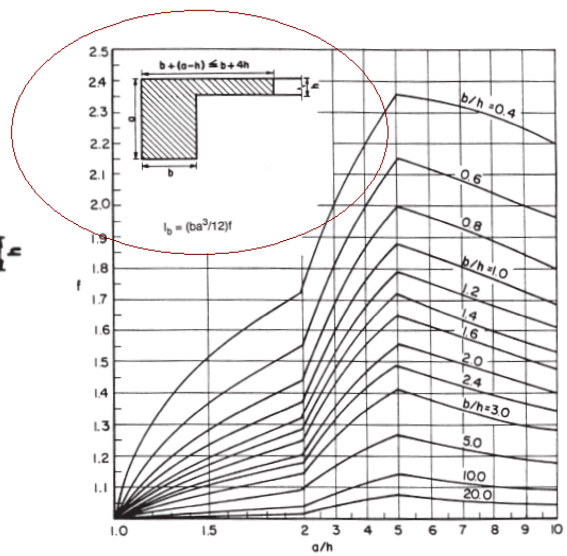


Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Beam Stiffness
  - Edge beams

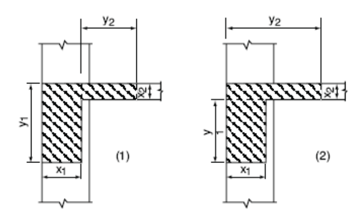


$I_b = (ba^3/12)f$

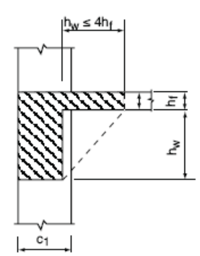


Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Computing C, Cross-Sectional Constant Defining Torsional Properties



Use larger value of C computed from (1) or (2)



Edge beam (ACI 13.2.4)

y	x* 4	5	6	7	8	9	10	12	14	16
12	202	369	592	868	1,118	1,538	1,900	2,557		
14	245	452	736	1,096	1,529	2,024	2,566	3,709	4,738	
16	288	534	880	1,325	1,871	2,510	3,233	4,861	6,567	8,083
18	330	619	1,024	1,554	2,212	2,996	3,900	6,013	8,397	10,813
20	373	702	1,167	1,782	2,553	3,482	4,567	7,165	10,226	13,544
22	416	785	1,312	2,011	2,895	3,968	5,233	8,317	12,055	16,275
24	458	869	1,456	2,240	3,236	4,454	5,900	9,459	13,885	19,005
27	522	994	1,672	2,583	3,748	5,183	6,900	11,197	16,628	23,101
30	586	1,119	1,888	2,926	4,260	5,912	7,900	12,925	19,373	27,197
33	650	1,243	2,104	3,269	4,772	6,641	8,900	14,653	22,117	31,293
36	714	1,369	2,320	3,612	5,284	7,370	9,900	16,381	24,860	35,389
42	842	1,619	2,752	4,298	6,308	8,828	11,900	19,837	30,349	43,581
48	970	1,869	3,183	4,984	7,332	10,286	13,900	23,293	35,836	51,773
54	1,098	2,119	3,616	5,670	8,356	11,744	15,900	26,749	41,325	59,965
60	1,226	2,369	4,048	6,356	9,380	13,202	17,900	30,205	46,813	68,157

* *Small side of a rectangular cross-section with dimensions x and y.*

Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Minimum Thickness

Two-Way Slab System	α_{fm}	β	Minimum h
Flat Plate	—	≤ 2	$\ell_n/30$
Flat Plate with Spandrel Beams[1] [Min. h = 5 in.]	—	≤ 2	$\ell_n/33$
Flat Slab		≤ 2	$\ell_n/33$
Flat Slab[2] with Spandrel beams[1] [Min. h = 4 in.]	—	≤ 2	$\ell_n/36$
Two-Way Beam-Supported Slab[3]	≤ 0.2	≤ 2	$\ell_n/30$
	1.0	1	$\ell_n/33$
		2	$\ell_n/36$
	≥ 2.0	1	$\ell_n/37$
		2	$\ell_n/44$
Two-Way Beam-Supported Slab[1,3]	≤ 0.2	≤ 2	$\ell_n/33$
	1.0	1	$\ell_n/36$
		2	$\ell_n/40$
	≥ 2.0	1	$\ell_n/41$
		2	$\ell_n/49$

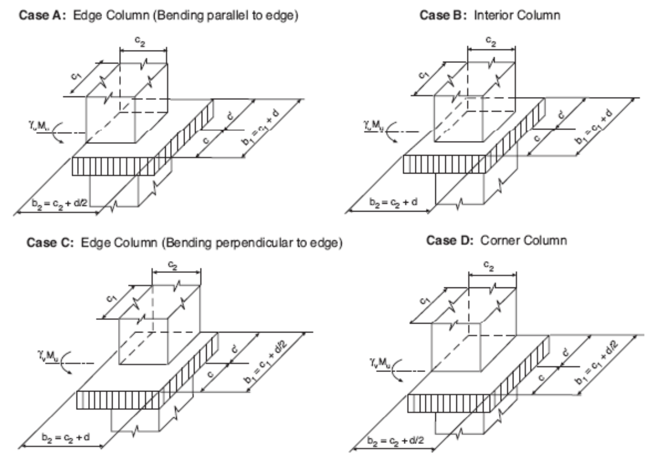
[1]Spandrel beam-to-slab stiffness ratio $\alpha_f \geq 0.8$ (9.5.3.3)

[2]Drop panel length $\geq \ell/3$, depth $\geq$ 1.25h (13.3.7)

[3]Min. h = 5 in. for $\alpha_{fm} \leq 2.0$; min. h = 3.5 in. for $\alpha_{fm} > 2.0$ (9.5.3.3)

Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Properties for Shear Stress Computations



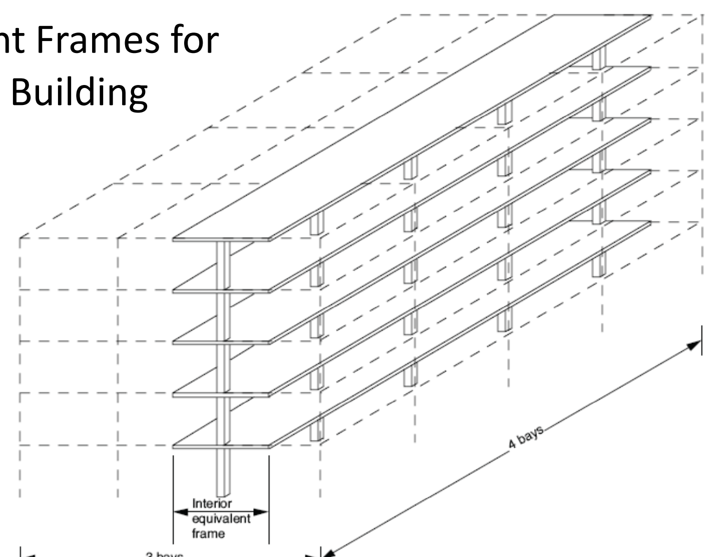
Design of Concrete Structures
Two-Way Slab Systems – Design Aids

- Properties for Shear Stress Computations (cont'd)

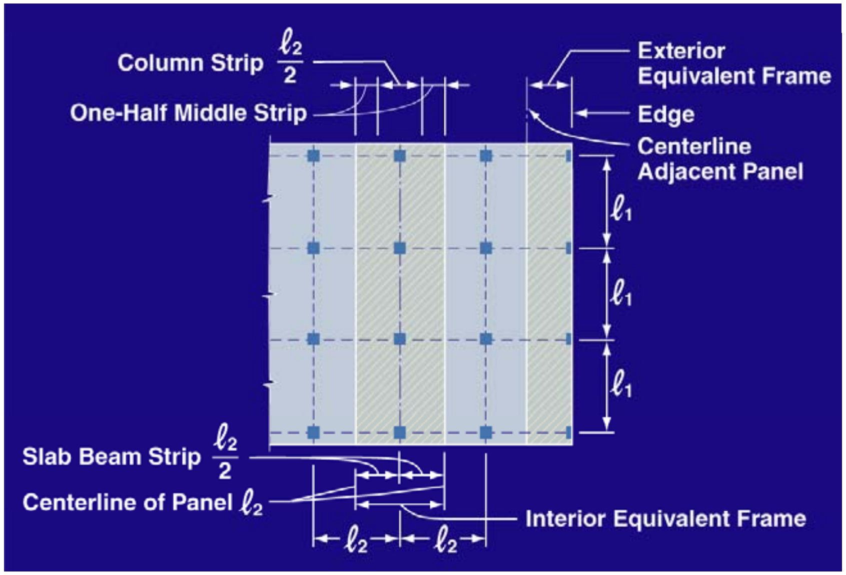
Case	Area of critical section, A_c	Modulus of critical section: J/c	Modulus of critical section: J/c'	c	c'
A	$(b_1+2b_2)d$	$\frac{b_1d(b_1+6b_2)+d^3}{6}$	$\frac{b_1d(b_1+6b_2)+d^3}{6}$	$\frac{b_1}{2}$	$\frac{b_1}{2}$
B	$2(b_1+b_2)d$	$\frac{b_1d(b_1+3b_2)+d^3}{3}$	$\frac{b_1d(b_1+3b_2)+d^3}{3}$	$\frac{b_1}{2}$	$\frac{b_1}{2}$
C	$(2b_1+b_2)d$	$\frac{2b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6b_1}$	$\frac{2b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6(b_1+b_2)}$	$\frac{b_1^2}{2b_1+b_2}$	$\frac{b_1(b_1+b_2)}{2b_1+b_2}$
D	$(b_1+b_2)d$	$\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6b_1}$	$\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6(b_1+2b_2)}$	$\frac{b_1^2}{2(b_1+b_2)}$	$\frac{b_1(b_1+2b_2)}{2(b_1+b_2)}$

Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method

- Equivalent Frames for a 5-Story Building



Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method



Design of Concrete Structures
Equivalent Frame Method – Operational Design Flowchart

Start

1. Input: Slab plan dimensions L_{EW}, L_{NS}, l_u, W_L, W_D, f'_c, f_y, E_c, E_s

2. Select preliminary thickness h for E-W, N-S using Eq. 11.9, and 11.10 for slabs with beams or slabs with drop panels, and Table 11.3 for flat plates. Chosen thicknesses h should always satisfy the minimum code requirements for thickness h. Increase h from Table 11.3 by at least 10% if no edge beams are used.

3. Compute E_{cc}, E_{cs}, E_{cb}

where c = column, s = slab, b = beam

$$K_c \simeq \frac{4EI}{L - 2h}$$

$$K_t = \Sigma \frac{9E_{cs}C}{L_2(1 - c_2/L_2)^3}$$

where $C = \Sigma(1 - 0.63x/y)\frac{x^3y}{3}$

Find $K_{ec} = \left(\frac{1}{K_c} + \frac{1}{K_t}\right)^{-1}$

cont'd

Design of Concrete Structures
Equivalent Frame Method – Operational Design Flowchart

cont'd

4. Compute $K_s \simeq \frac{4E_{cs}I}{(L_{n1} - c_1/2)}$

where L_{n1} = centerline span

c_1 = column depth

Find $DF = \frac{K_s}{\Sigma K}$

where $\Sigma K = K_{ec} + K_{s(\text{left})} + K_{s(\text{right})}$

5. Compute FEM for load intensity w_u and run a moment distribution where $\text{FEM} = \pm\frac{w_u(L_n)^2}{12}$. Adjust distributed M_u to support face values such that the adjusted M_u = distributed $M_u - \frac{V_c}{3}$

6. Convert the final M_u values to required nominal moment strengths $M_n = \frac{M_u}{\phi}$

7. Consider inelastic moment redistribution if necessary, not exceeding 20% but not more than $1000\,\epsilon_t$ percent as in Fig. 5.7

cont'd

Design of Concrete Structures
Equivalent Frame Method – Operational Design Flowchart

cont'd

8. Use appropriate factors in terms of $\alpha_1(l_2/l_1)$ of Section 11.4.2 to derive the column strip and middle strip moments.

9. Select the reinforcement. Compute available $M_n = A_s f_y(d - a/2)$ for support and midspan sections of the column and middle strips.

10. Available M_n < required M_n — No: Go to 9; Yes: continue

11. Shear-moment transfer

Compute V_u, select M_{uc} from step 5, compute g and unbalanced moment $M_n = M_{uc} + V_u g$.

Compute $\gamma_v = 1 - \frac{1}{1 + 2/3\sqrt{b_1/b_2}}$

where $b_1 = (c_1 + d/2)$ for exterior support

$= (c_1 + d)$ for interior support

$b_2 = c_2 + d$

$\gamma_f = 1 - \gamma_v$, compute $c_{AB} = \bar{x}$

γ_f can be increased to 1.0 at end columns provided V_u at support $\leq 0.75\ V_c$, and by 25% for interior columns if $V_u \leq 0.4\ V_c$ and $\rho \leq 0.375\rho_b$

cont'd

Design of Concrete Structures
Equivalent Frame Method – Operational Design Flowchart

cont'd

(12)

Determine $v_n = \dfrac{V_u}{\phi A_c} + \dfrac{\gamma_v c_{AB} M_n}{J_c}$ giving $V_n = v_n b_0 d$.

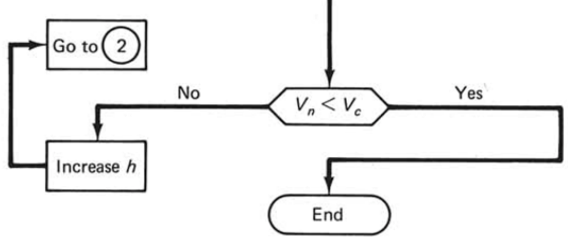
Maximum allowable nominal shear strength as the smallest of

(i) $V_c = \left(2 + \dfrac{4}{\beta}\right)\lambda\sqrt{f'_c}\, b_0 d$

(ii) $V_c = \left(\dfrac{\alpha_s d}{b_0} + 2\right)\lambda\sqrt{f'_c}\, b_0 d$

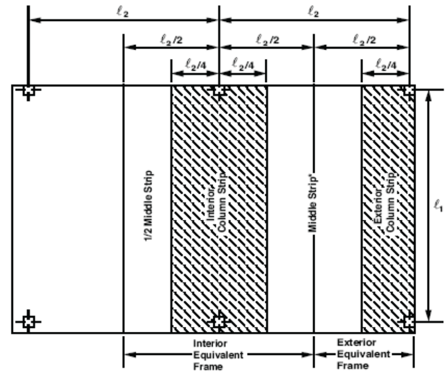
where α_s = 40 for interior columns, 30 for edge columns, and 20 for corner columns

(iii) $V_c = 4\lambda\sqrt{f'_c}\, b_0 d$



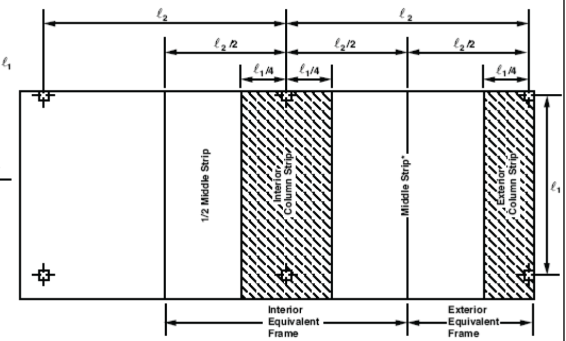
Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Design Strips of Equivalent Frame



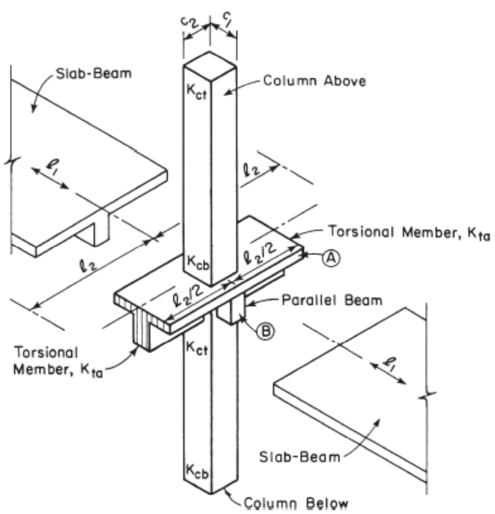
Design Strip for $\ell_2 \le \ell_1$

Design Strip for $\ell_1 \le \ell_2$



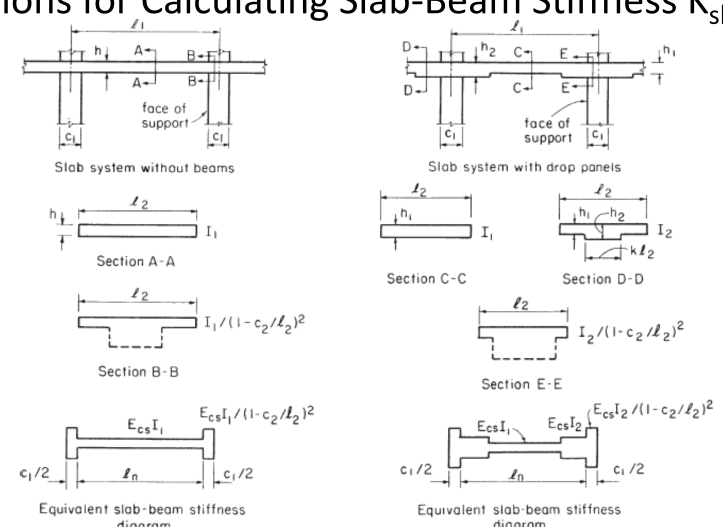
Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Equivalent Frame Members



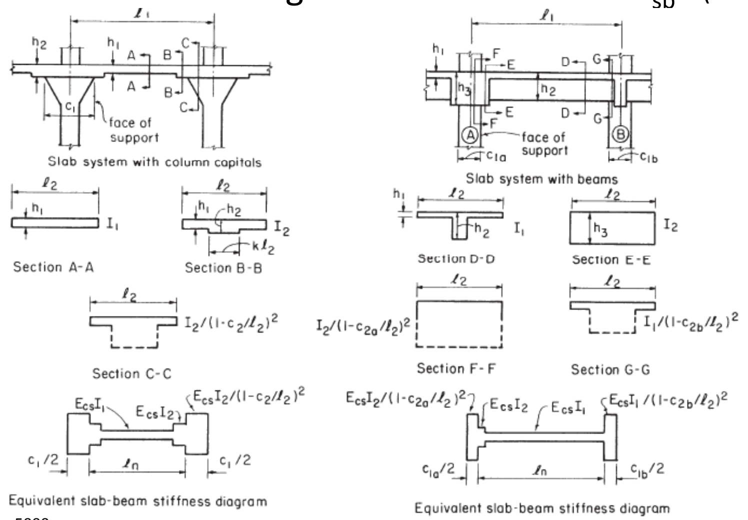
Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Sections for Calculating Slab-Beam Stiffness K_{sb}



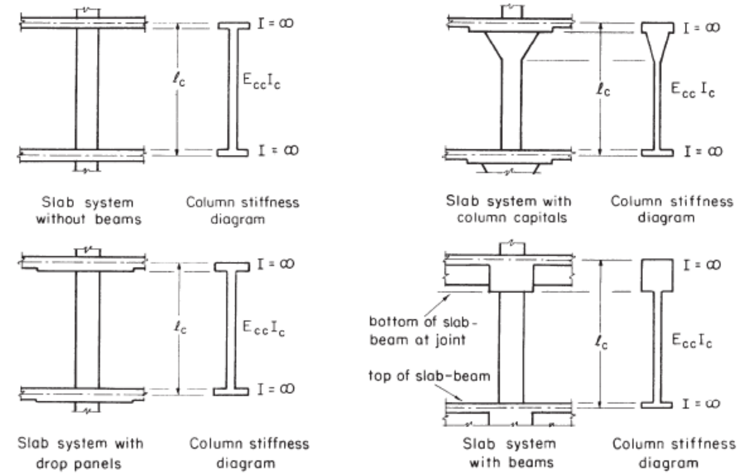
Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Sections for Calculating Slab-Beam Stiffness K_{sb} (cont'd)



Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Sections for Calculating Column Stiffness K_c



Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

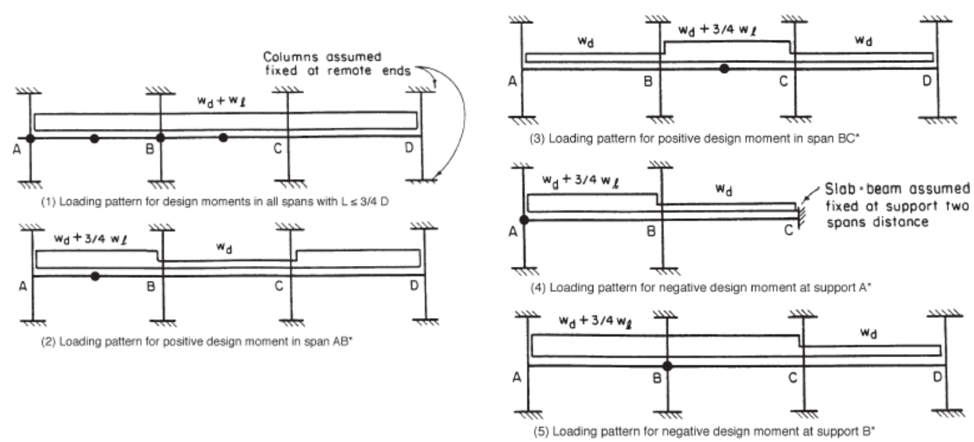
- Partial Frame Analysis for Vertical Loading
  - When exact loading pattern is not known, maximum factored moments are developed with the loading conditions:

a) When the service live load does not exceed ¾ of the service dead load, only loading pattern with full factored live load on all spans need be analyzed for negative and positive factored moments

(cont'd)

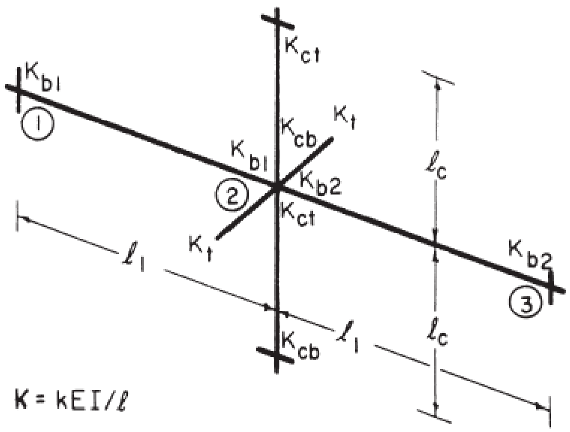
Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

b) When service live-to-dead load ratio exceeds 3/4 , these five loading patterns need to be analyzed



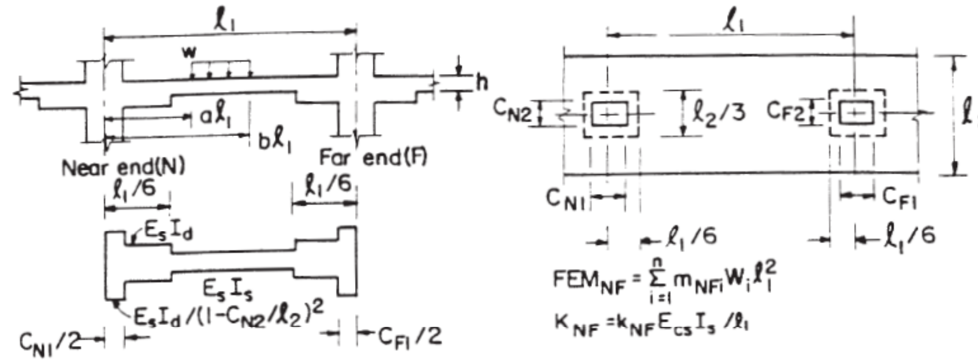
Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Moment Distribution Factors DF



Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Design Aids for Moment Distribution Constants



- Moment Distribution Constants for Slab-Beam Members (drop thickness = .25h)

Design of Concrete Structures
Two-Way Slabs – Equivalent Frame Method (cont'd)

- Design Aids for Moment Distribution Constants (cont'd)

C_{N1}/ℓ_1	C_{N2}/ℓ_2	Stiffness Factors k_{NF}	Carry Over Factors C_{NF}	Unif. Load Fixed end M. Coeff. (m_{NF})	Fixed end moment Coeff. (m_{NF}) for (b—a) = 0.2				
					a = 0.0	a = 0.2	a = 0.4	a = 0.6	a = 0.8
$C_{F1} = C_{N1}$; $C_{F2} = C_{N2}$									
0.00	—	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
0.10	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	4.99	0.55	0.0890	0.0160	0.0316	0.0266	0.0128	0.0020
	0.20	5.18	0.56	0.0901	0.0163	0.0322	0.0270	0.0127	0.0019
	0.30	5.37	0.57	0.0911	0.0167	0.0328	0.0273	0.0126	0.0018
0.20	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	5.17	0.56	0.0900	0.0161	0.0320	0.0269	0.0128	0.0020
	0.20	5.56	0.58	0.0918	0.0166	0.0332	0.0276	0.0126	0.0018
	0.30	5.96	0.60	0.0936	0.0171	0.0344	0.0282	0.0124	0.0016
0.30	0.00	4.79	0.54	0.0879	0.0157	0.0309	0.0263	0.0129	0.0022
	0.10	5.32	0.57	0.0905	0.0161	0.0323	0.0272	0.0128	0.0021
	0.20	5.90	0.59	0.0930	0.0166	0.0338	0.0281	0.0127	0.0019
	0.30	6.55	0.62	0.0955	0.0171	0.0354	0.0290	0.0124	0.0017

5100. Reinforced Concrete

- Objective and Scope Met
  - Provided introductory and intermediate level review of analysis and design of reinforced concrete design
  - Presented and discussed
    - Materials and Design Methods
    - Moment & Shear Design of Beams
    - Footing & Column Design
    - Development of Reinforcement
    - Strut & Tie Model, and Two-way Slabs