Uncertainty modeling of LOCA frequencies and break size distributions for the STP GSI-191 resolution

Elmira Popova, Alexander Galenko
The University of Texas at Austin

Background

- The Pilot Project for the STP GSI-191 resolution started in January 2011. One of the distinguishing characteristics of the proposed solution approach is the inclusion of uncertainties. Therefore, their proper modeling is required.
- Major input to the Pilot Project are the LOCA frequencies.
- The initial quantification used models and results from the EPRI "risk-informed in service inspection program."

Background

- The Pilot Project is an unique combination of existing PRA infrastructure and Casa Grande, a new simulator that models and executes events, physics, statistical sampling, i.e. performs everything that the existing PRA can't.
- Both of them have the same input – LOCA frequencies.
- The LOCA frequencies used in PRA are not location-specific, whereas Casa Grande needs them to be location-specific.
- In this presentation, location will be equivalent to weld locations inside STP.

NUREG-1829

- The expert elicitation study in NUREG-1829 gives the LOCA frequencies for 6 break size categories. Bellow is Table 1 from that study.

Plant Type	LOCA Size (gpm)	Eff. Break Size (inch)	Current-day Estimate (per cal. yr) (25 yr fleet average operation)			
			5th Per.	Median	Mean	95th Per.
PWR	>100	1/2	6.90E-04	3.90E-03	7.30E-03	2.30E-02
	>1,500	1 5/8	7.60E-06	1.40E-04	6.40E-04	2.40E-03
	>5,000	3	2.10E-07	3.40E-06	1.60E-05	6.10E-05
	>25K	7	1.40E-08	3.10E-07	1.60E-06	6.10E-06
	>100K	14	4.10E-10	1.20E-08	2.00E-07	5.80E-07
	>500K	31	3.50E-11	1.20E-09	2.90E-08	8.10E-08

NUREG-1829 results and our major challenges

- The LOCA frequencies are not plant specific or plant-location specific. In our analysis we need to “initiate” a break at a random location (weld) inside STP. (Call this Problem 1)
- The six break size categories (columns in the table) are ranges bounded by six discrete points. For a particular weld we need to be able to sample from the continuous range of break size values. (Call this Problem 2)
- The LOCA table provides four distributional characteristics (rows in the table) for each break size category: mean, median, 5^{th} and 95^{th} percentiles. We would like to sample from distributions that match these values. (Call this Problem 3)

Main objective

- In this communication we will discuss possible solutions to each of the three problems. We consider Problem 1 to be the critical one and we hope to find a mutual agreeable path.
- Main assumption – we conserve the values from NUREG-1829 and use them as input to both Case Grande and PRA analysis.

Proposed solution to Problem 1

- The LOCA frequencies are not plant specific or plant-location specific. In our analysis we need to “initiate” a break at a random location (weld) inside STP.
- The solution to Problem 1 in the initial quantification was a “bottom-up” approach: model the uncertainties at each weld location, and accumulate them to get the total LOCA frequency. By following this path one cannot hope to get the NUREG-1829 numbers as the resulting total cumulative frequencies.
- Our current methodology is “top-down”: we start with the NUREG-1829 numbers and distribute them to all weld locations. This way the conservation of the LOCA frequencies is guaranteed.

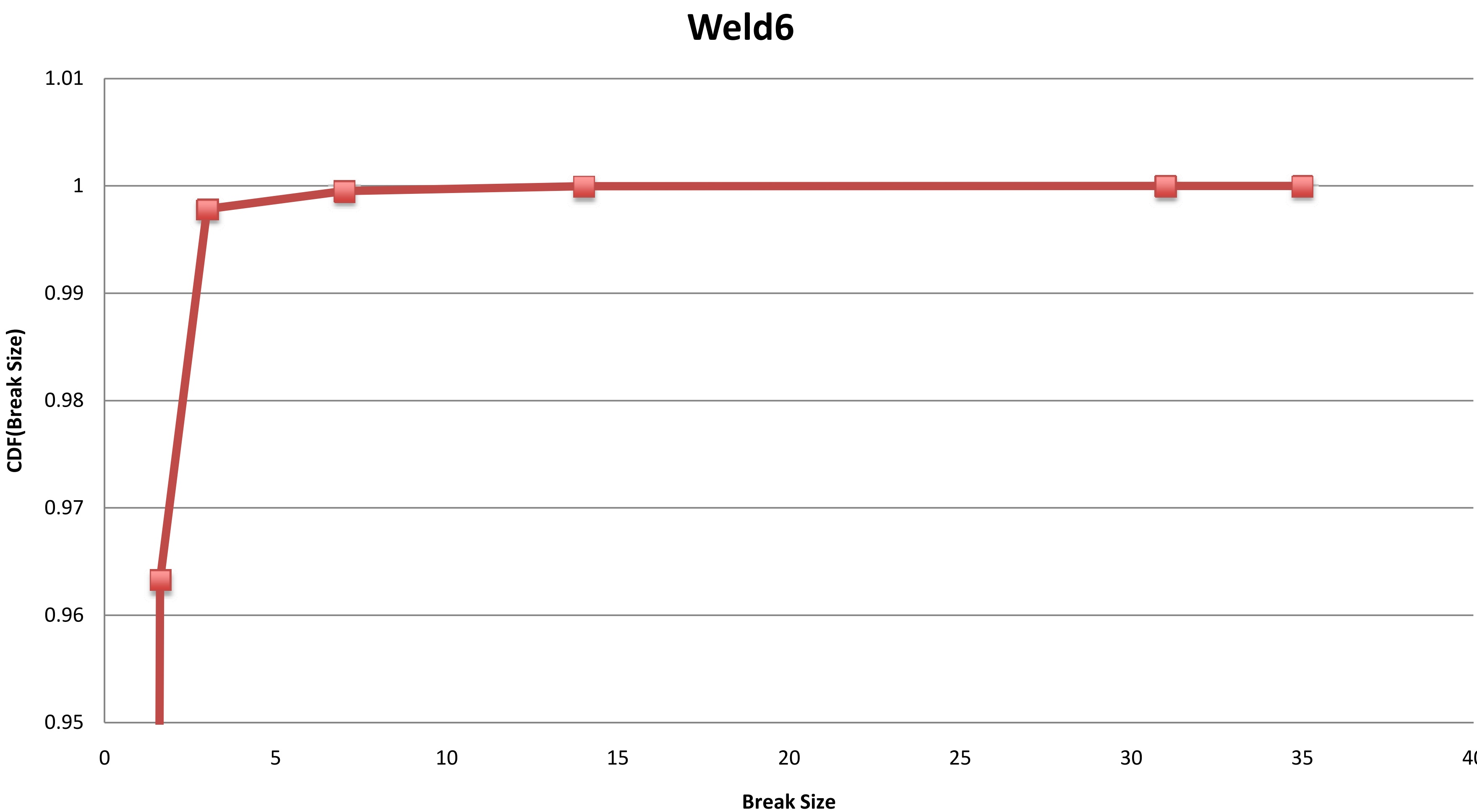
Proposed solution to Problem 1

- The distribution of the LOCA numbers will be equally likely.
- Small breaks are equally likely to occur in small, medium, or large welds; medium breaks are equally likely to occur in medium and large welds; large breaks can occur only in large welds.
- Each location (weld) is equally likely to be chosen.
- These assumptions and the law of total probability leads to the preservation of the total LOCA frequencies.

Proposed solution to Problem 2

- The six break size categories (columns in the NUREG-1829 table) are ranges bounded by six discrete points. For a particular weld we need to be able to sample from the continuous range of break size values.
- We will connect the bounding points with straight lines.
- This implies that the values within each range are equally likely. Hence we are modeling each break size as Uniform distribution over the range.

Resulting Break Size Cumulative Distribution Function for weld 6 from the Illustrative Example



Proposed solution to Problem 3

- The LOCA table provides four distributional characteristics (rows in the table) for each break size category: mean, median, 5th and 95th percentiles. We would like to sample from distributions that match these values.
- NUREG-1829 used two split Lognormal distributions.
- NUREG/CR 6828 used Gamma distribution.
- There are infinite number of distributions one can potentially fit.
- If we want to match the four characteristics as close as possible we need a parametric distribution with four parameters.
- The bounded Johnson distribution fits this requirement. Many different shapes: skewed, symmetric, bimodal, unimodal are all possible.

Johnson distribution

- CDF(x) of Bounded Johnson Distribution

$F[x] = \Phi\{\gamma + \delta f[(x - \xi) / \lambda]\}$,

$\Phi[x]$ – cdf of a standard Normal(0,1) random variable

γ, δ - shape parameters

ξ - location parameter (left bound)

λ - scale parameter

$\xi + \lambda$ – right bound

Description of the estimation procedure

- For each break size category we solve a nonlinear optimization problem.
- The objective function is the weighted squared error.
- Have four constraints that correspond to matching each of the NUREG-1829 characteristics: mean, median, 5^{th}, and 95^{th} percentiles.

Estimated Parameters of Johnson Distribution

	Johnson Parameters			
	gamma	**delta**	**xi**	**lambda**
Category1	0.7288246	0.3893326	0.000634494	0.02449228
Category2	6.95E-01	2.40E-01	7.41E-06	2.44E-03
Category3	7.24E-01	2.44E-01	2.06E-07	6.24E-05
Category4	7.14E-01	2.39E-01	1.36E-08	6.19E-06
Category5	4.73E-01	2.69E-01	1.87E-10	5.93E-07
Category6	4.75E-01	2.73E-01	1.77E-14	8.52E-08

Comparison with NUREG-1829

	NUREG-1829			Fitted Johnson			Error		
	5%	**Mean**	**95%**	**5%**	**Mean**	**95%**	**5%**	**Mean**	**95%**
Category1	6.90E-04	7.30E-03	2.30E-02	6.89E-04	7.30E-03	2.30E-02	0.08%	0.01%	0.00%
Category2	7.60E-06	6.40E-04	2.40E-03	7.56E-06	6.42E-04	2.40E-03	0.59%	0.38%	0.04%
Category3	2.10E-07	1.60E-05	6.10E-05	2.09E-07	1.60E-05	6.12E-05	0.40%	0.23%	0.38%
Category4	1.40E-08	1.60E-06	6.10E-06	1.40E-08	1.59E-06	6.08E-06	0.26%	0.65%	0.39%
Category5	4.10E-10	2.00E-07	5.80E-07	4.14E-10	1.98E-07	5.86E-07	0.94%	0.94%	0.98%
Category6	3.50E-11	2.90E-08	8.10E-08	3.59E-11	2.84E-08	8.41E-08	2.60%	2.03%	3.78%

Methodology summary

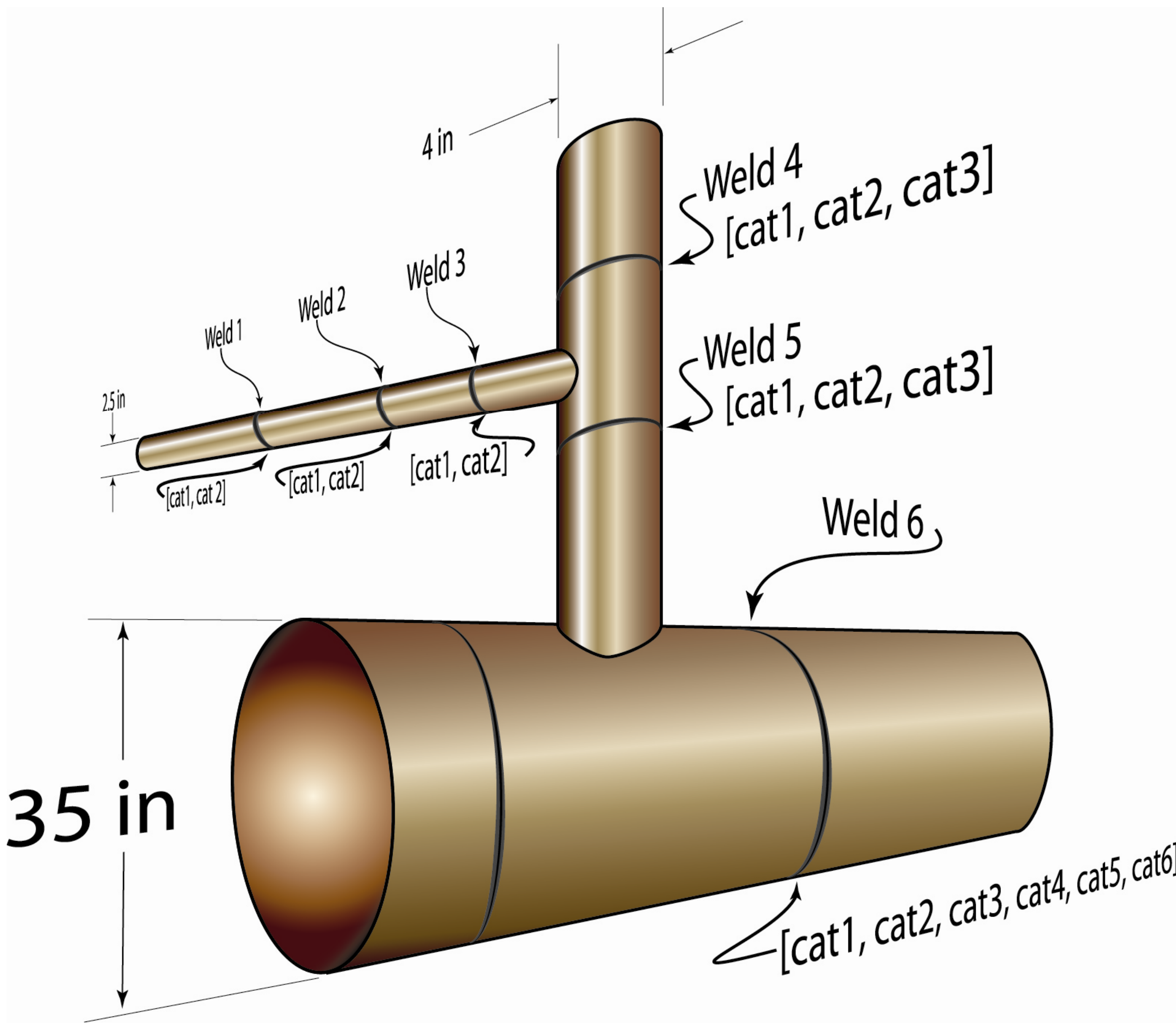
1. Set N - number of LOCA frequency samples and S - number of break size samples to generate.
2. Sample LOCA frequencies from the fitted Johnson distributions (or different distributions).
3. Distribute frequency across plant specific welds (equally likely or based on predefined weights).
4. Sample actual break size for each possible weld / break category combination using Uniform distribution (or a different distribution).
5. Estimate performance measures, store them.
6. If we ran S break sizes samples go to the next step, otherwise go to step 4.
7. Compute the performance measures summary, store them.
8. If we ran N LOCA frequencies samples go to the next step, otherwise, go to step 2.
9. Make aggregated performance measures summary.

Illustrative example – first four steps, N=1, S=1

- Welds:
  - Total number of welds: 6
  - Small welds (2.5 inches): 3
  - Medium welds (4 inches): 2
  - Large welds (35 inches): 1

- Failure categories:
  - Total number of failure categories: 6
  - Small welds: category1 and category2
  - Medium welds: category1, category2 and category3
  - Large welds: category1, category2, category3, category4, category5 and category6

Example: System Description



Example: Failure Categories

	BreakSize(inches)
Category1	0.5"-1.625"
Category2	1.625"-3"
Category3	3"-7"
Category4	7"-14"
Category5	14"-31"
Category6	31"-41"

Example: Global Failure frequencies

	NUREG-1829		
	5% quantile	**Mean**	**95% quantile**
Category1	6.90E-04	7.30E-03	2.30E-02
Category2	7.60E-06	6.40E-04	2.40E-03
Category3	2.10E-07	1.60E-05	6.10E-05
Category4	1.40E-08	1.60E-06	6.10E-06
Category5	4.10E-10	2.00E-07	5.80E-07
Category6	3.50E-11	2.90E-08	8.10E-08

Example: Global Failure Probabilities

Sampled Frequencies	Probabilities
3.90E-03	9.64E-01
1.40E-04	3.46E-02
3.40E-06	8.41E-04
3.10E-07	7.67E-05
1.20E-08	2.97E-06
1.20E-09	2.97E-07

$$P[cat_j] = Frequency[cat_j] / \sum_{l \in J} Frequency[cat_l]$$

Example: Global Failure Probabilities

- For example, the probability that a failure will fall into category 3 is given by

$$P[cat_{category3}] = \frac{3.40\text{E-}6}{3.90\text{E-}3 + 1.40\text{E-}4 + 3.40\text{E-}6 + 3.10\text{E-}7 + 1.20\text{E-}8 + 1.20\text{E-}9} =$$

$$= 8.41\text{E-}4$$

Example: Local Failure Probabilities

Weld	**weld1**	**weld2**	**weld3**	**weld4**	**weld5**	**weld6**	**Actual**	**Target**
Category1	1.61E-01	1.61E-01	1.61E-01	1.61E-01	1.61E-01	1.61E-01	9.64E-01	9.64E-01
Category2	5.77E-03	5.77E-03	5.77E-03	5.77E-03	5.77E-03	5.77E-03	3.46E-02	3.46E-02
Category3				2.80E-04	2.80E-04	2.80E-04	8.41E-04	8.41E-04
Category4						7.67E-05	7.67E-05	7.67E-05
Category5						2.97E-06	2.97E-06	2.97E-06
Category6						2.97E-07	2.97E-07	2.97E-07

$$P[cat_j \, at \, location_i] = P[cat_j] / M_j$$

Example: Local Failure Probabilities

- For example, the probability that weld4 will experience a break of category 3 is given by

$$M_{category3} = 3$$

$$P[cat_{category3}] = 8.41\text{E-}4$$

$$P[cat_{category3}\, at\, location_{weld4}] = P[cat_{category3}] / M_{category3} = \frac{8.41\text{E-}4}{3} = 2.80\text{E-}04$$

Example: Break Size Distribution

Weld	Parameters	Category1	Category2	Category3	Category4	Category5	Category6
weld1	**a**	0.5	1.625				
	b	1.625	**2.5**				
weld2	**a**	0.5	1.625				
	b	1.625	**2.5**				
weld3	**a**	0.5	1.625				
	b	1.625	**2.5**				
weld4	**a**	0.5	1.625	3			
	b	1.625	3	**4**			
weld5	**a**	0.5	1.625	3			
	b	1.625	3	**4**			
weld6	**a**	0.5	1.625	3	7	14	31
	b	1.625	3	7	14	31	**35**

$$breakSize_j^i \sim Uniform[\min Break_j^i, \max Break_j^i]$$

$$\min Break_j^i = cat_j^{\min Break}$$

$$\max Break_j^i = \min[cat_j^{\max Break}, weld_i^{size}]$$

Example: Break Size Distribution

- For example, the distribution that covers break size for weld4 in category 3 is given by

$$\min Break_{category3}^{wel4} = cat_{category3}^{\min Break} = 3$$

$$\max Break_{category3}^{weld4} = \min[cat_{category3}^{\max Break}, weld_{weld3}^{size}] = \min[7,4] = 4$$

$$breakSize_{category3}^{weld4} \sim Uniform[\min Break_{category3}^{weld4} \max Break_{category3}^{weld4}] \sim Uniform[3,4]$$

Example: Sampled Break Sizes

Weld	weld1	weld2	weld3	weld4	weld5	weld6
Category1	1.1	0.6	0.87	1.34	0.79	1.23
Category2	2.4	1.9	2.1	2.9	1.75	2.36
Category3				3.56	3.14	5.97
Category4						11.67
Category5						25.68
Category6						32.67

Presenters

- Elmira Popova, PhD, Professor, Robert and Jane Mitchell Endowed Faculty Fellowship in Engineering, Department of Mechanical Engineering, Operations Research and Industrial Engineering, The University of Texas at Austin
- Alexander Galenko, PhD, Research Associate, Department of Mechanical Engineering, Operations Research and Industrial Engineering, The University of Texas at Austin