

# STEEL-PLATE COMPOSITE (SC) DESIGN FOR NUCLEAR FACILITIES

## General Introduction



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# ACKNOWLEDGMENTS

- Purdue University
- Bechtel Corp.
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- Mitsubishi Heavy Industries
- URS
- AISC



## OUTLINE

- Overview of SC Construction, Examples, and Benefits
- Unique Issues, overview of past standardization activities
- Similarities and Differences w.r.t. RC Design Practice
- AISC SC Spec – Background, supporting work, and outline



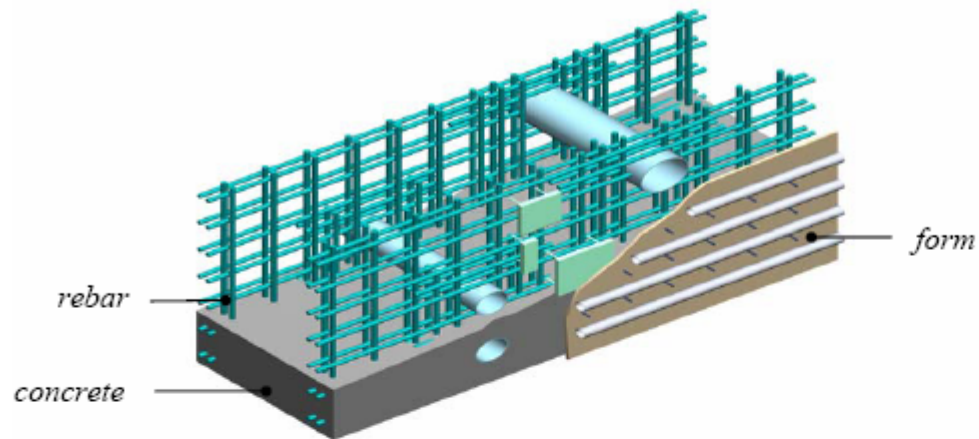
# Overview

- Nuclear structures often involve heavy concrete construction to provide adequate radiation shielding
- This results in longer construction durations and large field labor force requirement
- Nuclear industry is looking for ways to minimize schedule and labor requirements to make nuclear power competitive and to complete projects faster
- Generic modular construction, and especially modular steel-plate composite (SC) construction is seen as being key to achieving these goals
- Steel plates on the exterior eliminate formwork and serve as equivalent rebar when shear connectors are used

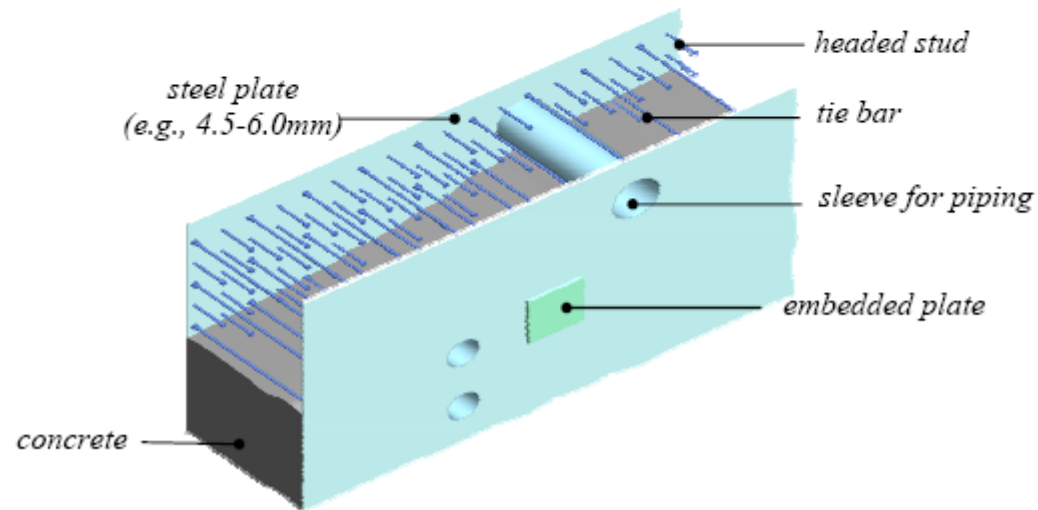




# OVERVIEW



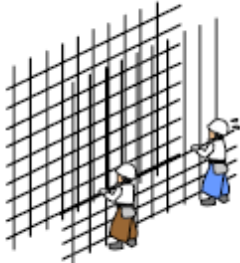
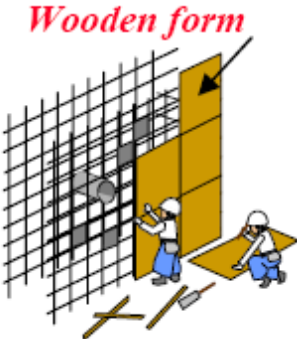
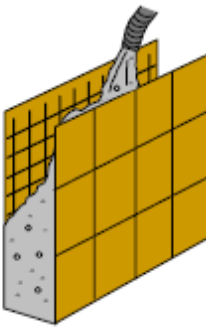

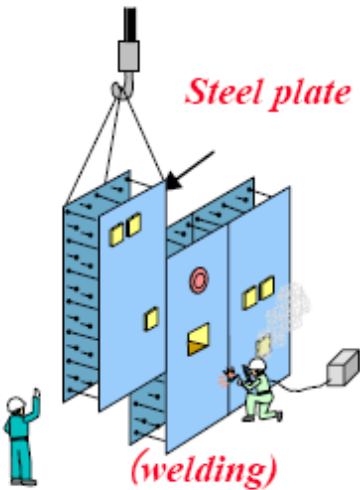
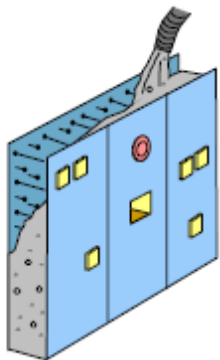
Reinforced Concrete



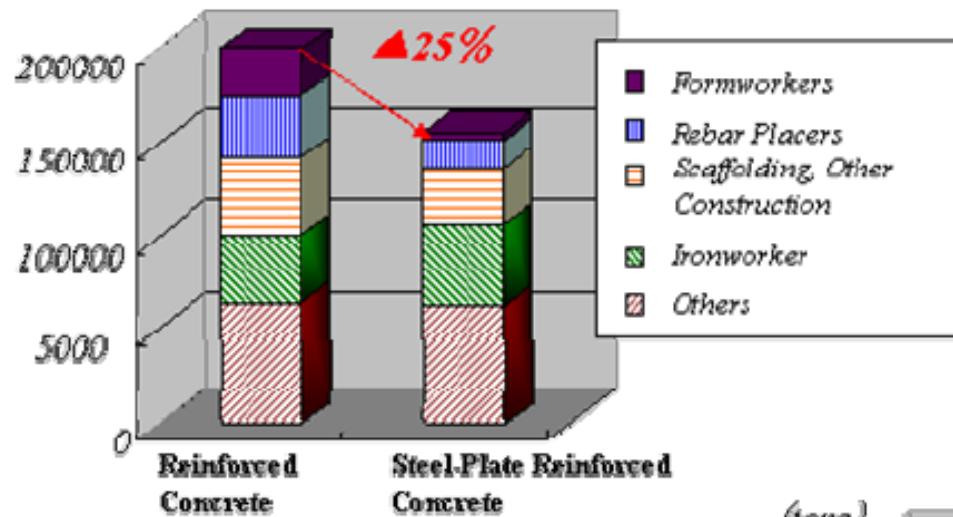
Steel-Plate Reinforced Concrete

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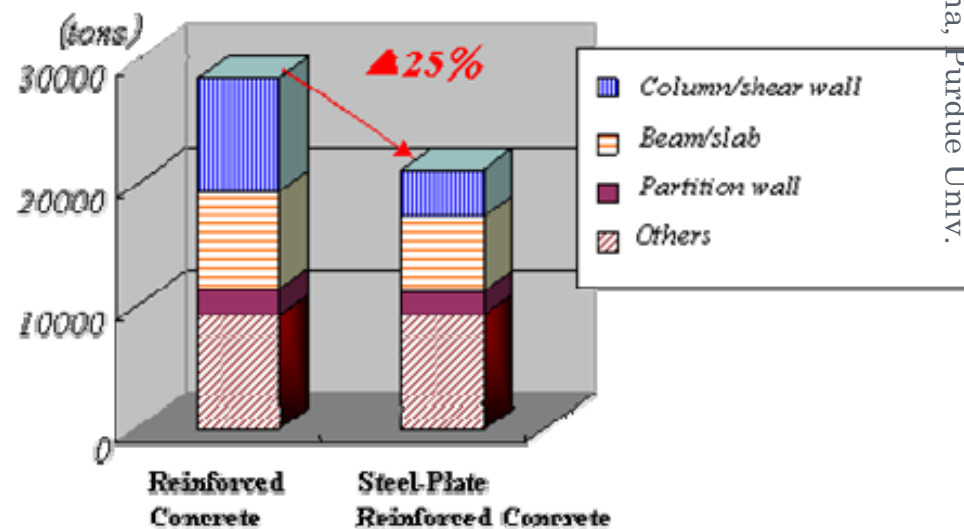
from MPR-2610, Report to DOE

<i>Work Structure</i>	<i>Rebar arrangement</i>	<i>Form work (assembling)</i>	<i>Placing concrete</i>	<i>Form work (removal)</i>
<i>RC</i>				
<i>28days</i>	<i>13days</i>	<i>7days</i>	<i>4days</i>	<i>4days</i>
<i>SC</i>	—			—
<i>14days</i>	—	<i>10days</i>	<i>4days</i>	—

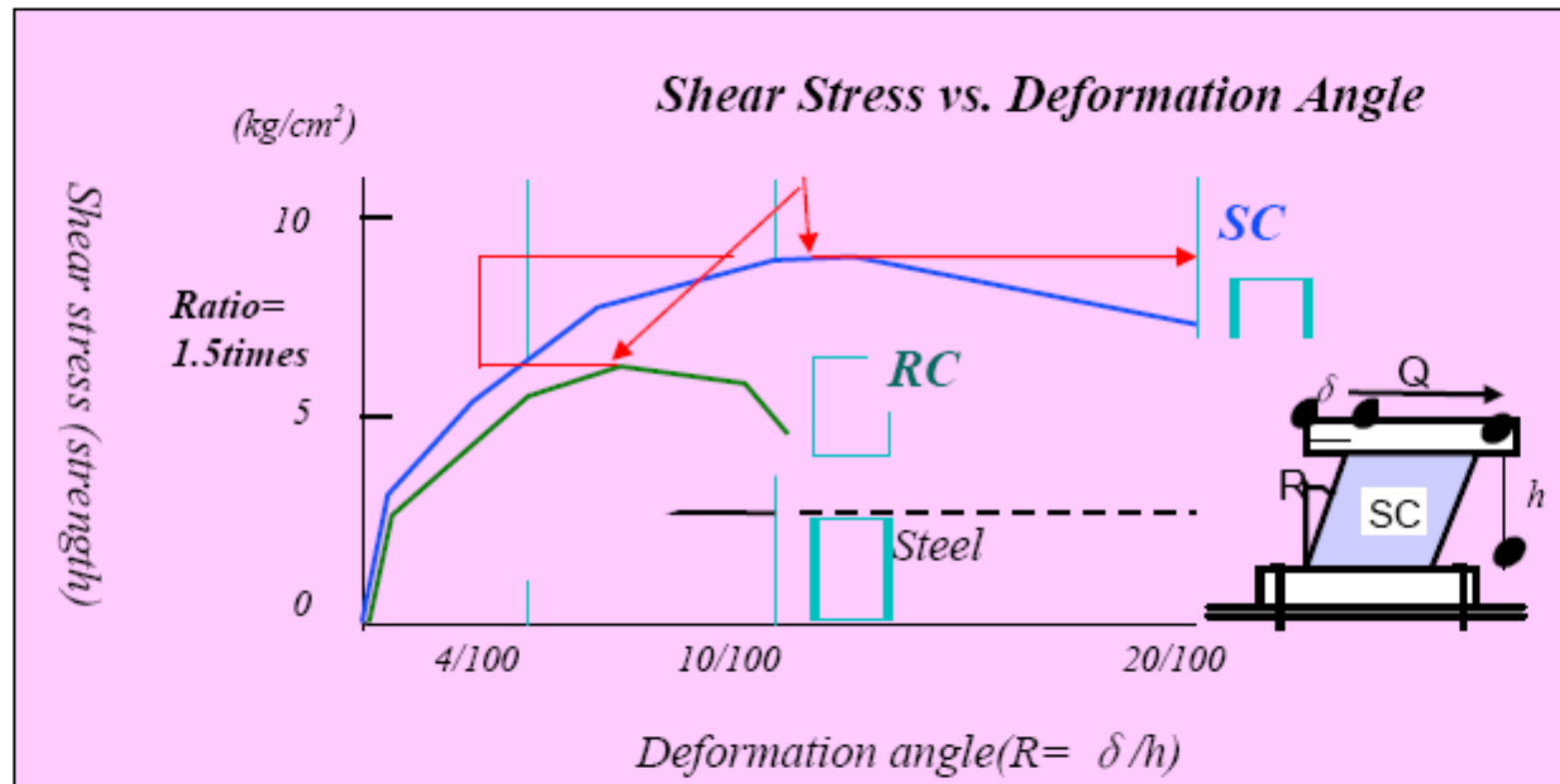
# OVERVIEW



from MPR-2610, Report to DOE



# OVERVIEW



Comparison of In-Plane Shear Behavior for RC and SC Walls

from MPR-2610, Report to DOE

## BENEFITS OF SC COMPOSITE CONSTRUCTION

- Reduced field labor
- Improved construction schedule
- Elimination of rebar and formwork
- Enables large-scale modularization
- Superior missile and blast resistance
- Superior strength and stiffness characteristics
- Superior ductility compared to RC construction

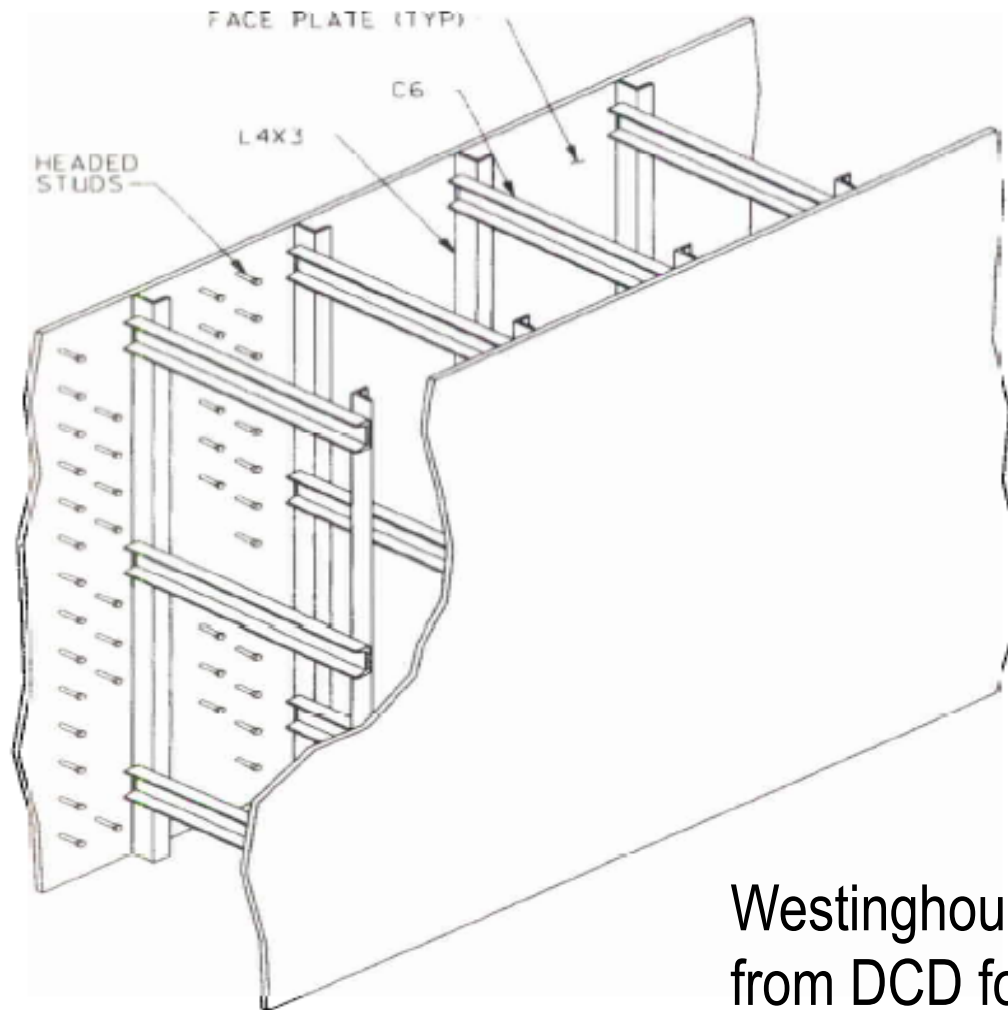


## BENEFITS OF SC COMPOSITE CONSTRUCTION

- Improved quality of placed concrete
- Penetrations can be handled relatively easily
- Increased shielding capability due to steel faceplates
- Minor commodity attachments can be handled easily
- Can accommodate activities by other trades by allowing the concrete placement activities lag module erection



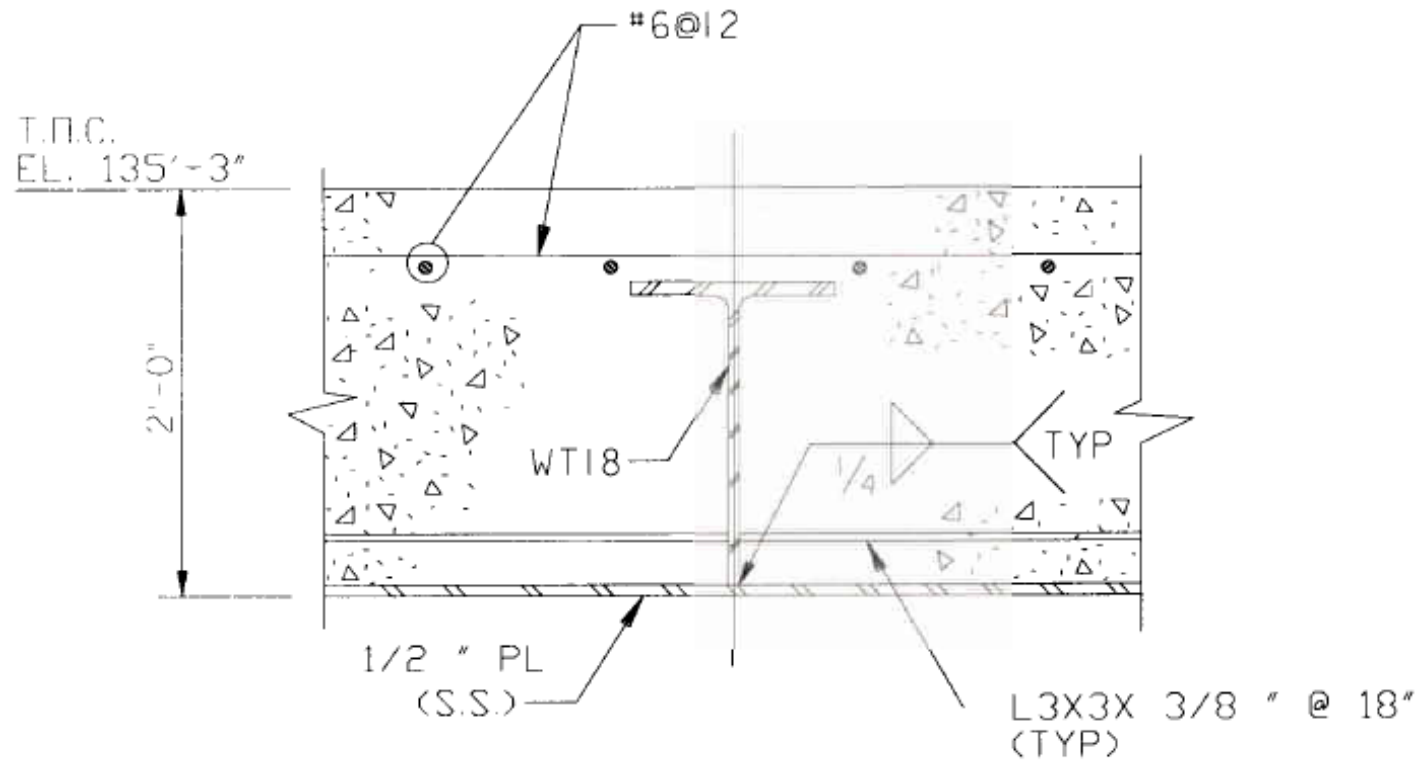
## SC EXAMPLES



Westinghouse SC Wall  
from DCD for AP1000



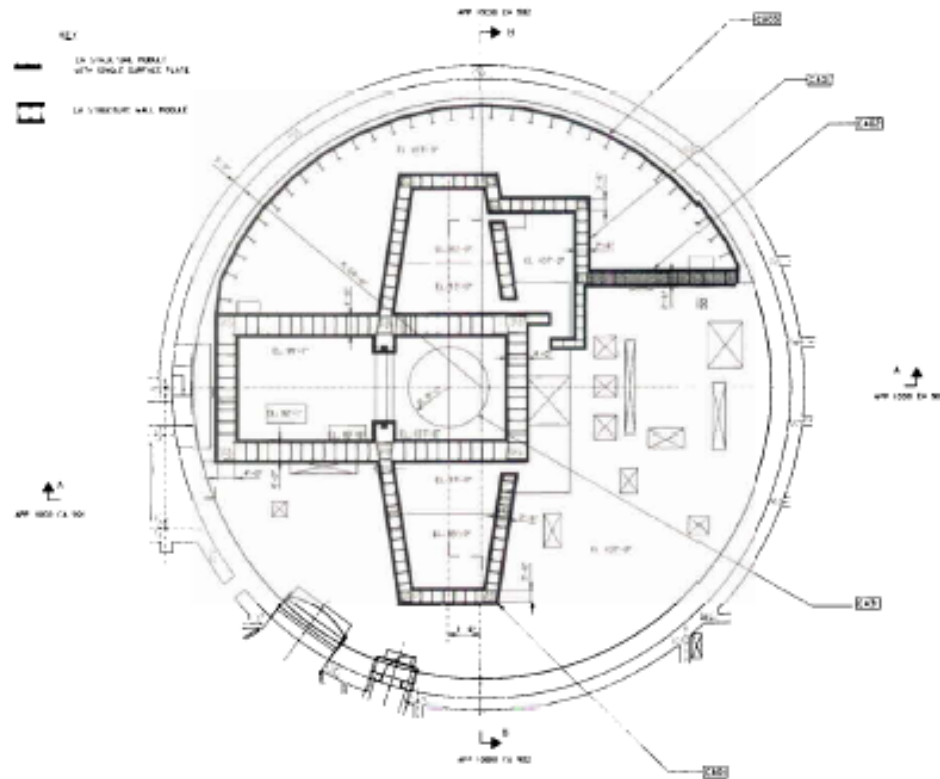
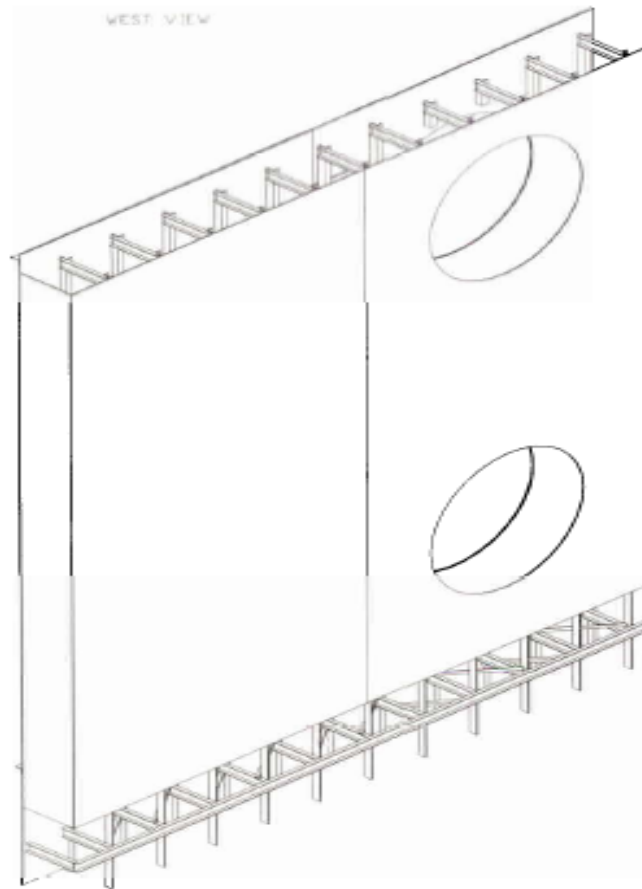
# SC EXAMPLES



Westinghouse SC Floor  
from DCD for AP1000



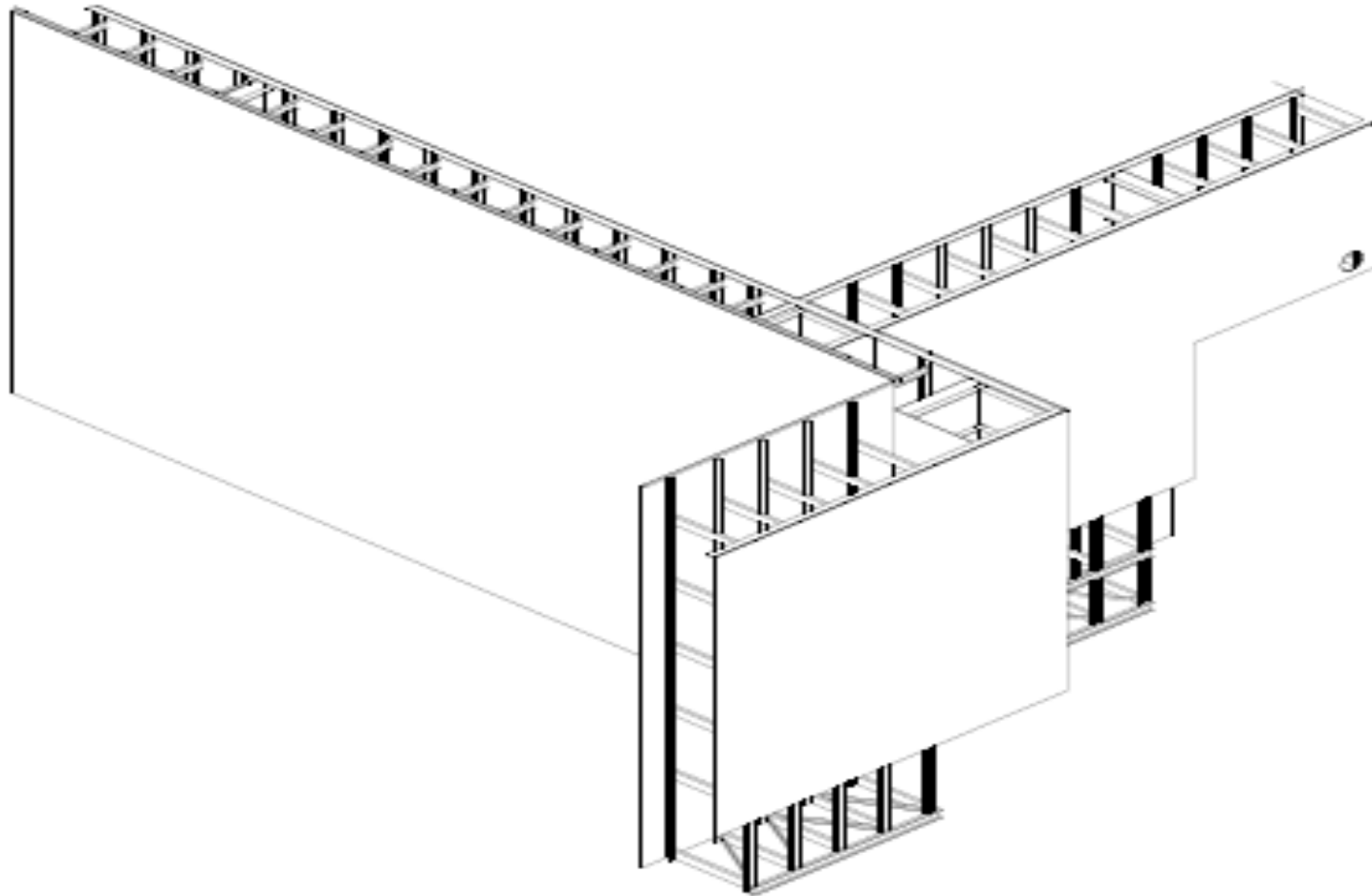
# SC EXAMPLES



from Westinghouse's DCD for AP1000

# EXAMPLE: INTERSECTING WALLS TO BUILD MODULES

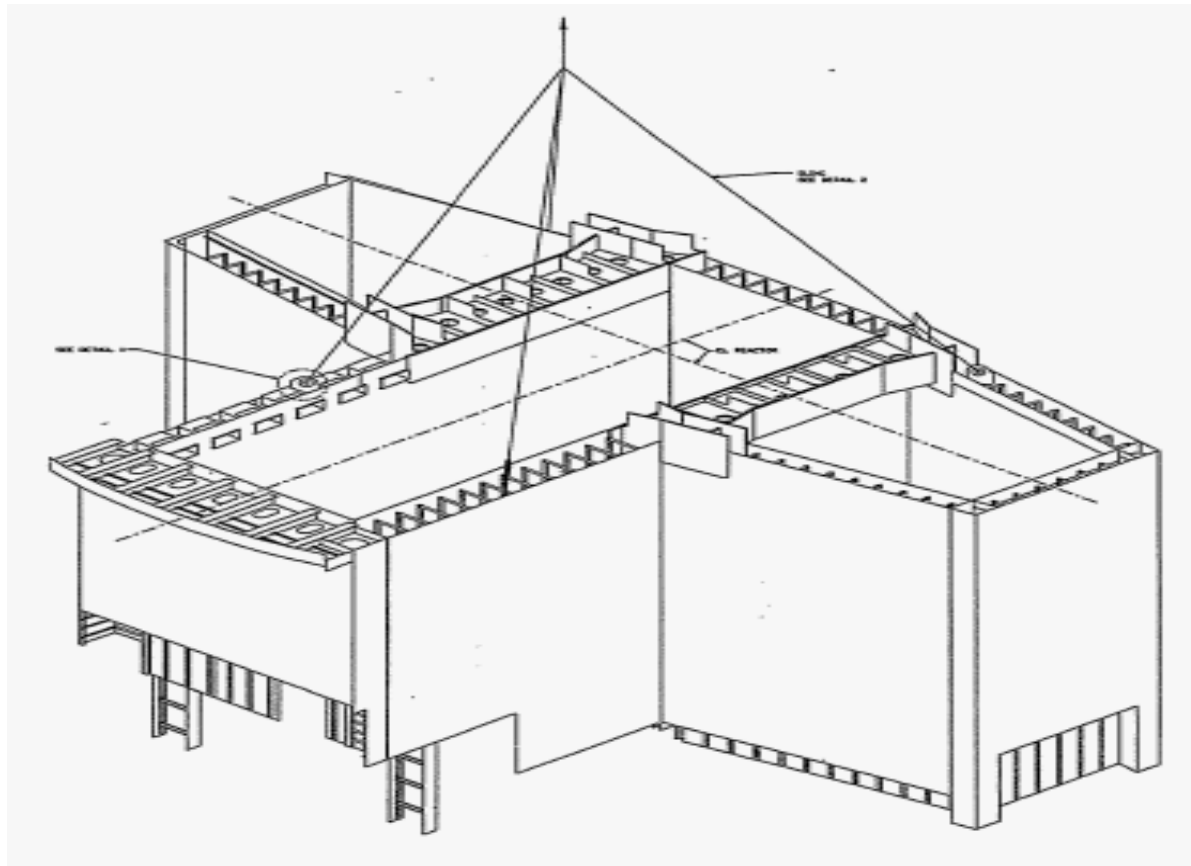
(REF: AP1000 DCD, REV. 18)



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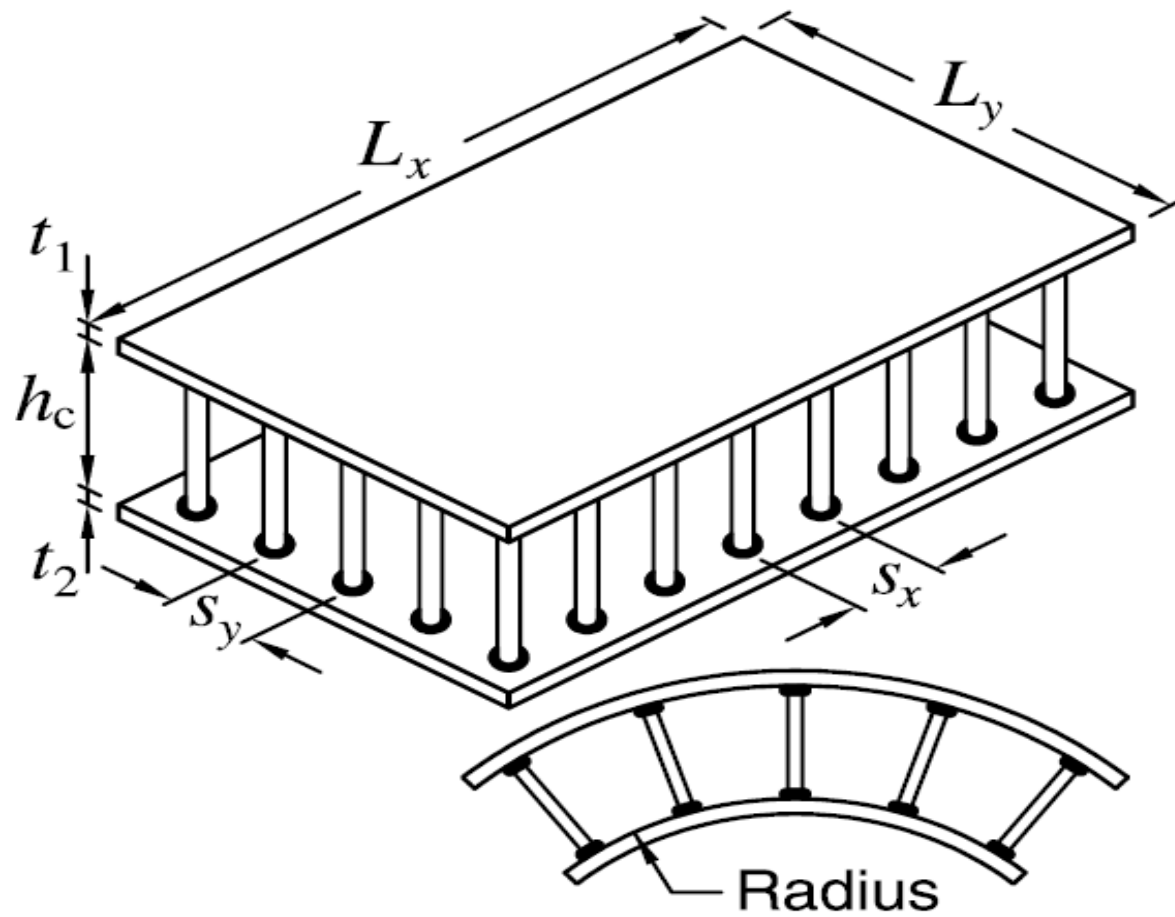
# ASSEMBLY OF INDIVIDUAL PANELS INTO LARGE MODULES



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## SC EXAMPLES



Bi-Steel Wall Panel from Corus

## UNIQUE ISSUES ABOUT SC DESIGN

- Basic SC premise is simple – use faceplates as stay-in-place formwork with shear connectors and tie-bars to enable composite action and structural integrity (section acting as a single unit)
- SC modules can thus be made from a wide variety of steel faceplates, shear connectors, tie bars, or embedded shapes, and concrete
- Various vendors and designers have come up with their own schemes for SC section detailing
- The challenge has been to develop a standard that is general enough to cover SC designs with various types of shear connectors and tie bars
- Additionally, SC design is used for wall and/or floor construction, possibly in combination with RC elements



## UNIQUE ISSUES ABOUT SC DESIGN

- Unlike the linear members covered in AISC Spec, the SC structural members are plate/shell elements such that appropriate analysis and capacity rules are warranted
- Besides composite action requirements, the structural integrity of empty and filled modules should be addressed
- Effect of residual stresses and initial imperfection due to concrete placement should be addressed
- Connection design should be properly performed!



# CONSIDERATIONS FOR ASSURING GOOD SC BEHAVIOR

- Maintain composite action
- Provide structural integrity
- Limit initial imperfection/locked-in Stress effect on local buckling
- Control reinforcement ratio
- Prevent non-ductile failure modes (e.g., oop shear)
- Strong connection performance, preferably stronger than that of the connected parts
- Limit too thin/thick faceplate and composite section thickness



# MAJOR ISSUES FACING SC ADOPTION IN USA

- Lack of US-based design code or consensus standard
- More research needed for interaction behavior under combined in-plane and out-of plane forces, connection design, system integrity requirements, analysis requirements, stiffness characterization
- Early missteps, insufficient vetting of technical issues requiring R&D, slow recognition of important technical issues
- Construction Issues – Availability/readiness of large fabrication shops, erection/fabrication tolerances, concrete inspection for voids, superior field welding techniques, etc
- Jointability – SC to SC and SC to RC Connections (at basemat, wall-to-wall and wall-to-slab interfaces) and associated rules for checking connection adequacy



# HISTORY OF REGULATORY/STANDARD DEVELOPMENT

- In 1997, DOE's Brookhaven National Lab issued a report NUREG/CR-6486 for the US Nuclear Regulatory Commission
- This report addressed several issues associated with design and analysis of SC structures
- It emphasized: proper understanding of the composite action, need for appropriate analysis techniques, connection methods, and need for testing as appropriate
- In the absence of any US code, US standard plant applicants argued that the faceplates were essentially the same as rebar and that ACI 349 was therefore generally applicable



# HISTORY OF REGULATORY/STANDARD DEVELOPMENT

- ACI 349 committee did not address design of SC structures in 1997, 2001, and 2006 versions (and the latest being worked on)
- As the prospect of new nuclear power plants became more certain, NRC began asking probing questions about design of SC members
- Outside the US, the Japanese introduced a guide document known as JEAG-4618 in 2005 (which became code in 2009)
- UK's Corus Corporation issued 2<sup>nd</sup> edition of Bi-Steel Design Manual in 2003 (not meant for nuclear and seismic applications)
- Korean Society of Steel Construction launched its own research and standardization effort in 2005; KSSC's standard was issued in December 2009



## SC STRUCTURES AND RC DESIGN ?

- Steel-concrete (SC) walls are somewhat similar to reinforced concrete (RC) walls. They both consist of thick concrete walls that are reinforced by steel.
- In SC, the concrete walls are reinforced with steel faceplates that are anchored to the concrete using shear connectors and connected to each other using steel tie bars.
- In RC, the concrete walls are reinforced with orthogonal grids of deformed steel rebar that are embedded into the concrete with clear cover, and arranged to provide curtains of steel reinforcement. These curtains may be connected to each other using deformed rebar stirrups or ties.



# SC STRUCTURES AND RC DESIGN ?

- In some aspects of structural behavior such as axial tension, compression, flexure, and out-of-plane shear, the behavior of SC walls is very similar to that of RC walls.
- For other aspects such as in-plane shear, combinations of in-plane and out-of-plane forces, thermal effects, etc. the behavior of SC walls can be quite different from that of RC walls.
- Additionally, some aspects of section detailing to address limit states of faceplate local buckling, interfacial shear transfer, section delamination failure, etc. are unique to SC
- In general, SC behaves as concrete filled steel structure in which steel faceplate yielding governs the design for almost all demands
- RC behaves as steel filled concrete structure in which concrete failure modes can govern the design

# SC SPECIFIC DESIGN ISSUES

## 1. Materials

- Different steel materials used for SC faceplates, ties, and shear connectors

## 2. Construction Load Effects

- Effect of concrete placement, rigging, etc. on steel faceplate stresses

## 3. Section Detailing Requirements

- Limitations w.r.t. section and faceplate thickness, reinforcement ratio, etc

## 4. Local Buckling

- Steel faceplate buckling at ambient and elevated temperatures

## 5. Composite Action

- Use of discrete shear connectors (ductile or non-ductile?)

## 6. Structural Integrity

- Tie-bars needed during construction and to prevent delamination failure



# SC SPECIFIC DESIGN ISSUES

## 7. Analysis Basis and Stiffness Modeling

- SC stiffness, FE model refined appropriately, thermal cracking effects

## 8. Member Design Basis

- Essentially elastic over 3T by 3T (rather than whole wall)

## 9. OOP Shear Behavior

- Account for size effect and ductile/non-ductile nature of tie-bars

## 10. In-Plane Shear Behavior

- Compression field action works with steel plates

## 11. Connection Design Basis

- Use of various connector types with clearly identifiable force transfer mechanisms

## 12. Connection Qualification

- Connection performance requirements specified, qualification needed



## PAST/ONGOING RESEARCH ACTIVITIES

- Combined Thermal/Mechanical Loading (Bechtel/Purdue/WEC)
- Damping in SC Structures (KSSC, Japan)
- Interaction Equation (KSSC, Purdue/Bechtel, Japan)
- Local Buckling – Corus (UK), Japan, Purdue/Bechtel
- Fire – Japan, KSSC, Purdue/Bechtel (plan analytical work)
- Missile Impact – Japan
- Out-of-Plane Behavior – Purdue/Bechtel/WEC, Japan, KSSC
- In-Plane Behavior – Purdue/WEC/Bechtel, Japan, KSSC
- Analysis Requirements – Purdue/Bechtel/WEC
- Stiffness Characterization – Purdue/Bechtel/WEC



## HISTORY OF AISC STANDARD DEVELOPMENT

- In 2005, Sanj Malushte began canvassing ASCE and AISC to form standard committee for SC structures
- AISC gave green signal in 2006 and a subcommittee was formed with Sanj as chairman and several members from the industry, academic, and fabricator community
- Prof. Varma of Purdue is the vice-chair
- The committee consists of many members from industry and academia as noted in the next two slides





# Introduction – AISC SC Subcommittee Roster

- Mark Holland, Paxton Vierling
- Ron Janowiak, Exelon
- William H. Johnson, Bechtel
- Shujin Fang, Sargent & Lundy
- Amit Varma, Purdue Univ.
- Michel Bruneau, Univ. at Buffalo
- Boza Stojadinovic, UC Berkeley
- Jerry Hajjar, Northeastern Univ.
- Sanj Malushte, Bechtel
- Erin Criste, AISC Secretary
- Jose Pires, Abdul Sheikh, NRC



# Introduction – AISC SC Subcommittee Roster

- Necip Akinci, Bechtel
- Taha Al-Shawaf, AREVA
- Keith Coogler, Westinghouse
- Brad Fletcher, Tata Steel
- Branko Galunic, URS
- Mark Guida, AREVA
- Wonki Kim, Hoseo Univ.
- Il-Hwan Moon, KEPCO
- Javad Moslemian, Sargent & Lundy
- Sujit Niogi, GE Nuclear
- Jason Redd, Southern Nuclear
- Matthew Van Liew, URS



## SC TOPICS ADDRESSED BY AISC COMMITTEE

- Loads and Load Combinations
- Local Buckling
- Composite Action
- Structural Integrity
- Analysis Requirements
- Design Basis
- Design for Tension, Compression, Flexure, OOP Shear, IP Shear
- Interaction Equation for Simultaneous Forces and Moments
- Effect of local attachments
- Effect of Combined Thermal and Mechanical Loads
- Fabrication and Erection Tolerances



## SC TOPICS ADDRESSED BY AISC COMMITTEE

- Resistance to Missile Impact
- Rules for Connection Design
- New materials, corrosion allowance, detailing around openings
- Treatment of SC Slabs, how different than SC Wall
- Commentary development, writing supporting papers



# AISC SC SPEC ORGANIZATION

- The SC spec has been developed as an appendix to AISC N690
- Appendix chapters are organized same as AISC 360, which is what is done for N690 as well (i.e., “dependent code” format)
- However, several chapters are very different in content because of the subject matter
- Special focus on general requirements with regard to local buckling, composite action, structural integrity, out-of-plane shear strength, etc
- Special focus on analytical requirements, incl. thermal
- Connection design requirements specified as performance requirements



# STEEL-PLATE COMPOSITE (SC) DESIGN FOR NUCLEAR FACILITIES

8/9/11

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## General Provisions and Requirements



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# INTRODUCTION

- In all the following slides, text that is blue in color is taken from the draft AISC Specification Ballot
- Text that is black in color is taken from associated commentary or explanation.



# GENERAL PROVISIONS (SECTION N9A)

## SCOPE

- Possible combinations of steel-plate composite (SC) walls with SC slabs, and RC walls / slabs, acting together as a unit.
- The SC walls shall consist of two steel faceplates acting compositely with structural concrete sandwich.
- Additional steel grade materials will be introduced as needed
- Welding Requirements:
  - Welds in the connection regions shall be made with filler metals with minimum CVN toughness of 20 ft-lb at 0°F.
  - All other welds shall be made with filler metals with minimum CVN toughness of 20 ft-lb at 40°F





# LOADS AND LOAD COMBINATIONS (Section N9B2)

## LOAD AND RESISTANCE FACTORED DESIGN

### Extreme Environmental Load Combinations

Replace with the following:

$$D + 0.8L + C + T_o + R_o + E_s + F + H \quad (\text{N9B2.5.3-1})$$

$$D + 0.8L + T_o + R_o + W_t + F + H \quad (\text{N9B2.5.3-2})$$

$$D + 0.8L + C + 1.2P_a + R_a + T_a + F + H \quad (\text{N9B2.5.3-3})$$

$$D + 0.8L + (P_a + R_a + T_a) + (Y_r + Y_j + Y_m) + E_s + F + H \quad (\text{N9B2.5.3-4})$$

SC walls can be subject to soil and fluid pressures.

The pertinent load combinations come from ACI 349, SRP 3.8.3 and 3.8.4, and RG 1.142.

F and H are treated the same way as in ACI 349 and RG 1.142




## N9B2 – LOADS AND LOAD COMBINATIONS

Inclusion of F and H is needed because, unlike the N690 main spec body that deals with linear steel elements (beams, columns, braces, etc), SC walls can be subject to soil and fluid pressures. The pertinent load combinations come from ACI 349, SRP 3.8.3 and 3.8.4, and RG 1.142.

For the “Normal Load Combinations” set, load F is treated like dead load, and load H is treated like live load. In the other sets of load combinations, F and H are treated the same way as in ACI 349 and RG 1.142.

F shall be treated in the same manner as D and H shall be treated in the same manner as L when stability evaluations are performed.




## GENERAL REQUIREMENTS (SECTION N9B)

- Minimum and Maximum Section Thickness
- Maximum and Minimum Plate Thickness
- Maximum and Minimum Reinforcement Ratio
- Maximum and Minimum Plate Yield Strength
- Minimum Concrete Compressive Strength
- Faceplate Compactness Requirement
- Composite Action Requirement
- Structural Integrity Requirement
- Relative Parity of Faceplate Yield Strength
- Strength and ductility of Splices in Faceplates



## GENERAL REQUIREMENTS (SECTION N9B)

- Requirement #1 (Minimum Thickness): The minimum section thickness for exterior SC walls shall be 18 in. The minimum section thickness for interior SC walls shall be 12 in.
  - Basis for Exterior Wall: The minimum thickness for exterior walls is based on Table 1 of SRP Section 3.5.3 Revision 3. It requires minimum 16.9 in thick 4,000-psi RC walls to resist tornado missile (the SC wall is conservatively treated as RC wall for missile loading).
  - Basis for Interior Wall: The minimum thickness for interior walls is based on 4.5% maximum steel ratio and 0.25 in minimum faceplate thickness ( $T_{\min} = 2 \times 0.25 / 0.045 = 11.1$  in)
- 

## GENERAL REQUIREMENTS (SECTION N9B)

- Requirement #2 (Maximum Thickness): The maximum section thickness for SC walls shall be 60 in.
- Basis: Concern about section behaving as a unit, lack of test data, thicker sections will involve thick plate/shell behavior such that the theory for defining member capacity is different compared to thin shell theory that has been considered in this standard development



# GENERAL REQUIREMENTS (SECTION N9B)

- Requirement #7 (Faceplate Compactness Requirement): The steel faceplates of SC walls shall be compact as specified in Section N9B4.
- Basis: The requirement to maintain the faceplates to be compact enables them to withstand compressive stress of nearly  $F_y$  without incurring local buckling.



## GENERAL REQUIREMENTS (SECTION N9B)

- Requirement #8 (Composite Action Requirement):  
Composite action shall be provided between steel faceplates and concrete using shear connectors in accordance with Section N9B5.
- Basis: Requirements of Section N9B5 ensure that steel faceplates have adequate shear connectors to be able to develop full composite action over less than three times the wall thickness.



## GENERAL REQUIREMENTS (SECTION N9B)

- Requirement #9 (Structural Integrity Requirement): The steel faceplates shall be tied to each other in accordance with the structural integrity requirements specified in Section N9B6.
- Basis: The structural integrity provisions of Section N9B6 help guard against splitting (delamination) failure within connection and interior regions, and provide compression tying reinforcement spacing that is consistent with ACI 318/349.





# SC WALLS NOT MEETING GENERAL REQUIREMENTS (SECTION N9B)

- It is possible that some designs may involve very thick walls, HSC walls, or walls with high reinforcement ratio.
- These situations constitute as outliers relative to the general requirements that are based on considerations of existing test data, and an expectation of thin shell behavior and ductile performance.
- Two options are presented: (1) Development of project-specific design method that is based on pertinent test data, or (2) use of ACI 349 code provided that the faceplate is treated merely as a liner and conventional reinforcement is provided for design as an RC section.



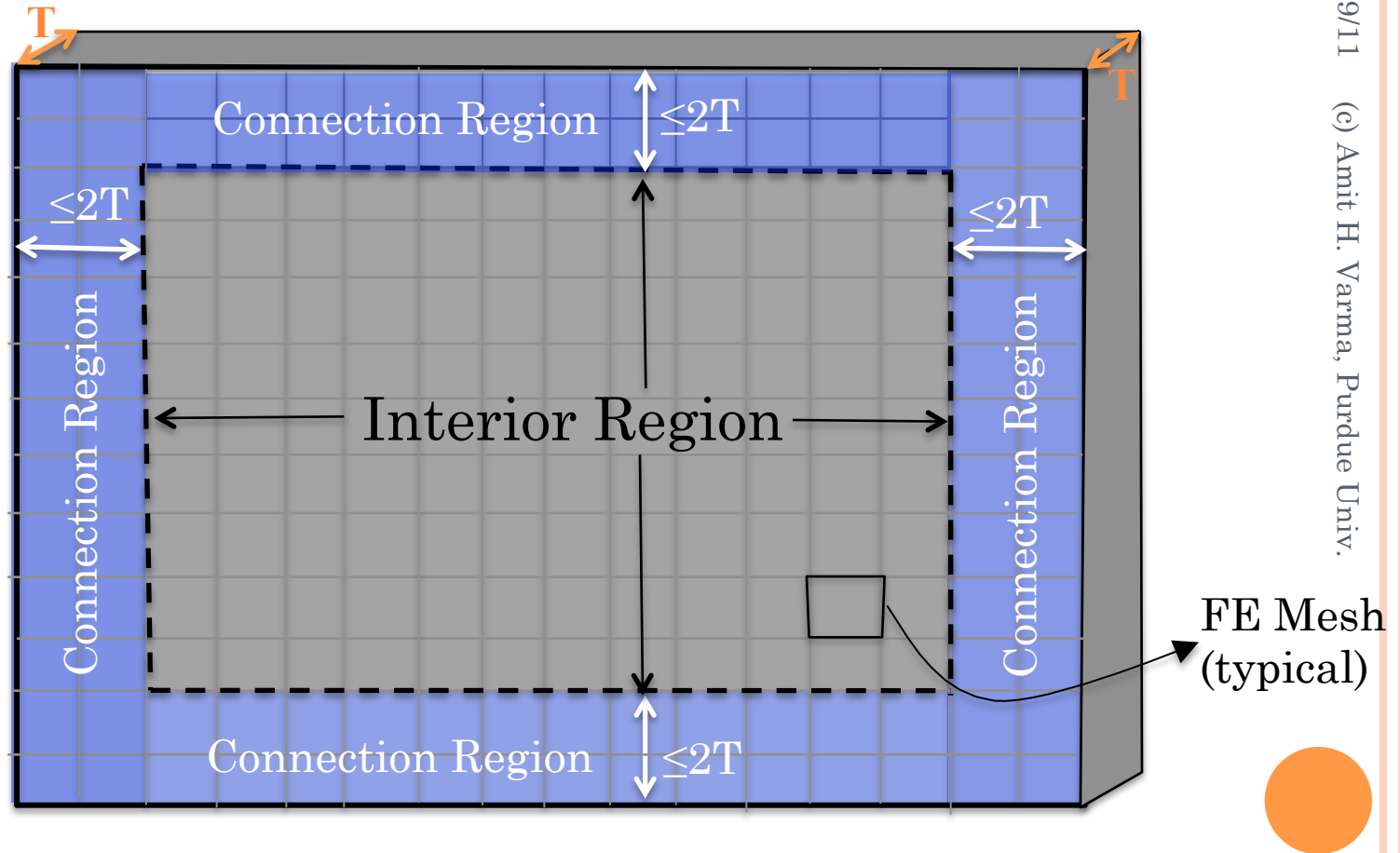
## N9B3 DESIGN BASIS (SPEC)

- SC walls and their connections shall be designed and detailed according to the provisions for *Load and Resistance Factor Design* (LRFD) or to the provisions for *Allowable Strength Design* (ASD).
- The expanse of the SC walls shall be separated into: (i) *SC wall connection regions*, and (ii) *SC wall interior wall regions*. The *SC wall connection regions* shall consist of perimeter strips that are no more than two times the section thickness (2T) from the connected edges.
- The *SC wall connection regions* shall be designed using Section N9J to accomplish complete force transfer. The *SC wall interior regions* shall be designed using Sections N9D, N9E, N9F, N9G, and N9H.



# N9B3 DESIGN BASIS

- The expanse of SC walls are separated into *connection regions* and *interior regions*



## N9B3 DESIGN BASIS

- SC walls are connected to other walls and slabs (SC or RC) along their edges.
- Force transfer to the SC walls occurs over a finite length.
- Composite action between the steel plates and the concrete infill also develops over this finite length.
- This finite length over which force transfer occurs and composite action develops is designated as the *connection region*.



## N9B3 DESIGN BASIS

- The length of the *connection region* is required to be less than or equal to two times the section thickness ( $2T$ ) from the connected edges:
  - This requirement is based on the typical development length of #11 to #18 steel reinforcing bars, which can typically amount to about 2 times the RC wall thickness in nuclear structures.
  - Experience shows that SC walls with typical steel plate thickness and shear connector spacing will also need a force transfer region length that is comparable to the development length of rebars in RC walls.
  - Limiting the force transfer region to  $2T$  ensures that the interior regions of the SC walls are of sufficient expanse to develop ductility through steel plate yielding for beyond SSE loading.



## N9B3 DESIGN BASIS

- The *connection regions* are designed to achieve adequate force transfer and composite action in accordance with the requirement of Section N9J.
- The *interior regions* are designed using the SC wall design strength equations for individual demands (Section N9D for tension, N9E for compression, N9F for flexure, N9G for shear), and for combinations of demands (Section N9H).



## N9B3.5 DESIGN FOR STABILITY (SPEC)

- Second-order analyses of structures with straight SC walls need not be performed if the conditions of Section 10.10.1 of ACI 318-08 are satisfied.
- If the conditions of Section 10.10.1 of ACI 318-08 are not satisfied, then second order effects may be addressed using the *first order analysis method* of the *Specification* Appendix 7.3.
- If the limitations of Appendix 7.3 are not met, then second order analysis shall be performed.

8/9/11

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# N9B3.5 DESIGN FOR STABILITY

## ACI 318-08 Section 10.10.1 (Excerpt)

**10.10.1** — Slenderness effects shall be permitted to be neglected in the following cases:

(a) for compression members not braced against sidesway when:

$$\frac{k\ell_u}{r} \leq 22 \quad (10-6)$$

(b) for compression members braced against sidesway when:

$$\frac{k\ell_u}{r} \leq 34 - 12(M_1/M_2) \leq 40 \quad (10-7)$$

where  $M_1/M_2$  is positive if the column is bent in single curvature, and negative if the member is bent in double curvature.

It shall be permitted to consider compression members braced against sidesway when bracing elements have a total stiffness, resisting lateral movement of that story, of at least 12 times the gross stiffness of the columns within the story.

**10.10.1.1** — The unsupported length of a compression member,  $\ell_u$ , shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered. Where column capitals or haunches are present,  $\ell_u$  shall be measured to the lower extremity of the capital or haunch in the plane considered.

**10.10.1.2** — It shall be permitted to take the radius of gyration,  $r$ , equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it shall be permitted to compute  $r$  for the gross concrete section.





# N9B3.5 DESIGN FOR STABILITY

## AISC 360-10 Appendix 7.3 (Excerpt)

### Limitations

The use of the *First-Order Analysis Method* shall be limited to the following conditions:

- (1) The structure supports gravity loads primarily through nominally-vertical columns, walls, or frames.
- (2) The ratio of maximum second-order drift to maximum first-order drift (both determined for *LRFD load combinations* or 1.6 times *ASD load combinations*) in all stories is equal to or less than 1.5.

**User Note:** The ratio of second-order drift to first-order drift in a story may be taken as the  $B_2$  multiplier, calculated as specified in Appendix 8.

- (3) The required compressive strengths of all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure satisfy the limitation:

$$\alpha P_r \leq 0.5 P_y \quad (\text{A-7-2-1})$$

where

$$\alpha = 1.0 \text{ (LRFD)} \quad \alpha = 1.6 \text{ (ASD)}$$

$P_r$  = required axial compressive strength under LRFD or ASD load combinations, kips (N)

$P_y$  = member yield strength ( $=F_y A$ ), kips (N)

### Required Strengths

The *required strengths* of components shall be determined from a *first-order analysis*, with additional requirements (1) and (2) below. The analysis shall consider flexural, shear, and axial member deformations, and all other deformations that contribute to displacements of the structure.

- (1) All load combinations shall include an additional lateral load,  $N_i$ , applied in combination with other loads at each level of the structure:

$$N_i = 2.1\alpha(\Delta/L)Y_{il} \geq 0.0042Y_{il} \quad (\text{A-7-2-2})$$

where

$$\alpha = 1.00 \text{ (LRFD)} \quad \alpha = 1.60 \text{ (ASD)}$$

$Y_{il}$  = gravity load at level  $i$  from the LRFD load combination or ASD load combination, as applicable, kips (N)

$\Delta/L$  = the maximum ratio of  $\Delta$  to  $L$  for all stories in the structure

$\Delta$  = first-order inter-story drift due to the LRFD or ASD load combination, as applicable, in. (mm). Where  $\Delta$  varies over the plan area of the structure,  $\Delta$  shall be the average drift weighted in proportion to vertical load or, alternatively, the maximum drift.

$L$  = story height, in. (mm)

The additional lateral load at any level,  $N_i$ , shall be distributed over that level in the same manner as the *gravity load* at the level. The additional lateral loads shall be applied in the direction that provides the greatest destabilizing effect.

## N9B3.5 DESIGN FOR STABILITY

- SC walls in nuclear application will generally exceed 2ft in thickness. Their typical height-to-thickness ratio are expected to meet the requirements of ACI 318-08 Section 10.10.1(b) such that second-order analysis will generally be unnecessary for the labyrinthine box structures where SC walls will be used.
- In the rare situation that the ACI requirements are not satisfied, it is expected that the structure will still meet the limitations of AISC 360-10 Appendix 7.3, allowing first order analysis to be performed with notional lateral loads in lieu of second-order analysis.



## N9B4 COMPACTNESS REQUIREMENT (SPEC)

- SC wall sections shall be *compact* for compression, flexure, and shear. For an SC wall section to classify as *compact*, its steel faceplates shall be anchored to the concrete using either *steel headed stud anchors*, structural shapes, or tie bars.
- The *width-to-thickness ratio* of the steel faceplates shall be calculated as the largest clear spacing ( $s$ ) of the *steel headed stud anchors*, structural shapes, or tie bars divided by the faceplate thickness ( $t_p$ ), and the  $s/t_p$  ratio of the steel faceplates shall not exceed the limiting width-to-thickness ratio  $\lambda_p$  given by Equation N9B4.1:

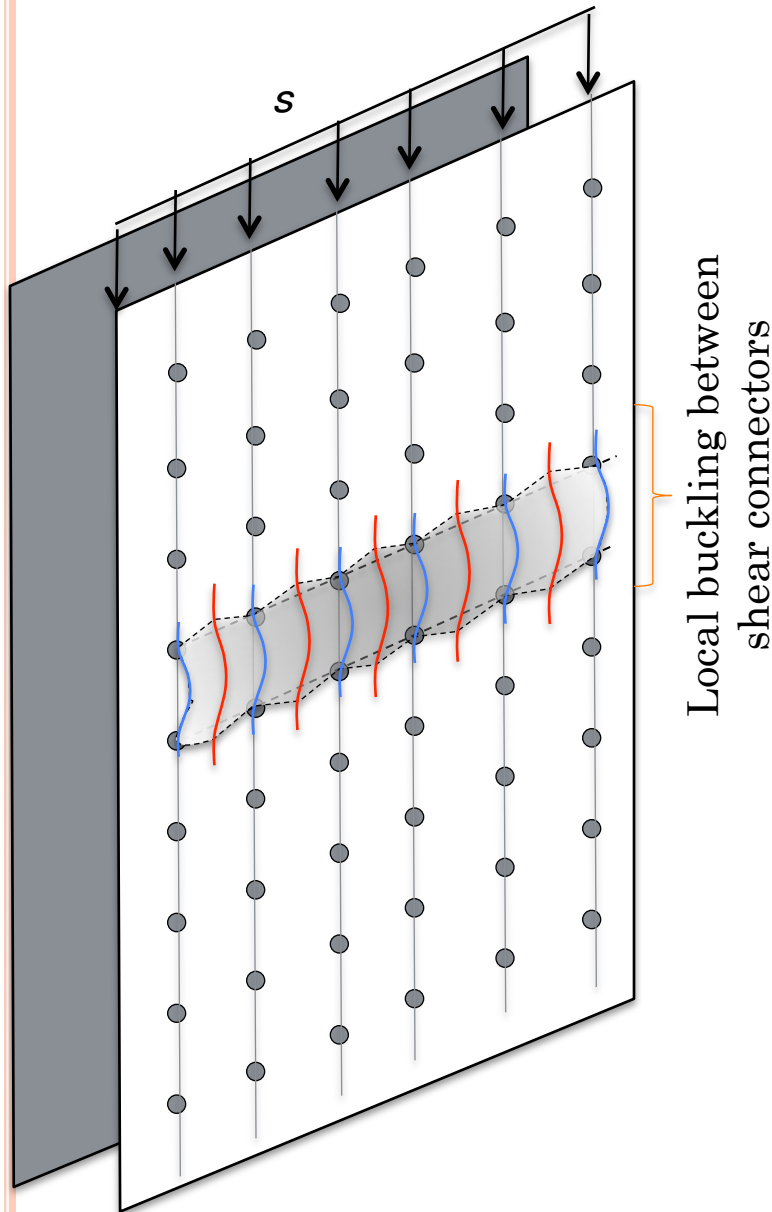
$$\lambda_p = 1.0 \sqrt{\frac{E}{F_y}}$$

- Where,  $E_s$  = elastic modulus of steel = 29000 ksi
- $F_y$  = steel plate nominal yield stress in ksi



# N9B4: COMPACTNESS REQUIREMENT

## LOCAL BUCKLING LIMIT STATES

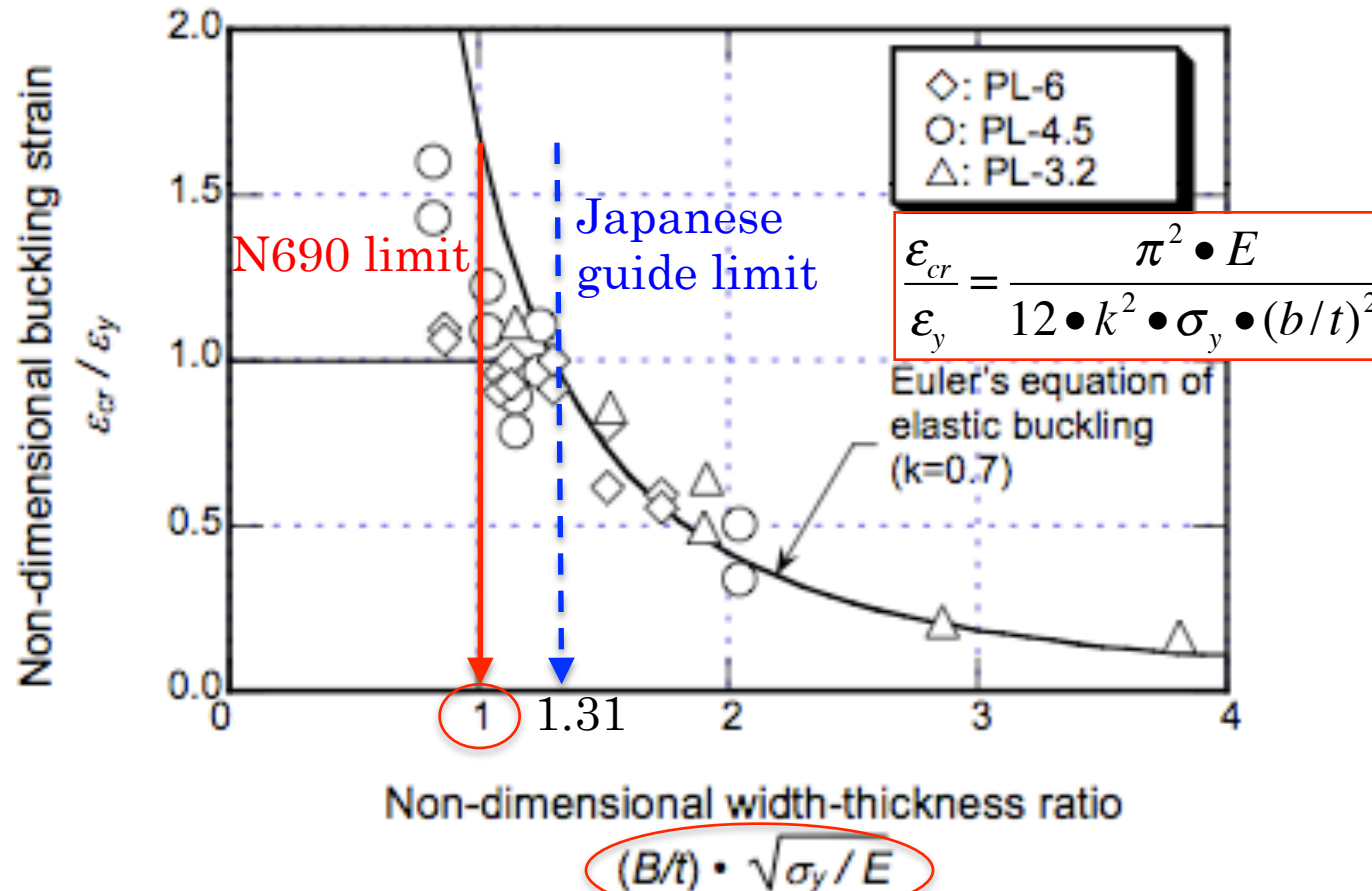


- When the steel faceplate is subjected to compressive stresses across the width, it undergoes local buckling between the shear connectors
- The horizontal lines joining the shear connectors act as fold lines, and local buckling occurs as shown
- The buckling mode indicates fix-ends along the vertical lines with shear connectors, and partially fix ends for the regions in between (red lines in figure)

# N9B4: COMPACTNESS REQUIREMENT

## LOCAL BUCKLING LIMIT STATE

This figure shows the relationship between buckling strain ( $\epsilon_{cr}$ ) of the steel plate and its normalized slenderness. It was obtained from compression experiments of SC structure wall panels by Japanese.  $\epsilon_{cr}$  is almost consistent with Euler's curve with partially fixed ( $k=0.7$ ) end condition.



## N9B4: COMPACTNESS REQUIREMENT LOCAL BUCKLING LIMIT STATE

- For steel faceplates with nominal yield stress greater than or equal to 50 ksi, no additional limits are placed on locked-in stresses or displacements due to concrete casting.
- The use of steel faceplates with nominal yield stress less than 50 ksi is not permitted for the following reasons:
  - (1) Local buckling can occur prematurely (before yielding) because the locked in stresses due to concrete casting can be a much higher proportion of the yield stress
  - (2) Accident thermal loading conditions can cause compression yielding (and potential local buckling) of the steel plates.




## N9B5: REQUIREMENTS FOR COMPOSITE ACTION

- Shear connectors shall be used to enable composite action between the steel faceplates and concrete infill of the SC walls as specified in Section N9B5.2 and N9B5.3.
- **N9B5.1 CLASSIFICATION OF SHEAR CONNECTORS**
  - Shear connectors shall be classified as either yielding or non-yielding type.
  - Steel headed stud anchors, meeting the requirements of Chapter I of the Specification, shall be classified as yielding shear connectors. The design strength of steel headed stud anchors ( $\phi Q_n$ ) shall be in accordance with Chapter I of the Specification.
  - All other types of shear connectors shall be tested to determine their design shear strength ( $\phi Q_n$ ) and interfacial slip capability. An interfacial slip capability of at least 0.20-inch before reduction of the shear strength to 90% of  $\phi Q_n$  is required for classification as yielding shear connectors. Shear connectors not meeting this requirement shall be classified as non-yielding type.
  - When combinations of yielding and non-yielding shear connectors are used, the resulting shear connector system shall be classified as non-yielding type.



## N9B5.1 CLASSIFICATION OF SHEAR CONNECTORS

- The shear connectors used in SC construction can consist of steel headed stud anchors, embedded steel shapes, or tie bars (smooth or deformed), etc., which may be attached to the steel plates using welding or bolting.
  - The shear strength of the connectors refers to its ability to resist interfacial shear flow before failure
  - Shear connectors that have a ductile interfacial slip behavior can re-distribute the interfacial shear flow equally over several connectors. Such connectors are referred as 'yielding' type, e.g., steel headed stud anchors.
  - Shear connectors that have a non-ductile interfacial slip behavior cannot re-distribute the shear force equally over several connectors. Such connectors are referred as 'non-yielding' type.
- 



## N9B5.1 CLASSIFICATION OF SHEAR CONNECTORS

- Steel headed stud anchors are typically capable of sustaining at least 0.20 in. of interfacial slip in a ductile manner
- All other types of shear connectors need to be tested to determine their design shear strength and interfacial slip capability.
- An interfacial slip capability of at least 0.20 in. before reduction in shear strength to 90% of the design shear strength is required to qualify as 'yielding' type



## N9B5.2 REQUIREMENTS FOR YIELDING SHEAR CONNECTORS

The spacing ( $s$ ) of yielding shear connectors shall be the smallest of the following:

- (1) The spacing required to prevent local buckling of steel faceplates in accordance with Equation N9B4.1.
- (2) The spacing required to develop the yield strength of the steel faceplates over development length ( $L_d$ )

$$s \leq \sqrt{\frac{(\phi Q_n) L_d}{t_p F_y}} \quad (\text{N9B5.2-1})$$

where,

$\phi Q_n$  = Design shear strength of shear connector determined in accordance with Section N9B5.1 of the Specification

$L_d$  = development length, shall be taken as up to three times the panel thickness

- (3) The spacing required to develop interfacial shear strength greater than the out-of-plane shear strength of the section

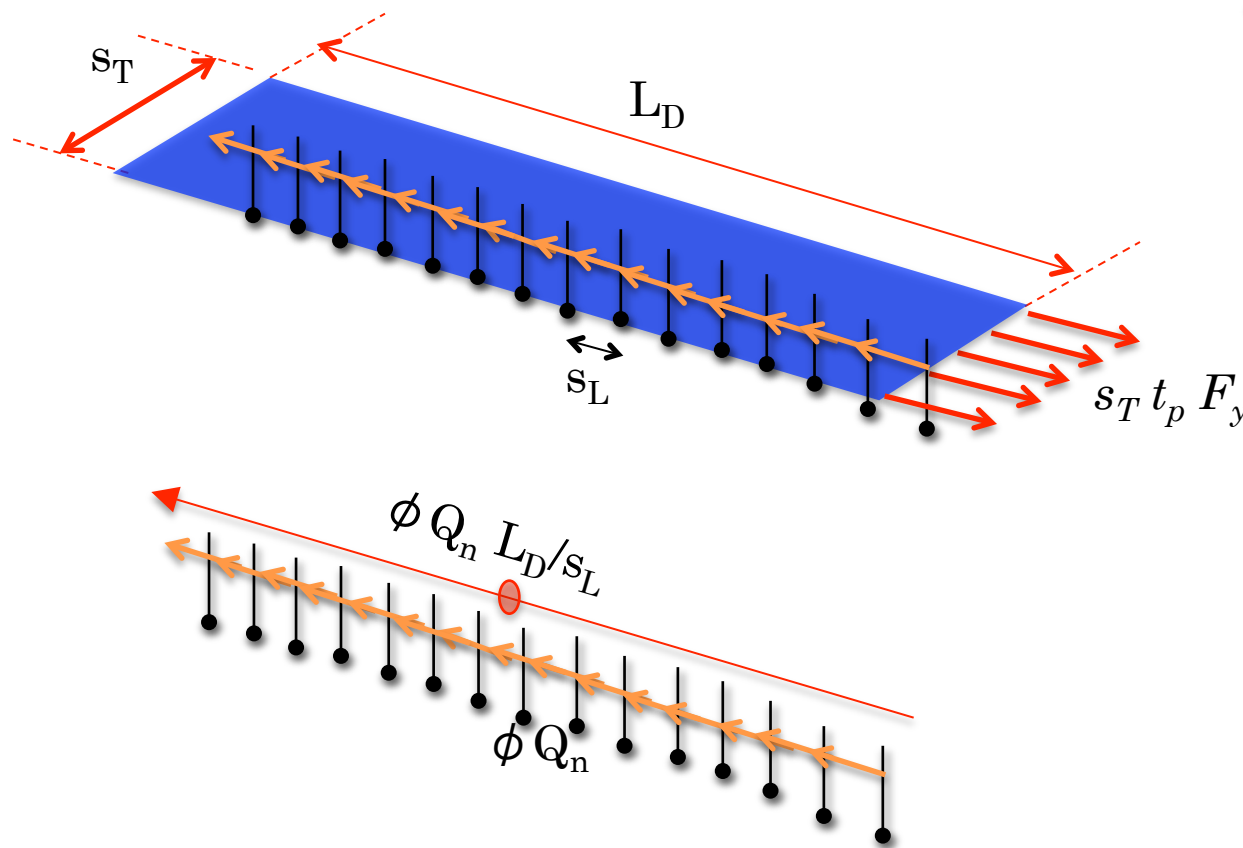
$$s \leq \sqrt{\frac{(\phi Q_n)(0.9 T)}{\phi_{vo} V_{no}}} \quad (\text{N9B5.2})$$

where,

$\phi_{vo} V_{no}$  = Design out-of-plane shear strength calculated in accordance with Section N9G2

# N9B5.2 REQUIREMENTS FOR YIELDING SHEAR CONNECTORS

Requirement (2): Plate yielding over development length



$$\phi Q_n \frac{L_D}{s_L} \geq s_T t_p F_y$$

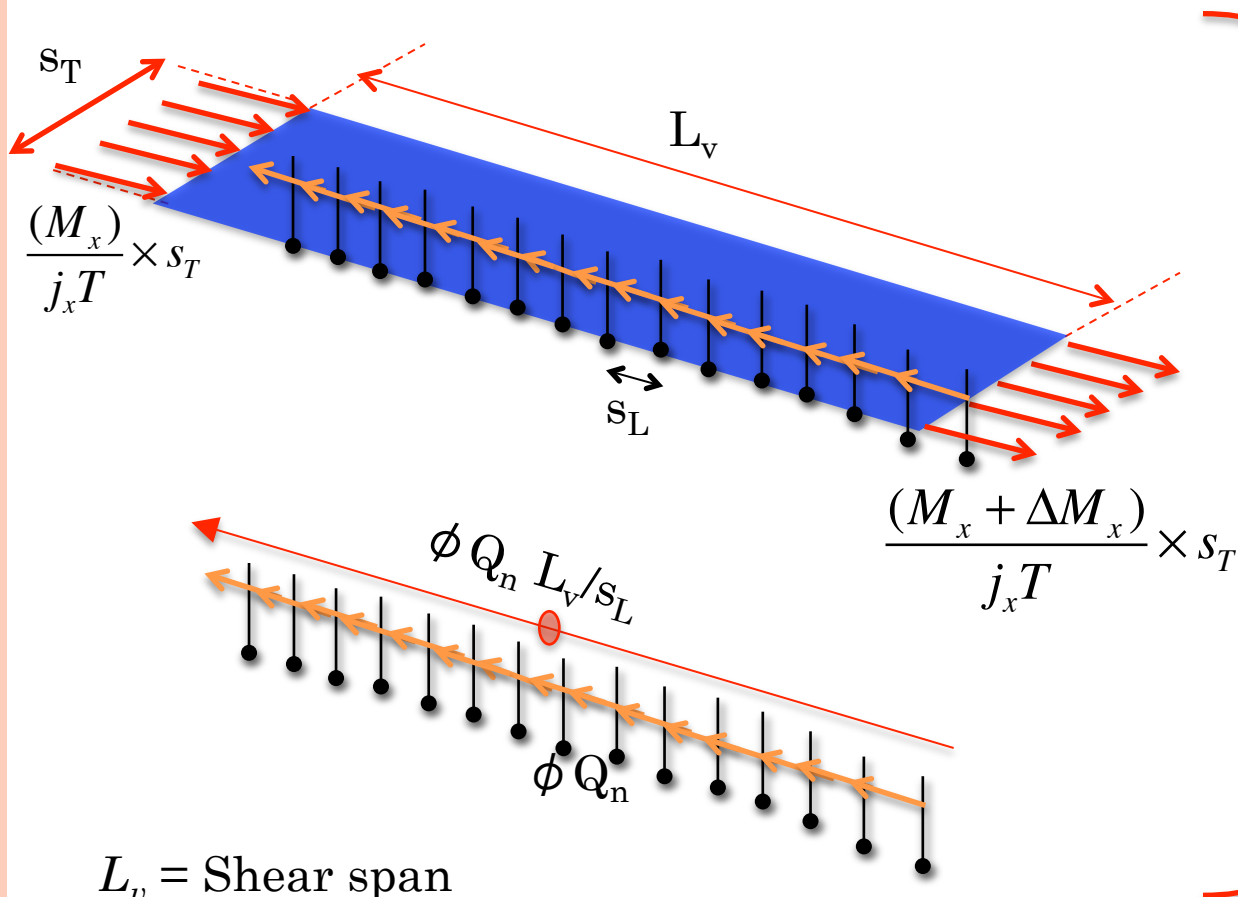
$$\therefore \frac{\phi Q_n L_D}{t_p F_y} \geq s_T s_L$$

$$\therefore s \leq \sqrt{\frac{\phi Q_n L_D}{t_p F_y}}$$

$s_T$  = transverse spacing of shear connectors  
 $s_L$  = longitudinal spacing of shear connectors

# N9B5.2 REQUIREMENTS FOR YIELDING SHEAR CONNECTORS

(3) Interfacial shear strength greater than out-of-plane shear strength



$$\phi Q_n \frac{L_v}{s_L} \geq \frac{\Delta M_x}{j_x T} S_T$$

$$\therefore s_T s_L \leq \phi Q_n \left( \frac{L_v}{\Delta M_x} \right) j_x T$$

$$\therefore s \leq \sqrt{\frac{\phi Q_n (j_x T)}{V_{no}}}$$

Substituting;

$$\frac{\Delta M_x}{L_v} = V \leq V_{no}$$

$L_v$  = Shear span

$j_x T$  = effective arm length for bending moment

$\Delta M_x$  = change in bending moment over shear span

## N9B5.3 REQUIREMENTS FOR NON-YIELDING SHEAR CONNECTORS

The spacing ( $s$ ) of non-yielding shear connectors shall be the smaller of the following:

- (1) Spacing required to prevent local buckling of steel faceplates in accordance with Section N9B4.
- (2) The spacing required to develop the yield strength of the steel faceplates over development length ( $L_d$ )

$$s \leq 0.7 \sqrt{\frac{(\phi Q_n) L_d}{t_p F_y}} \quad (\text{N9B5.3-1})$$

where,

$\phi Q_n$  = Design shear strength of shear connector determined in accordance with Section B5.1

$L_d$  = development length, shall be taken as up to three times the panel thickness

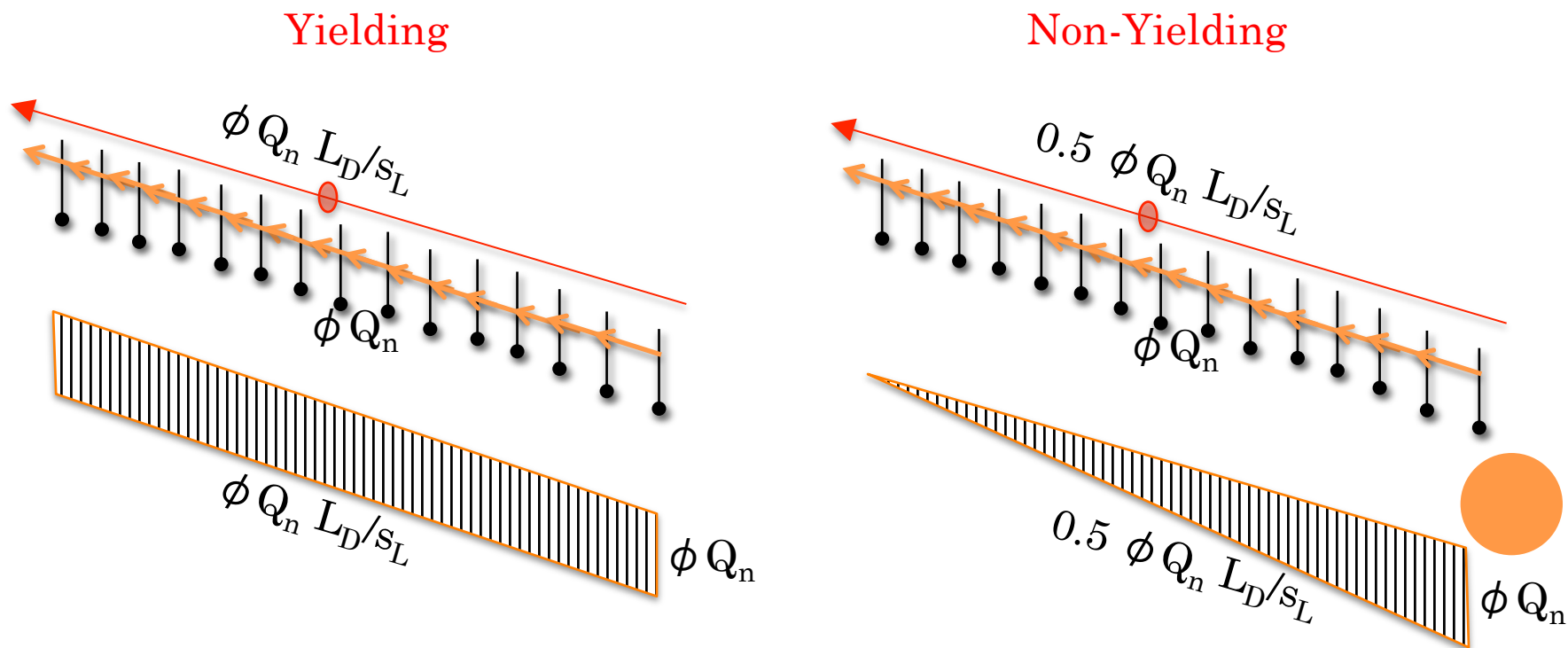
- (3) The spacing required to develop interfacial shear strength greater than the out-of-plane shear strength of the section

$$s \leq 0.7 \sqrt{\frac{(\phi Q_n)(0.9 T)}{\phi_{vo} V_{no}}} \quad (\text{N9B5.3-2})$$

$\phi_{vo} V_{no}$  = Design out-of-plane shear strength calculated in accordance with Section N9G2

## N9B5.3 REQUIREMENTS FOR NON-YIELDING SHEAR CONNECTORS

- For non-yielding shear connectors, the resistance is not divided equally between all connectors. Instead, a triangular distribution occurs with the maximum value for the first or last connector.

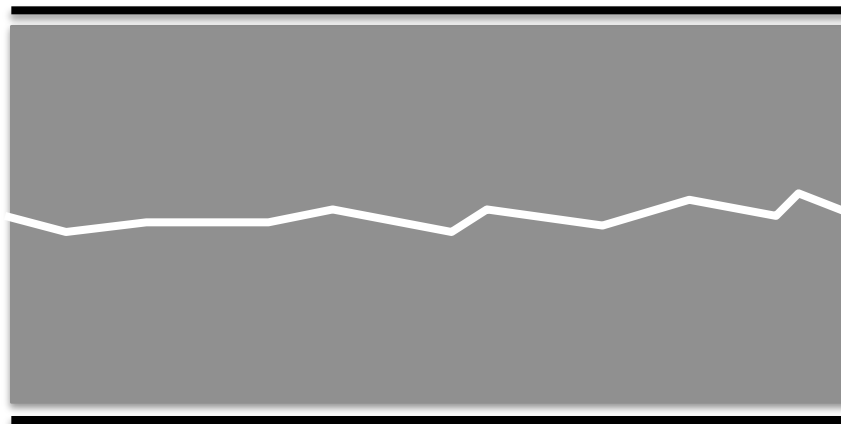


# N9B6 SC WALL INTEGRITY REQUIREMENTS

- The opposite steel faceplates of SC walls shall be connected to each other using *integrity systems* consisting of individual components such as structural shapes, frames, or tie bars. The individual components shall have spacing no greater than the section thickness. They shall be classified as *yielding shear reinforcement* or *non-yielding shear reinforcement* in accordance with Section N9B6.1.
- In the *SC wall interior regions*, they shall be designed to resist: (i) the minimum required forces specified in Section N9B6.2, and (ii) the required out-of-plane shear strength in Section N9G2.
- In the *SC wall connection regions*, they shall be designed to resist: (i) the minimum required forces specified in Section N9B6.2 for integrity, (ii) the required out-of-plane shear strength in Section N9G2, and (iii) the required forces determined in Section N9J8.1 based on their participation in connection *force transfer mechanisms* in Section N9J7.

# N9B6 SC WALL INTEGRITY REQUIREMENTS

- SC walls consist of steel faceplates with concrete sandwiched in between. The ability of the steel faceplates to interact with each other through the concrete sandwich is very important.
- This connectivity is required for the SC section to act as a unit with the steel and the concrete acting in unison.
- There is a potential failure plane through the section thickness that can result in delamination or splitting failure of the wall section





## N9B6 STRUCTURAL INTEGRITY REQUIREMENT

- Ties are not needed to prevent local buckling of plates. That is achieved using shear connectors
- Ties are needed to provide out-of-plane shear strength as per Section N9G2, which may be needed to resist calculated force demands or required strengths.
- Ties are also needed to provide out-of-plane shear strength, which may be needed for indirect (non-calculated) force demands resulting from eccentricities within the section due to force transfer, disparity between the steel faceplate strengths, and second-order effects
- Tie spacing can be as large as section thickness  $T$ , or 48 times the tie bar diameter (See Art. 7.10.5.2 in ACI)

## N9B6.1 DESIGN TENSION STRENGTH AND CLASSIFICATION OF INTEGRITY SYSTEM COMPONENTS

- The design uniaxial tensile strength,  $\phi_t F_t$ , of individual components of integrity systems and the allowable uniaxial tensile strength shall be the lowest value of the following:
  - *Tensile yielding* in the gross section of individual components determined as per the Specification Section D2.1 and using the corresponding  $\phi$  factor
  - *Tensile rupture* in the net section of individual components determined as per the Specification Section D2.2 and using the corresponding  $\phi$  factor
  - Limiting strength of the associated connection(s) determined as per the Specification Chapter J and using the corresponding  $\phi$  factor
- The individual components of *integrity systems* governed by limit state (1) shall be classified as *yielding shear reinforcement*. The individual components of *integrity systems* governed by limit state (2) and (3) shall be classified as *non-yielding shear reinforcement*.

## N9B6.1 DESIGN TENSION STRENGTH AND CLASSIFICATION OF INTEGRITY SYSTEM COMPONENTS

- The design tensile strength of the integrity system components considers the limit states of: (1) gross yielding of the component, (2) net section fracture of the component, and (3) fracture failure of the component to faceplate connections
- This design tensile strength will be needed in Section N9G2 to compute the out-of-plane shear strength
- If the limit state of gross yielding governs, then the integrity component is classified as yielding shear reinforcement, otherwise as non-yielding shear reinforcement
- This information will be used in Section N9G2 to compute out-of-plane shear strength



## N9B6.2 REQUIRED TENSION STRENGTH FOR COMPONENTS OF INTEGRITY SYSTEMS

- The minimum required tensile strength for the individual components classified as *yielding shear reinforcement* or *non-yielding shear reinforcement* shall be as follows:

$$F_{req} = \frac{t_p F_y T}{4 \left( \frac{L_{TR}}{3S_L} + \frac{1}{2} \right) \left( \frac{L_{TR}}{S_T} \right)} \quad (\text{N9B6.1})$$

- where,
  - $t_p$  = steel faceplate thickness, in.
  - $F_y$  = steel plate yield stress, ksi
  - $T$  = SC panel section thickness
  - $S_L, S_T$  = spacing of individual components in orthogonal directions
  - $L_{TR}$  = required transfer length to develop the full membrane strength of the panel section without jeopardizing delamination of the composite section
  - $L_{TR}$  shall be taken as three times the SC panel thickness in *SC wall interior regions*. For *SC wall connection regions*,  $L_{TR}$  shall be taken as two times the SC panel thickness.



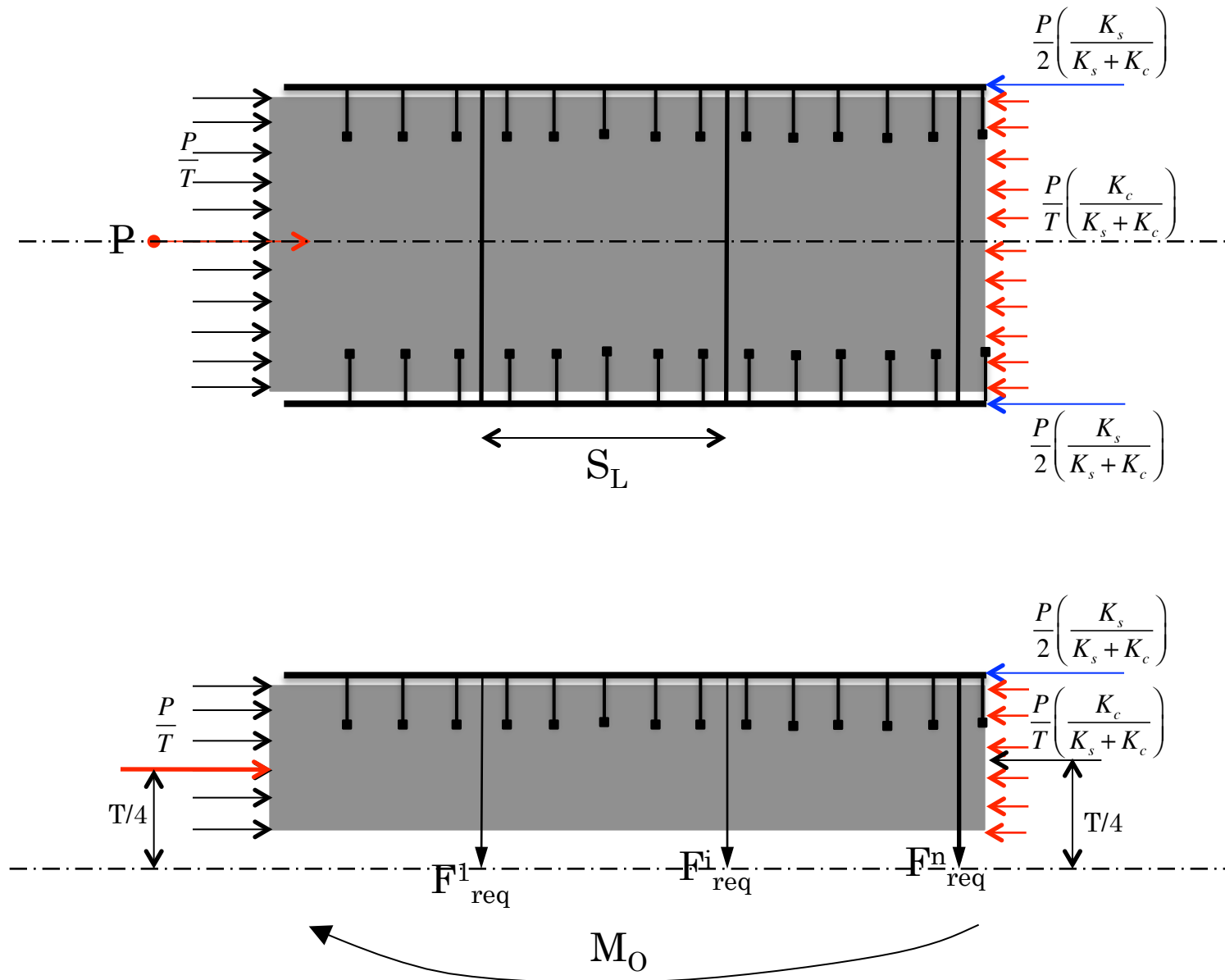
## N9B6.2 REQUIRED TENSION STRENGTH FOR COMPONENTS OF INTEGRITY SYSTEMS

- Eccentric moment causing splitting failure
  - Case 1, if the compressive forces are applied only to the concrete, then they will slowly transfer over to the composite section over the transfer length  $L_{tr}$ . However, over this transfer length, there will be an eccentric moment  $M_o$  that will have to be resisted by the cross-section without splitting
  - Case 2, if the tensile forces are applied only to the steel, then they will slowly transfer over to the composite section until concrete cracking occurs after the transfer length  $L_{tr}$ . Over this transfer length, there will be an eccentric moment  $M_o$  that will have to be resisted by the cross-section without splitting



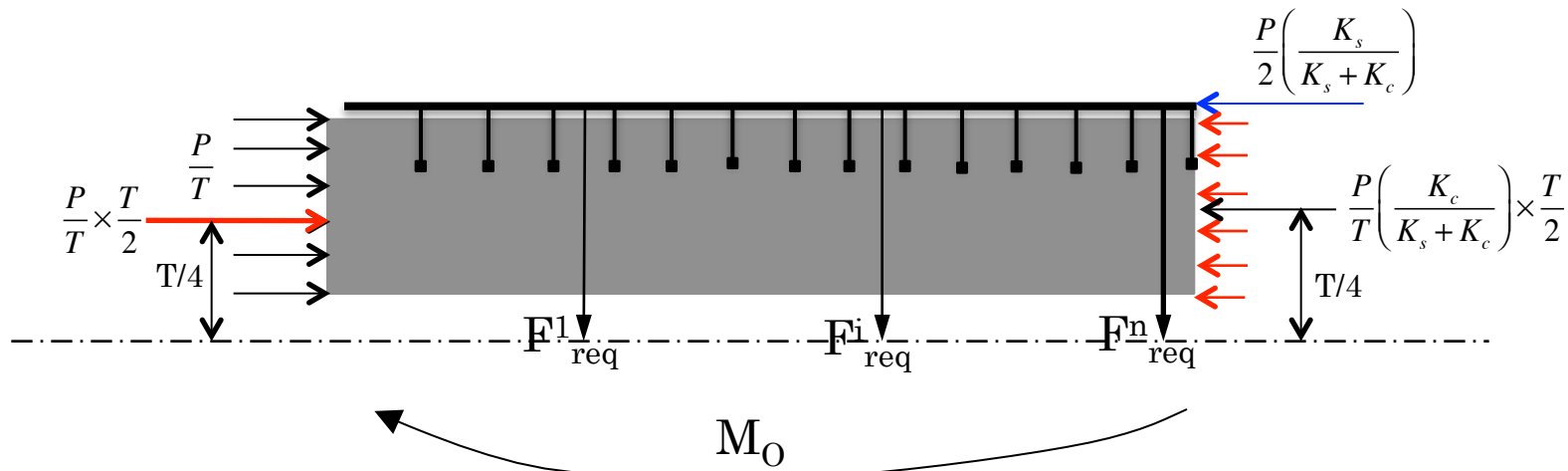
# INTEGRITY REQUIREMENT: CASE 1

LOAD APPLIED TO CONCRETE ONLY, RESISTED BY COMPOSITE SECTION



# INTEGRITY REQUIREMENT: CASE 1

## ECCENTRIC MOMENT $M_o$ ACTING TO SPLIT SECTION



$$+\sum M_{o-o} = -\left(\frac{P}{T} \times \frac{T}{2} \times \frac{T}{4}\right) + \frac{P}{2} \left(\frac{K_s}{K_s + K_c}\right) \times \frac{T}{2} + \frac{P}{T} \left(\frac{K_c}{K_s + K_c}\right) \times \frac{T}{2} \times \frac{T}{4} - M_o = 0$$

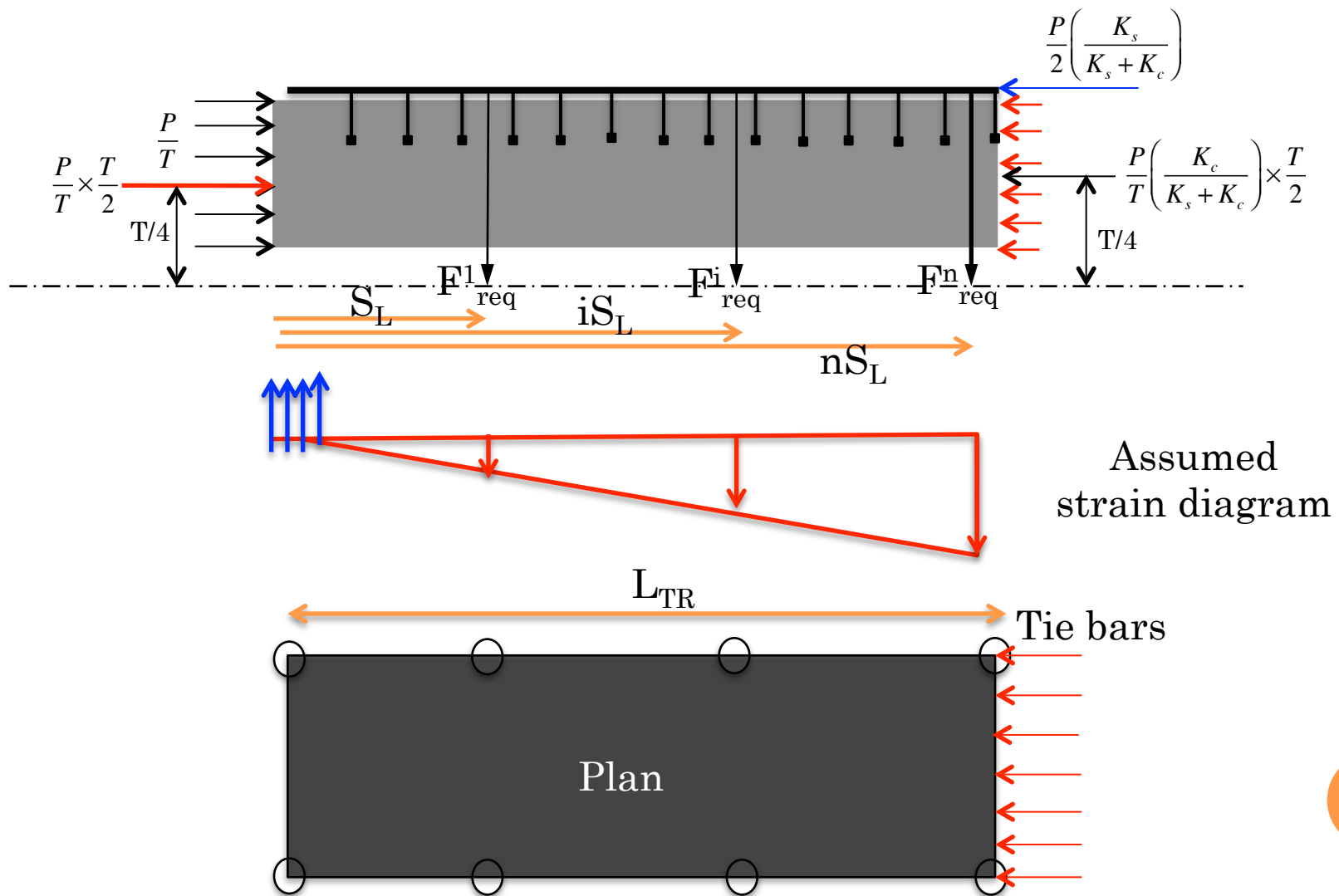
$$\therefore M_o = -\left(\frac{PT}{8}\right) + \frac{PT}{4} \left(\frac{K_s}{K_s + K_c}\right) + \frac{PT}{8} \left(\frac{K_c}{K_s + K_c}\right)$$

$$\therefore M_o = \frac{PT}{8} \times \left(\frac{K_s}{K_s + K_c}\right)$$

$$\therefore M_o = \frac{P}{2} \times \left(\frac{K_s}{K_s + K_c}\right) \times \frac{T}{4} = \text{steel plate force} \times \frac{T}{4}$$

# INTEGRITY REQUIREMENT: CASE 1

## RESISTING MOMENT $M_R$





# INTEGRITY REQUIREMENT: CASE 1

## RESISTING MOMENT > ECCENTRIC MOMENT

For adequate structural integrity

$$M_R \geq M_O$$

$$\therefore F_{req}^n L_{TR} \times \left( \frac{L_{TR}}{3S_L} + \frac{1}{2} \right) \geq S_T t_p F_y \frac{T}{4}$$

$$\therefore F_{req} \geq \frac{t_p F_y \frac{T}{4}}{\left( \frac{L_{TR}}{3S_L} + \frac{1}{2} \right) \times \frac{L_{tr}}{S_T}}$$

*Simplify*

$$\therefore F_{req} \approx \frac{t_p F_y \frac{T}{4}}{\left( \frac{L_{TR}}{3S_L} + \frac{1}{2} \right) \times \frac{L_{tr}}{S_T}}$$

Example:

$$t_p = 0.5 \text{ in.}$$

$$F_y = 50 \text{ ksi}$$

$$T = 30 \text{ in.}$$

$$S_L = S_T = T$$

$$L_{TR} = 3T$$

$$\frac{L_{TR}}{S_L} = \frac{L_{TR}}{S_T} = 3$$

$$\therefore F_{req} = \frac{0.5 \times 50 \times \frac{30}{4}}{1.5 \times 3} = 41.7 \text{ kips}$$



# STEEL-PLATE COMPOSITE (SC) DESIGN FOR NUCLEAR FACILITIES

## Analysis Requirements



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# N9C1: GENERAL PROVISIONS

- SC walls shall be modeled using elastic, three-dimensional, thick-shell or solid finite elements. Regions around section penetrations larger than half the wall thicknesses shall be modeled explicitly with appropriately refined mesh.
- The input (geometric and material) parameters for the finite elements shall be specified according to Section N9C3 to achieve the equivalent stiffness properties specified in Section N9C2. Second order-effects shall be addressed according to Section N9B3.5. Finite element analyses involving accident thermal conditions shall be conducted according to Section N9C4.
- The viscous damping ratio for safe shutdown earthquake (SSE) seismic analysis shall not exceed 5% for determination of required strengths for SC walls.

## N9C1: GENERAL PROVISIONS

- SC wall structures are modeled using linear elastic finite elements, as explained earlier in Section N9B3.1
- These finite elements can be thick-shell finite elements or solid finite elements. Finer meshes are used around section penetrations larger than the wall thickness
- The viscous damping ratios for SSE seismic analysis can be assumed to not exceed 5%. This is based on 1/10<sup>th</sup> scale tests of the entire CIS structure consisting of SC modules (Japanese research)
- When using shell elements to model the expanse of the SC walls, it is recommended to use meshes consisting of at least 4-6 elements along the short direction and 6-8 elements along the long direction. These numbers are based on recommendations of the ASCE 4-09, and will adequately capture local modes of vibration.



## N9C1: GENERAL PROVISIONS

- Finite elements larger than  $3T$  are not recommended for the interior regions.
- Finite elements larger than  $1.5T$  are not recommended for connection regions and regions around section penetrations
- These element size limits are recommended based on the design capacity equations that are deemed appropriate up to  $3T \times 3T$ , i.e., the equations do not apply to the whole wall.



## N9C2.1 EFFECTIVE FLEXURAL STIFFNESS

The effective flexural stiffness of SC walls shall be as follows:

$$(EI)_{eff} = [E_s I_s + c_1 E_c I_c] \left[ 1 - \frac{\Delta T_s}{150 F} \right] \geq E_s I_s \quad (\text{N9C2-1})$$

where,

$$c_1 = 0.48 \rho' + 0.10$$

$\rho' = \rho n = \text{stiffness adjusted reinforcement ratio}$

$E_s$  = elastic modulus of the steel plates, 29000 ksi

$I_s$  = moment of inertia of the fully cracked section =  $12[t_p(T-t_p)^2/2]$  in<sup>4</sup>/ft

$T$  = section thickness in in.

$\rho$  = steel plate reinforcement ratio =  $2t_p/T$

$n$  = modular ratio of steel and concrete =  $E_s/E_c$

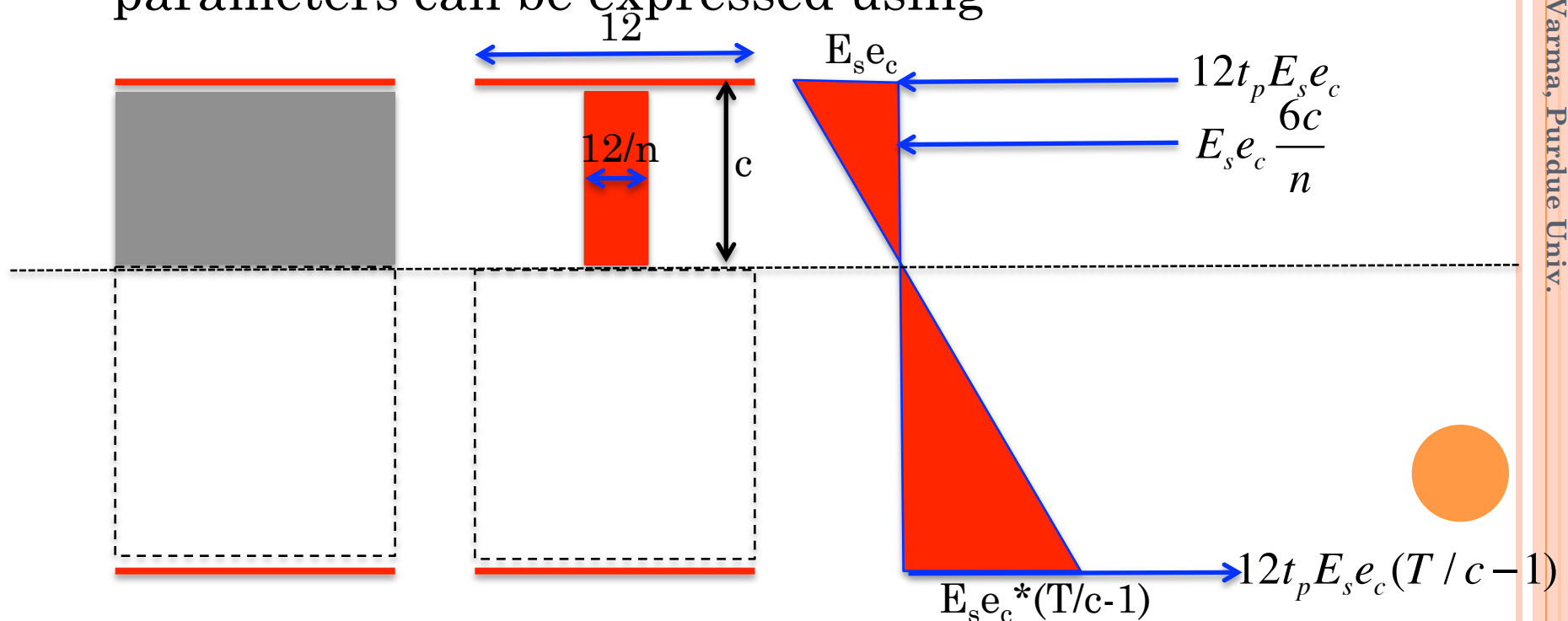
$E_c$  = elastic modulus of concrete, ACI 349-06 Section 8.3.1

$I_c$  = moment of inertia of concrete =  $12(T^3/12)$  in<sup>4</sup>/ft

$\Delta T_s$  = maximum temperature difference in °F between steel faceplates, or maximum temperature increase in °F for steel faceplates subjected to thermal loading due to accident conditions.

## N9C2.1 FLEXURAL STIFFNESS OF SC WALLS

- Experimental data indicates that there is no uncracked flexural stiffness manifest in SC walls due to effects of shrinkage, partial composite action, and reduced bond parameter due to discrete stud locations
- The flexural stiffness of a wide range of SC wall parameters can be expressed using



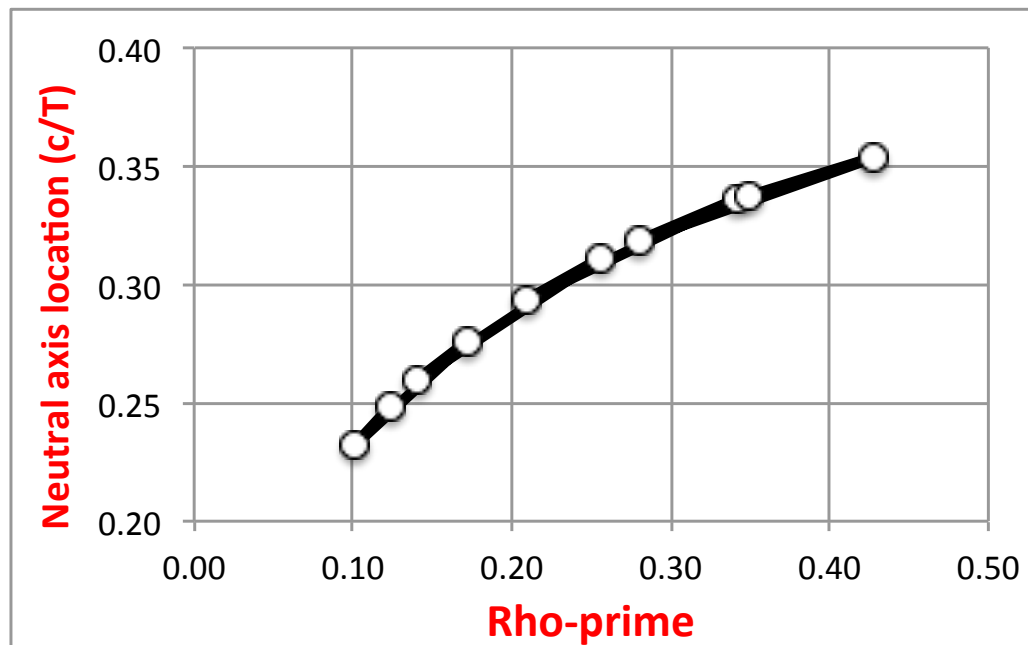
## N9C2.1 FLEXURAL STIFFNESS OF SC WALL

- Define a new parameter  $\rho'$  as:

$$\rho' = \frac{2t_p}{T} \frac{E_s}{E_c}$$

- The neutral axis distance from the top plate can be defined as  $c$ , and it can be calculated as:

$$\frac{c}{T} = \sqrt{\rho'^2 + \rho'} - \rho'$$





## N9C2.1 FLEXURAL STIFFNESS OF SC WALL

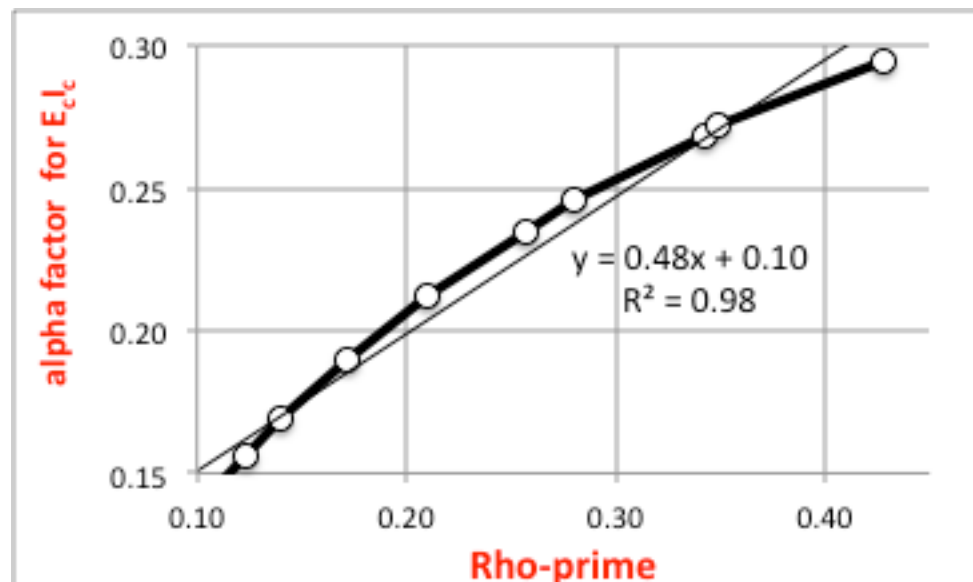
- The corresponding flexural stiffness can be calculated approximately as  $(EI)_{cr-tr}$  per ft. width:

$$(EI)_{cr-tr} = E_s \left[ 12t_p T^2 \left[ 1 + 2 \left( \frac{c}{T} \right)^2 - 2 \frac{c}{T} - \frac{t}{T} \right] + \frac{4T^3}{n} \left( \frac{c-t}{T} \right)^3 \right]$$

- Or, more simply,  $(EI)_{cr-tr}$  can be computed as:

$$(EI)_{cr-tr} = E_s I_s + \alpha E_c I_c$$

Where,  $\alpha = 0.48 \rho' + 0.10$



## N9C2.1 FLEXURAL STIFFNESS RECOMMENDATION

- Experimental data shows that accident thermal loading increases the steel plate temperature rapidly, while the concrete lags behind
- A nonlinear temperature gradient develops through the composite cross-section because of the significantly lower thermal conductivity of concrete
- The nonlinear thermal gradient results in cracking of the concrete due to its low tensile stress ( $f'_t$ ).
- The flexural stiffness recommendation accounts for the potential cracking of the concrete due to the accident thermal gradient through the composite section



## N9C2.1 FLEXURAL STIFFNESS RECOMMENDATION

- Temperature increases on the steel plate greater than 150 F are assumed to result in full (through section) concrete cracking, i.e., the flexural stiffness will be equal to that of the steel ( $E_s I_s$ ) alone
- A linear interpolation from the cracked-transformed flexural stiffness ( $E_s I_s + c_1 E_c I_c$ ) to the steel section stiffness alone ( $E_s I_s$ ) is assumed over a steel surface temperature difference or increase from 0 – 150 F.
- **Note:** The decreases in flexural stiffness are used only for situations with accident thermal loading.
- **Note:** Ambient thermal loading conditions are assumed to produce linear thermal gradients, which develop gradually over time. As a result, there will be little to no additional concrete cracking due to ambient thermal loading.



## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

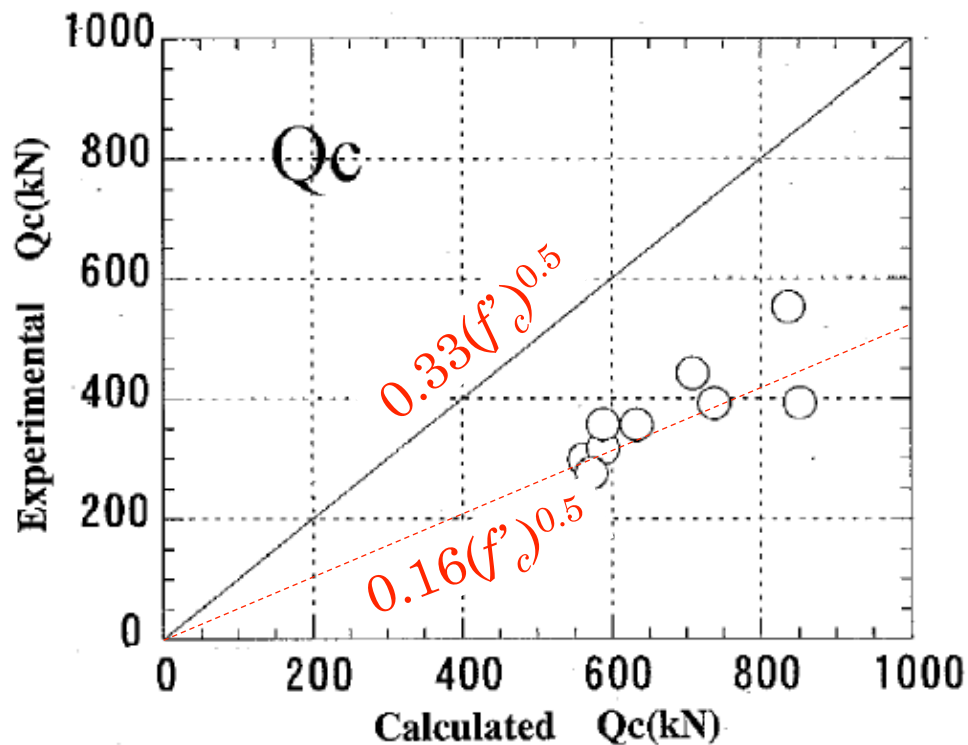
- The in-plane shear behavior of SC module walls is governed by the plane-stress behavior of the steel plates and orthotropic cracked behavior of the concrete infill.
  - Composite (uncracked) behavior is assumed to occur when the in-plane shear force is less than or equal to:

$$V_{ck} = 2\sqrt{f'_c} \left( A_c + \frac{G_s}{G_c} A_s \right)$$

- The pre-cracking shear stiffness can be estimated as the composite shear stiffness ( $G_s A_s + G_c A_c$ ). It is important to understand that the composite action between the steel plates and the concrete infill (through the studs, ties, etc.) is discrete and not perfect.



## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS



Plot of experimental vs. calculated values of cracking strength by Ozaki, et al. ("Study on steel plate reinforced concrete panels subjected to cycle in-plane shear", Nuclear Engineering and Design, 2004.)

- Calculated  $Q_c$  (or  $V_{ck}$ ) in terms of concrete stress of:

- $0.33(f'_c)^{0.5}$  MPa =  $4(f'_c)^{0.5}$  psi

- Experimental values observed fall closer to

- $0.16(f'_c)^{0.5}$  MPa =  $2(f'_c)^{0.5}$  psi

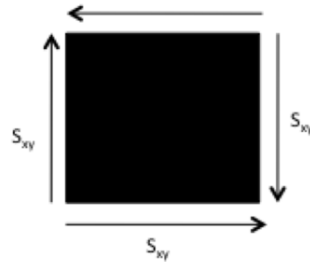
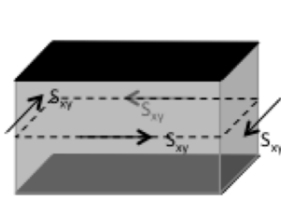
## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

- After cracking, the tangent stiffness reduces significantly to that of the cracked composite section
  - The tangent stiffness  $K_{cr}$  can be estimated as  $K_s + K_c$ , where
  - $K_s = G_s A_s$  and  $K_c = \frac{1}{\frac{4}{0.7E_c A_c} + \frac{2(1-\nu)}{E_s A_s}}$
- The secant stiffness can be estimated as a function of the applied shear force ( $V$ ).
  - The secant stiffness  $K_{sec}$  can be normalized with respect to the uncracked stiffness  $K_{uncr}$



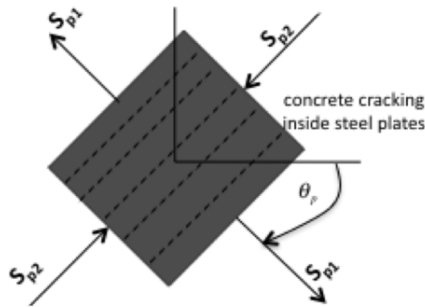
# N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

(a) Stress block diagrams



(a) SC Composite Section Subjected to In-Plane Shear

(b) Plan View of SC Composite Section

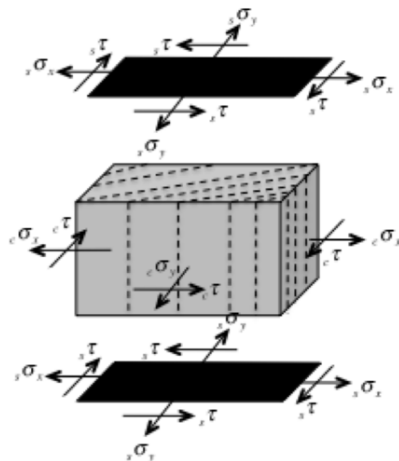


$$\tan(2\theta_p) = \frac{2S_{xy}}{S_x - S_y} = \infty$$

$$\therefore \theta_p = 45^\circ$$

$$S_{p1,2} = \pm S_{xy}$$

(c) Principal Forces in SC Composite Section



(d) Steel and concrete stresses due to in-plane shear

(b) Equations

Force Equilibrium

$$\begin{Bmatrix} 0 \\ 0 \\ S_{xy} \end{Bmatrix} = \begin{Bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{Bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix} + \frac{E_c T_c}{4} \begin{Bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{Bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$S_x = S_y = \frac{S_{xy}}{2t_s} \frac{K_\beta}{K_\alpha + K_\beta}$$

Steel normal stresses

$$\tau = \frac{S_{xy}}{2t_s} \frac{K_\alpha}{K_\alpha + K_\beta}$$

Steel shear stress

$$S_x = S_y = -\tau = \frac{S_{xy}}{T_c} \frac{-K_\beta}{K_\alpha + K_\beta}$$

Concrete normal and shear stresses

$$K_\alpha = G_s 2t_s \quad K_\beta = \frac{1}{\frac{4}{E_c T_c} + \frac{2(1-\nu)}{E_s 2t_s}}$$

Steel and concrete stiffness contributions

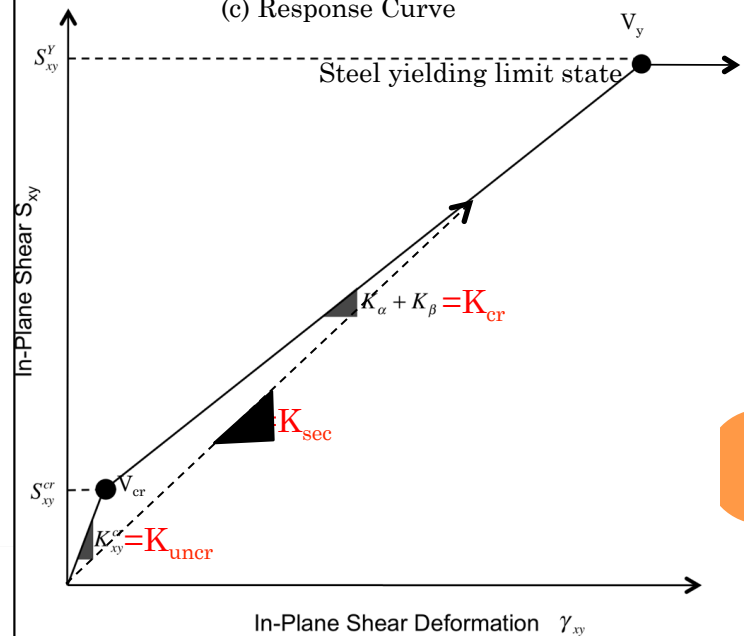
$$S_{xy} = (K_\alpha + K_\beta) \gamma_{xy}$$

Post-cracking shear stiffness

$$S_{xy}^y = \frac{K_\alpha + K_\beta}{\sqrt{3K_\alpha^2 + K_\beta^2}} (2t_s F_y)$$

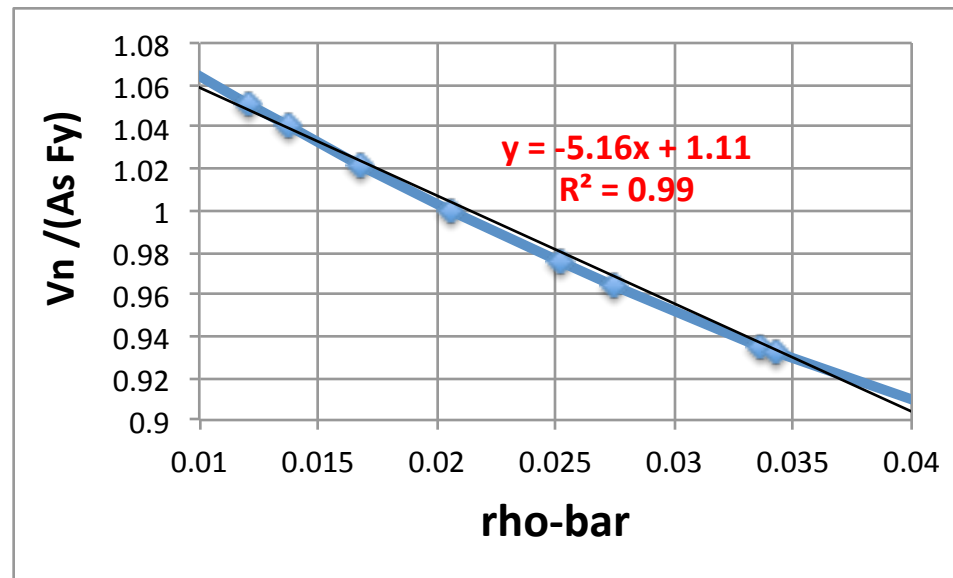
In-plane shear strength (steel yielding limit state)

(c) Response Curve



## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

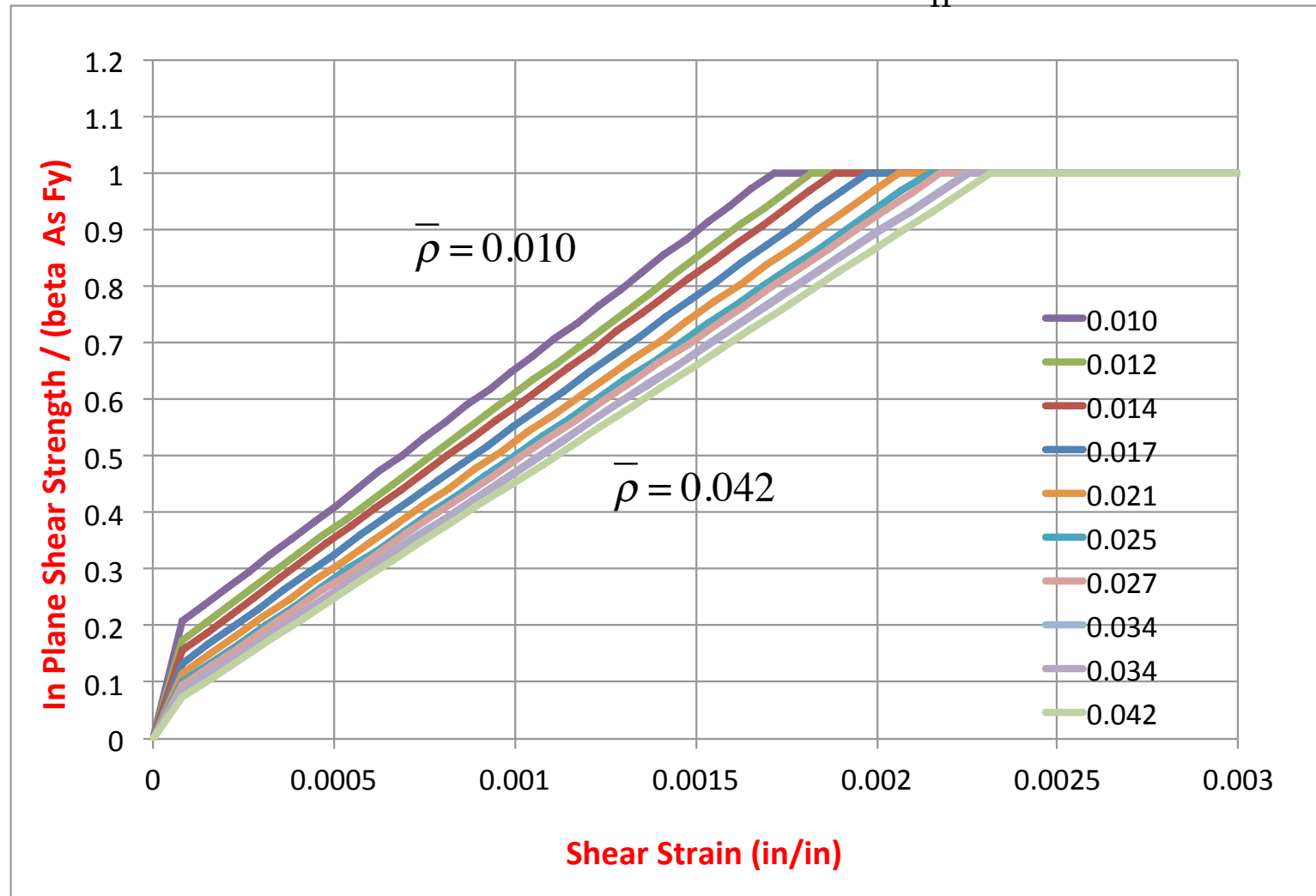
- The applied shear force (V) can be normalized with respect to the nominal shear strength ( $V_n = \beta A_s F_y$ )
- Where,  $\beta = 1.11 - 5.16 \times \bar{\rho}$
- And the new parameter rho-bar is as follows:  $\bar{\rho} = \frac{A_s F_y}{\sqrt{f'_c} A_c}$
- The parameter rho-bar is non-dimensional, where  $F_y$  is in ksi and  $f'_c$  is in psi.
- The values of rho-bar are between 0.01 and 0.04 for nuclear structures and for APWR





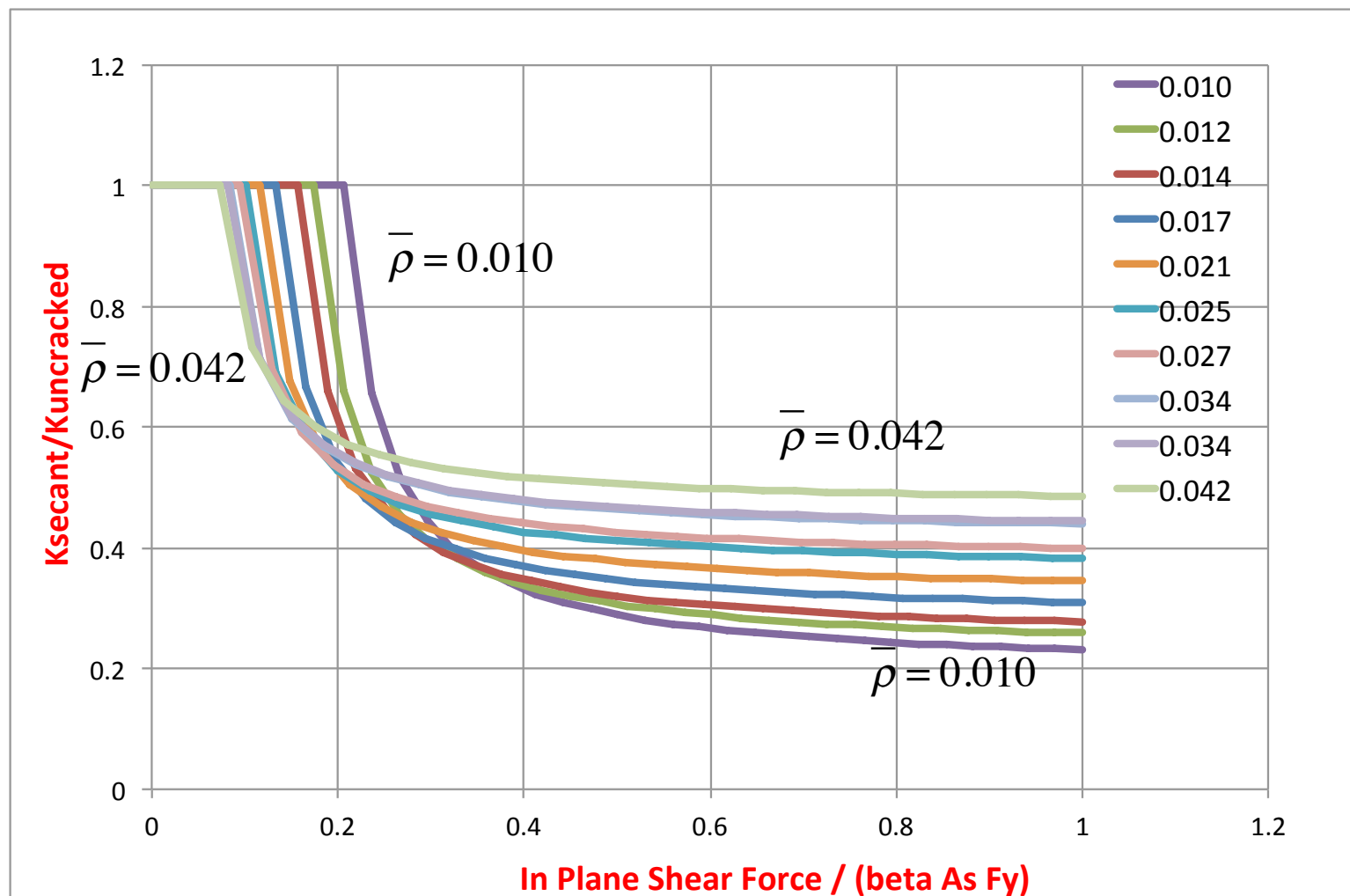
## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

- Shear force-shear strain behavior for different values of rho-bar. Force normalized w.r.t.  $V_n$



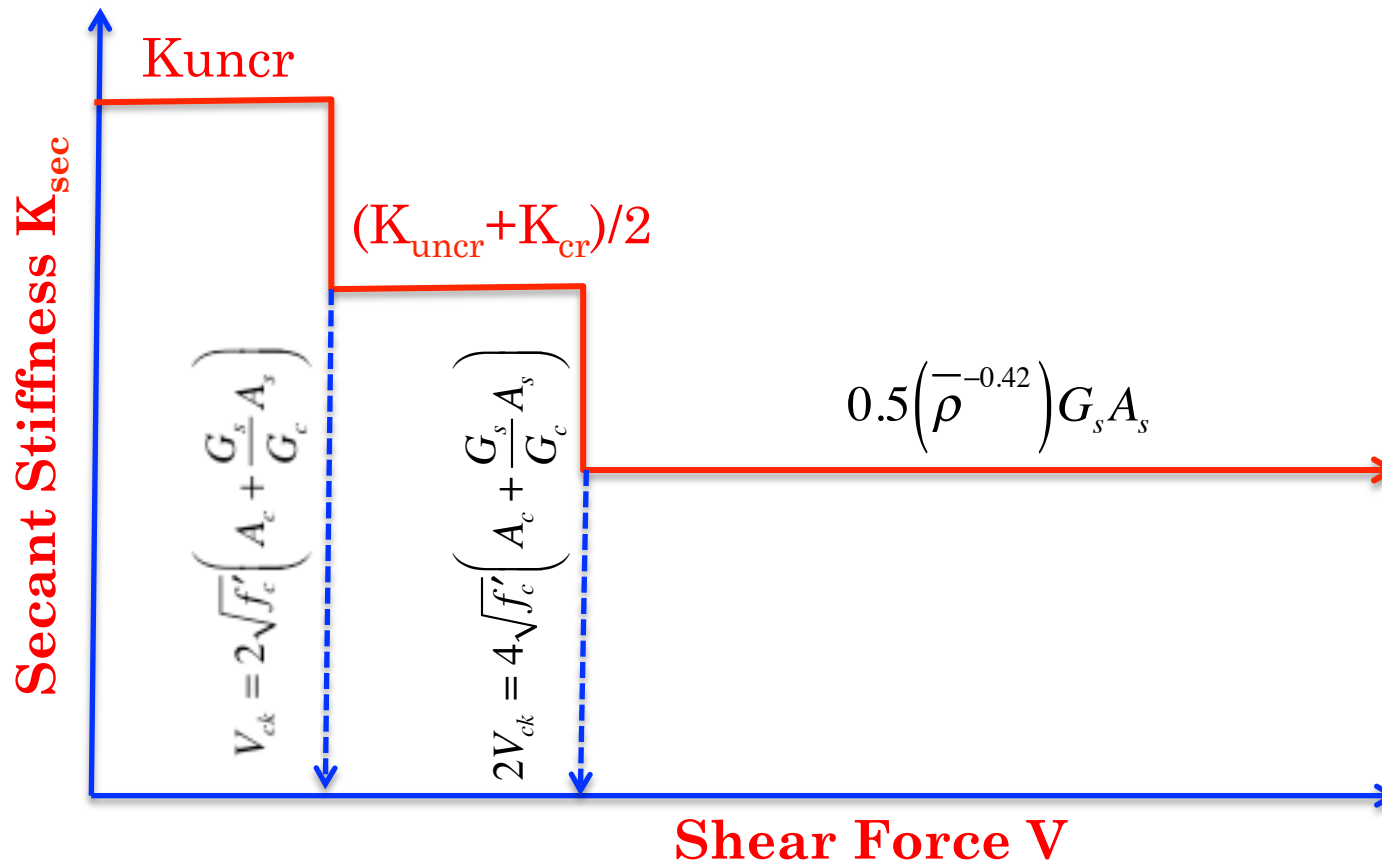
## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

- Variation of  $K_{\text{sec}}/K_{\text{uncr}}$  with respect to the shear force  $V/V_n$  for different values of  $\bar{\rho}$ .

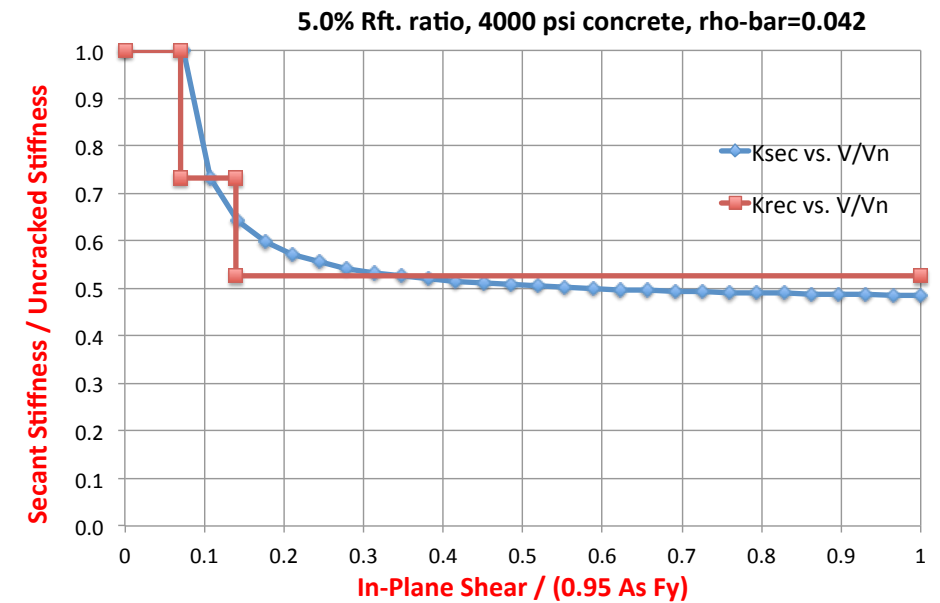
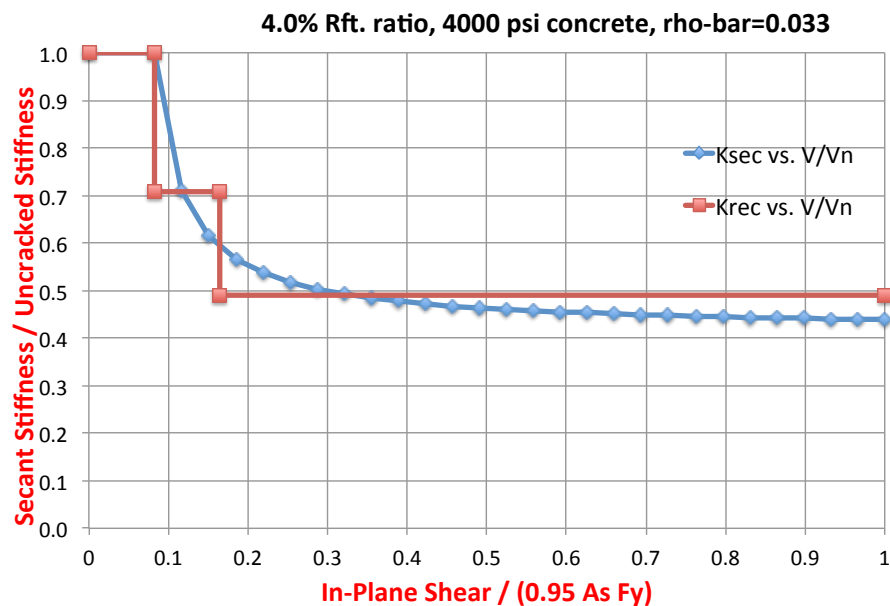
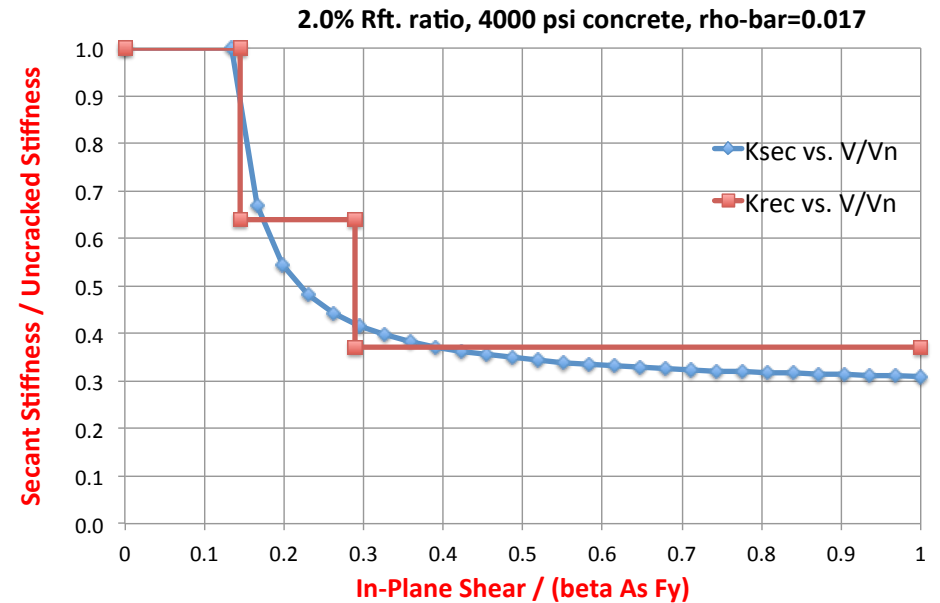
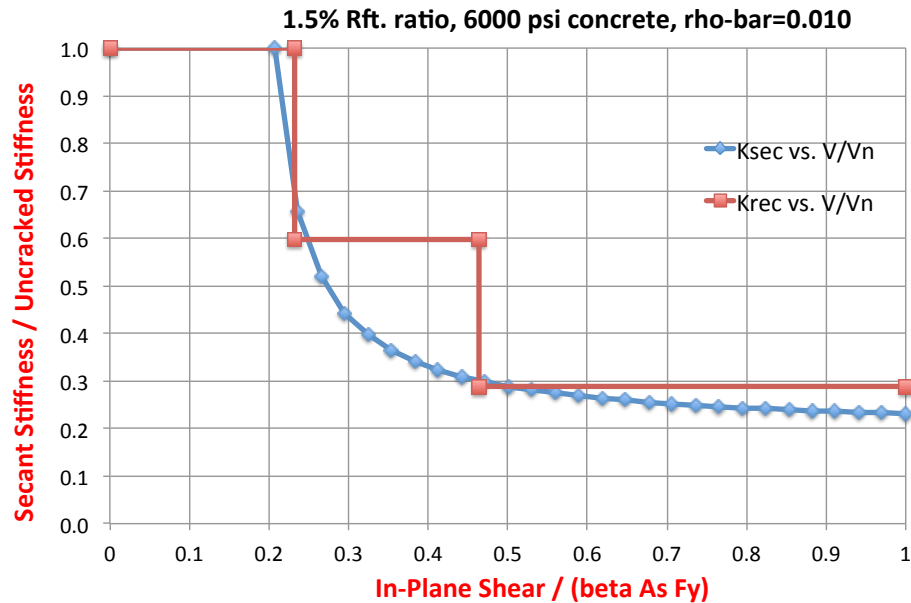


# A SIMPLE MODEL FOR SECANT STIFFNESS

- Looking at the variation in secant stiffness w.r.t.  $V/V_n$  at different values of  $\rho$ -bar, the following simple model was developed for SC walls.



# COMPARISON OF RECOMMENDED SECANT STIFFNESS WITH BEHAVIOR



## N9C2.2 EFFECTIVE IN-PLANE SHEAR STIFFNESS

- **Accident Thermal Loading Combinations**

For all loading combinations involving accident thermal conditions, the in-plane shear stiffness shall account for the effects of concrete cracking using Equation N9-C2-5, i.e.,  $(GA)_{\text{eff}} = (GA)_{\text{cr}}$ , irrespective of the corresponding required membrane in-plane shear strength per unit width ( $S_{\text{rxy}}$  in Section N9C5).



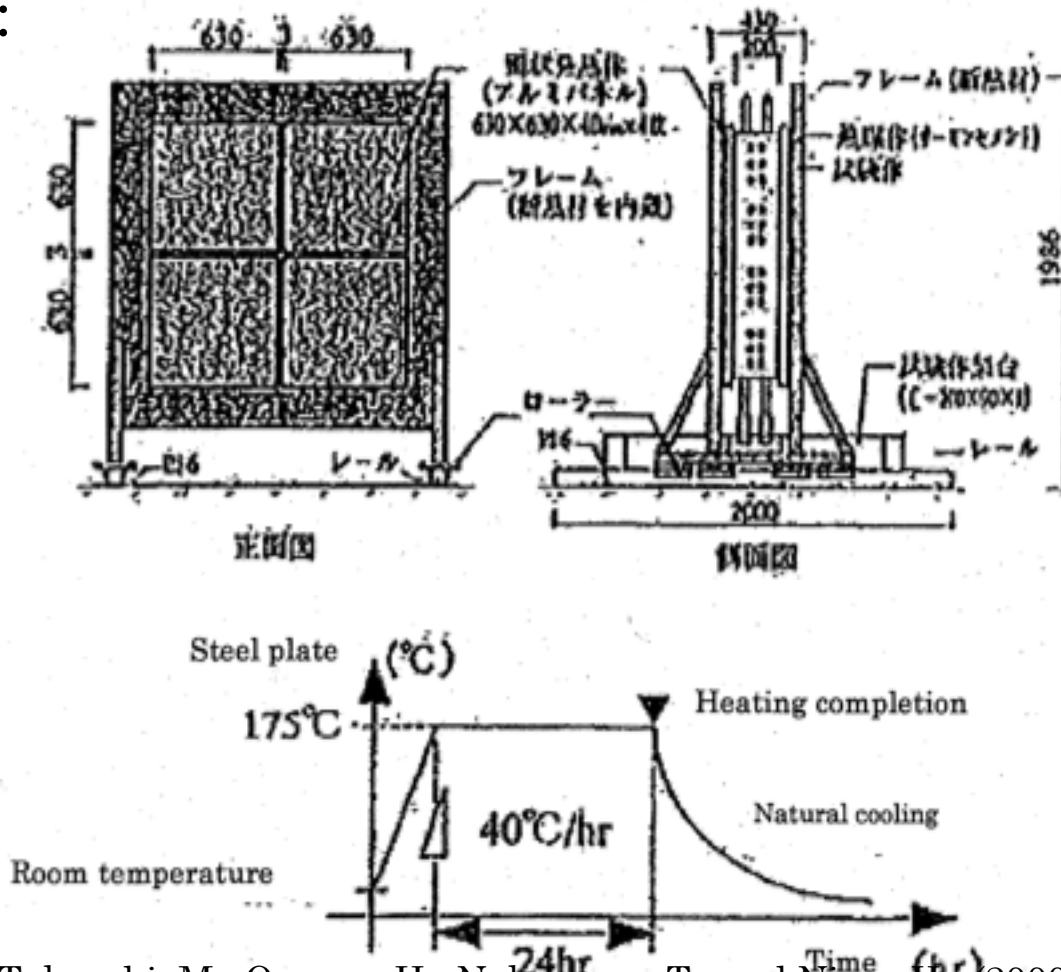
# EFFECTS OF THERMAL LOADING ON STIFFNESS

- The in-plane shear stiffness of SC walls after accident thermal loading was evaluated experimentally by researchers in Japan
- As expected, accident thermal loading induces concrete cracking in two orthogonal directions due to the thermal gradient through the composite section and low cracking threshold of concrete
- The in-plane shear stiffness of SC walls after accident thermal loading can be estimated as the the post-cracking shear stiffness of the composite section ( $K_s + K_c$ )
- The in-plane shear stiffness after accident thermal cracking can be estimated as  $K_{cr} = 0.5 \left( \bar{\rho}^{-0.42} \right) G_s A_s$
- No change in shear strength after thermal loading



# EFFECTS OF THERMAL LOADING ON STIFFNESS

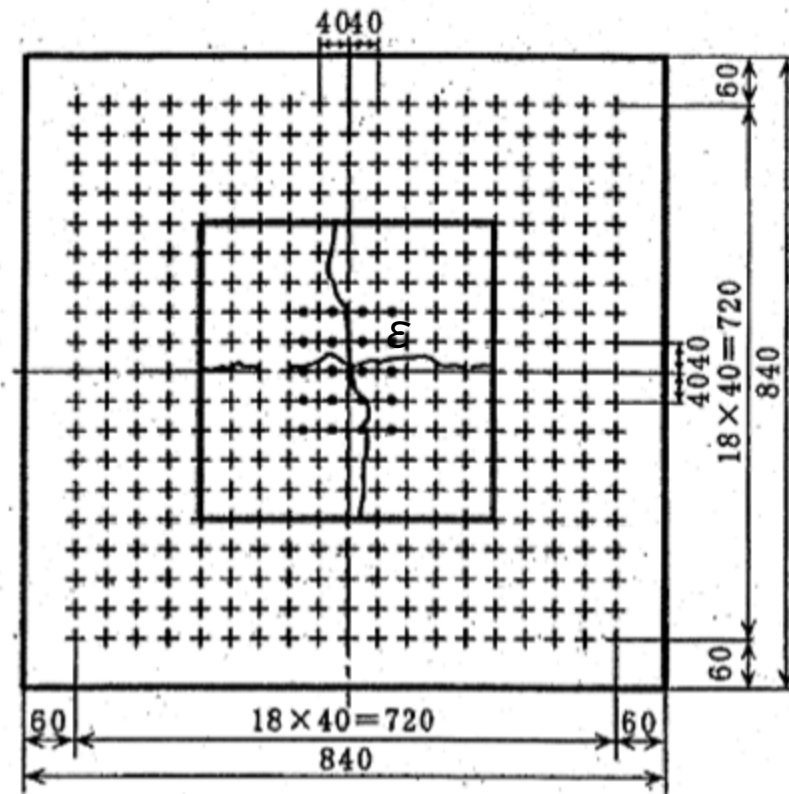
- The experimental verification was conducted as follows:



Ozaki, M., Akita, S., Takeuchi, M., Oosuga, H., Nakayama, T., and Niwa, H., (2000). "Experimental Study on Steel-plate-reinforced Concrete Structure Part 41: Heating Tests (Outline of Experimental Program and Results), Annual Conference of Architectural Institute of Japan, 2000, Part 41-43, pp. 1127-1132.

# EFFECTS OF THERMAL LOADING ON STIFFNESS

- Example of orthogonal cracking of concrete due to thermal gradient



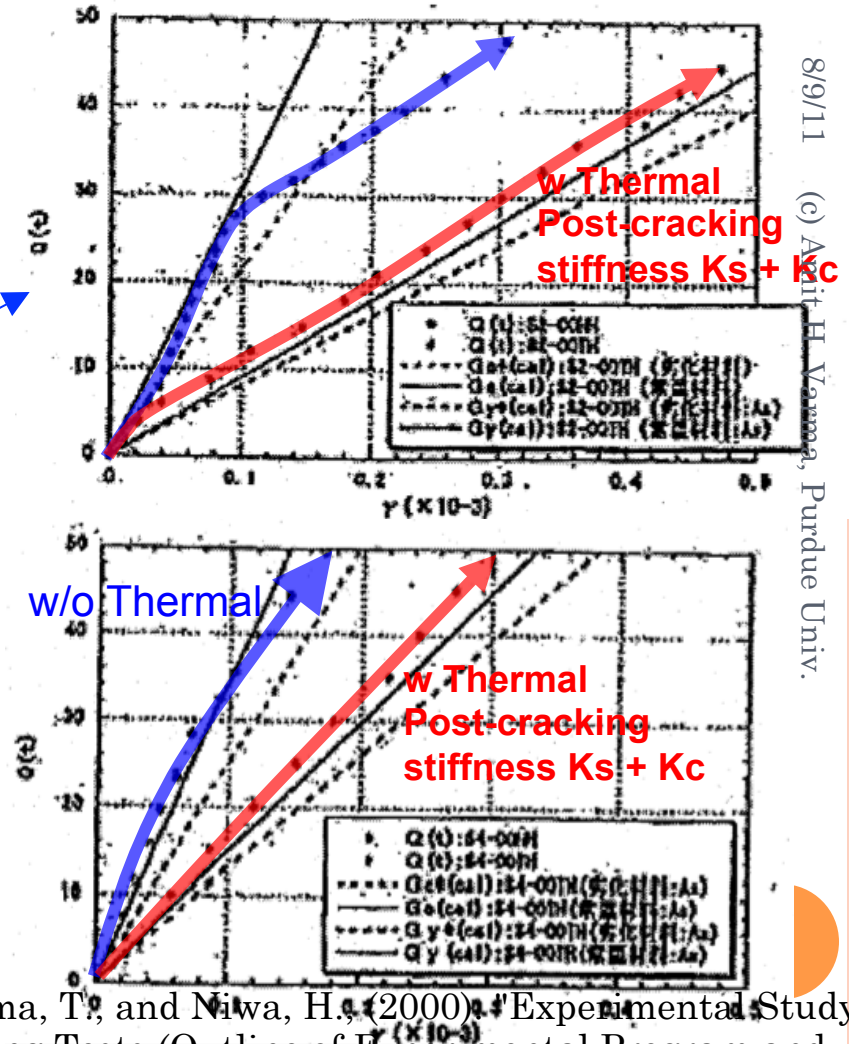
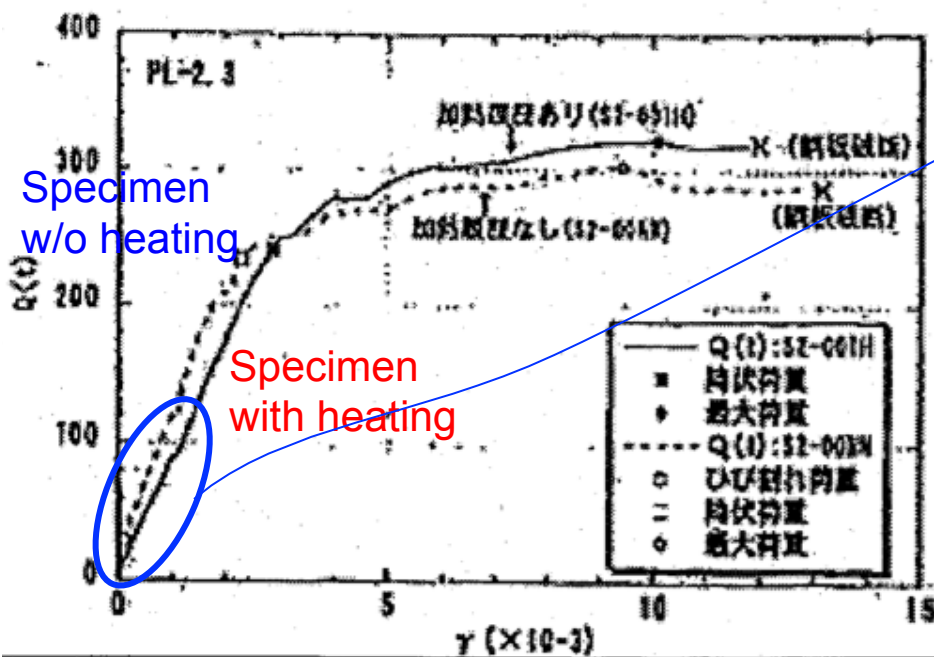
8/9/11 (c) Amit H. Varma, Purdue Univ.

Ozaki, M., Akita, S., Takeuchi, M., Oosuga, H., Nakayama, T., and Niwa, H., (2000). "Experimental Study on Steel-plate-reinforced Concrete Structure Part 41: Heating Tests (Outline of Experimental Program and Results), Annual Conference of Architectural Institute of Japan, 2000, Part 41-43, pp. 1127-1132.



# EFFECTS OF THERMAL LOADING ON STIFFNESS

## ○ Shear Force – Strain Behavior



8/9/11 (c) Anit H. Varma, Purdue Univ.

Ozaki, M., Akita, S., Takeuchi, M., Oosuga, H., Nakayama, T., and Niwa, H., (2000) "Experimental Study on Steel-plate-reinforced Concrete Structure Part 41: Heating Tests (Outline of Experimental Program and Results), Annual Conference of Architectural Institute of Japan, 2000, Part 41-43, pp. 1127-1132.

## N9C3: INPUT PARAMETERS FOR FINITE ELEMENT MODELS

- The input parameters required to define the finite element models for the SC walls include the section thickness ( $T_m$ ), material elastic modulus ( $E_m$ ), Poisson ratio ( $\nu_m$ ), material density ( $\rho_m$ ), thermal expansion coefficient ( $\alpha_m$ ), thermal conductivity ( $k_m$ ), and the material specific heat ( $c_m$ ). These shall be specified as follows:
  - (1) The Poisson ratio ( $\nu_m$ ), thermal expansion coefficient ( $\alpha_m$ ), and thermal conductivity ( $k_m$ ) for the finite element models shall be assumed to be that for the concrete material.
  - (2) The section thickness ( $T_m$ ) and the material elastic modulus ( $E_m$ ) for the finite element models shall be calibrated to match the recommended effective stiffness values,  $(EI)_{eff}$  and  $(GA)_{eff}$ , in Section N9C2.



## N9C3: INPUT PARAMETERS FOR FINITE ELEMENT MODELS

- (3) The material density ( $\gamma_m$ ) for the models shall be calibrated after obtaining the section thickness ( $T_m$ ) to match the mass of the SC composite section.
- (4) The material specific heat ( $c_m$ ) for the models shall be calibrated after obtaining its density ( $\gamma_m$ ) to match the specific heat of the concrete section.



## N9C3: INPUT PARAMETERS TO FINITE ELEMENT MODELS

- A linear elastic finite element model of the composite SC section will be developed using one material. As mentioned earlier this model is used for dynamic SSI and subsequent analysis
- For this single material linear elastic model, recommended to:
  - Match the Poisson's ratio, thermal expansion coefficient, and thermal conductivity of the concrete material because these parameters will govern the thermally induced displacements of the structure.
  - Calibrate the section thickness and material elastic modulus to match the effective stiffness (flexure and in-plane shear) of the section
  - Calibrate the material density to match the mass of the section



- Calibrate the material specific heat to match that of the concrete.
- Calibrating the specific heat will allow transient heat transfer analysis to be conducted using the linear elastic, single material, finite element model with accuracy



## N9C4: ANALYSES INVOLVING ACCIDENT THERMAL CONDITIONS

- Analyses for load combinations involving accident thermal conditions shall include sequential heat transfer and structural analyses.
- The heat transfer analyses shall be conducted using the input parameters specified in Section N9C3. These analyses shall be used to estimate the temperature histories and through-section temperature profiles produced by the thermal accident conditions.
- The structural finite element analyses shall be conducted in accordance with N9C1, N9C2, and N9C3. These models shall use the temperature histories and through-section temperature profiles calculated by the heat transfer analyses as thermal loading.

## N9C4: ANALYSES INVOLVING ACCIDENT THERMAL CONDITIONS

- The out-of-plane moment demands ( $M_{rx}$  and  $M_{ry}$ ) in the *SC wall interior regions* caused by the thermal gradients need not exceed  $M_{r-th}$  calculated as follows:

$$M_{r-th} = (EI)_{eff} \times \left( \frac{\alpha_s \Delta T_s}{T} \right)$$

where,

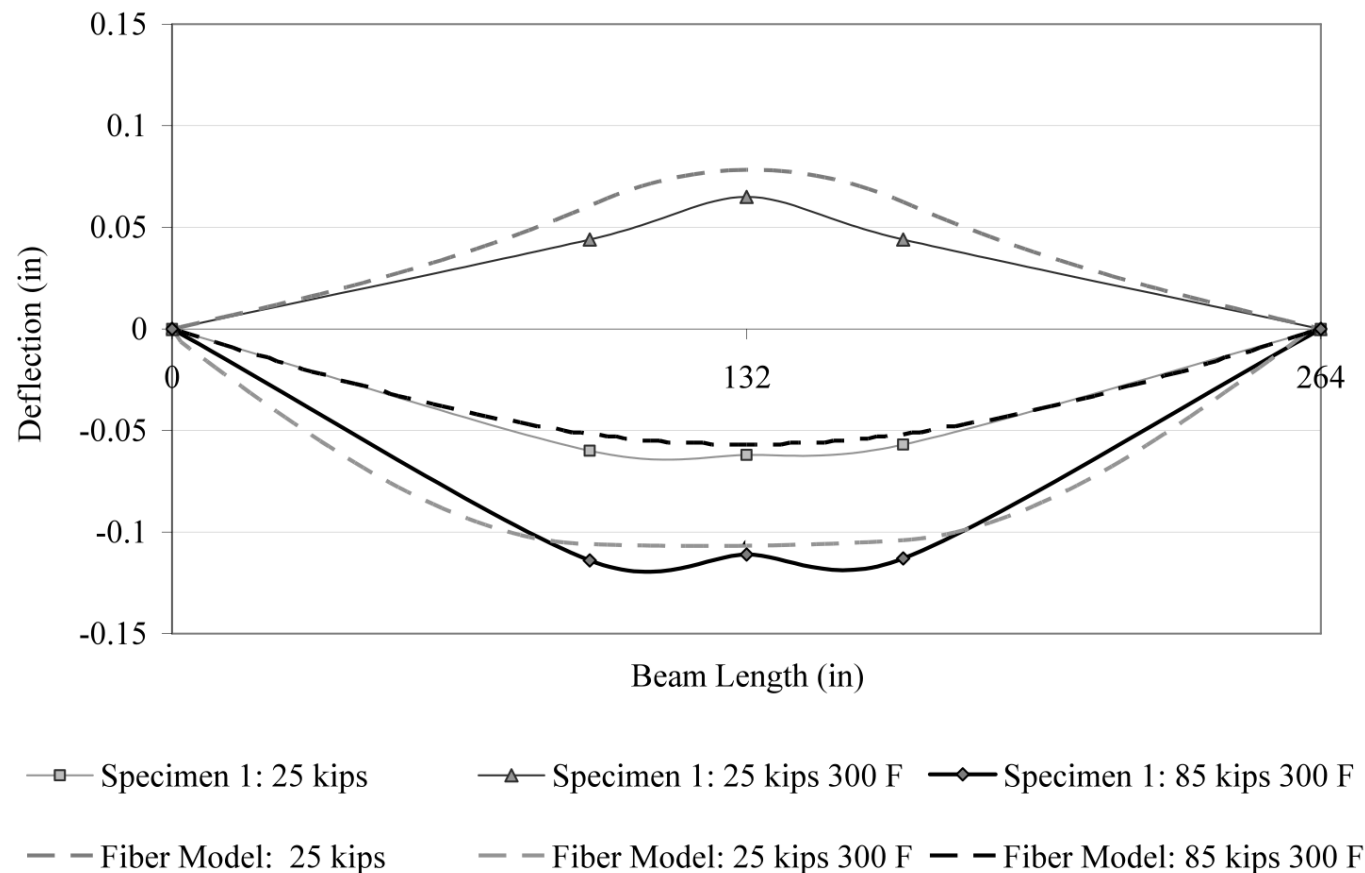
$\alpha_s$  = thermal expansion coefficient of steel faceplate in °F

$\Delta T_s$  = maximum temperature difference in °F between steel faceplates

**User Note:** Analysis for thermal loads may predict moments higher than the  $M_{r-th}$  values defined above, because it does not directly account for the self-limiting effect due to concrete cracking. The  $M_{r-th}$  value above considers full flexural restraint, and accounts for the relief from concrete cracking on limiting the thermally induced moments for design.

## N9C4: LIMIT ON $M_{R-TH}$

- Good comparisons between experimental and predicted behavior (Booth et al. 2007)

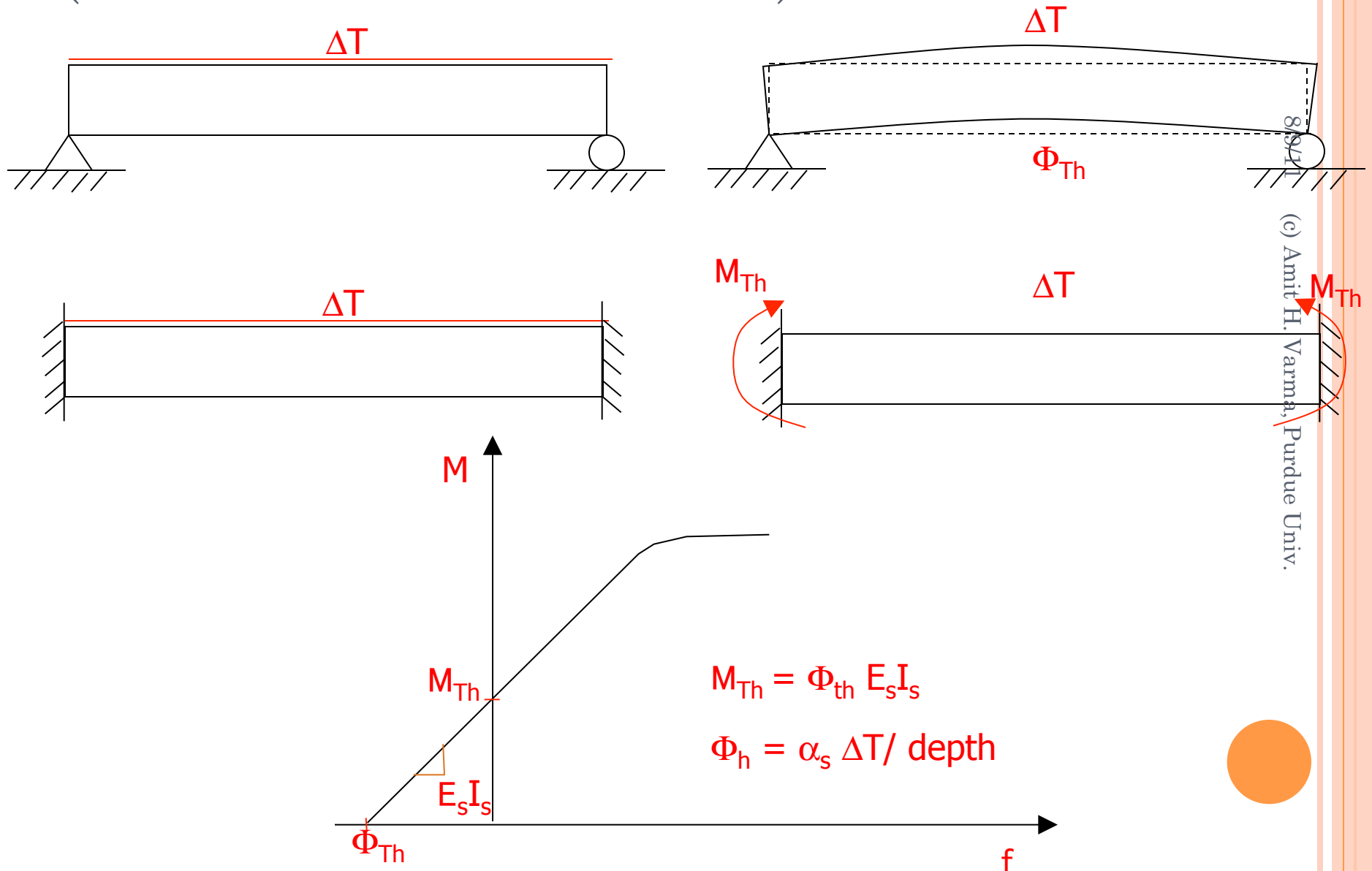


8/9/11 (c) Amit H. Varma, Purdue Univ.

Booth, P.N., Varma, A. H., Malushte, S. R., Johnson, W. H., "Response of Modular Composite Walls to Combined Thermal & Mechanical Load." *Trans. of the Internal Assoc. for Struct. Mech. in Reactor Tech. Conf., SMiRT-19, Paper No. H01/4.* Toronto, Canada, Aug. 2007, 10 pp

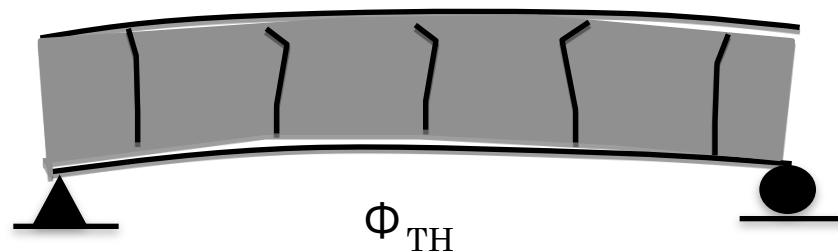
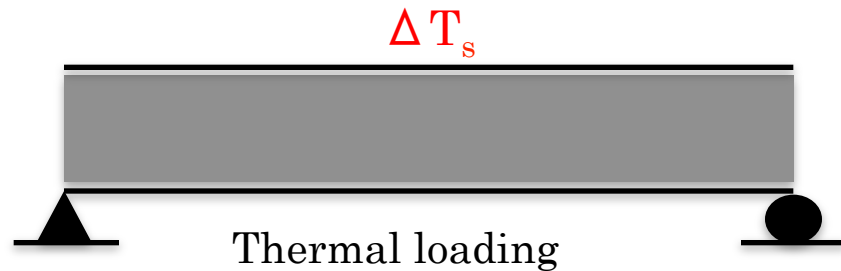


# N9C4: LIMIT ON $M_{R-TH}$ (AFTER BOOTH ET AL. 2007)



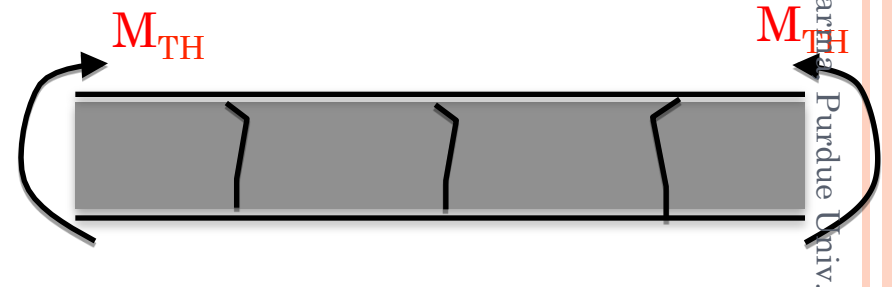
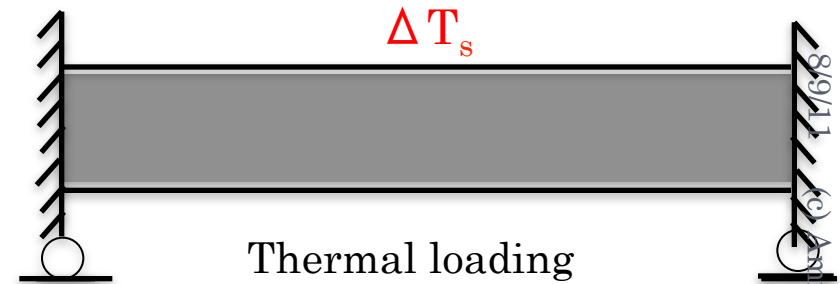
# N9C4: LIMIT ON $M_{R-TH}$

(a) Externally unrestrained beam



$$\phi_{TH} = \frac{\alpha \Delta T_s}{d}$$

(b) Externally restrained beam



$$M_{TH} = (EI)_{cr} \phi_{th} = (EI)_{cr} \frac{\alpha \Delta T}{d}$$

## N9C4: LIMIT ON $M_{R-TH}$

### CONCLUSIONS FROM BOOTH ET AL. (2007)

- Ambient stiffness of the composite walls can be predicted using cracked transformed section properties.
- The stiffness of the composite wall subjected to heating (to 300 F) can be predicted using fully cracked (steel only) section properties.
- The fiber model approach predicts the experimental results with reasonable accuracy.
- A simple equation was developed (based on the analytical results) to predicts the effects of combined thermal and mechanical loading.
  - This equation is generally applicable for SC systems

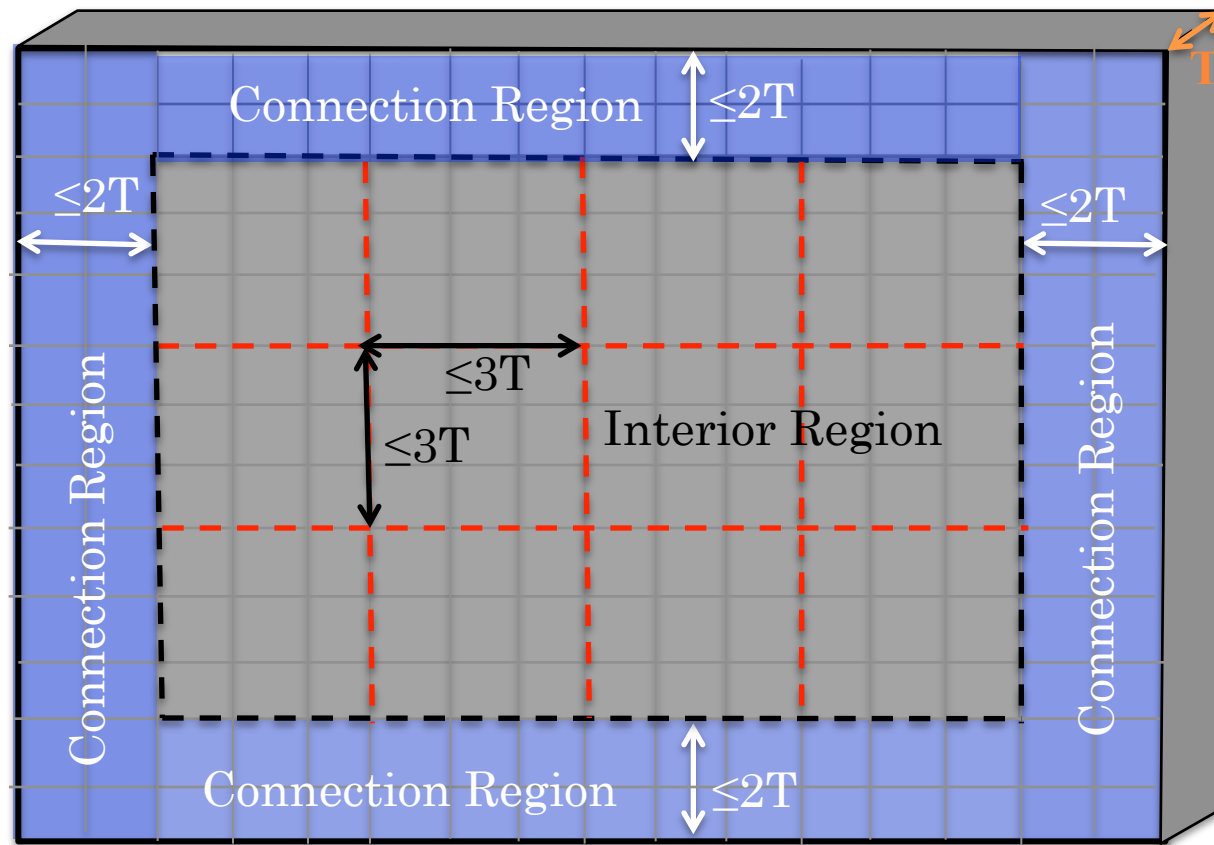


## N9C5: REQUIRED STRENGTHS FOR INTERIOR WALL REGIONS

- The results from the finite element analyses shall be classified for the *SC wall interior regions* by *demand type* (in-plane membrane forces, out-of-plane moments, and out-of-plane shear forces).
- Each *demand type* shall be averaged over *interior panel sections* that are no larger than three times the wall thickness in length and width. In the vicinity of openings or penetrations, each *demand type* shall be averaged over *interior panel sections* no larger than the wall thickness in length and width.



# N9C5 Required Strengths for Interior Wall Regions



Averaging and Design Assessment over  $3T \times 3T$  panels because development length is  $3T$ . Size represents reasonable but not extensive yielding (first onset of significant inelastic deformation at SSE)

## N9C6: DESIGN OF SC WALL INTERIOR REGIONS

- The design of *SC wall interior regions* shall be evaluated over the *interior panel sections* identified in Section N9C5. For all loading combinations in Section N9B2, the design of the SC wall *interior panel sections* shall be evaluated as follows:
- For each *demand type*, the *available strength* shall be greater than or equal to the corresponding *required strength*. The *available strength* for each demand type shall be calculated in accordance with the applicable provisions of Section N9D, N9E, N9F, and N9G.
- For simultaneous combinations of various demand types and their corresponding *required strengths*, the *interior panel section* adequacy shall be verified using the interaction equations provided in Section N9H.



## ANALYSIS AND EVALUATION BASIS

- Perform elastic finite element analysis with FE sizes less than 3T or 9-ft, whichever smaller
- Demand is evaluated for sections up to 3T by 3T or 9-ft by 9-ft, whichever smaller
- Section capacities for individual demands and combined interaction are based on section size no greater than the smaller of 3T by 3T or 9-ft by 9-ft, whichever smaller
- The sections are maintained essentially elastic; the  $\phi$  factors are accorded on the basis of nature of failure mode and degree of variability in the capacity



## RESULTS FROM ANALYSIS: IN-PLANE AND OUT-PLANE DEMANDS

- The FE analysis of the structure will result in force and deformation demands on the components.
- In the spirit of LRFD, the analysis will be elastic that accounts for the effects of: (a) concrete cracking, (b) slip between the steel plates and concrete infill, and (c) connection flexibility.
- The results from the analysis will include the following demands:
  - In plane Force demands ( $S_x$ ,  $S_y$ , and  $S_{xy}$ )
  - Moment ( $M_x$ ,  $M_y$ , and  $M_{xy}$ )
  - Out of Plane shear ( $V_{xz}$  and  $V_{yz}$ )





# STEEL-PLATE COMPOSITE (SC) DESIGN FOR NUCLEAR FACILITIES

## Design for Individual Demands



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## N9D → DESIGN OF SC WALLS FOR TENSION ¶

### 1. → GENERAL PROVISIONS ¶

The design of SC walls for the required membrane axial tension demands ( $S_{rx}$  or  $S_{ry}$ ) shall be evaluated over the *interior panel sections* defined in Section N9C5. ¶

Concrete shall have no contribution to the *design tensile strength* of SC walls. The steel ribs shall have no contribution to the *design tensile strength* of SC walls. The effects of holes in steel plates shall be accounted for in accordance with Section N9D2. ¶

### 2. → TENSILE STRENGTH OF SC WALLS ¶

The *design uniaxial tensile strength* per unit width ( $\phi_t T_n$ ) and the *allowable uniaxial tensile strength* per unit width ( $T_n/\Omega_t$ ) of *interior panel sections* shall be determined according to the *limit state* of *tensile yielding* in the gross section and *tensile rupture* in the net section. ¶

(a) For tensile yielding in the gross section: ¶

$$\begin{array}{ccccccc} T_n = F_y A_s & \rightarrow & & \rightarrow & \rightarrow & \rightarrow & \rightarrow \text{ (N9-D2-1) } \\ \phi_t = 0.90 \text{ (LRFD)} & \rightarrow & & \rightarrow & \rightarrow & \rightarrow & \rightarrow \Omega_t = 1.67 \text{ (ASD)} \end{array}$$

(b) For tensile rupture in the net section: ¶

$$\begin{array}{ccccccc} T_n = F_u A_{sn} & \rightarrow & & \rightarrow & \rightarrow & \rightarrow & \rightarrow \text{ (N9-D2-2) } \\ \phi_t = 0.75 \text{ (LRFD)} & \rightarrow & & \rightarrow & \rightarrow & \rightarrow & \rightarrow \Omega_t = 2.00 \text{ (ASD)} \end{array}$$

where, ¶

→  $A_s$  → = gross area of steel plates in panel section per unit width,  $\text{in}^2/\text{ft}$  ¶

→  $A_{sn}$  → = net area of steel plates in panel section per unit width,  $\text{in}^2/\text{ft}$  ¶


→  $F_y$  → = specified minimum yield stress of the steel faceplates,  $\text{ksi}$  ¶

→  $F_u$  → = specified minimum tensile strength of the steel faceplates,  $\text{ksi}$  ¶



## N9E: DESIGN OF SC WALLS FOR COMPRESSION

- **Commentary:**

- The SC walls are designed by calculating their axial compressive strength on a per foot basis. This calculation uses the clear length of the wall (vertical or horizontal, whichever is larger) and an  $(EI)'$  which is based on the  $(EI)_{\text{eff}}$  of composite CFT columns in the Specification Chapter I.
  - The equation for  $(EI)_{\text{eff}}$  for composite CFT columns has been simplified conservatively to  $E_s I_s + 0.65 E_c I_c$ .
  - The more accurate equation in Specification Chapter I, which is a function of the reinforcement ratio can also be use.
- 

## ■ **N9F → DESIGN OF SC WALLS FOR FLEXURE ¶**

### ■ **1. → GENERAL PROVISIONS ¶**

The design of SC walls for the required out-of-plane flexural demands ( $M_{rx}$  or  $M_{ry}$ ) shall be evaluated over the *interior panel sections* defined in Section N9C5. ¶

The tensile strength of the concrete shall be neglected in the determination of the *nominal flexural strength* of *interior panel sections*. The steel ribs shall have no contribution to the *nominal flexural strength* of SC panel sections. ¶

### ■ **2. → DESIGN FLEXURAL STRENGTH OF SC WALLS ¶**

The *design flexural strength* ( $\phi_b M_n$ ) per unit width and the allowable flexural strength ( $M_n/\Omega_b$ ) per unit width of *interior panel sections* shall be determined for the limit state of steel faceplate tension yielding as follows: ¶

$$\phi_b = 0.90 \text{ (LRFD)} \rightarrow \rightarrow \rightarrow \Omega_b = 1.67 \text{ (ASD)} ¶$$

$$\rightarrow \rightarrow \rightarrow \rightarrow M_n = F_y (A_{sw}/2) (0.90 T) \rightarrow \rightarrow \rightarrow \text{(N9-F2-1)} ¶$$

where, ¶

→  $A_{sw}$  → = net area of steel plates in panel section,  $\text{in}^2 / \text{ft}$  ¶

→  $T$  → = total thickness of SC wall panel section, in ¶

→  $F_y$  → = specified yield stress of face plate, ksi ¶



## N9G2.1 GENERAL PROVISIONS

- The design of SC walls for the required out-of-plane shear demands ( $V_{rx}$  and  $V_{ry}$ ) shall be evaluated over *interior panel sections* defined in Section N9C5.
- The out-of-plane shear reinforcement in the form of *integrity system* components, namely, structural steel shapes, frames or tie bars embedded in the concrete infill shall be classified as *yielding shear reinforcement* or *non-yielding shear reinforcement* in accordance with N9B6.1.
- The *nominal out-of-plane shear strength* per unit width ( $V_{no}$ ) for SC walls with yielding and non-yielding shear reinforcement shall be estimated using Section N9G2.2 and N9G2.3, respectively
- The *design out-of-plane shear strength* per unit width  $\Phi_{vo} V_{no}$ , or the allowable out-plane shear strength per unit width  $V_{no}/W_{vo}$ , of interior panel sections shall be calculated using:

$$\phi_{vo} = 0.75 \text{ (LRFD)}$$

$$\Omega_{vo} = 2.00 \text{ (ASD)}$$



## N9G2.1 GENERAL PROVISIONS

- The specifications requires classification of the shear reinforcement, which are the same as the integrity system components, as yielding or non-yielding type. Currently, both types of shear reinforcement are permitted
- The strength factors ( $\phi$  and  $\Omega$ ) for out-of-plane shear reflect the non-ductile nature of the failure mode





## N9G2.2 NOMINAL OUT-OF-PLANE SHEAR STRENGTH OF SC WALLS WITH YIELDING SHEAR REINFORCEMENT

### ○ N9G2.2.1 Spacing Less than or Equal to Half of Wall Thickness

- The *nominal out-of-plane shear strength* per unit width  $V_{no}$  shall be calculated as follows:

$$V_{no} = V_c + V_s \quad (\text{N9-G2-6})$$

where,

$$V_c = 0.0015 (f'_c)^{0.5} T_c \quad (\text{N9-G2-7})$$

$$V_s = F_t (T_c/S_L) (12/S_T) \quad (\text{N9-G2-8})$$

$T_c$  = concrete sandwich thickness =  $T - 2t_p$

$F_t$  = nominal uniaxial tensile strength of individual components of *integrity systems* (acting as shear reinforcement) determined as per N9B6.1

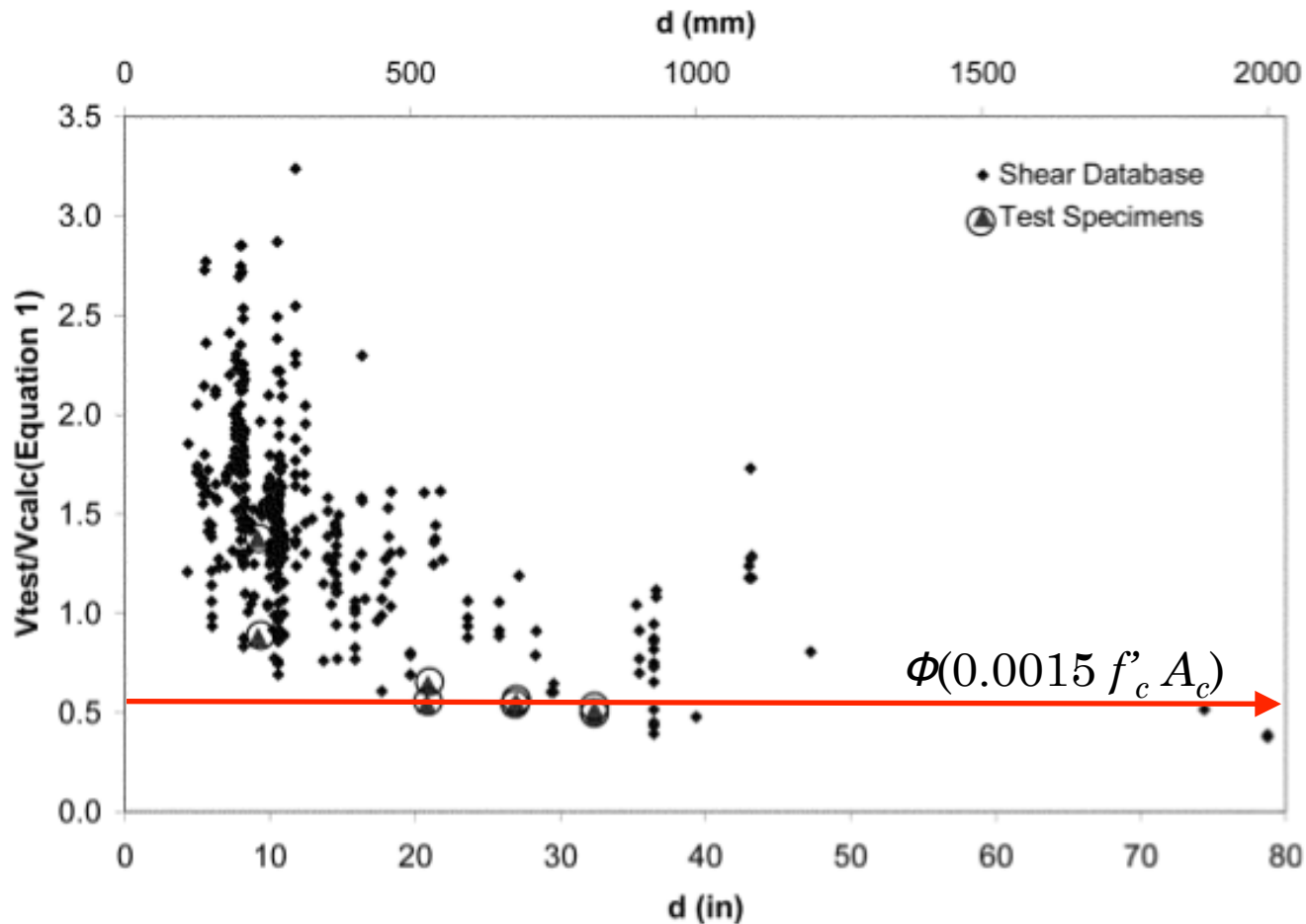
$f'_c$  = concrete compressive strength in psi

$S_L$  = spacing of shear reinforcement along the direction of one-way shear

$S_T$  = spacing of shear reinforcement transverse to the direction of one-way shear




## N9G2.2.1: $V_c$ CONTRIBUTION AS FUNCTION OF SIZE



*Fig. 9—Ratio of  $V_{test}$  to  $V_{calc}$  per Eq. (1) (ACI 318-08)*

Sneed, L.H., and Ramirez, J. (2010). "Influence of Effective Depth on Shear Strength of Concrete Beams – Experimental Study." *ACI Structural Journal*, Vo. 107, No. 5, Sept-Oct.2010

## N9G2.2. $V_s$ CONTRIBUTION

- The out-of-plane shear behavior and strength of SC walls is similar to that of RC walls with some differences associated with crack spacing, width, etc. due to the more discrete nature of the bond (via shear connectors) in SC walls
  - The  $V_s$  contribution is based on the well-known mechanism of shear or flexure-shear crack passing through several yielding type shear reinforcement, and engaging them in axial tension
  - The classification of the shear reinforcement (or integrity system component) as yielding and the determination of its design axial tension strength are important for this calculation
- 

## N9G2.2 YIELDING SHEAR REINFORCEMENT

- Shear Reinforcement with Spacing Greater than Half of Wall Thickness
  - The *nominal out-plane shear strength* per unit width  $V_{no}$  shall be the larger of the following:
    - (1)  $V_{no} = V_c = 0.0015 (f'_c)^{0.5} T_c 12$  (N9-G2-4)
    - (2)  $V_{no} = V_s = F_t (12/S_T)$  (N9-G2-5)

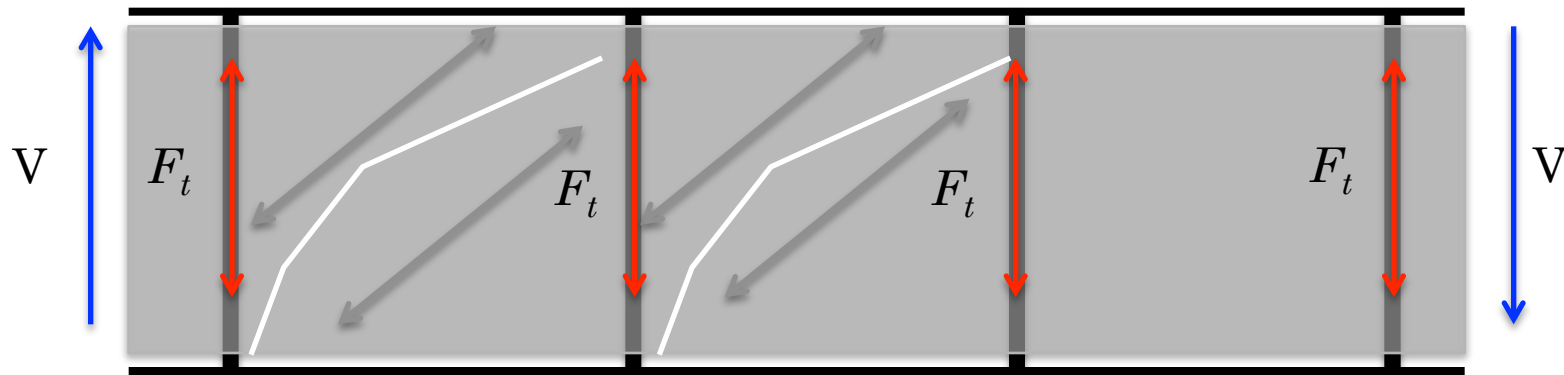


## YIELDING SHEAR REINFORCEMENT: SPACING IS GREATER THICKNESS / 2

- When the spacing of the shear reinforcement is greater than the section thickness divided by two, the maximum out-of-plane shear strength is limited to the larger of: (i) the concrete shear strength contribution, or (ii) the steel contribution alone.
- This is based on the ability of the SC beam to develop an internal truss mechanism for equilibrium. The strength of this truss mechanism is limited to that of the tie (shear reinforcement).
- The concrete and steel contributions cannot be added for shear reinforcement spacing greater than thickness divided by two because the shear or flexural-shear crack may not pass through more than one shear reinforcement



# YIELDING SHEAR REINFORCEMENT: SPACING IS GREATER THICKNESS / 2



## N9G2.3 NOMINAL OUT-OF-PLANE SHEAR STRENGTH OF SC WALL PANELS WITH NON-YIELDING SHEAR REINFORCEMENT

- For *non-yielding shear reinforcement*, the *nominal out-of-plane shear strength* shall be established by: (i) conducting project-specific large-scale out-of-plane shear tests, or (ii) using applicable test results available in published literature.
- In the absence of such test results, the following shall be used to provide conservative estimates:
- The *nominal out-of-plane shear strength* per unit width  $V_{no}$  shall be calculated as follows:

$$V_{no} = V_c + V_s \quad (\text{N9-G2-6})$$

- where,

- $V_c = 0.0015 (f'_c)^{0.5} T_c 12 \quad (\text{N9-G2-7})$

- $V_s = F_t (0.5 T_c / S_L) (12 / S_T) \quad (\text{N9-G2-8})$



## NON-YIELDING SHEAR REINFORCEMENT SPACING LESS THAN THICKNESS / 2

- For non-yielding type shear reinforcement, the steel contribution to the out-of-plane shear strength is reduced to half of that for yielding type shear reinforcement for the following reasons.
- When the concrete shear or flexure shear cracks form, they will activate all the individual shear reinforcements that it will pass through.
- However, it is unclear whether these individual shear reinforcements will be able to develop their individual axial design strengths before one of them (the one with largest axial force) fails in a non-ductile manner.
- While it is feasible that the concrete shear or flexure-shear crack will activate all the individual shear reinforcements that it will pass through, it is unclear whether

## NON YIELDING SHEAR REINFORCEMENT SPACING GREATER THAN THICKNESS / 2

- Requirements are same as that for yielding shear reinforcement with spacing greater than thickness / 2. The reasoning is also the same.





# N9G1 DESIGN OF SC WALLS FOR IN-PLANE SHEAR

- N9G1.1 General Provisions
  - The design of SC walls for the required in-plane membrane shear demands ( $S_{rx}$ ) shall be evaluated over the *interior panel sections* defined in Section N9C5.
  - The steel ribs shall have no contribution to the *design in-plane shear strength* of SC walls.
- The *design in-plane strength* ( $\phi_{vi} V_{ni}$ ) per unit width and the *allowable in-plane shear strength* ( $V_{ni} / \Omega_{vi}$ ) per unit width of *interior panel sections* shall be determined for the limit state of steel faceplate tension yielding as follows:



# N9G1 DESIGN OF SC WALLS FOR IN-PLANE SHEAR

- $V_{ni} = \kappa F_y A_{sn}$  (N9-G1-1)

- where,

$$\kappa = 1.11 - 5.16 \underline{\rho}$$

$\underline{\rho}$  = Strength-adjusted reinforcement ratio

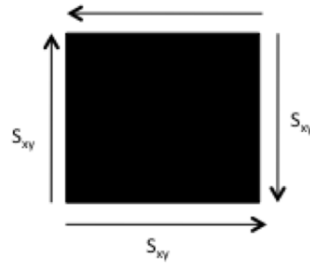
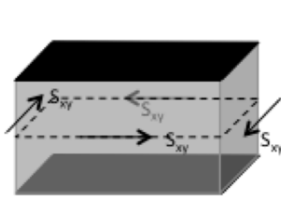
$F_y$  = yield strength of steel faceplates in ksi

$A_{sn}$  = net area of steel faceplates, in<sup>2</sup>/ft



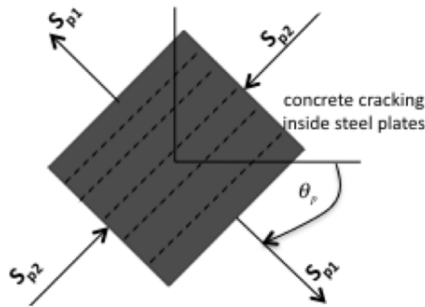
# IN-PLANE SHEAR BEHAVIOR

(a) Stress block diagrams



(a) SC Composite Section Subjected to In-Plane Shear

(b) Plan View of SC Composite Section

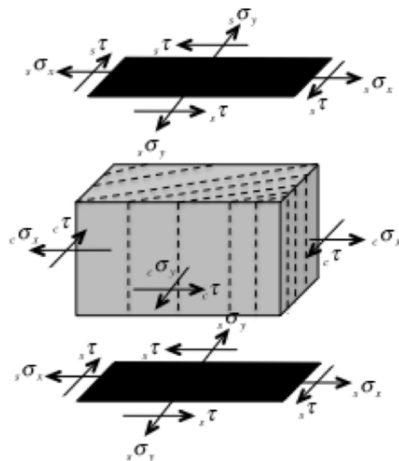


$$\tan(2\theta_p) = \frac{2S_{xy}}{S_x - S_y} = \infty$$

$$\therefore \theta_p = 45^\circ$$

$$S_{p1,2} = \pm S_{xy}$$

(c) Principal Forces in SC Composite Section



(d) Steel and concrete stresses due to in-plane shear

(b) Equations

Force Equilibrium

$$\begin{Bmatrix} 0 \\ 0 \\ S_{xy} \end{Bmatrix} = \begin{Bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{Bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix} + \frac{E_c T_c}{4} \begin{Bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{Bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$S_x = S_y = \frac{S_{xy}}{2t_s} \frac{K_\beta}{K_\alpha + K_\beta}$$

Steel normal stresses

$$\tau = \frac{S_{xy}}{2t_s} \frac{K_\alpha}{K_\alpha + K_\beta}$$

Steel shear stress

$$S_x = S_y = -\tau = \frac{S_{xy}}{T_c} \frac{-K_\beta}{K_\alpha + K_\beta}$$

Concrete normal and shear stresses

$$K_\alpha = G_s 2t_s \quad K_\beta = \frac{1}{\frac{4}{E_c T_c} + \frac{2(1-\nu)}{E_s 2t_s}}$$

Steel and concrete stiffness contributions

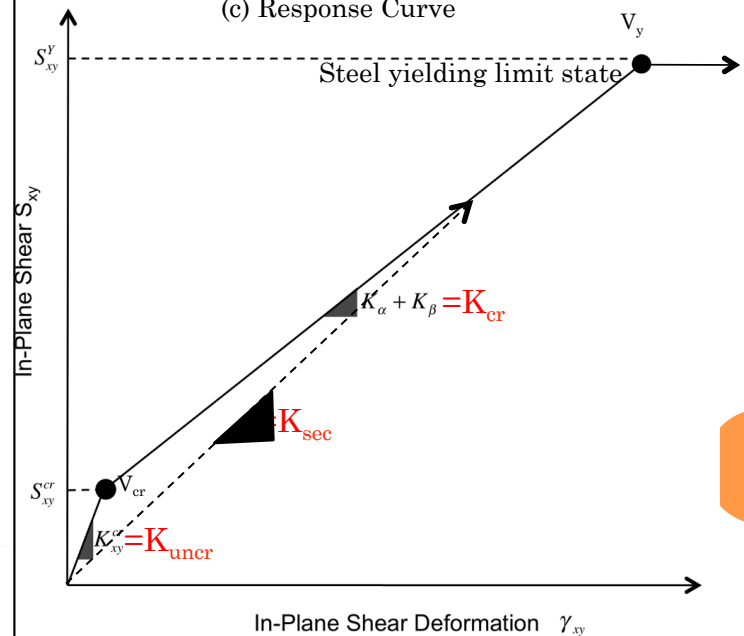
$$S_{xy} = (K_\alpha + K_\beta) \gamma_{xy}$$

Post-cracking shear stiffness

$$S_{xy}^y = \frac{K_\alpha + K_\beta}{\sqrt{3K_\alpha^2 + K_\beta^2}} (2t_s F_y)$$

In-plane shear strength (steel yielding limit state)

(c) Response Curve



# IN-PLANE SHEAR BEHAVIOR

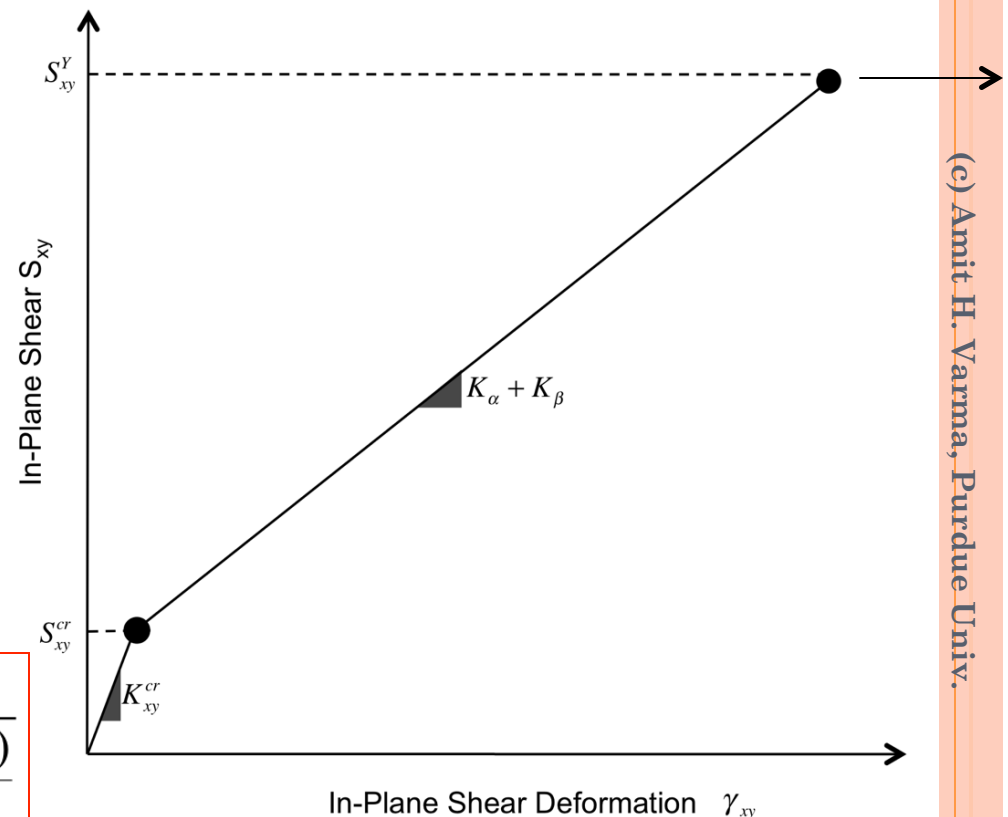
- Use the stresses to calculate von-Mises stress and establish the yielding of the steel plate.
- The yield load corresponds to:

$$S_{xy}^Y = \frac{K_\alpha + K_\beta}{\sqrt{3K_\alpha^2 + K_\beta^2}} (2t_s F_y)$$

- The shear stiffness post-cracking corresponds to;

$$S_{xy} = (K_\alpha + K_\beta) \gamma_{xy}$$

$$K_\alpha = G_s 2t_s \quad K_\beta = \frac{1}{\frac{4}{E'_c T_c} + \frac{2(1-\nu)}{E_s 2t_s}}$$



# IN-PLANE SHEAR BEHAVIOR

- The in-plane shear strength can be expressed as:

$$V_n = S_{xy}^Y = \frac{K_\alpha + K_\beta}{\sqrt{3K_\alpha^2 + K_\beta^2}} (2t_s F_y) = \beta A_s F_y$$

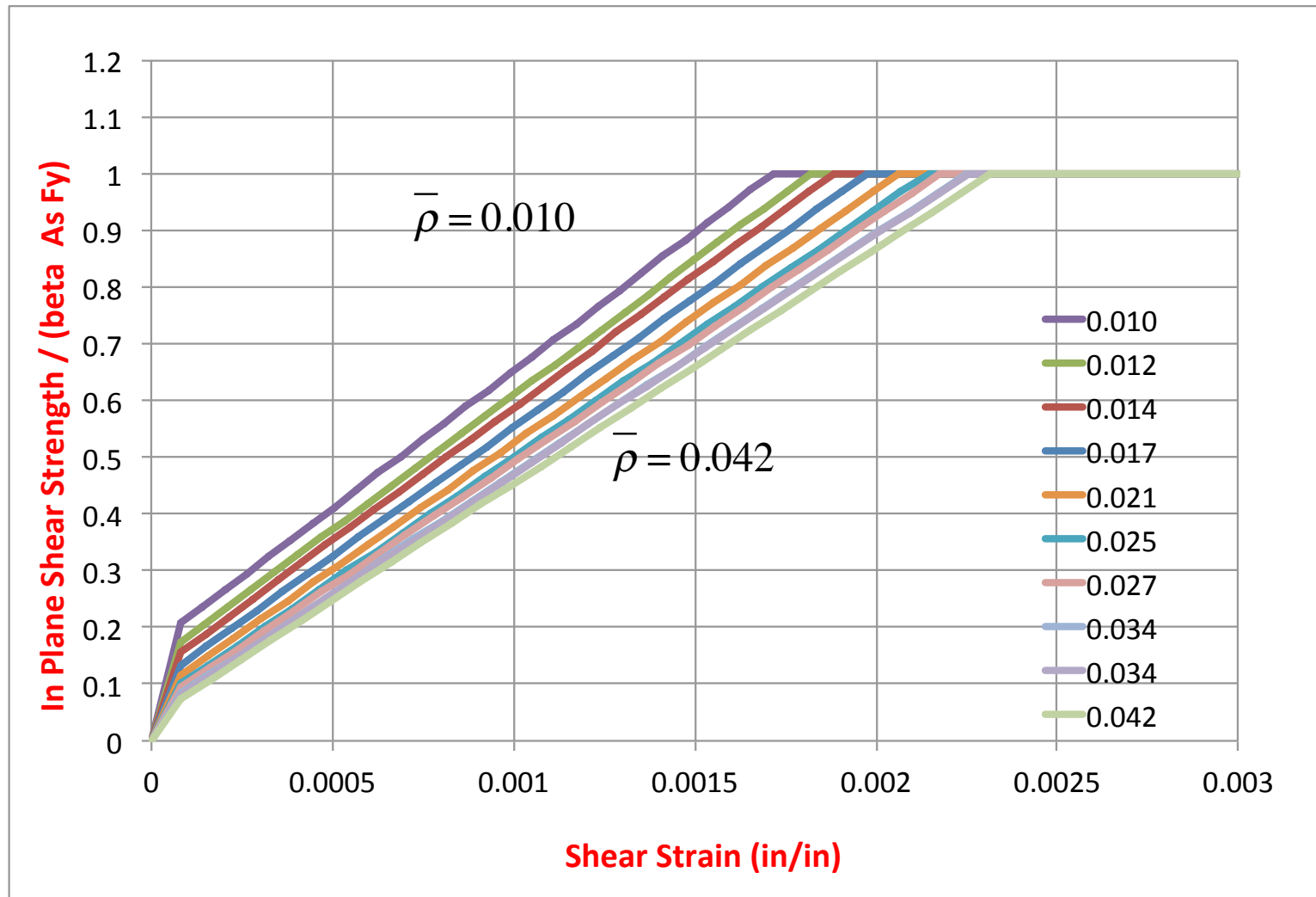
- Where,  $\beta = 1.11 - 5.16 \bar{\rho}$
- And the new parameter rho-bar is as follows:

$$\bar{\rho} = \frac{A_s F_y}{\sqrt{f'_c} A_c}$$

- The parameter rho-bar is non-dimensional, where  $F_y$  is in ksi and  $f'_c$  is in psi.
- The in-plane shear behavior is a function of rho-bar, the values of which are between 0.01 and 0.04 for nuclear structures

# IN-PLANE SHEAR BEHAVIOR OF SC WALLS

- Shear force-shear strain behavior for different values of  $\bar{\rho}$ . Force normalized w.r.t.  $V_n$





# **N9H DESIGN OF SC WALL FOR COMBINED FORCES**

# DESIGN OF SC WALL FOR COMBINED FORCES

- N9H1 General Provisions
  - The results from elastic finite element analysis conducted according to Sections N9C1, N9C2, N9C3, and N9C4 shall be used to determine three required in-plane membrane forces ( $S_{rx}$ ,  $S_{ry}$ ,  $S_{rxy}$ ), three out-of-plane moments ( $M_{rx}$ ,  $M_{ry}$ ,  $M_{rxy}$ ), and two out-of-plane shear demands ( $V_{rx}$  and  $V_{ry}$ ) in accordance with Section N9C5.





## N9H2 INTERACTION EQUATIONS FOR OUT-OF-PLANE SHEAR FORCES

- The design adequacy of the *interior panel sections* subjected to the two out of plane shear demands ( $V_{rx}$  and  $V_{ry}$ ) shall be evaluated as follows:

(1) If  $V_{rx}$  and  $V_{ry}$  are both greater than  $\phi_{vo} V_c$ , and the shear reinforcement spacing is less than or equal to section thickness divided by two

$$\frac{V_{rx} - \phi_{vo} V_c}{\phi_{vo} (V_{no}^x - V_c)} + \frac{V_{ry} - \phi_{vo} V_c}{\phi_{vo} (V_{no}^y - V_c)} \leq 1.0$$

where,  $\phi_{vo} V_{no}^x$  and  $\phi_{vo} V_{no}^y$  are the *design out-of-plane shear strengths* in the x and y directions estimated using Section N9G2.2.1.



## N9H2 INTERACTION EQUATIONS FOR OUT-OF-PLANE SHEAR FORCES

(2) If spacing is greater than or equal to section thickness divided by two, and the shear strength ( $\phi_{vo} V_{no}$ ) is governed by the steel contribution alone ( $\phi_{vo} V_s$ ):

$$\frac{V_{rx}}{\phi_{vo} V_{no}^x} + \frac{V_{ry}}{\phi_{vo} V_{no}^y} \leq 1.0$$

- **User Note:** These interaction equations are for the steel shear reinforcement that will have to resist both the out-of-plane shear demands ( $V_{rx}$  and  $V_{ry}$ ) when they are greater than  $\phi_{vo} V_c$



## COMMENTARY N9H2

- The out-of-plane shear forces  $V_{rx}$  and  $V_{ry}$  both rely on using the same steel shear reinforcement for their steel contributions ( $V_s$ ).
- The steel shear reinforcement is subjected to axial tension after the concrete cracks and its contribution ( $V_c$ ) is exceeded.
- Both  $V_{rx}$  and  $V_{ry}$  subject the steel shear reinforcement to axial tension demand after they exceed their respective concrete contributions  $V_c$ .
- Therefore, a linear interaction is assumed, and the steel shear reinforcement is checked to ensure that it is not overstressed (yielded) by the combinations of demands.

## COMMENTARY N9H2

- In the linear interaction equations, the numerators are the portion of the demands greater than the corresponding concrete contributions  $V_c$ . The denominators are the contributions of the steel shear reinforcements  $V_s$



## N9H2 COMMENTARY

- Currently, there is no interaction between axial tension or compression and the concrete contribution to the shear strength  $V_c$ .
- This is different from the equations in ACI 349, however, it is well known and understood in the concrete research community
- Proprietary experiments have confirmed that in the presence of axial tension, concrete cracks perpendicular to the steel plates and through the section thickness. The  $V_c$  contribution of this concrete (cracked due to axial tension) is the similar to that of uncracked concrete, because the shear and shear flexural cracks still have to form (or grow out from) the through section cracks.

# N9H3 DESIGN FOR COMBINED IN-PLANE AND OUT-OF-PLANE DEMANDS

- Please see draft spec for extensive description of the spec.
- Excerpts on next 3 slides



# INTERACTION EQUATIONS

$S'_{rx}$ ,  $S'_{ry}$ , and  $S'_{rxy}$  are the required in-plane membrane strengths per unit width calculated for each *notional half of the SC section* using Equations N9-H1-6 to N9-H1-8.

$$S'_{rx} = \frac{S_{rx}}{2} \pm \frac{M_{rx}}{\beta_x T} \quad (\text{N9-H1-6})$$

$$S'_{ry} = \frac{S_{ry}}{2} \pm \frac{M_{ry}}{\beta_y T} \quad (\text{N9-H1-7})$$

$$S'_{rxy} = \frac{S_{rxy}}{2} \pm \frac{M_{rxy}}{\beta_{xy} T} \quad (\text{N9-H1-8})$$

$S'_{rx}$  = *required membrane axial strength* in direction x for each notional half of SC section, kips/ft

$S'_{ry}$  = *required membrane axial strength* in direction y for each notional half of SC section, kips/ft

$S'_{rxy}$  = *required membrane in-plane shear strength* for each notional half of SC section, kips/ft

$\beta_i$  = parameter for distributing required flexural strengths ( $M_{rx}$ ,  $M_{ry}$ ,  $M_{rxy}$ ) into the corresponding membrane force couples acting on each notional half of SC section

$\beta_x$  = 0.9 if  $S_{rx} > -0.6P_{co}$       and       $\beta_x = 0.67$  if  $S_{rx} < -0.6P_{co}$

$\beta_y$  = 0.9 if  $S_{ry} > -0.6P_{co}$       and       $\beta_y = 0.67$  if  $S_{ry} < -0.6P_{co}$

$\beta_{xy}$  = 0.67

# INTERACTION EQUATIONS

$$S_{rp1,2} = \frac{S'_{rx} + S'_{ry}}{2} \pm \sqrt{\left(\frac{S'_{rx} - S'_{ry}}{2}\right)^2 + (S'_{rxy})^2} \quad (\text{N9-H1-5})$$

Range of $S_{\max}$ and $S_{\min}$	Interaction Equation	Equation Number
$S_{r,\max} \geq 0$ and $S_{r,\min} \geq 0$	$\frac{S_{r,\max}}{T_{ci}} \leq 1.0$	N9-H1-1
$S_{r,\min} < 0$ and $S_{r,\max} + S_{r,\min} \geq 0$	$\frac{S_{r,\max} + S_{r,\min}}{T_{ci}} - \frac{S_{r,\min}}{V_{ci}} \leq 1.0$	N9-H1-2
$S_{r,\max} > 0$ and $S_{r,\max} + S_{r,\min} < 0$	$\frac{S_{r,\max}}{V_{ci}} - \frac{S_{r,\max} + S_{r,\min}}{P_{ci}} \leq 1.0$	N9-H1-3
$S_{r,\max} \leq 0$ and $S_{r,\min} \leq 0$	$\frac{-S_{r,\min}}{P_{ci}} \leq 1.0$	N9-H1-4

Where,  $S_{r,\max}$  and  $S_{r,\min}$  are the larger and smaller of the two *required principal in-plane membrane strengths* ( $S_{rp1,2}$ ) per unit width for each *notional half of the SC section* calculated using Equation N9-H1-5.



# INTERACTION EQUATIONS

## **Additionally, In Equations N9-H1-1 to N9-H1-4:**

$T_{ci}$  = *available tensile strength* per unit width for each notional half of SC section, kips/ft

$V_{ci}$  = *available in-plane shear strength* per unit width for each notional half of SC section, kips/ft

$P_{ci}$  = *available compressive strength* per unit width for each notional half of SC section, kips/ft

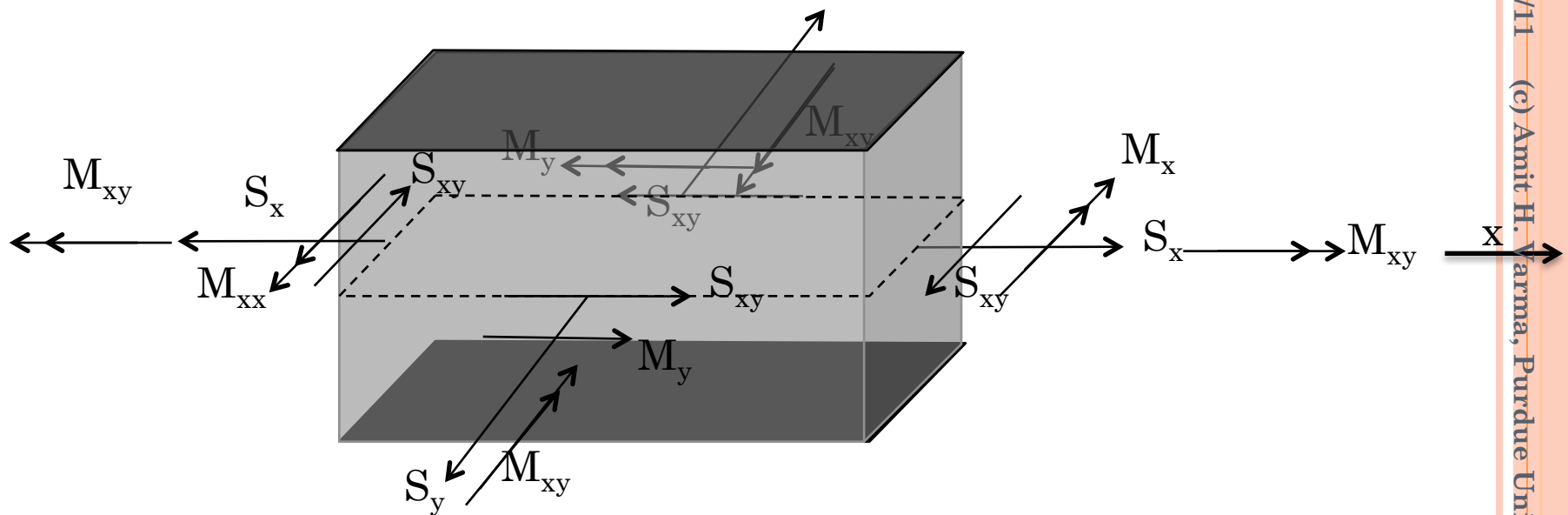
## **For design according to N9-B.3 (LRFD)**

$T_{ci} = \phi_{ti} T_{ni}/2$ , where  $T_{ni}$  is calculated using the nominal tensile strength according to Section N9-D and  $\phi_{ti} = 0.95$

$V_{ci} = \phi_{vs} V_{ni}/2$ , where  $V_{ni}$  is calculated using the nominal in-plane shear strength according to Section N9-G and  $\phi_{ti} = 0.95$

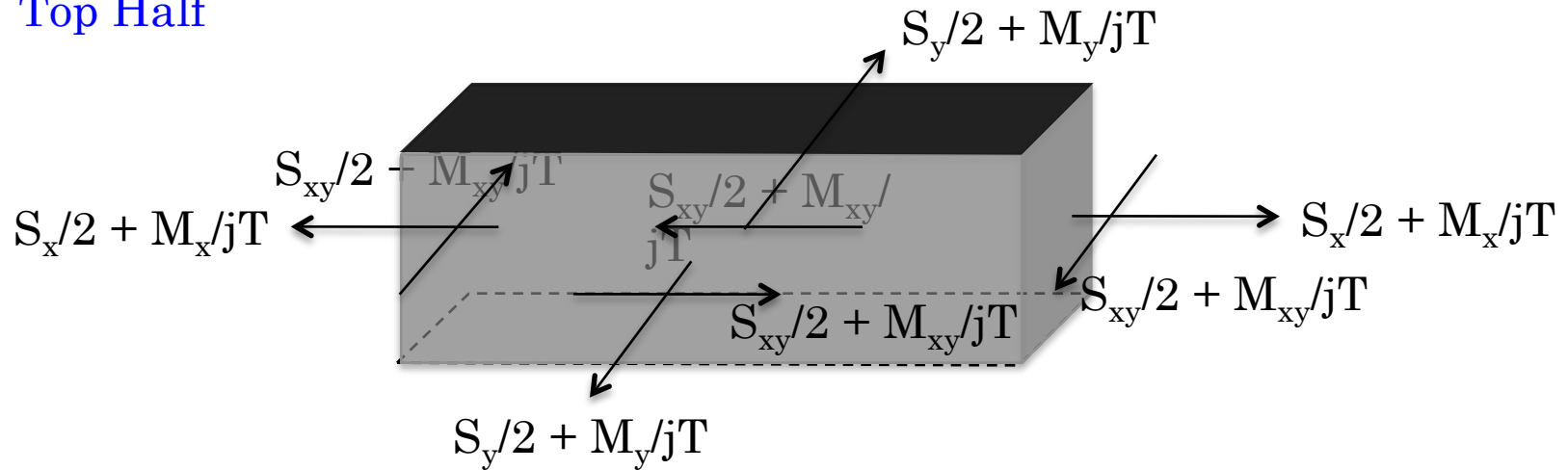
$P_{ci} = \phi_{ci} P_{no}/2$ , where  $P_{no}$  is calculated using the nominal section compressive strength according to N9-E and  $\phi_{ci} = 0.80$

# COMMENTARY ON INTERACTION EQUATIONS & APPROACH

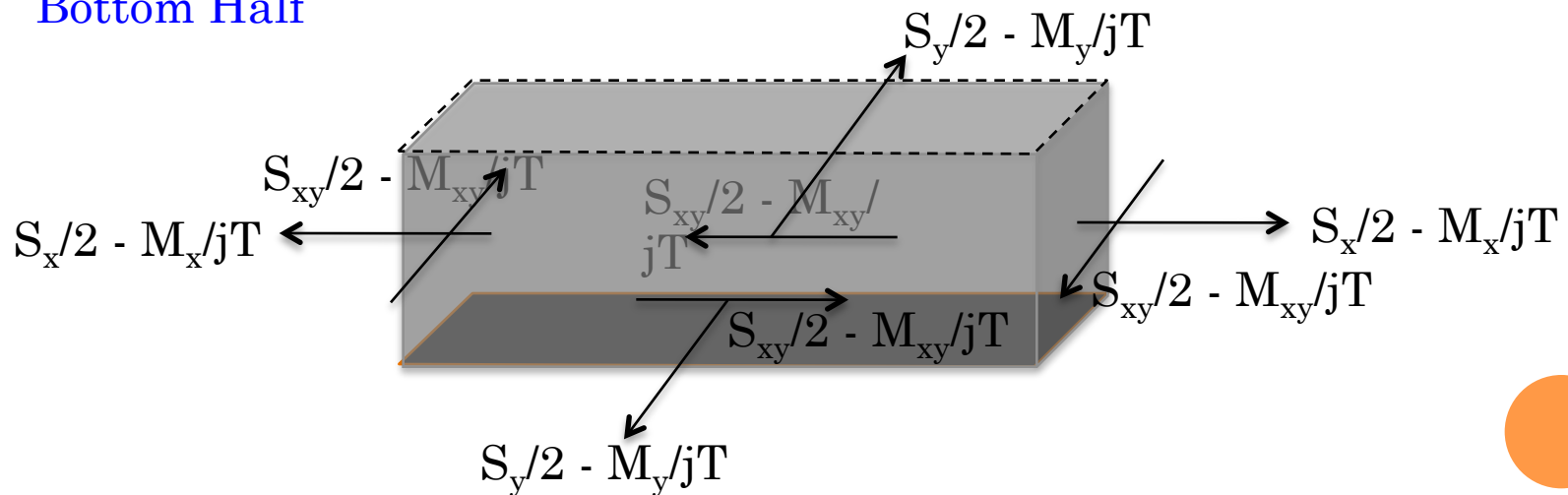


# TWO NOTIONAL HALVES

## Top Half

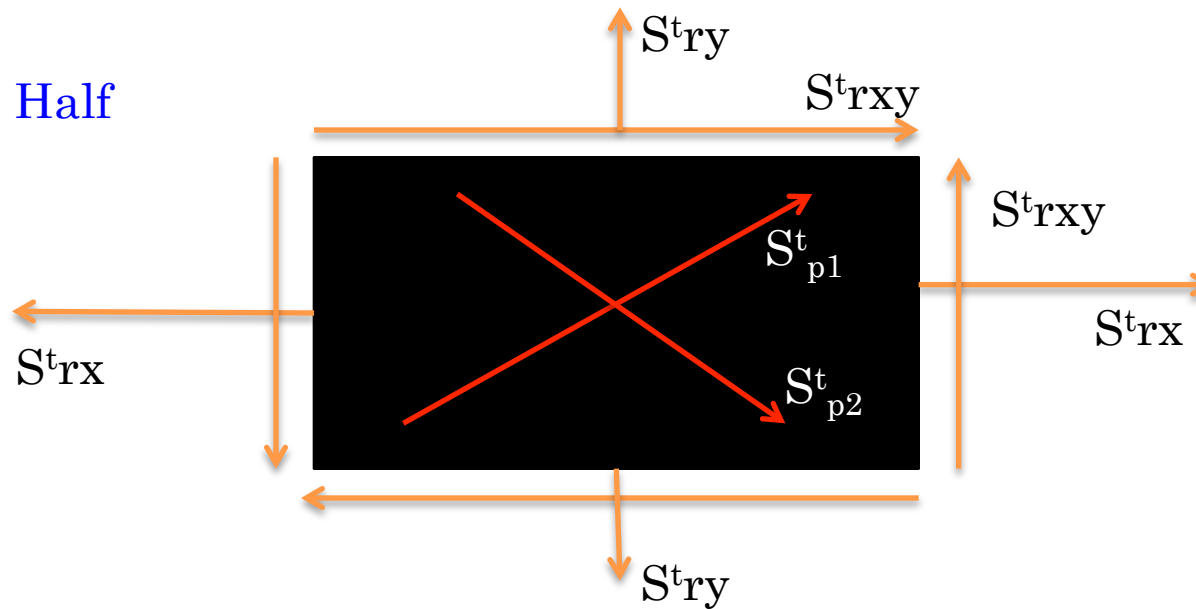


## Bottom Half

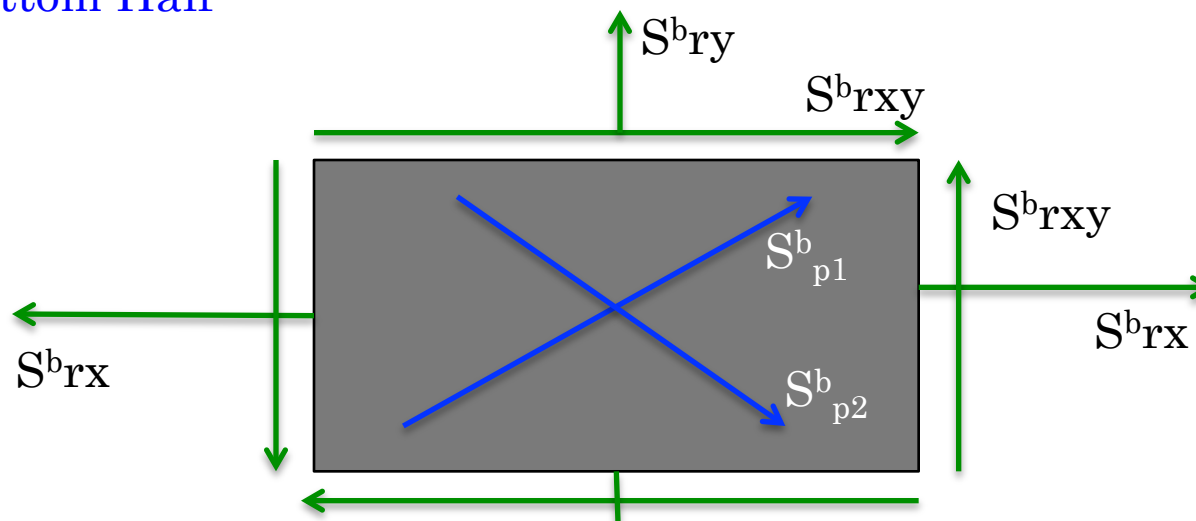


# TWO NOTIONAL HALVES (CHECK ON EACH)

Top Half

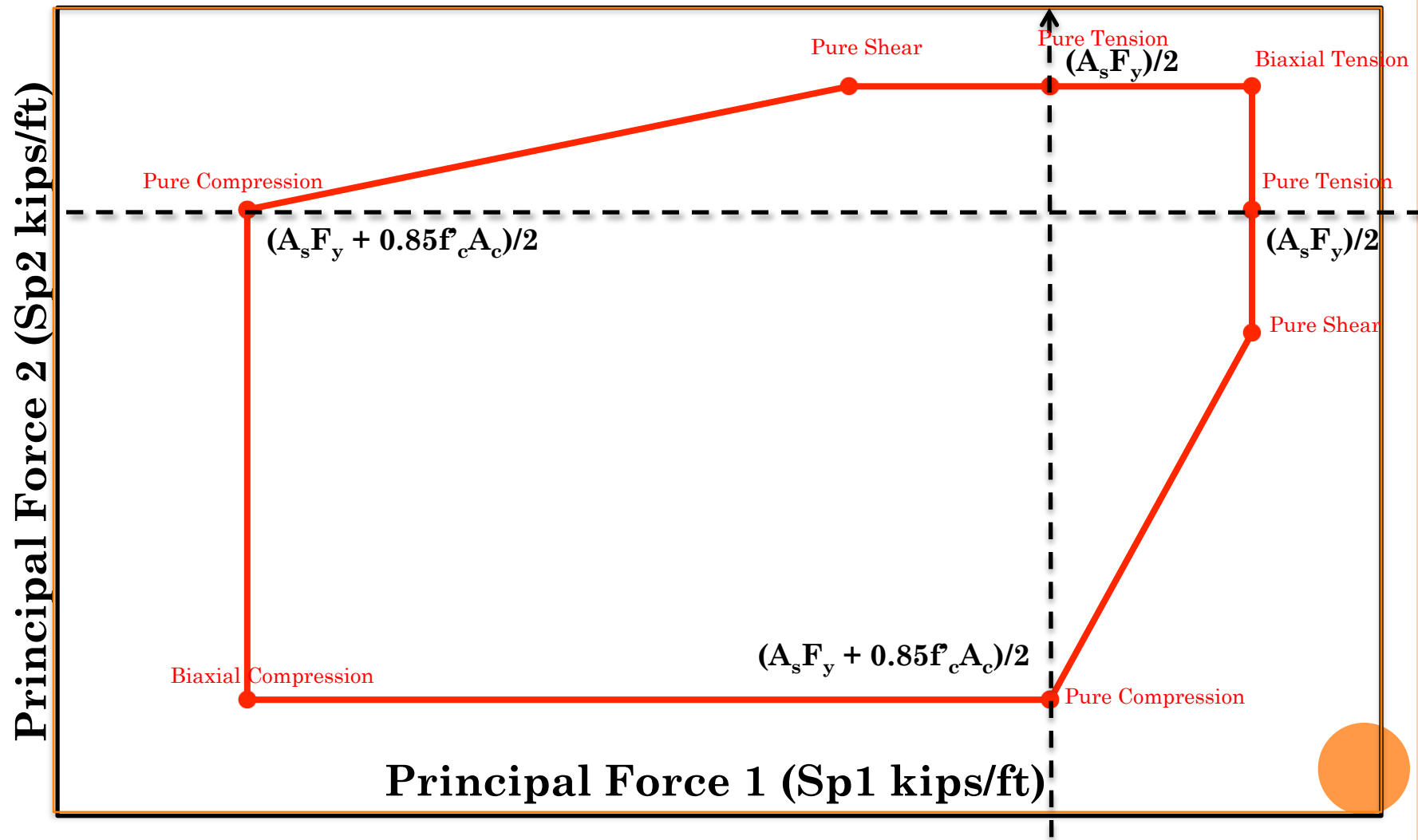


Bottom Half



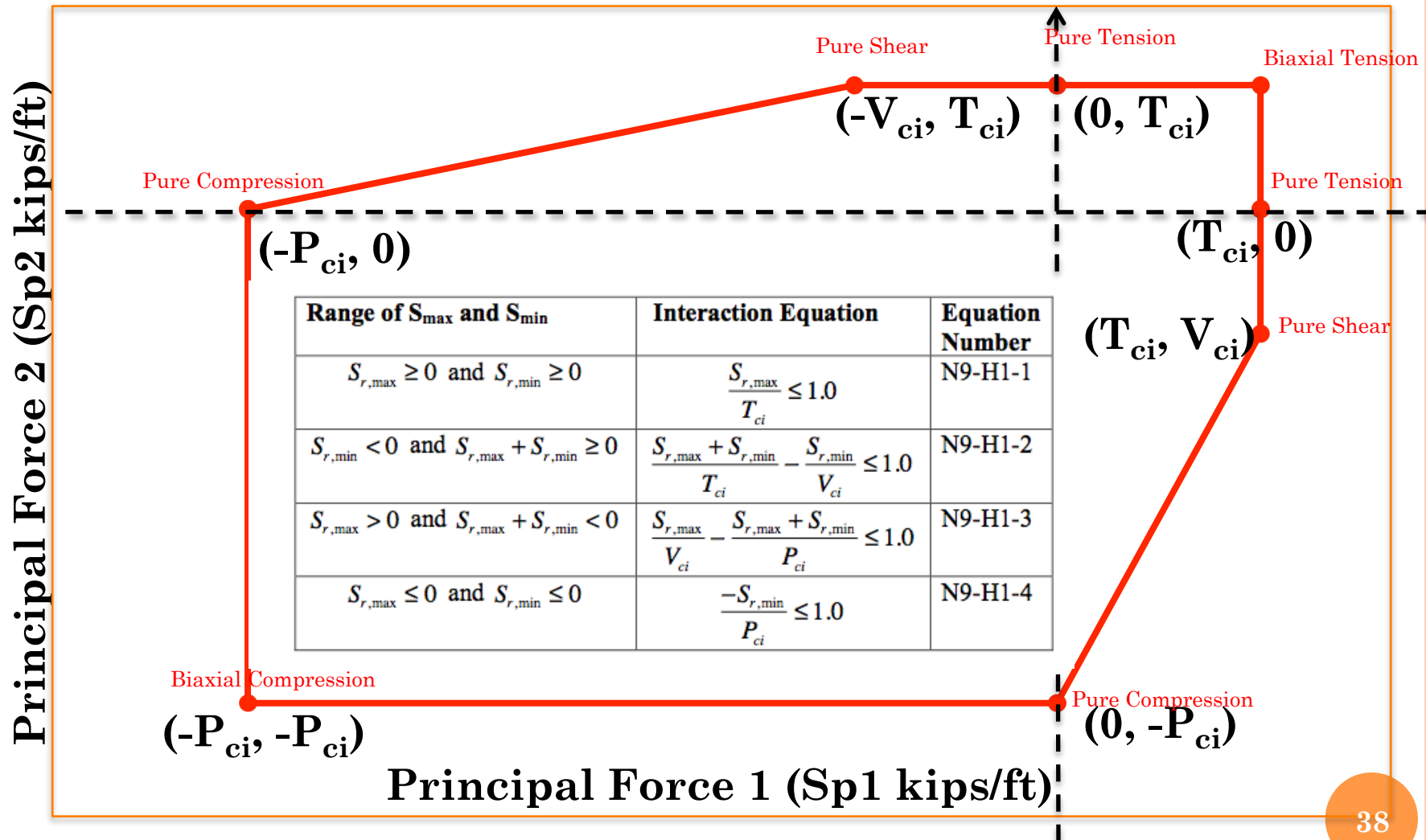
# INTERACTION SURFACE FOR EACH NOTIONAL HALF

## Failure Surface for In-Plane Forces in Principal Forces



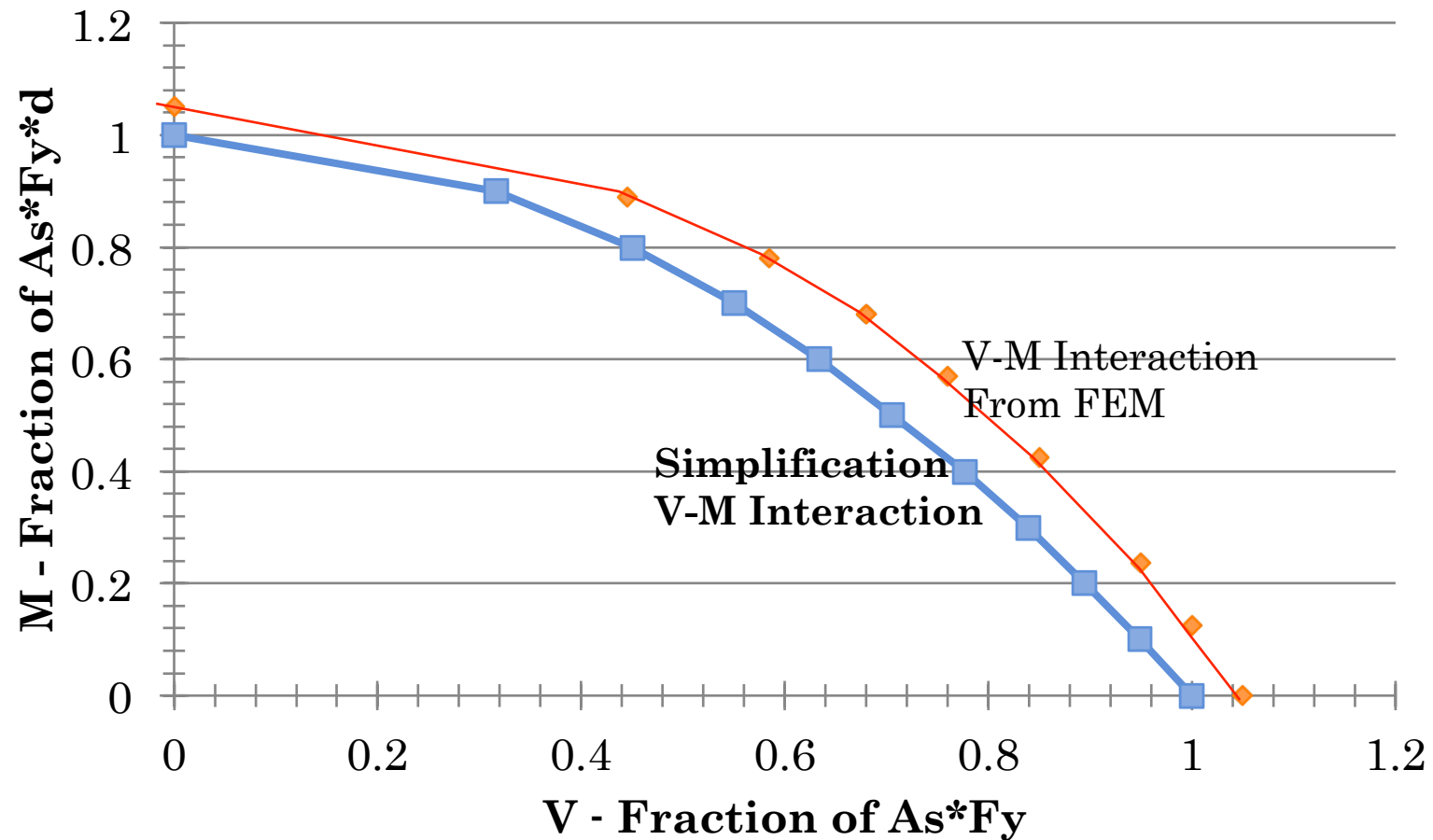
# INTERACTION SURFACE FOR EACH NOTIONAL HALF

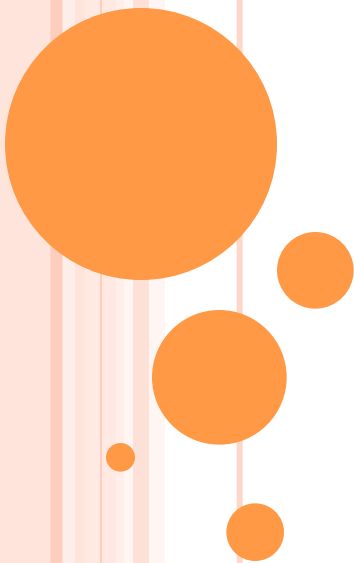
## Failure Surface for In-Plane Forces in Principal Forces



# EXAMPLE VERIFICATION USING FINITE ELEMENT MODELS

## M-V INTERACTION





# N9J CONNECTION DESIGN FOR SC WALLS



# INTRODUCTION

- The following connection types are possible: SC wall to SC or RC wall, SC wall to basemat, SC wall to SC or RC slab; splices between co-planar SC and RC walls are also possible
- Unlike pure steel or RC structures, jointability requires careful consideration in SC and composite structures
- Bolting and welding is used as connection elements in steel structures; column anchorages involve baseplates, anchor rods and shear lugs... Well established rules and methods exist for sizing these connections
- Embedded rebars (dowels), shear-friction rebar, use of joint ties, etc, are used as connection elements across RC to RC joints (often construction joints)... Again,

# UNIQUE ISSUES INVOLVING SC CONNECTIONS

- More complicated than connections involving linear composite members as multiple types of demands exist on plate/shell type SC elements
- Unlike RC walls, SC walls have very high in-plane shear strength; use of shear friction reinforcement alone may not be sufficient to match the SC wall strength
- Various types of connecting elements may be brought to bear to resist various demands; however, often the same type of connecting element may resist different types of demands simultaneously
- Unlike RC member connections, it is not easy to “embed” the “rebar” in SC because it is in the form of continuous faceplates



# UNIQUE ISSUES INVOLVING SC CONNECTIONS

- Beyond SSE performance needs to be considered, especially if the connection involves brittle failure mode or if design needs to satisfy a “Review Level Earthquake”
- It is possible that the connection will need to be designed as weaker than the connected elements (particularly for in-plane shear)... Adequate inelastic deformation capacity will need to be specified
- Interaction due to various types of demands will need to be accounted for, preferably on a small element basis (say two times the SC element thickness) rather than considering the entire SC wall (or SC slab) as one unit



## DRAFT SC CONNECTION PROVISIONS IN AISC SPEC

- Distinguishes between “design demands” and “required strengths” for connections

### N9J3 CONNECTION DESIGN DEMAND

- For all loading combinations in Section N9B2, the *connection design demands* shall be calculated from *elastic finite element analysis* conducted according to Sections N9C1, N9C2, N9C3, and N9C4, and shall be averaged over the *connection regions* defined in Section N9B3.
- These *connection design demands* consist of membrane axial force, in-plane shear force, out-of-plane moment, and out-of-plane shear force.

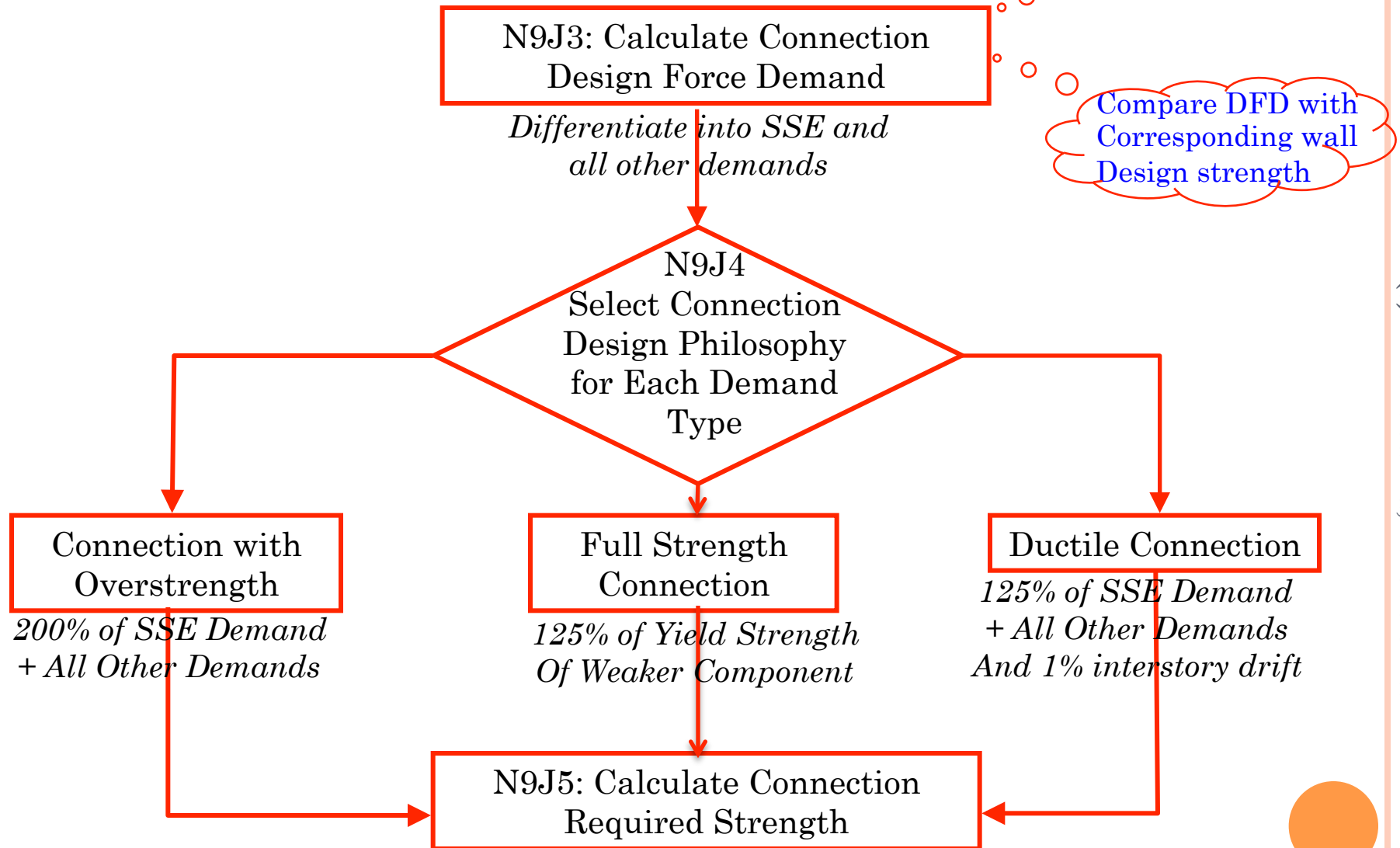
# N9J4 CONNECTION DESIGN PHILOSOPHY

- Each *connection design demand* type shall be differentiated into two parts: (i) those due to seismic loads and, (ii) those due to all other non-seismic loads (i.e., static, thermal, and dynamic loads). For each demand type, a *connection design philosophy* shall be selected from those outlined below.
- *Full strength connection*: Develop the full strength of the weaker of the connected parts.
- *Connection with Overstrength*: Develop significant overstrength with respect to the design demands, while non-ductile failure modes may govern connection strength.
- *Ductile connection*: Develop nominal overstrength with respect to the design demands, while assuring that ductile failure modes govern the connection strength, and the wall is capable of achieving 1% interstory drift capacity.

# N9J5 CONNECTION REQUIRED STRENGTHS

- The *connection required strengths* for each demand type shall be as follows:
  1. For *full strength connections*, the *connection required strength* shall be 125% of the smaller of the corresponding nominal capacities of the connected parts.
  2. For connections with overstrength, the connection required strength shall be 200% of the connection design demand due to seismic loads plus 100% of the connection design demand due to non-seismic loads.
  3. For *ductile connections*, the *connection required strength* shall be 125% of the *connection design demand* due to seismic loads plus 100% of the *connection design demand* due to non-seismic loads.

## Calculation of Connection Required Strength



## N9J6 CONNECTION AVAILABLE STRENGTH

- The *connection available strength* for each demand type shall be calculated using the applicable *force transfer mechanism* identified in accordance with Section N9J7 and the *available strength* of its contributing *connectors* calculated in accordance with Section N9J8.





## N9J7 CONNECTION FORCE TRANSFER MECHANISM

- For each particular demand type, a clearly identifiable *force transfer mechanism* shall be identified and provided. Each *force transfer mechanism* shall involve *connectors* of the same type in the entire *connection region*. If more than one *force transfer mechanisms* are possible for resisting a particular demand type, then the one that provides the largest *connection available strength* shall be the governing *force transfer mechanism*.
- *Connectors* shall consist of steel-headed stud anchors, anchor rods, tie bars, reinforcing bars and dowels, post-tensioning bars, shear lugs, embedded steel shapes, welds and bolts, rebar mechanical couplers, and direct bearing in compression.
- Direct bond transfer between the steel plate and concrete shall not be considered as a valid *connector* or *force transfer mechanism*.



# N9J8 DESIGN OF CONNECTORS

- *Connectors* shall be designed such that their *available strength* exceeds their *total required strength* as calculated below:
- N9J8.1 REQUIRED STRENGTH FOR CONNECTORS
- The *force transfer mechanism* for each demand type shall be used to compute the *required strength* for its contributing *connectors*.
- The *total required strength* for each *connector* shall be computed as the superposition of *required strengths* from all demand types.



## N9J8.2 AVAILABLE STRENGTH OF CONNECTORS

- The available strength for connectors shall be determined as follows:
  1. For steel headed stud anchors, the available strength shall be determined in accordance with *Specification* Section I.8.
  2. For welds and bolts, the available strength shall be determined in accordance with *Specification* Section J.
  3. For compression transfer via direct bearing in concrete, the available strength shall be determined in accordance with *Specification* Section I.6.
  4. For shear friction load transfer mechanism, the available strength shall be determined in accordance with ACI 349.
  5. For embedded shear lugs and shapes, the available strength shall be determined in accordance with ACI 349.
  6. For anchor rods, the available strength shall be determined from ACI 349.
  7. For post-tensioning bars, the available strength shall be determined from the PCI Design Handbook.

## Calculation of Connection Available Strength

N9J7 Provide Connection Force Transfer Mechanism

For each demand type, provide a clear FTM using the same connector type in the entire connection region

If more than one FTM are possible, use the one with the largest available strength as the governing FTM

N9J8.1 Compute Required Strengths for Connectors of FTM

For each demand type, use FTM and connection required strengths to...

Combine connector required strengths for all demands

N9J8.2 Determine Connector Available Strength from References

For each demand type

N9J6 Connection Available Strength

Use corresponding FTM and connector strength

## N9J9 CONNECTION QUALIFICATION

- The *connection available strength* for each individual demand type and for combinations of various demand types shall be reviewed by a *independent peer-review panel*.
- **User Note:** Peer review is required because this Specification does not provide: (1) prescriptive guidelines for detailing the connection region (for example, clear distance between steel headed stud anchors and steel dowel bars or steel faceplates, spacing of dowel bars, etc.), and (2) interaction equations for designing connections for combinations of various demands.



# COMMENTARY N9J9 CONNECTION QUALIFICATION

## CONNECTION QUALIFICATION FOR EACH INDIVIDUAL DEMAND TYPE

- The *connection available strength* for each individual demand type, as determined in Section N9J6, reviewed by an *independent peer-review panel*.
- **If deemed necessary** by the *independent peer-review panel*, connection qualification for a particular demand type **may be** performed using (i), (ii) and (iii) on next slide, and reviewed by the *independent peer review panel* :



# COMMENTARY N9J9 CONNECTION QUALIFICATION

- i. Nonlinear inelastic finite element analysis results that account for various failure modes, and have been benchmarked using either (ii) or (iii).

AND

- ii. Benchmarking of finite element models and analysis approaches shall be performed using existing test results, no smaller than 1/3-scale, reported in research literature or documented tests that are relevant for the *connectors* and failure mechanisms.

OR

- iii. When suitable test data is not available, benchmarking of finite element models and analysis approaches shall be performed using project-specific data for tests conducted on specimens that are representative of the connection configurations, *connector* sizes, material strengths, fabrication processes, and no smaller than 1/3 scale in size.



# COMMENTARY

- N9J9.2 Connection Qualification for Combinations of Demands
  - The connection adequacy for combinations of demands, i.e., combined in-plane and out-of-plane forces, reviewed by an *independent peer-review panel*.
  - **If deemed necessary** by the *independent peer review panel*, the connection adequacy for combinations of demands **may be further verified** by results of nonlinear inelastic finite element analyses conducted using the benchmarked nonlinear finite element models developed in N9J9.1..





N9J9 Connection  
Qualification

IF DEEMED NECESSARY  
BY PEER REVIEW PANEL

N9J9.1 Connection  
Qualification for each  
Individual Demand Type

N9J9.2 Connection  
Qualification for  
Combinations of Demands

IPRP Review of  
connection available strength  
For each demand type

IPRP Review of  
connection adequacy for  
combinations of demands

*If required by  
IPRP*

*If required by  
IPRP*

Nonlinear FEA of  
connection for  
individual demand

Nonlinear FEA of  
connection for  
combined demand

*Models  
benchmarked  
using existing test  
data*

*Benchmarked  
models*

# STEEL-PLATE COMPOSITE (SC) DESIGN FOR NUCLEAR FACILITIES

Behavior for In-Plane Demands  
and Combinations



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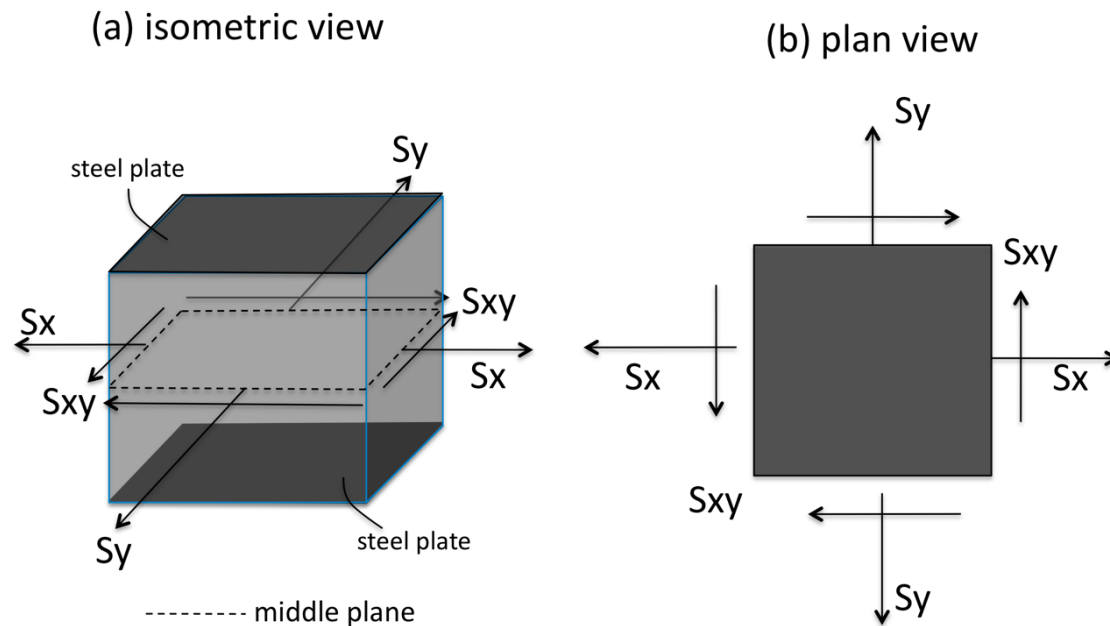
## IN PLANE BEHAVIOR

- The effects of these demands on the behavior of the steel-concrete composite (SC) structure was investigated
- To begin, let's focus on the behavior for in-plane forces  $S_x$ ,  $S_y$ , and  $S_{xy}$ .
- These are forces per unit length typically in kips/ft



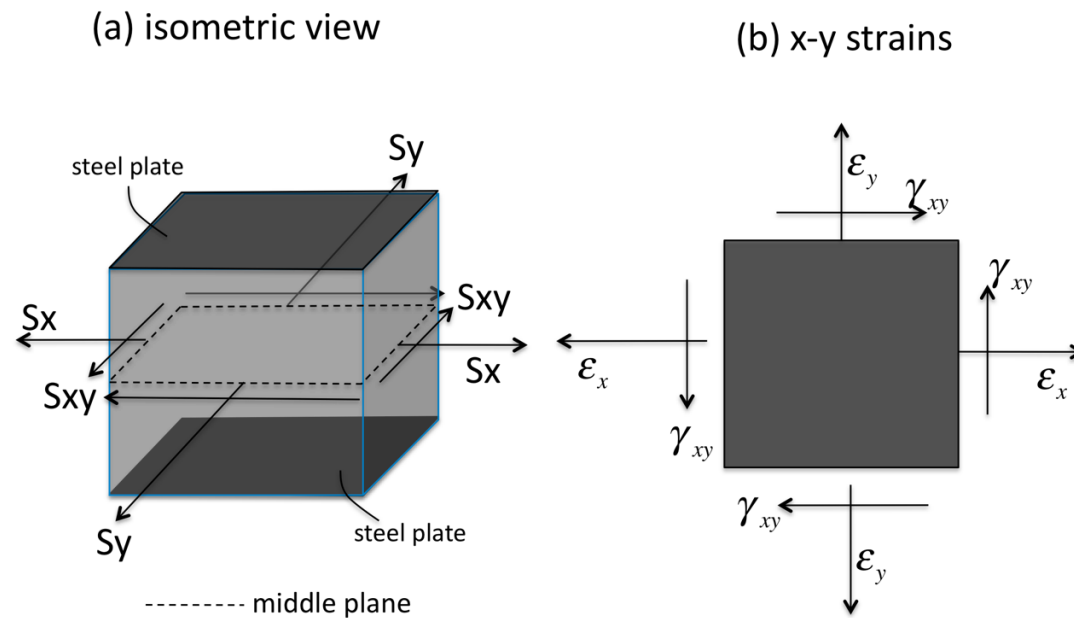
## IN PLANE BEHAVIOR

- Consider a concrete filled steel (CFS) element taken from the SC structure
- It is subjected to in-plane forces  $S_x$ ,  $S_y$ , and  $S_{xy}$  per unit length (in.).



# IN PLANE BEHAVIOR

- These in-plane forces will cause some deformations in the element. These deformations can be denoted by the strain terms:  $\epsilon_x$ ,  $\epsilon_y$ ,  $\gamma_{xy}$



## TWO FACTS

- If  $\theta_p$  is the principal direction
- Stress Transformation relates the principal stresses to the x-y stresses,

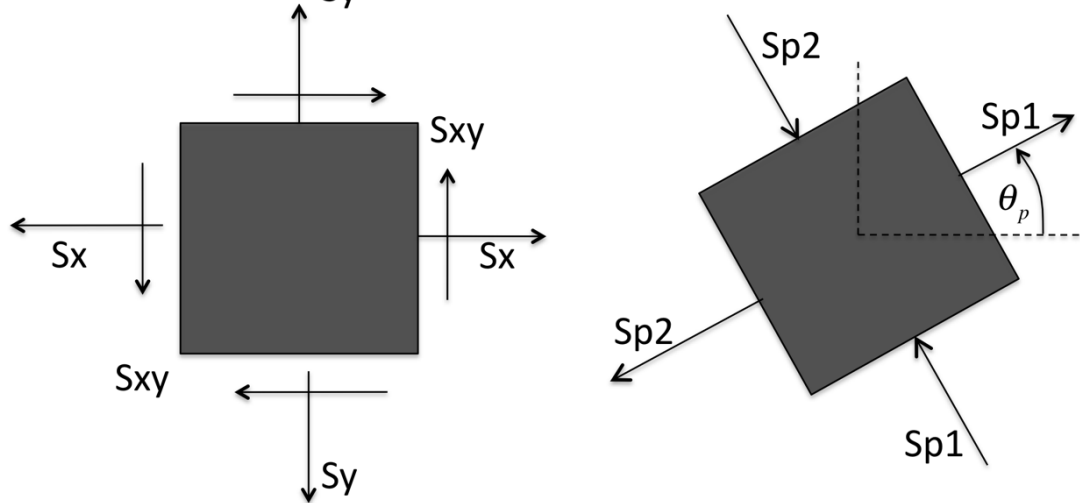
$$[T]_{\sigma} = \frac{1}{2} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & 2 \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -2 \sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}$$

- Strain Transformation relates the principal strains to the x-y strains

$$[T]_{\epsilon} = \frac{1}{2} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -\sin(2\theta_p) \\ -2 \sin(2\theta_p) & 2 \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}$$

## PRINCIPAL IN-PLANE FORCES ( $S_{p1}$ , $S_{p2}$ )

- The in-plane forces ( $S_x$ ,  $S_y$ , and  $S_{xy}$ ) can be used to compute the principal direction ( $\theta_p$ ) and principal membrane forces  $S_{p1}$  and  $S_{p2}$  per unit length using Equations below:



$$\tan(2\theta_p) = \frac{2S_{xy}}{S_x - S_y}$$

$$S_{p1} = \frac{(1 + \cos(2\theta_p))}{2} S_x + \frac{(1 - \cos(2\theta_p))}{2} S_y + \sin(2\theta_p) S_{xy}$$

$$S_{p2} = \frac{(1 - \cos(2\theta_p))}{2} S_x + \frac{(1 + \cos(2\theta_p))}{2} S_y - \sin(2\theta_p) S_{xy}$$

# CONCRETE CRACKING

- Before concrete cracking, the principal forces are resisted by the composite section (steel and concrete)
- Concrete cracking occurs after the principal membrane force ( $S_{p1}$  or  $S_{p2}$ ) exceeds  $S_{ct}$ .

$$S_{ct} = \left( \frac{4\sqrt{f'_c}}{1000} - \varepsilon_{sh} E_c \right) \left( T_c + \frac{E_s}{E_c} 2t_s \right) \quad \text{kips / in.}$$

- $S_{ct}$  can be reduced to account for the effects of concrete shrinkage tensile strain ( $\varepsilon_{sh}$ )
- This point is variable because the tensile strength of concrete can be highly variable and shrinkage effects are not easy to characterize.
- Cracking can occur due to one or both principal forces. The cracks will be oriented in the plane perpendicular to the principal force that exceeds  $S_{ct}$ .



## POST-CRACKING BEHAVIOR

- Post-cracking the concrete will offer very little stiffness and stress capacity in the principal direction perpendicular to the plane of cracking.
- However, it will have stiffness and strength in the principal direction parallel to the plane of cracking. This can be referred as cracked orthotropic behavior
- Japanese researchers recommend that the stiffness in the direction parallel to the plane of cracking can be assumed to be 70% of the uncracked stiffness ( $E_c$ ).
- Lets assume the stiffness and strength perpendicular to the direction of cracking are assumed to be zero, which is a conservative assumption.



## PRINCIPAL STRAINS

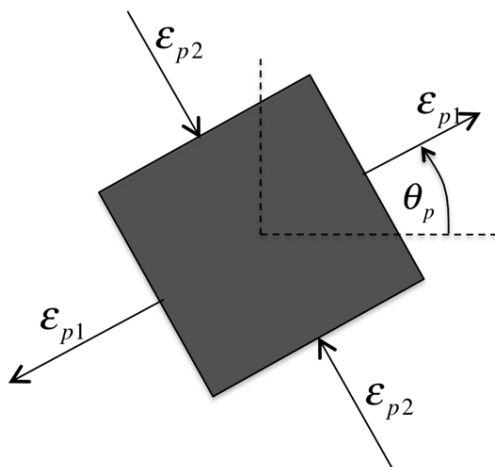
- Assume that the principal strains ( $\epsilon_{p1}$  and  $\epsilon_{p2}$ ) are in the same direction as the principal forces ( $S_{p1}$  and  $S_{p2}$ ).
- They are related to the x-y strain ( $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$ ) by the strain transformation rules. In these equations  $\theta_p$  is the principal direction computed earlier.

$$\begin{Bmatrix} \epsilon_{p1} \\ \epsilon_{p2} \\ 0 \end{Bmatrix} = [T]_{\epsilon} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad [T]_{\epsilon} = \frac{1}{2} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -\sin(2\theta_p) \\ -2\sin(2\theta_p) & 2\sin(2\theta_p) & 2\cos(2\theta_p) \end{bmatrix}$$

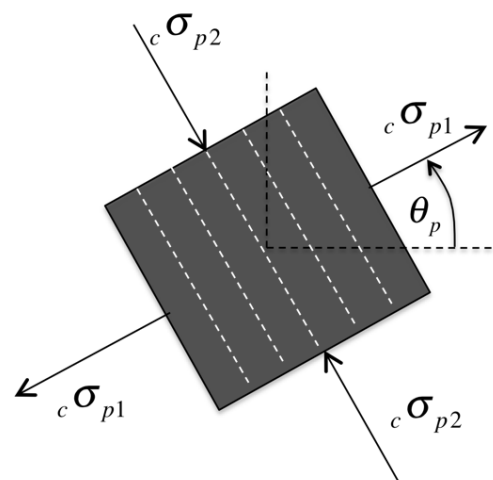
# CRACKED CONCRETE STRESS-STRAIN RELATIONSHIP

- The concrete infill will have principal stresses ( ${}_c\sigma_{p1}$  and  ${}_c\sigma_{p2}$ ) corresponding to the principal strains ( $\epsilon_{p1}$  and  $\epsilon_{p2}$ ). The principal stresses will be related to the strains via the reduced elastic modulus ( $E'_c$ )

(a) principal strains



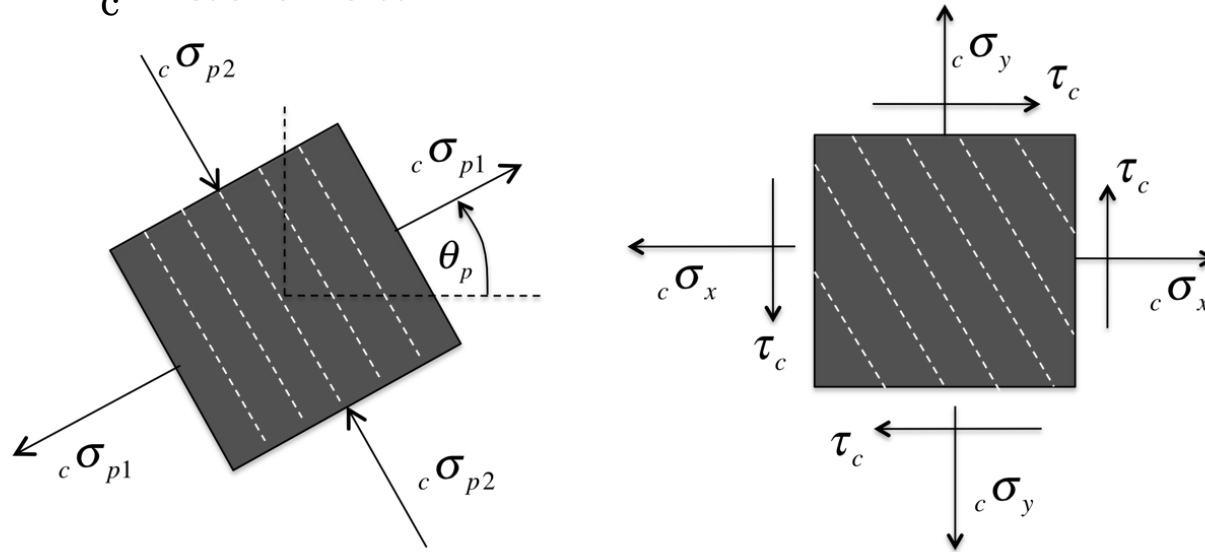
(b) principal stresses



$$\begin{Bmatrix} {}_c\sigma_{p1} \\ {}_c\sigma_{p2} \\ 0 \end{Bmatrix} = E'_c \begin{bmatrix} 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 \text{ or } 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_{p1} \\ \epsilon_{p2} \\ 0 \end{Bmatrix}$$

# CRACKED CONCRETE STRESS-STRAIN RELATIONSHIP

- The concrete principal stresses ( ${}_c\sigma_{p1}$  and  ${}_c\sigma_{p2}$ ) can be transformed back to stresses in the x-y directions  ${}_c\sigma_x$ ,  ${}_c\sigma_y$ , and  ${}_c\tau$  as shown



$$\begin{Bmatrix} {}_c\sigma_x \\ {}_c\sigma_y \\ {}_c\tau \end{Bmatrix} = [T]_{\sigma}^{-1} \begin{Bmatrix} {}_c\sigma_{p1} \\ {}_c\sigma_{p2} \\ 0 \end{Bmatrix} \quad [T]_{\sigma} = \frac{1}{2} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & 2 \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -2 \sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}$$

# CRACKED CONCRETE STRESS-STRAIN RELATIONSHIP

- Collecting all the terms that we have developed so far we get the stress-strain relationship for cracked concrete:

$$\begin{Bmatrix} {}_c\sigma_x \\ {}_c\sigma_y \\ {}_c\tau \end{Bmatrix} = [K]_c \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$[K]_c = [T]_\sigma^{-1} \begin{bmatrix} 0 \text{ or } E'_c & 0 & 0 \\ 0 & 0 \text{ or } E'_c & 0 \\ 0 & 0 & 0 \end{bmatrix} [T]_\epsilon$$

$$[K]_c = \frac{E'_c}{4} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & 2 \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -2 \sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}^{-1} \begin{bmatrix} 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 \text{ or } 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -\sin(2\theta_p) \\ -2 \sin(2\theta_p) & 2 \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}$$

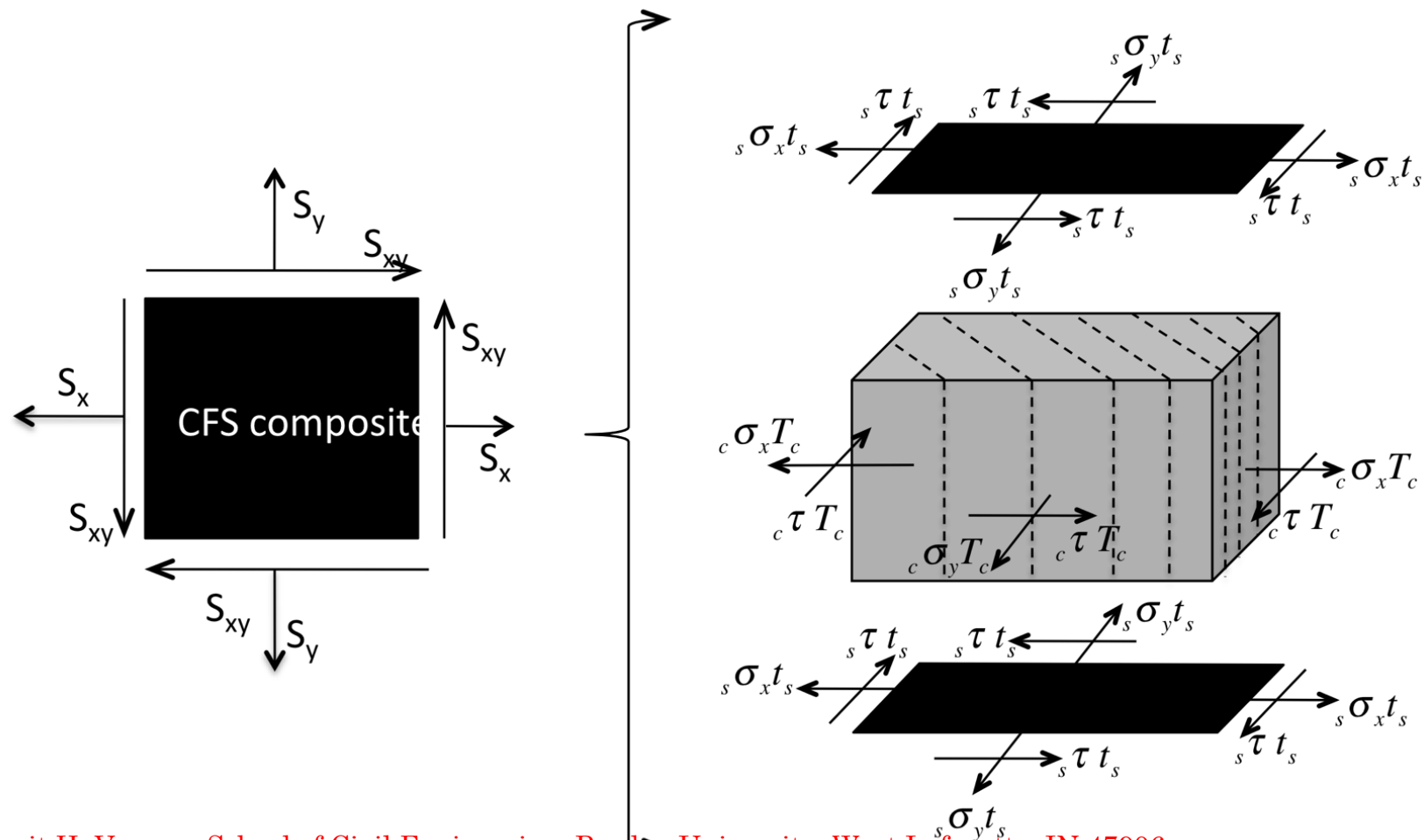
# STEEL PLATE STRESS-STRAIN RELATIONSHIP

- The steel plates will have plane stress behavior, and the corresponding elastic isotropic behavior can be used to relate the steel plate membrane stresses ( $\sigma_x$ ,  $\sigma_y$ , and  $\tau$ ) to the composite section strains ( $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$ )

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau \end{Bmatrix} = [K]_s \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad [K]_s = \frac{E_s}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

# FORCE EQUILIBRIUM OF COMPOSITE SECTION

- Static force equilibrium diagram for the CFS composite panel / element subjected to membrane forces ( $S_x$ ,  $S_y$ , and  $S_{xy}$ ) per unit length. (in.)



# FORCE EQUILIBRIUM OF COMPOSITE SECTION

$$\begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau \end{Bmatrix}_s \times 2t_s + \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau \end{Bmatrix}_c \times T_c$$

$$\begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix} = 2t_s [K]_s \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} + T_c [K]_c \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = [2t_s [K]_s + T_c [K]_c]^{-1} \begin{Bmatrix} S_x \\ S_y \\ S_{xy} \end{Bmatrix}$$

- For the applied  $S_x$ ,  $S_y$ , and  $S_{xy}$ , solve the equation above to get  $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$ .



# FORCE EQUILIBRIUM OF COMPOSITE SECTION

- After obtaining the strains, calculate the steel plates stresses and concrete stresses using the following equations:

$$\begin{Bmatrix} {}_s\sigma_x \\ {}_s\sigma_y \\ {}_s\tau \end{Bmatrix} = [K]_s \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \longrightarrow \begin{Bmatrix} {}_s\sigma_{p1} \\ {}_s\sigma_{p2} \\ 0 \end{Bmatrix} = [T]_\sigma \bullet \begin{Bmatrix} {}_s\sigma_x \\ {}_s\sigma_y \\ {}_s\tau \end{Bmatrix}$$

$$\begin{Bmatrix} {}_c\sigma_{p1} \\ {}_c\sigma_{p2} \\ 0 \end{Bmatrix} = E'_c \begin{bmatrix} 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 \text{ or } 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} [T]_\epsilon \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

# DESIGN CHECKS AND LOAD RATIOS

- Establish yielding of the steel plates. Calculate the Von Mises stress caused by the loading

$$\sigma_{VM} = \sqrt{\sigma_{p1}^2 + \sigma_{p2}^2 - \sigma_{p1} \sigma_{p2}}$$

- The load ratio to cause yielding =  $F_y / \sigma_{VM}$
- Since the concrete was assumed to remain elastic, check that the minimum principal stress (compressive) is still in the elastic range, i.e.,

$$\sigma_{p2, p1} \leq 0.70 f'_c$$

- This entire process can be automated easily to determine the behavior of CFS panels subjected to a variety of in-plane membrane force combinations

final-interaction.html

$$ts := 0.3875$$

$$T := 30$$

$$Es := 29000$$

$$\nu := 0.3$$

$$fc := 6000$$

$$Fy := 50$$

$$Ec := 57 \cdot 6000^{0.5} \rightarrow 4415.2010146764552491$$

Material Properties Section

$$\text{Unitload} := ts$$

$$Sxy := \text{Unitload}$$

$$Sx := 0.0000000001$$

$$Sy := 0$$

Applied Loads

$$a := \left( 2 \cdot \frac{Sxy}{Sx - Sy} \right) \rightarrow 7.75e9$$

$$\theta := \frac{1}{2} \text{atan}(a) \rightarrow 0.78539816333293218058$$

Fixed crack orientation

$$Sxp1 := \frac{(Sx + Sy)}{2} + \sqrt{\left[ \frac{(Sx - Sy)}{2} \right]^2 + Sxy^2} \rightarrow 0.38750000005$$

$$Sxp2 := \frac{(Sx + Sy)}{2} - \sqrt{\left[ \frac{(Sx - Sy)}{2} \right]^2 + Sxy^2} \rightarrow -0.38749999995$$

$$a := \begin{cases} 0 & \text{if } Sxp1 > 0 \\ 1 & \text{otherwise} \end{cases}$$

$$b := \begin{cases} 0 & \text{if } Sxp2 > 0 \\ 1 & \text{otherwise} \end{cases}$$

final-interaction.html

$$\text{Strains} := \text{Totstiffness}^{-1} \cdot \begin{pmatrix} S_x \\ S_y \\ S_{xy} \end{pmatrix} \rightarrow \begin{pmatrix} 0.00001341364243137941753194 \\ 0.00001341364242623966630965 \\ 0.00003983307197549749994407 \end{pmatrix}$$

Strains from Force Equilibrium

$$\text{Steelstress} := \frac{\text{Steelstiff}}{t_s} \cdot \text{Strains} \rightarrow \begin{pmatrix} 0.555708043536580445407787788 \\ 0.555708043421924456602857018 \\ 0.444291956649779807062344912 \end{pmatrix}$$

Steel Stresses

$$\text{Steeltrans} := \text{Stresstrans} \cdot \text{Steelstress} \rightarrow \begin{pmatrix} 1.00000000012903225807151014 \\ 0.1114160868294726439391346661 \\ 6.98147263681860780342587348\text{e-}21 \end{pmatrix}$$

Steel Principal Stresses

$$\text{Concretestress} := \frac{\text{Concretetiffness}}{\frac{T}{2}} \cdot \text{Strains} \rightarrow \begin{pmatrix} -0.0143557911180283281729035685 \\ -0.0143557911217330484621370571 \\ 0.0143557911198806883173634277 \end{pmatrix}$$

Concrete Stresses

$$\text{Concprinstress} := \text{Stresstrans} \cdot \text{Concretestress} \rightarrow \begin{pmatrix} -3.2717216691711313940852610208\text{e-}23 \\ -0.028711582239761376635007908383 \\ -1.8311088432742040945524135676\text{e-}23 \end{pmatrix}$$

Concrete Principal Stresses

$$\text{Steeffstress} := \sqrt{(\text{Steeltrans}_0)^2 + (\text{Steeltrans}_1)^2 - \text{Steeltrans}_0 \cdot \text{Steeltrans}_1} \rightarrow 0.94920885890230090630320192014$$

Calculating the von-mises stress

$$\text{Loadratio} := \frac{F_y}{\text{Steeffstress}} \rightarrow 52.6754460107144273242833455978$$

Load ratio to failure by yielding

12/23/09

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Purdue University, West Lafayette, IN 47906



# PURE SHEAR BEHAVIOR

$S_{xy}$  – based on theory

## PURE SHEAR BEHAVIOR

- For pure in-plane shear loading, the applied membrane forces  $S_x$  and  $S_y$  will be equal to zero, and the in-plane shear loading will be equal to  $S_{xy}$ .
- The principal direction ( $\theta_p$ ) =  $\tan^{-1}(\infty) = 45^\circ$
- The principal forces will be equal to  $S_{xy}$  in magnitude;  $S_{p1}$  will be in tensile and  $S_{p2}$  will be compressive.
- Concrete cracking will occur when the applied in-plane shear ( $S_{xy}$ ) exceeds the tensile cracking strength ( $S_{ct}$ ) of the composite CFS section

$$S_{ct} = \left( \frac{4\sqrt{f'_c}}{1000} - \epsilon_{sh} E_c \right) \left( T_c + \frac{E_s}{E_c} 2t_s \right) \times 12 \quad kips / ft$$

## PURE SHEAR BEHAVIOR

- The in-plane shear stiffness before the concrete cracking limit state can be calculated as:

$$K_{xy}^{cr} = G_s A_s + G_c A_c = \frac{E_s t_s}{2(1 + \nu_s)} + \frac{E_c T_c}{2(1 + \nu_c)} \text{ kips/in}$$

- The concrete cracking planes will be perpendicular to direction of the principal stress  $S_{p1}$ , i.e., at  $135^\circ$ .
- The principal direction  $\theta_p$  will be equal to  $45^\circ$ . Substituting this value in the equation for  $[K]_c$

# PURE SHEAR BEHAVIOR

$$[K]_c = \frac{E'_c}{4} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & 2 \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -2 \sin(2\theta_p) \\ -\sin(2\theta_p) & \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}^{-1} \begin{bmatrix} 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 \text{ or } 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 + \cos(2\theta_p) & 1 - \cos(2\theta_p) & \sin(2\theta_p) \\ 1 - \cos(2\theta_p) & 1 + \cos(2\theta_p) & -\sin(2\theta_p) \\ -2 \sin(2\theta_p) & 2 \sin(2\theta_p) & 2 \cos(2\theta_p) \end{bmatrix}$$

- Substituting  $\theta_p = 45^\circ$  The concrete  $[K]_c$  becomes

$$[K]_c = \frac{E'_c}{4} \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \quad \left\{ \begin{matrix} {}_c \sigma_x \\ {}_c \sigma_y \\ {}_c \tau \end{matrix} \right\} = [K]_c \left\{ \begin{matrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{matrix} \right\}$$



# PURE SHEAR BEHAVIOR

## Force Equilibrium for Pure Shear

$$\begin{Bmatrix} 0 \\ 0 \\ S_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{2E_s t_s}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} + \frac{E'_c T_c}{4} \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \end{Bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

## Solve simultaneous equations to get the strains and calculate stresses etc. for pure shear case

$$_s \sigma_x = _s \sigma_y = \frac{S_{xy}}{2t_s} \frac{K_\beta}{K_\alpha + K_\beta}$$

$$_s \tau = \frac{S_{xy}}{2t_s} \frac{K_\alpha}{K_\alpha + K_\beta}$$

$$_c \sigma_x = _c \sigma_y = -_c \tau = \frac{S_{xy}}{T_c} \frac{-K_\beta}{K_\alpha + K_\beta}$$

$$K_\alpha = G_s 2t_s \quad K_\beta = \frac{1}{\frac{4}{E'_c T_c} + \frac{2(1-\nu)}{E_s 2t_s}}$$

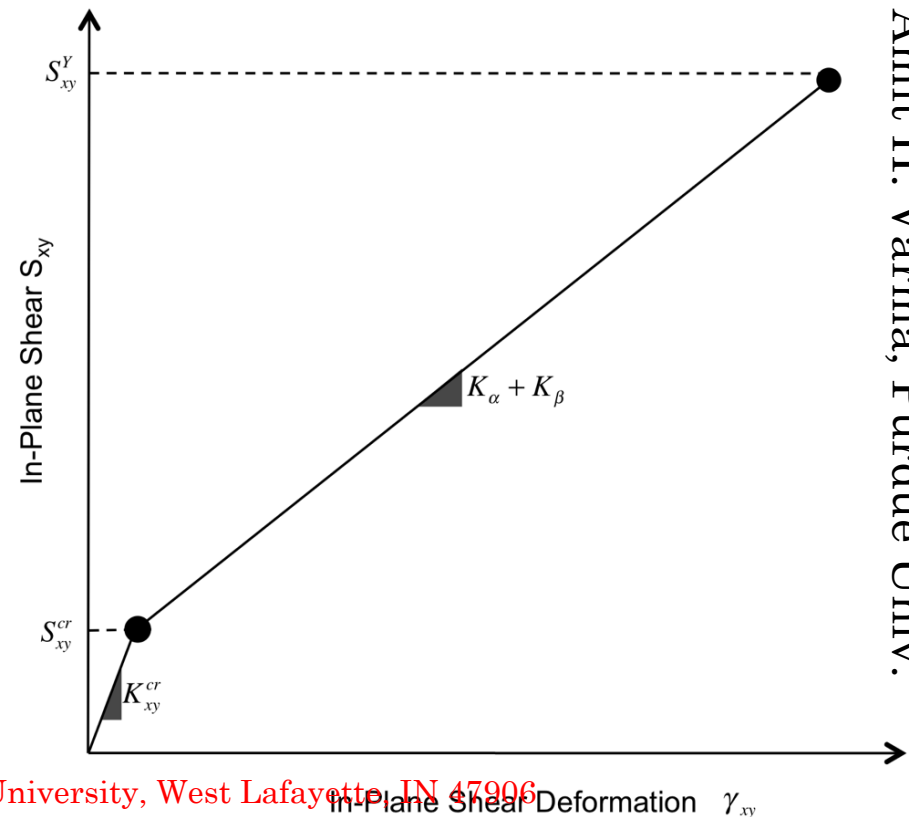
# PURE SHEAR BEHAVIOR

- Use the stresses to calculate von-mises stress and establish the yielding of the steel plate.
- The yield load corresponds to:

$$S_{xy}^Y = \frac{K_\alpha + K_\beta}{\sqrt{3K_\alpha^2 + K_\beta^2}} (2t_s F_y)$$

- The shear stiffness post-cracking corresponds to;

$$S_{xy} = (K_\alpha + K_\beta) \gamma_{xy}$$



# PURE SHEAR BEHAVIOR – PARAMETRIC STUDY

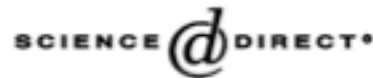
Effect of Concrete Thickness on In-plane Shear Strength and Stresses							
Assumed 0.5 in. steel plate thickness with $F_y = 50$ ksi							
Concrete $T_c$ (in.)	$K_a$ (kips-ft/ft)	$K_b$ (kips-ft/ft)	$\frac{S_{nv}}{A_s F_y}$	$\frac{s \sigma_x}{F_y} = \frac{s \sigma_y}{F_y}$	$\frac{s \tau}{F_y}$	$\frac{c \sigma_x}{f'_c} = \frac{c \sigma_y}{f'_c} = \frac{-\tau_c}{f'_c}$	$\frac{c \sigma_p}{f'_c}$
18	133846	99852	0.93	0.40	0.53	0.24	0.48
20	133846	106206	0.94	0.42	0.52	0.23	0.45
22	133846	112040	0.95	0.44	0.52	0.21	0.43
24	133846	117414	0.97	0.45	0.52	0.20	0.41
26	133846	122381	0.98	0.47	0.51	0.19	0.39
28	133846	126986	0.99	0.48	0.51	0.19	0.37
30	133846	131266	1.00	0.49	0.50	0.18	0.36
32	133846	135256	1.00	0.50	0.50	0.17	0.34
34	133846	138983	1.01	0.51	0.50	0.16	0.33
36	133846	142472	1.02	0.52	0.49	0.16	0.32

# COMPARISON WITH EXPERIMENTAL RESULTS

- Japanese Tests by Ozaki et al. (2004)



Available online at [www.sciencedirect.com](http://www.sciencedirect.com)



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[www.elsevier.com/locate/nuceng](http://www.elsevier.com/locate/nuceng)

## Study on steel plate reinforced concrete panels subjected to cyclic in-plane shear

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# COMPARISON WITH EXPERIMENTAL RESULTS

## Japanese tests of Ozaki et al. (2004)

Table 2

Test specimen (research program I)

12/23/09

Specimens	Surface steel plate ( $t$ ) (mm)	Headed stud bolt			Nodal force (MPa)	Partitioning web
		Pitch in welding ( $B$ ) (mm)	Diameters (mm)	$B/t$		
S2-00NN	2.3	70	4	30	0.0	
S2-15NN					1.47	
S2-30NN					2.94	
S3-00NN	3.2	100	5	31	0.0	
S3-15NN					1.47	
S3-30NN					2.94	–
S3-00PS					0.0	Studs were welded
S3-00PN						Without studs
S4-00NN	4.5	135	9	30		–

Table 4

Experimental results

Specimen	Steel		Concrete	Elastic shear modulus	Post-cracking shear modulus	Cracking strength		Yield strength		Maximum strength	
	Yield stress, Young's modulus (MPa)	$A_w \times A_p$ (cm <sup>2</sup> )	Compressive strength, tangential stiffness (MPa)	$G_e$ ( $\times 10^3$ MPa)	$G_y$ ( $\times 10^3$ MPa)	$Q_c$ (kN)	$\gamma_c$ ( $\times 10^{-3}$ )	$Q_y$ (kN)	$\gamma_y$ ( $\times 10^{-3}$ )	$Q_u$ (kN)	$\gamma_u$ ( $\times 10^{-3}$ )
S2-00NN	340 ( $1.97 \times 10^5$ )	53.5 (17.1)	42.2 ( $2.72 \times 10^4$ )	12.4	4.16	293	0.115	2290 (–2110)	2.50 (–1.99)	2960 (–2780)	9.41 (–6.12)
S2-15NN			41.6 ( $2.77 \times 10^4$ )	13.2	4.14	433	0.133	2330 (–2290)	2.71 (–2.21)	3110 (–2930)	10.00 (–6.02)
S2-30NN			42.0 ( $2.79 \times 10^4$ )	16.4	3.69	542	0.168	2490 (–2570)	3.01 (–2.41)	3110 (–3200)	10.48 (–6.03)
S3-00NN	351 ( $1.99 \times 10^5$ )	75.4 (16.9)	41.9 ( $2.71 \times 10^4$ )	12.9	4.88	311	0.134	3070 (–3070)	3.01 (–2.00)	3610 (–3430)	6.05 (–6.03)
S3-15NN			41.6 ( $2.67 \times 10^4$ )	13.1	4.29	384	0.141	3130 (–3120)	2.99 (–3.01)	3760 (–3330)	7.99 (–6.01)
S3-30NN			40.1 ( $2.70 \times 10^4$ )	11.9	4.67	385	0.186	3170 (–3080)	2.80 (–2.96)	3730 (–3550)	5.57 (–5.63)
S3-00PS		75.4 (25.4)	41.9 ( $2.71 \times 10^4$ )	13.1	5.81	350	0.141	2680 (–2640)	1.93 (–1.97)	3580 (–3220)	10.87 (–5.98)
S3-00PN			39.9 ( $2.72 \times 10^4$ )	16.4	4.92	271	0.113	2350 (–2390)	2.01 (–2.03)	3510 (–3060)	17.00 (–6.02)
S4-00NN	346 ( $2.07 \times 10^5$ )	104.9 (36.7)	42.8 ( $2.76 \times 10^4$ )	16.4	8.22	349	0.103	3510 (–3560)	2.00 (–2.00)	4100 (–3790)	5.67 (–4.00)

# COMPARISON WITH EXPERIMENTAL RESULTS

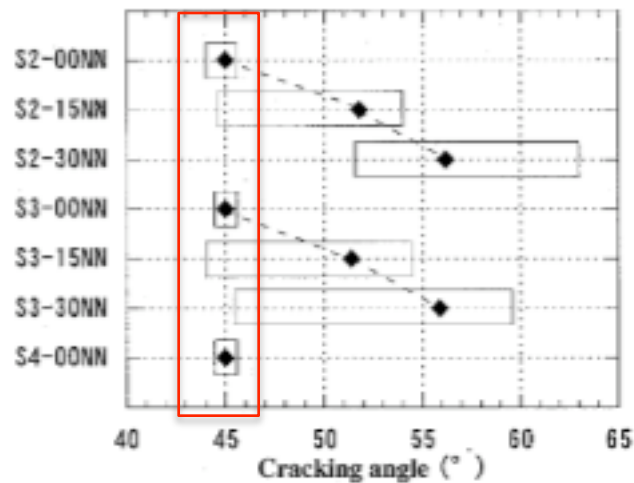
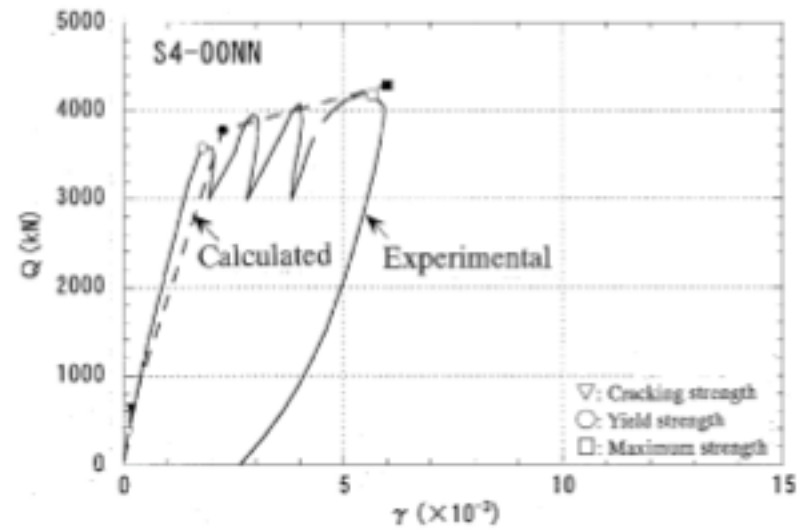
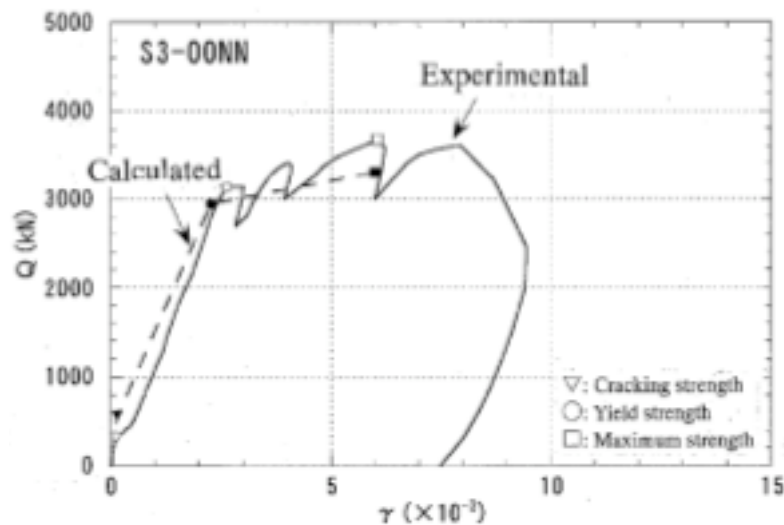
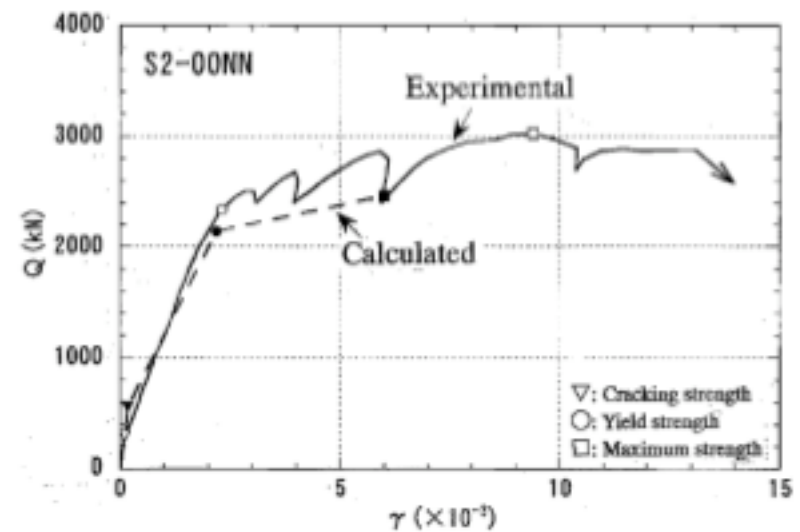
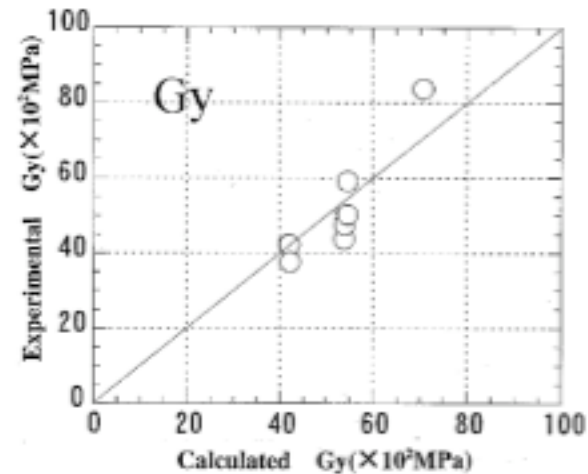
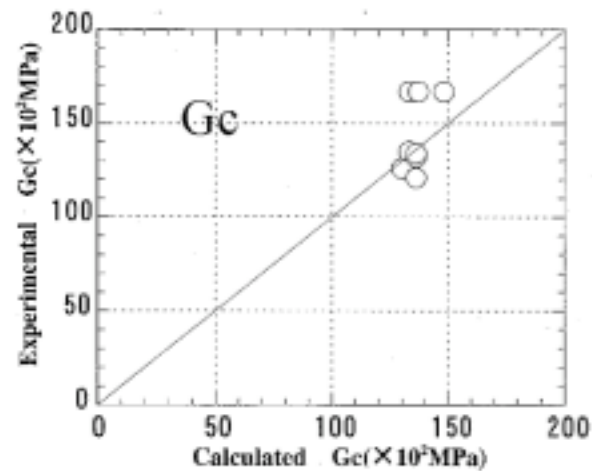
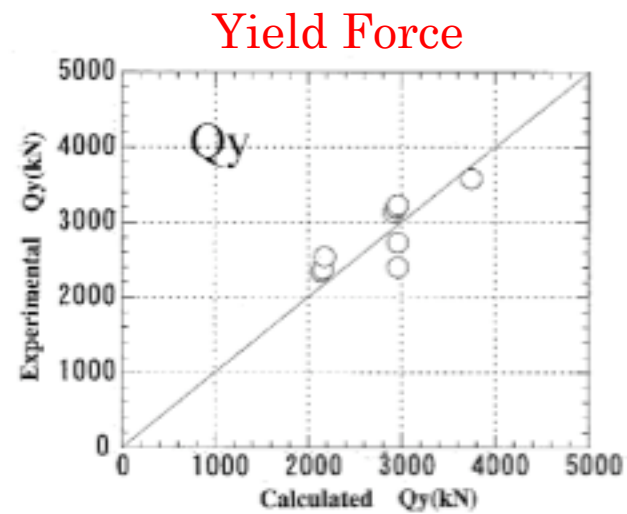
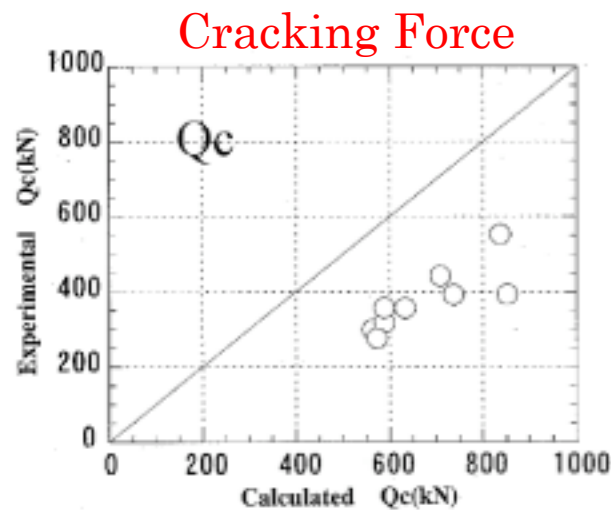


Fig. 7. Cracking angles.



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# COMPARISON WITH EXPERIMENTAL RESULTS





# COMPARISON WITH EXPERIMENTAL RESULTS

- Tests conducted by Lee, Choi, Hong , and Lee in Korea

*The 5<sup>th</sup> International Symposium on Steel Structures  
March 12-14, 2009, Seoul, Korea*

## In-Plane Shear Behavior of Composite Steel Concrete Walls

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Department of Architecture, Seoul National University, Seoul, Korea

Designation	Thickness (mm)	Diameter of Stud ( $\phi$ -mm)	Spacing (mm)	Axial Force Ratio (%)	Axial Force (kN)
S30/400 F3.2 N00-1				0	0
S30/400 F3.2 N00-2	3.2	6-57	100	0	0
S30/400 F3.2 N08				8	705.6



# COMPARISON WITH EXPERIMENTAL RESULTS

- Setup used by these researchers tries to simulate a large panel zone in beam-to-column cruciform
- The setup worked ok. First specimen failed due to welding failure

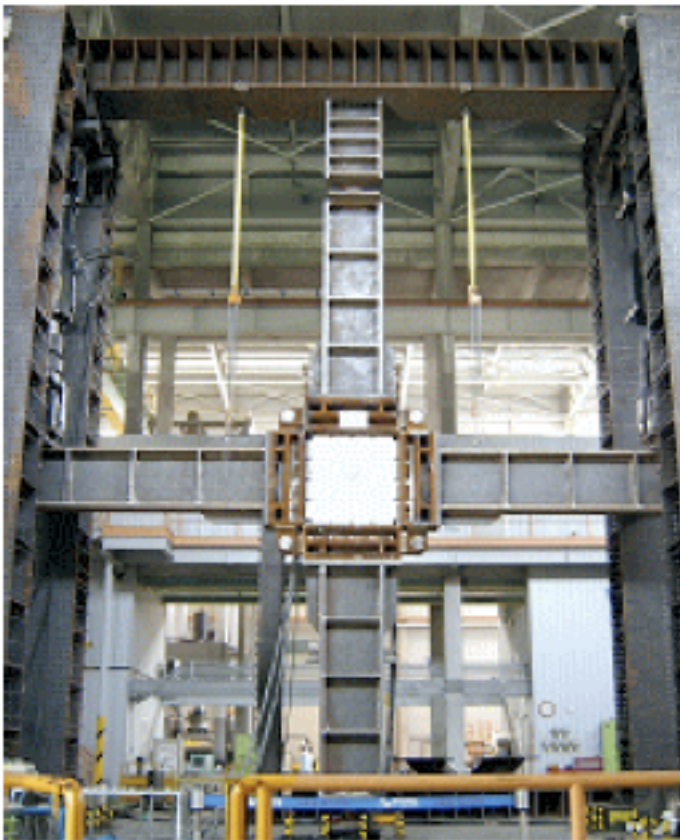


Figure 1. Loading Set-up for Pure Shear

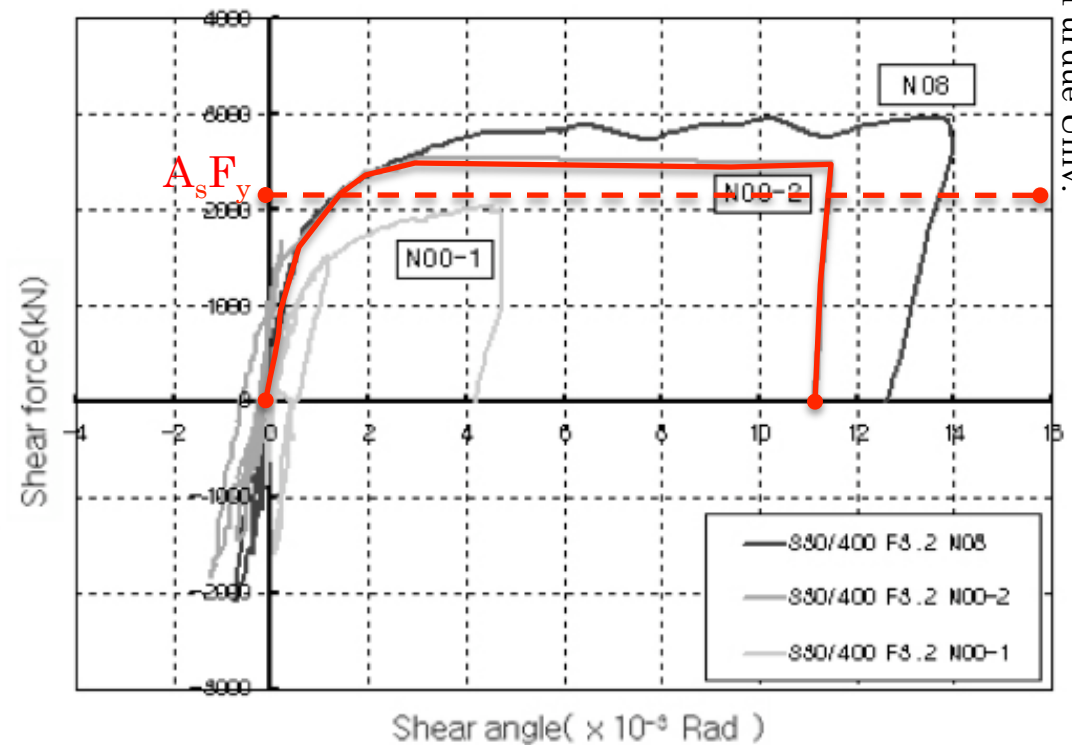


Figure 3. Shear Behavior of SC walls without stiffener

## COMPARISON WITH EXPERIMENTAL RESULTS

Photograph showing 45 deg. cracks in concrete for the pure shear specimen



Figure 5. Diagonal Cracks in Concrete



# COMPARISON WITH EXPERIMENTAL RESULTS

Table 3. Strength of Non-Stiffened Steel Concrete panel

Specimen	Ultimate Shear (Q kN)	Max Shear Angle (rad $\times 10^{-3}$ )	Yield Strength (kN)	Specification (kN)	Material Strength	
					steel (MPa)	conc. (MPa)
S30/400 F3.2 N00-1	2019	4.39	1601	2233	297	36
S30/400 F3.2 N00-2	2597	6.46	1601	2233	297	36
S30/400 F3.2 N08	2900	13.6	1601	2262	297	36

As  $F_y = 1200 \text{ mm} \times 3.2 \text{ mm} \times 2 \times 297 \text{ N/mm}^2 = 2280 \text{ kN}$

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# IN PLANE BEHAVIOR

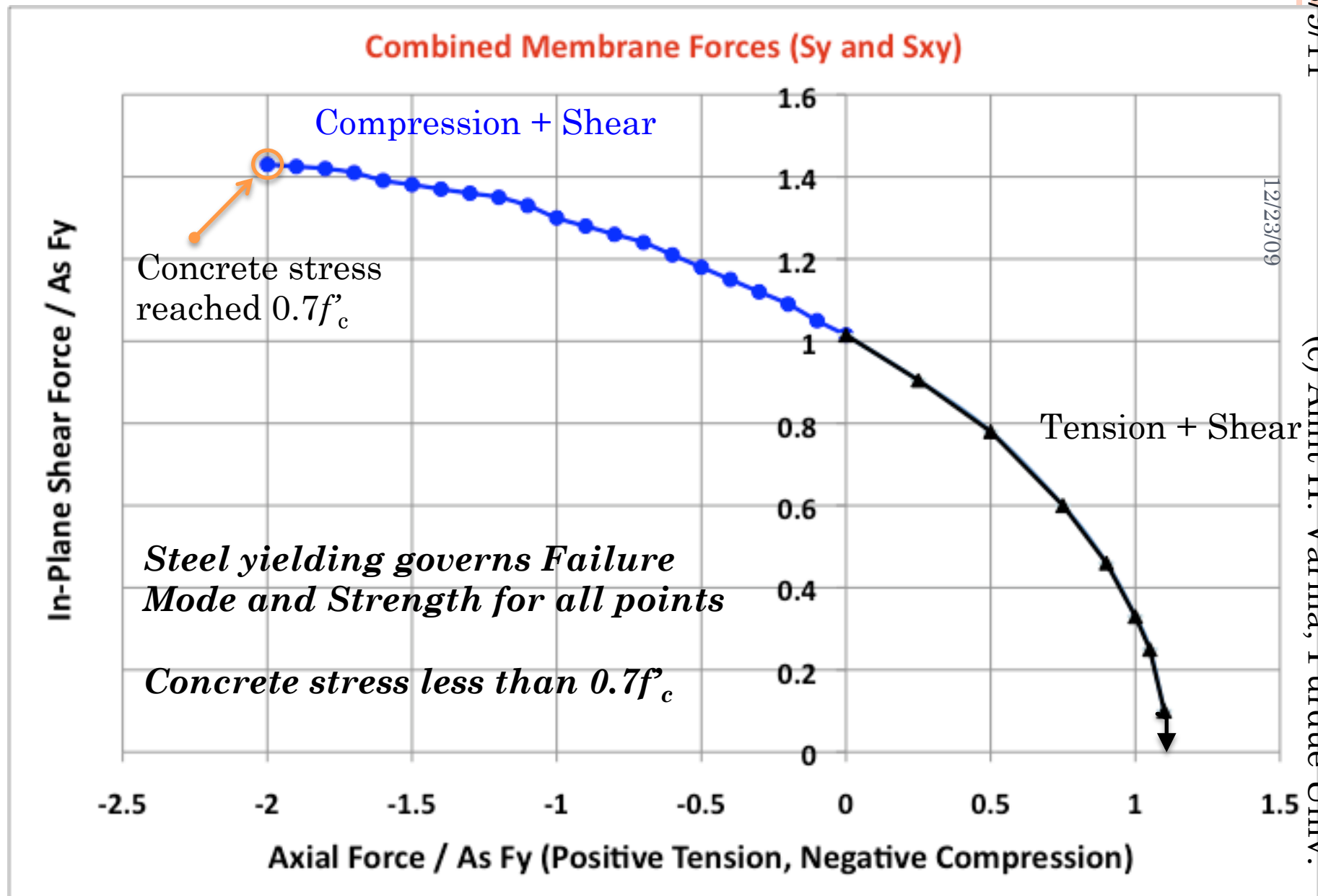
TENSION or COMPRESSION + IN PLANE SHEAR

## IN-PLANE BEHAVIOR

- Used the Mathcad program to analyze the CFS element for different combinations of membrane force (tension or compression) and shear force.
- The limit state was always yielding of the steel plate, and the concrete stresses were limited to the elastic range (less than  $0.70 f'_c$ )
- The last point on the compression + shear portion of the curve had concrete stress equal to  $0.70 f'_c$



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## IN-PLANE BEHAVIOR WITH $S_y$ AND $S_{xy}$

- Combined effects of axial force ( $S_y$ ) and in-plane force  $S_{xy}$ .
- Results obtained using Mathcad sheet.
  - Assumptions – Concrete remains elastic (checked by making sure that concrete principal stress is less than  $0.7f'_c$ ).
  - Concrete has no tension strength
- Note the change in shear strength with axial tension and axial compression.
- Axial compression increases the in-plane shear strength.
- Need more sophisticated analysis to investigate the behavior of CFS panels elements to membrane forces.

## LIMITATIONS OF MATHCAD APPROACH

- The concrete material model was elastic with no compression yielding possible. This limitation becomes significant for cases with larger compression + shear.
- The mathcad program assumes zero tension stiffness and strength. This is conservative assumption but needs refinement
- Theory is not verified using independent finite element analyses



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# IN-PLANE BEHAVIOR

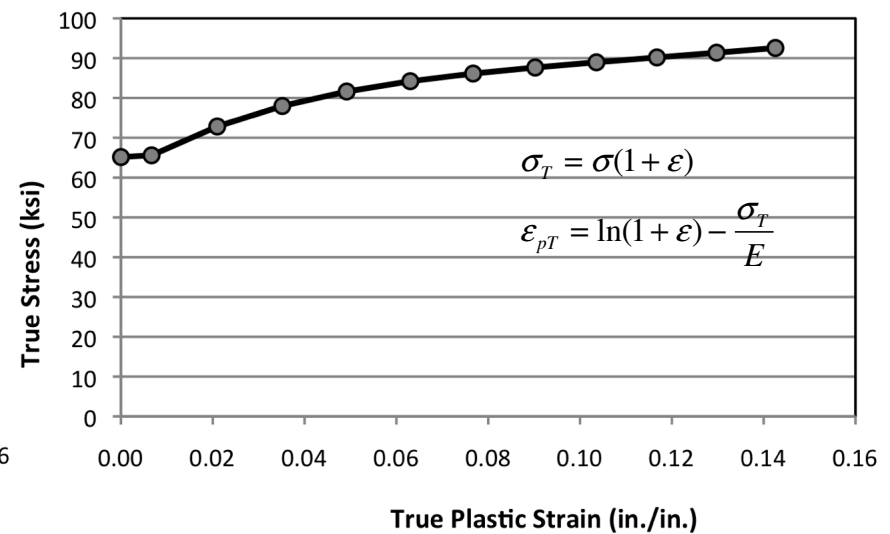
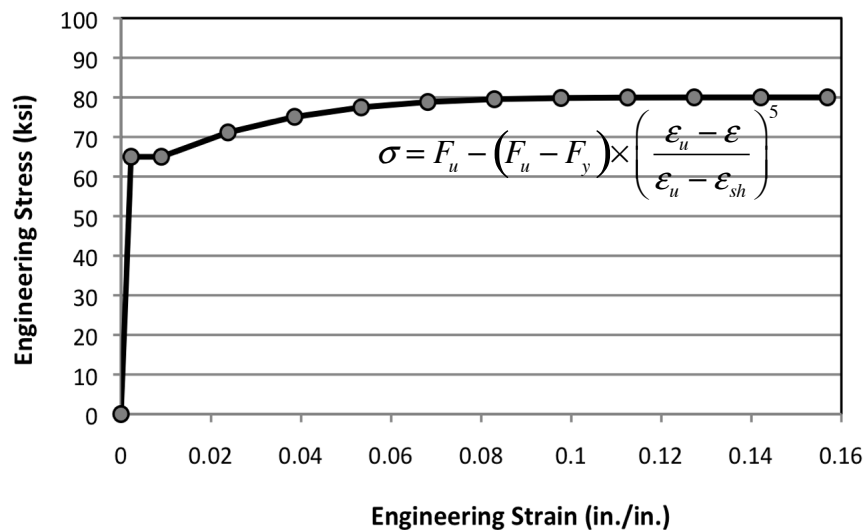
## Finite Element Models

# NONLINEAR FINITE ELEMENT ANALYSIS OF CFS PANELS TO IN-PLANE FORCES

- Three different types of models
  - (1) Shell Solid (SSo) Model
  - (2) Shell Shell (SSh) Model
  - (3) Layered Composite (LCS) Shell Model
  
- Three different concrete material models
  - (1) Smeared cracking – linear Drucker-Prager – associated flow rule concrete model
  - (2) Concrete damaged plasticity model - smeared cracking – parabolic Drucker – Prager - non-associated flow rule – and damaged elasticity
  - (3) Linear elastic orthotropic model for cracked concrete

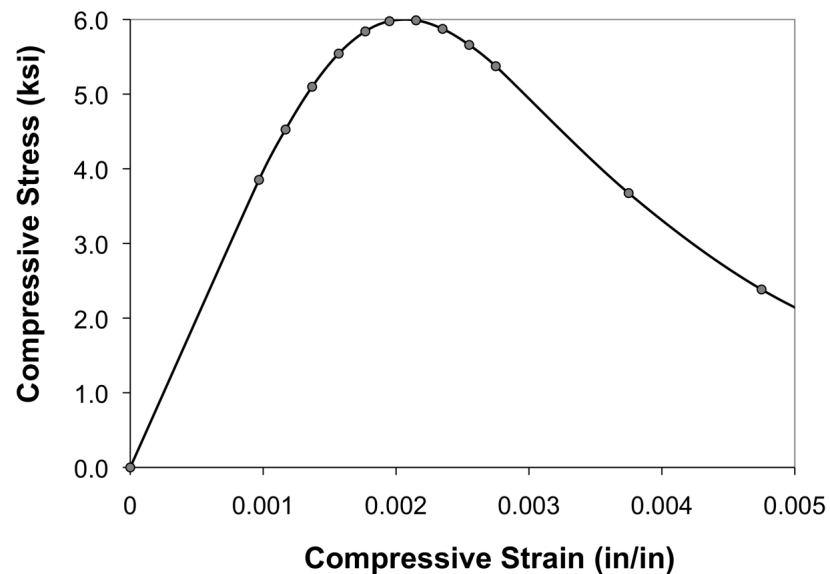
# INPUT PARAMETERS

## Steel stress-strain curve

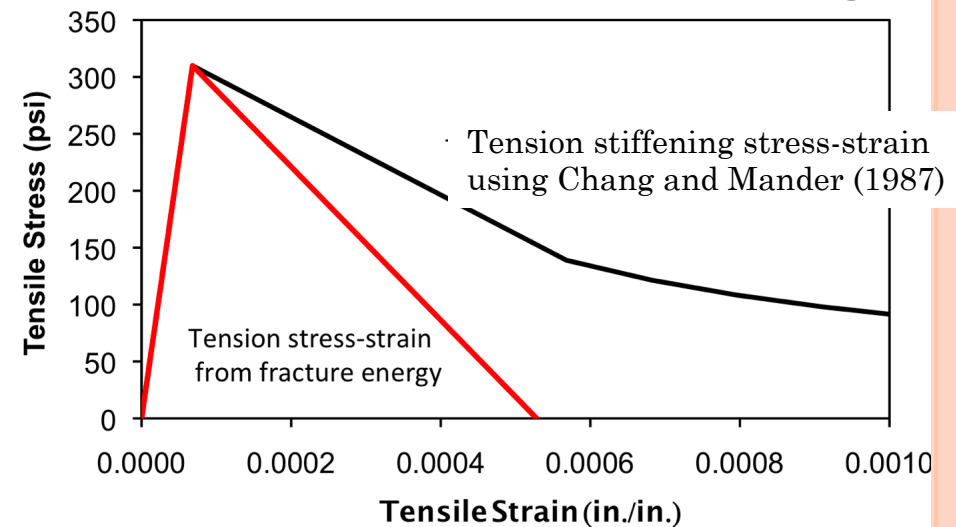


# INPUT PARAMETERS

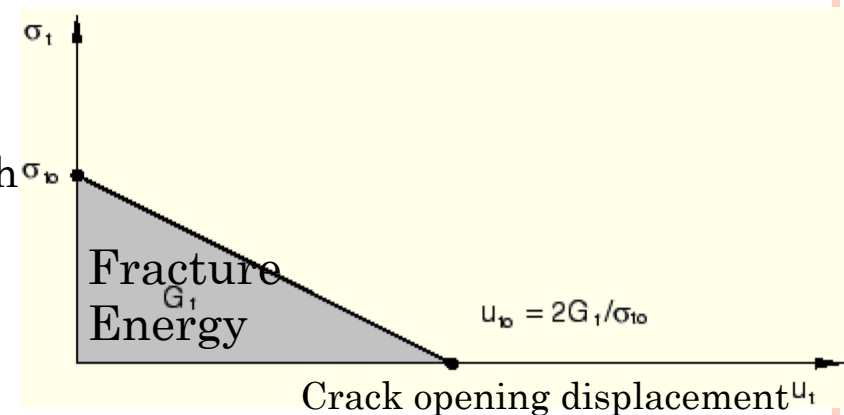
## Concrete compression



## and Tension behavior



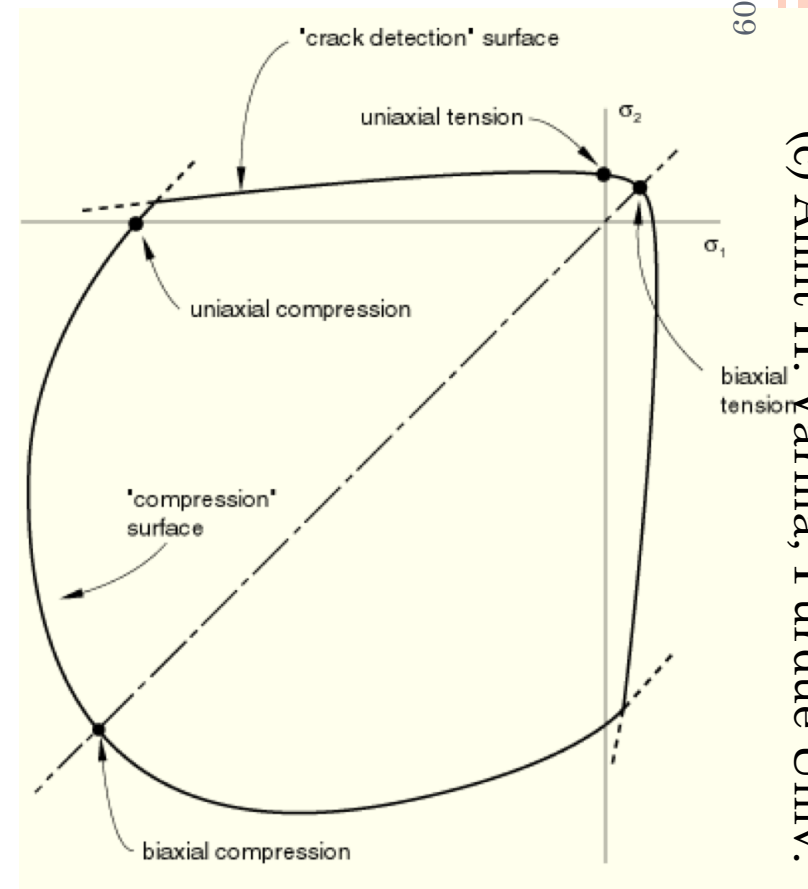
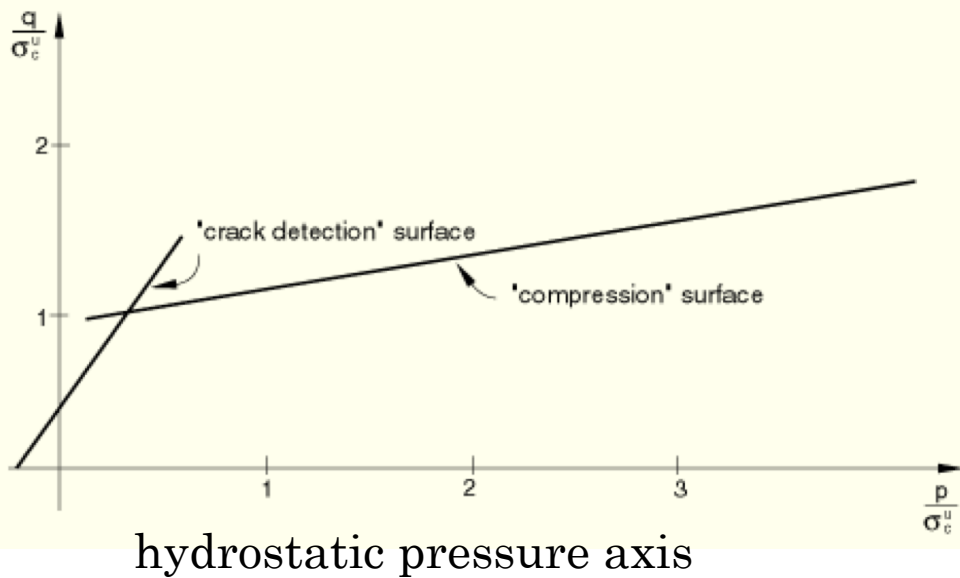
Tensile Strength  $\sigma_b$



# INPUT PARAMETERS

## Concrete Smeared Cracking Model

Failure Surfaces in p-q space



# INPUT PARAMETERS

## Concrete Damaged Plasticity

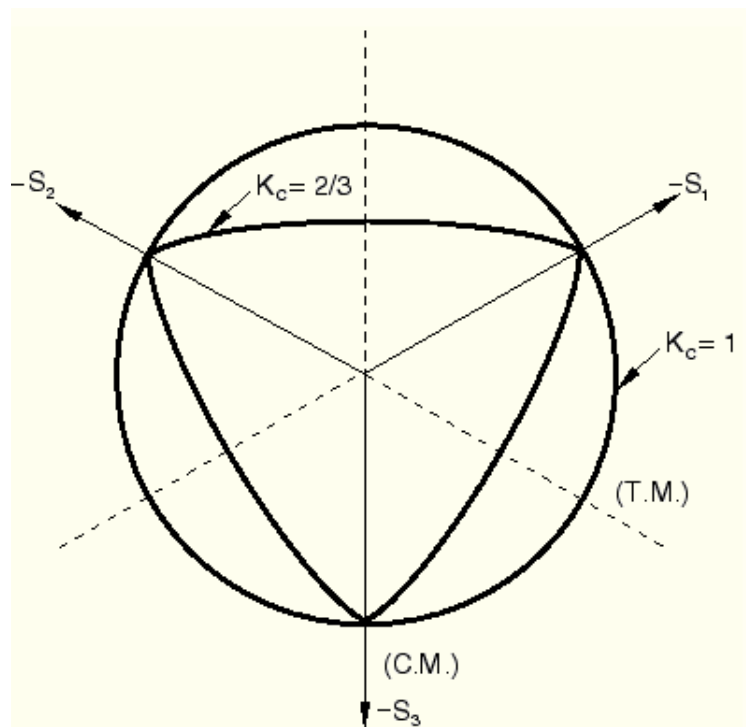
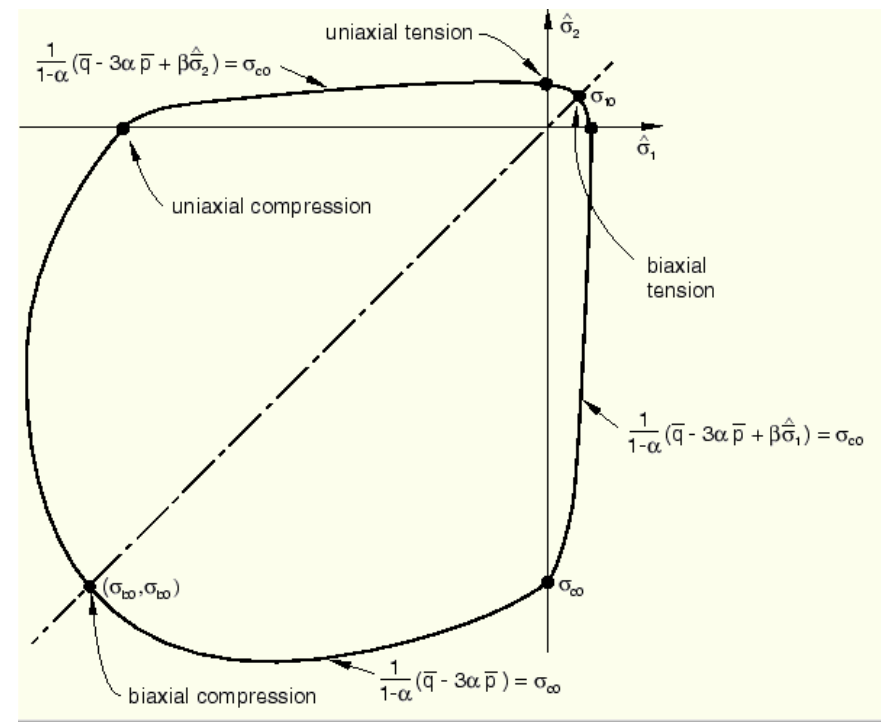
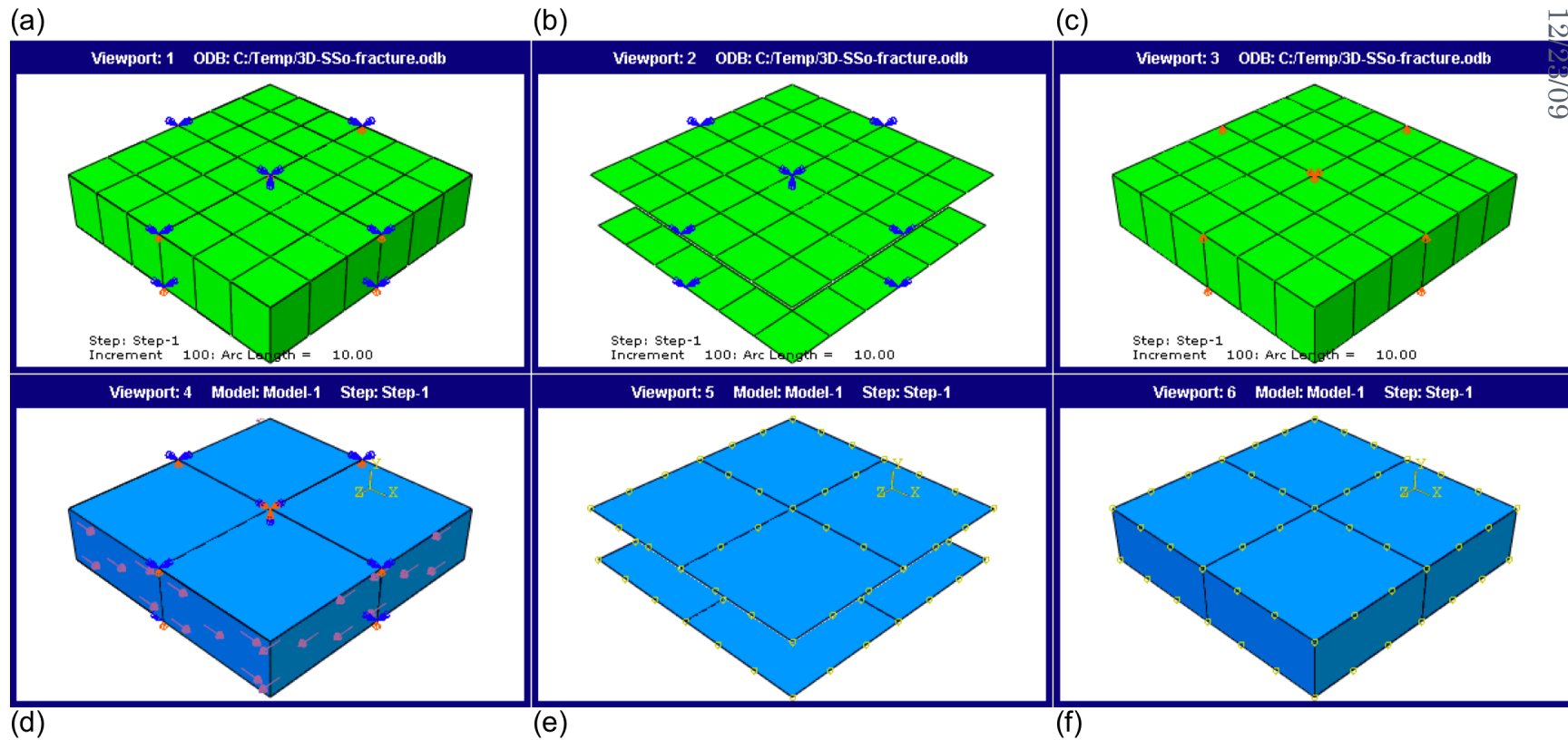


Figure 4.5.2-5 Yield surface in plane stress.

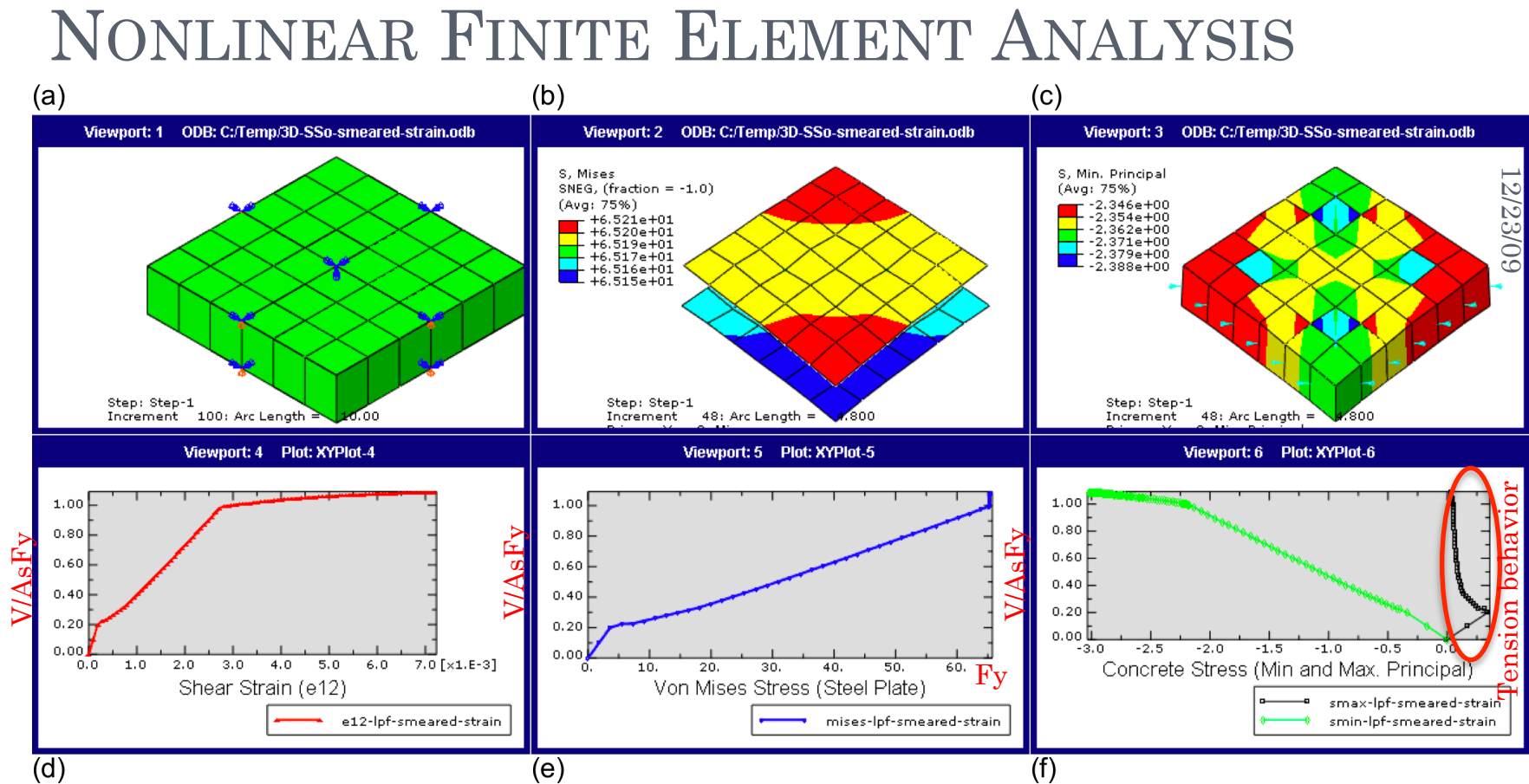


# NONLINEAR FINITE ELEMENT ANALYSIS



**Figure 5-7.** Details of SSo model:

- (a) Model with mesh and boundary conditions.
- (b) Steel plates with shell (S4R) elements, fully tied to concrete elements
- (c) Concrete infill modeled using solid (C3D8R) elements
- (d) Loading applied as uniformly distributed shear traction on the concrete element surfaces
- (e) Steel plates tied to concrete
- (f) Concrete surfaces tied to steel plates



**Figure 5-8.** Results from SSo model with smeared concrete cracking model (tension stiffening stress strain curve):

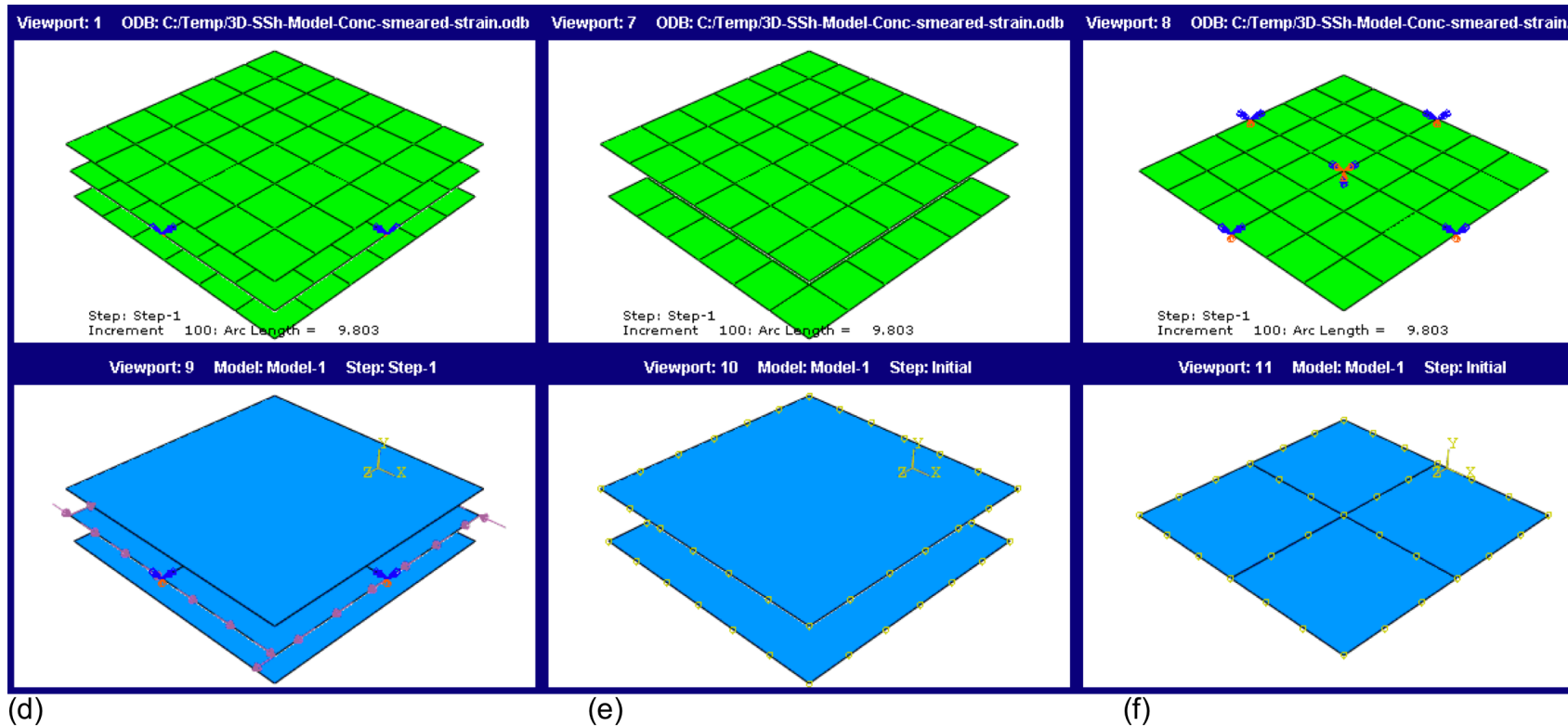
- (a) Model with boundary conditions,
- (b) Von Mises stresses in steel plate when shear force ratio ( $S_{xy}/A_s F_y$ )=1.0,
- (c) Minimum principal stress in the concrete when shear force ratio=1.0,
- (d) Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. shear strain response,
- (e) Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. Von Mises stress in steel plate, and

(f) Shear force ratio vs. minimum and maximum principal stress in concrete.



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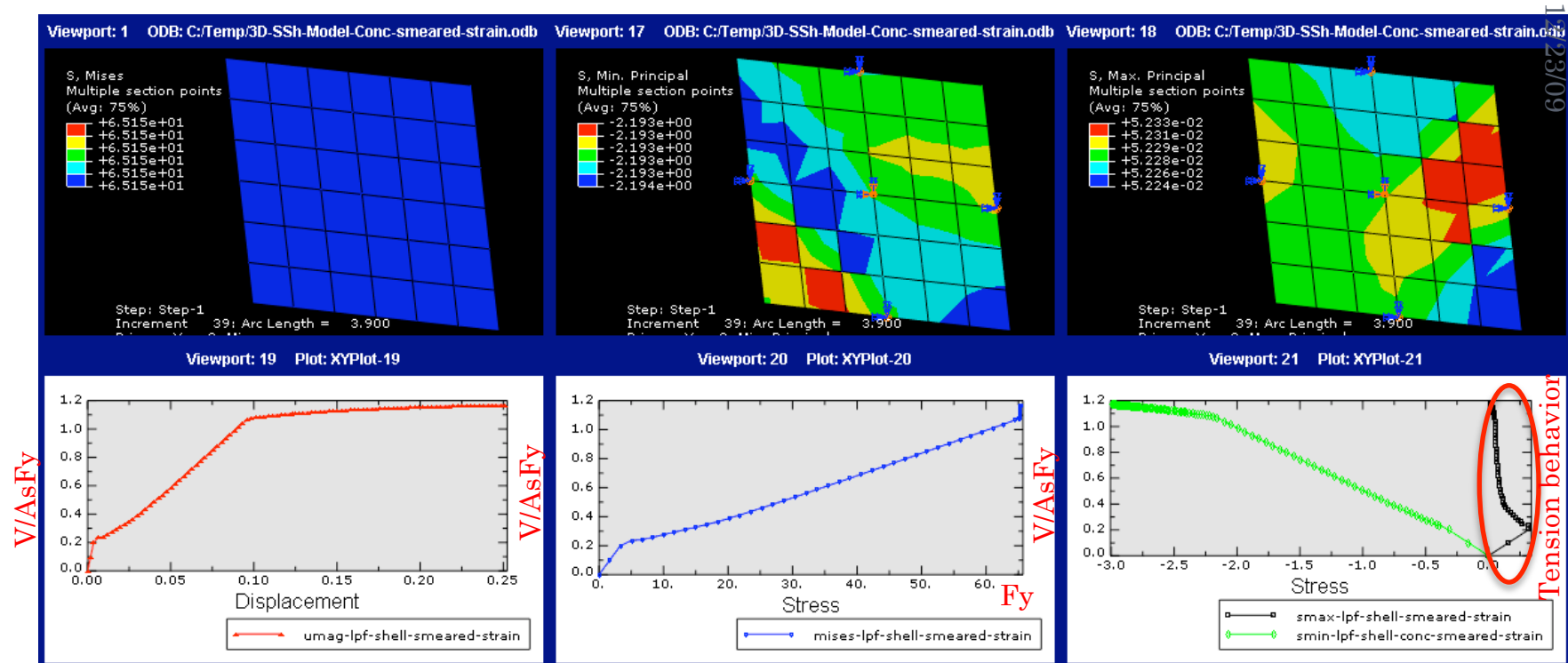
# NONLINEAR ANALYSIS – SSH MODEL



**Figure 5-11.** Details of SSH model:

- (a) Model with mesh and boundary conditions.
- (b) Steel plates with shell (S4R) elements, fully tied to concrete elements
- (c) Concrete infill modeled using shell (S4R) elements, boundary conditions included
- (d) Loading applied as uniformly distributed shear traction on the concrete shell element edges
- (e) Steel plates tied to concrete
- (f) Concrete surfaces tied to steel plates

# NONLINEAR ANALYSIS – SSH MODEL

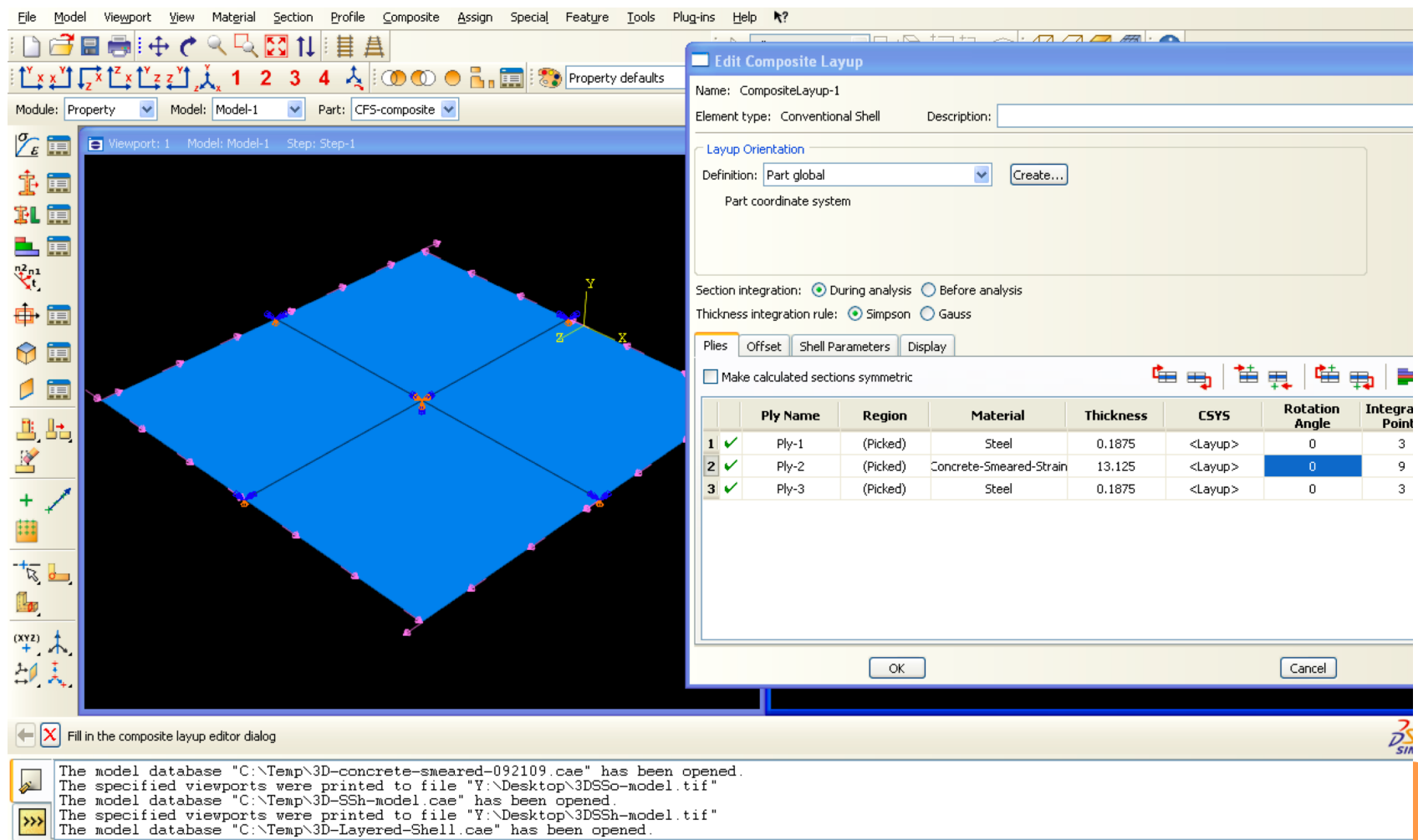


**Figure 5-13.** Results from SSH model with concrete smeared cracking model (tension stiffening stress-strain curve):

- Von Mises stresses in steel plate when shear force ratio ( $S_{xy}/A_s F_y$ )=1.0,
- Minimum principal stress in the concrete when shear force ratio=1.0,
- Maximum principal stress in the concrete when shear force ratio=1.0
- Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. shear strain response,
- Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. Von Mises stress in steel plate, and
- Shear force ratio vs. minimum and maximum principal stress in concrete.

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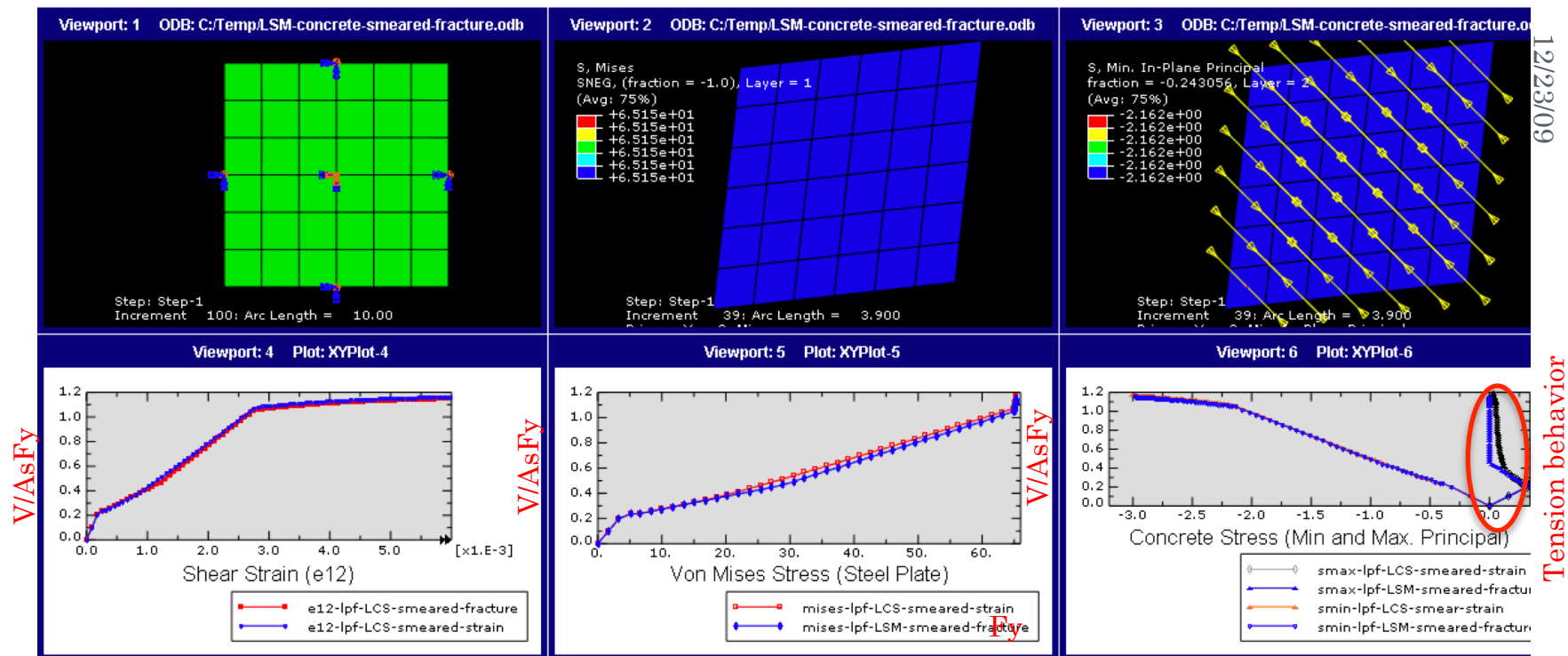
# NONLINEAR ANALYSIS – LCS MODEL



**Figure 5-15.** Layered Composite Shell (LCS) model with ply details, boundary conditions, and shell edge loading.

# NONLINEAR ANALYSIS – LCS MODELS

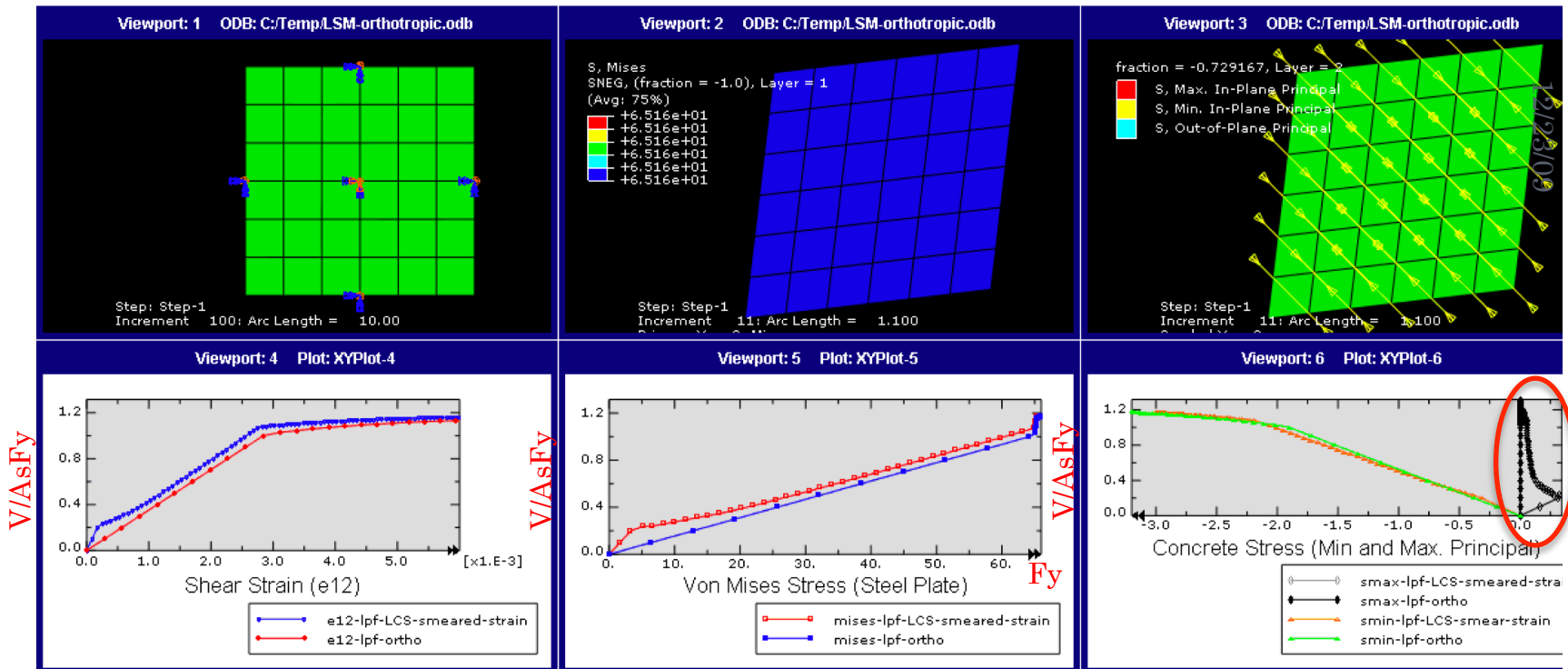
## TENSION STIFFENING AND FRACTURE ENERGY



**Figure 5-16.** Results from LCS model with concrete smeared cracking model (tension stiffening and fracture energy):

- Model with boundary conditions
- Deformed shape and Von Mises stresses in steel plate when shear force ratio ( $S_{xy}/A_s F_y$ )=1.0
- Minimum principal stress in the concrete when shear force ratio=1.0
- Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. shear strain response
- Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. Von Mises stress in steel plate
- Shear force ratio vs. minimum and maximum principal stress in concrete.

# NONLINEAR ANALYSIS – LCS MODEL WITH ORTHOTROPIC ELASTIC MODEL

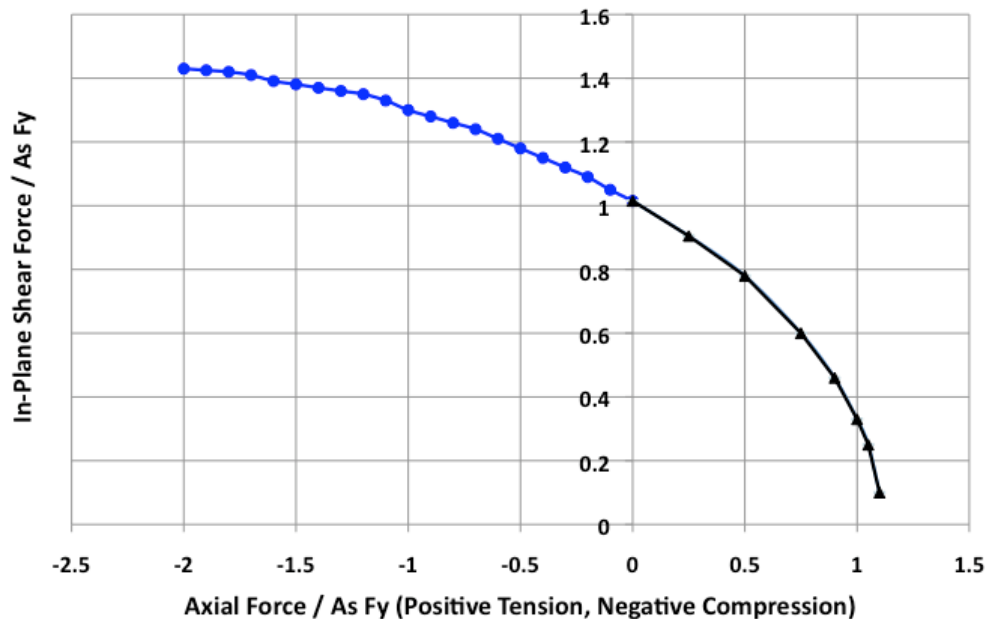


**Figure 5-18.** Results from LCS model with orthotropic elastic concrete model:

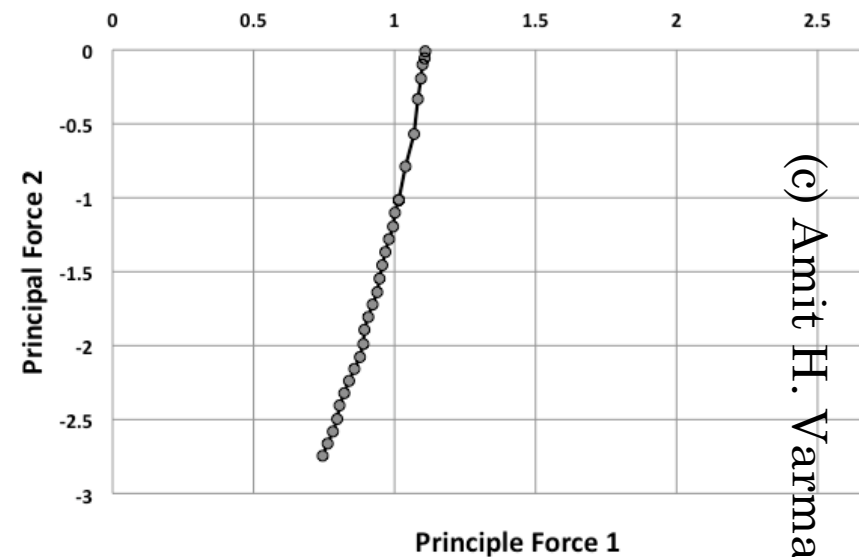
- Model with boundary conditions
- Deformed shape and Von Mises stresses in steel plate when shear force ratio ( $S_{xy}/A_s F_y$ )=1.0
- Minimum principal stress in the concrete when shear force ratio=1.0
- Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. shear strain response
- Shear force ratio ( $S_{xy}/A_s F_y$ ) vs. Von Mises stress in steel plate
- Shear force ratio vs. minimum and maximum principal stress in concrete.

# NONLINEAR ANALYSIS USING ANALYTICAL MODEL WITH ORTHOTROPIC CRACKED CONCRETE

Combined Membrane Forces ( $S_y$  and  $S_{xy}$ )



In-Plane Interaction in  $Sp1 - Sp2$  Space



$$\tan(2\theta_p) = \frac{2S_{xy}}{S_x - S_y}$$

$$S_{p1} = \frac{(1 + \cos(2\theta_p))}{2} S_x + \frac{(1 - \cos(2\theta_p))}{2} S_y + \sin(2\theta_p) S_{xy}$$

$$S_{p2} = \frac{(1 - \cos(2\theta_p))}{2} S_x + \frac{(1 + \cos(2\theta_p))}{2} S_y - \sin(2\theta_p) S_{xy}$$



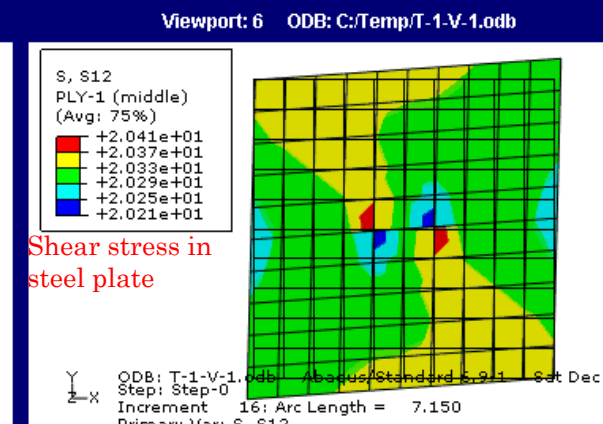
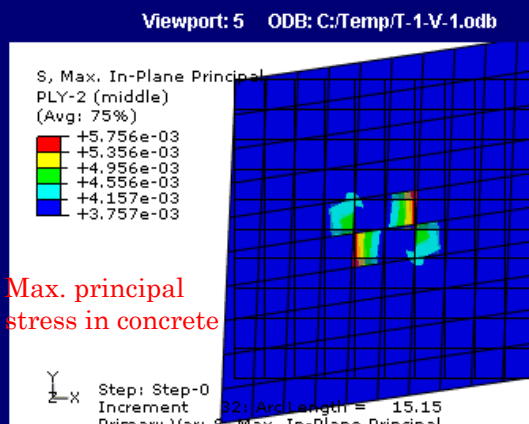
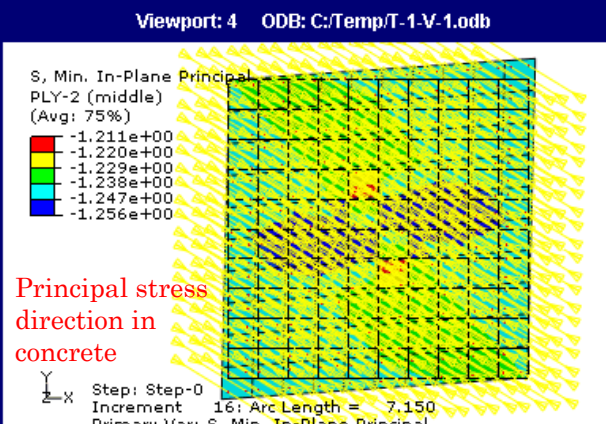
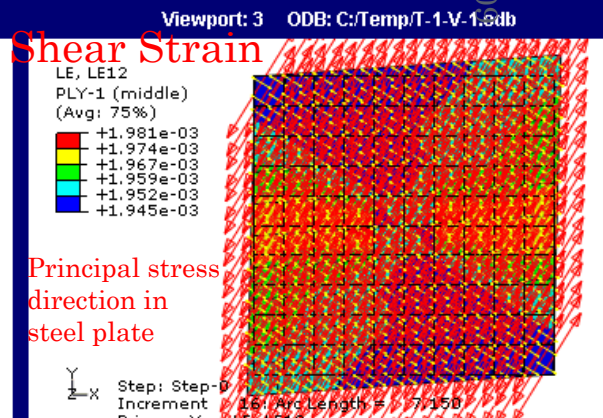
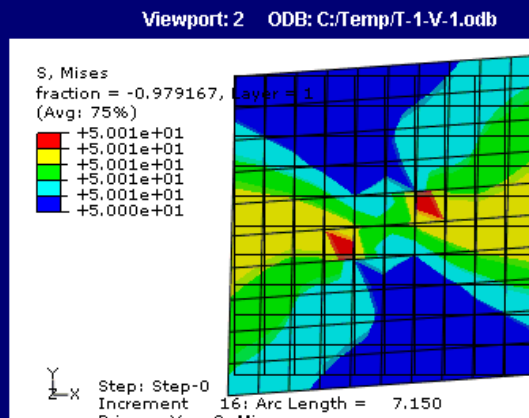
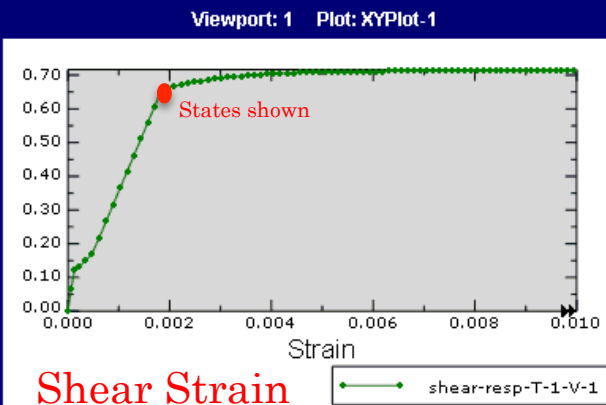
# FEM ANALYSIS FOR TENSION + SHEAR

Applied Tension = Shear =  $A_s F_y$ : Analysis using modified Riks method

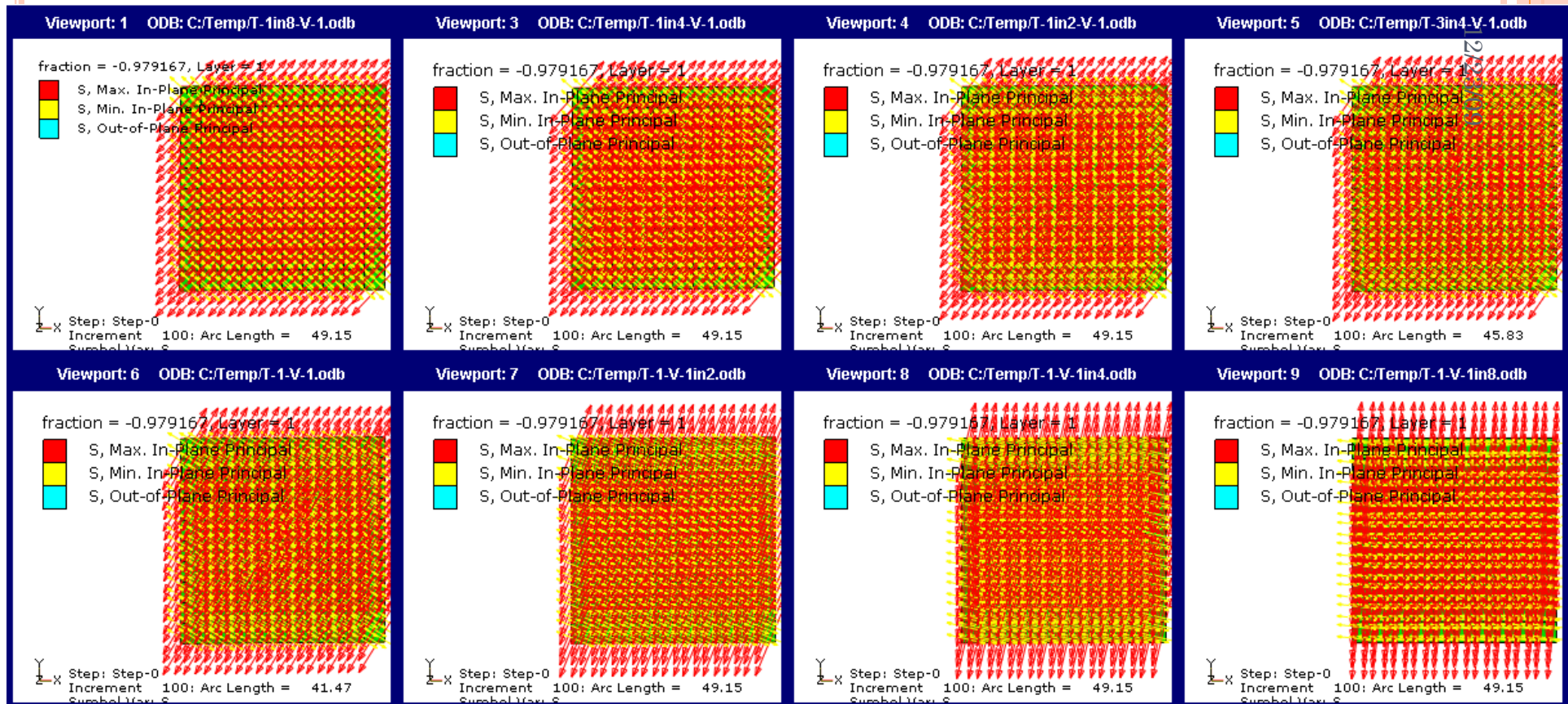
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Load Factor

Shear Strain

Principal stress  
direction in  
steel plateShear stress in  
steel plate

# FEM ANALYSIS FOR TENSION + SHEAR CRACK ORIENTATION FOR CASES (PROPORTIONAL LOADING)

 $T/V=1/8$ 
 $T/V=0.75$ 
 $T/V=0.5$ 
 $T/V=0.75$ 

 $T=V$ 
 $T=2V$ 
 $T=4V$ 
 $T=8V$ 

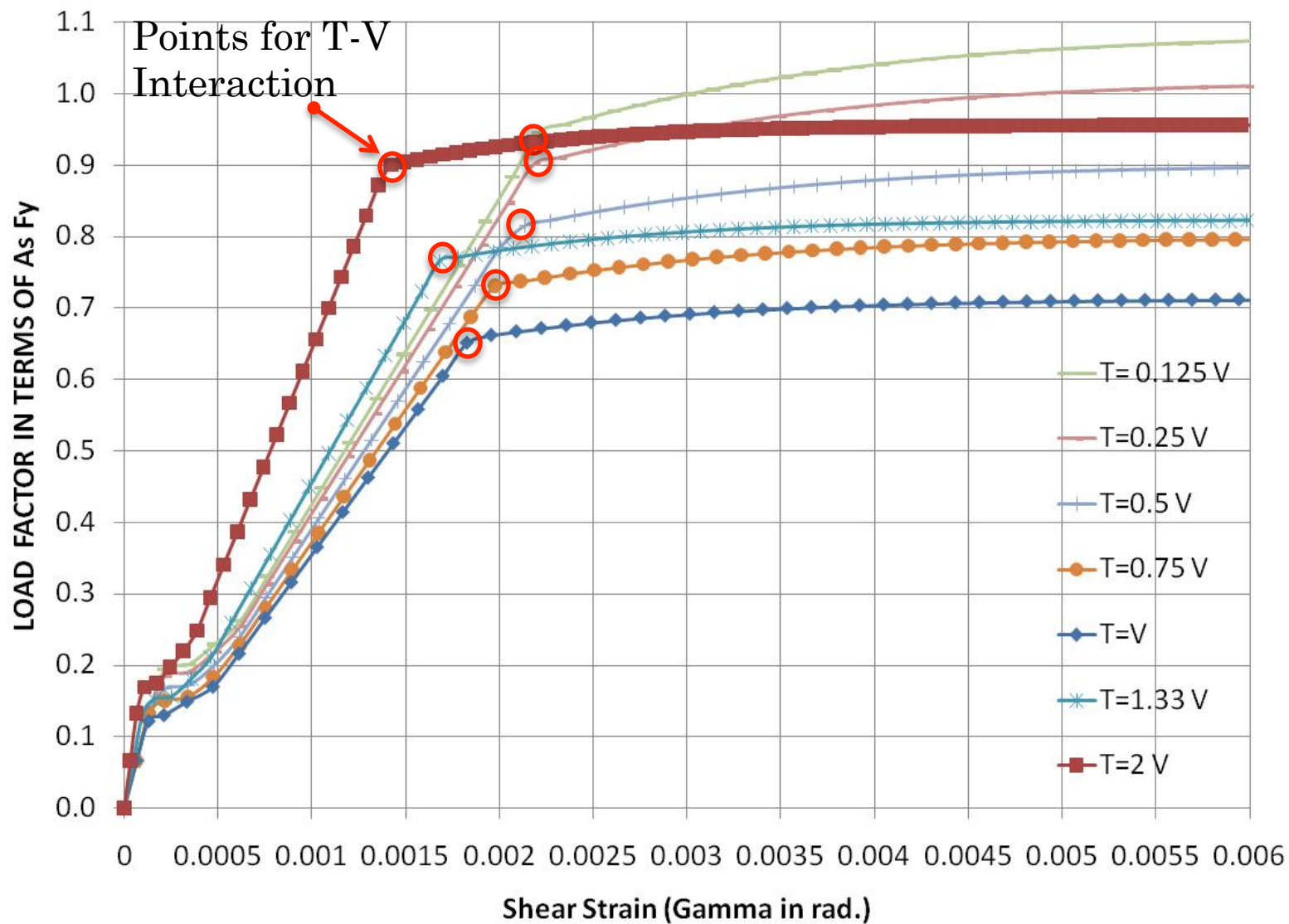
As shown, the crack orientation changes slowly with increasing  $T$ .

As  $T/V$  changes from 0 to 1, there is little change in crack orientation.

Crack orientation changes more rapidly as the  $T/V$  ratio increases from 1 to inf.



# RESULTS FROM FINITE ELEMENT ANALYSIS TENSION + SHEAR (PROPORTIONAL LOADING)



# RESULTS FROM TENSION + SHEAR ANALYSIS

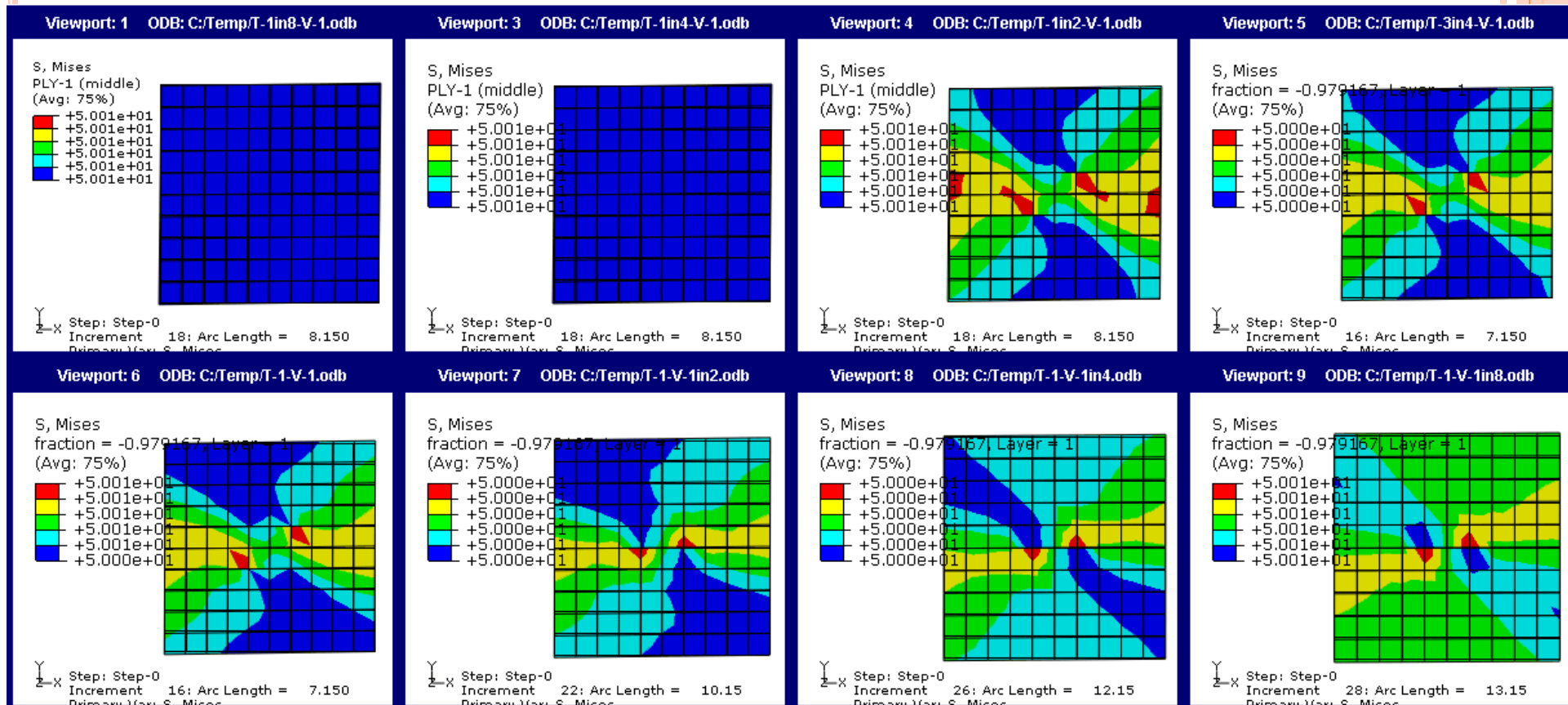
Von Mises stress in steel plates corresponding to the points of yielding i.e., turning points in the shear force-strain responses on previous slide

$T/V=1/8$

$T/V=0.75$

$T/V=0.5$

$T/V=0.75$



$T=V$

$T=2V$

$T=4V$

$T=8V$

# RESULTS FROM TENSION + SHEAR ANALYSIS

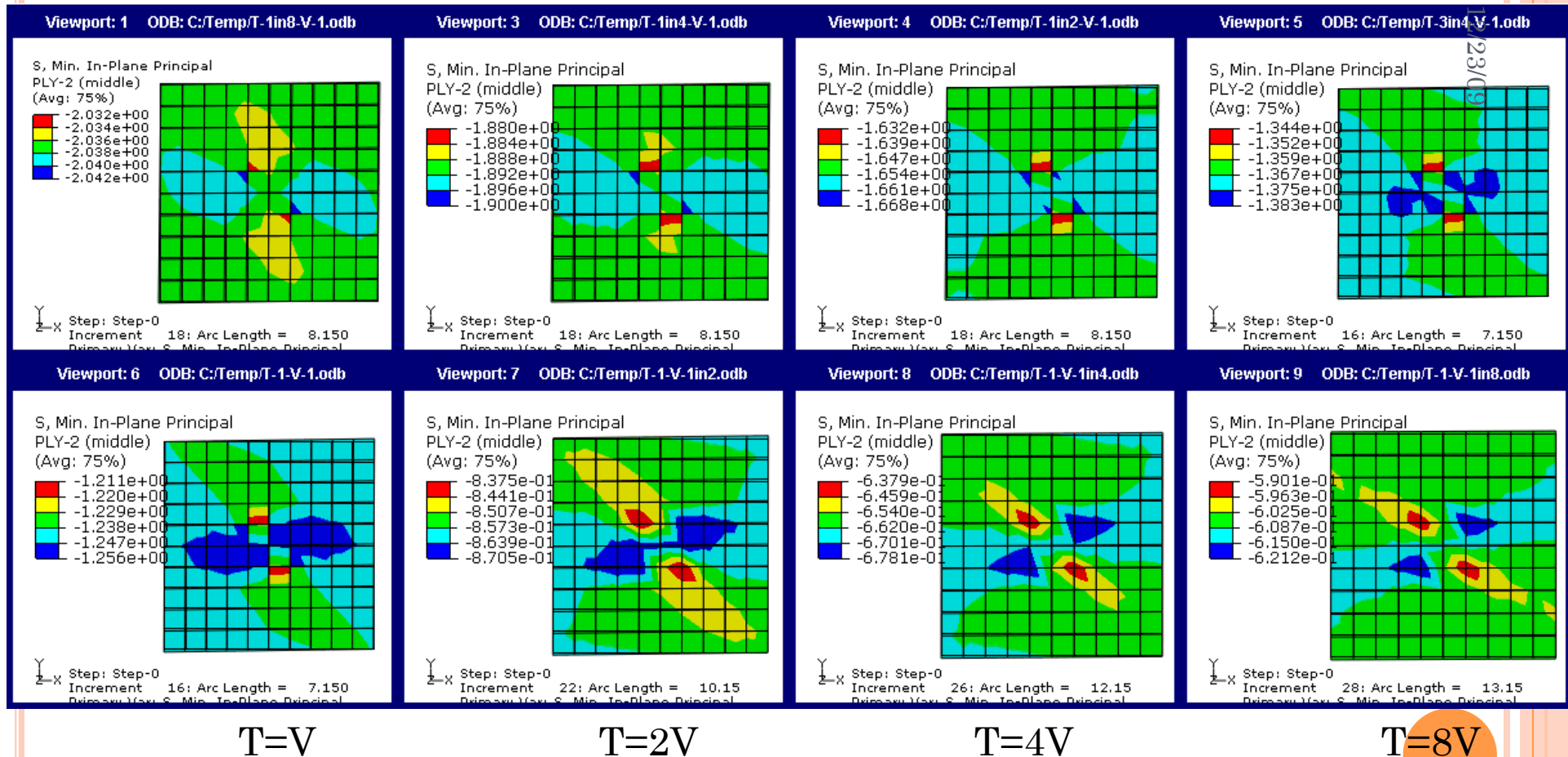
Minimum principal stress in concrete at the onset of yielding

$T/V=1/8$

$T/V=0.75$

$T/V=0.5$

$T/V=0.75$

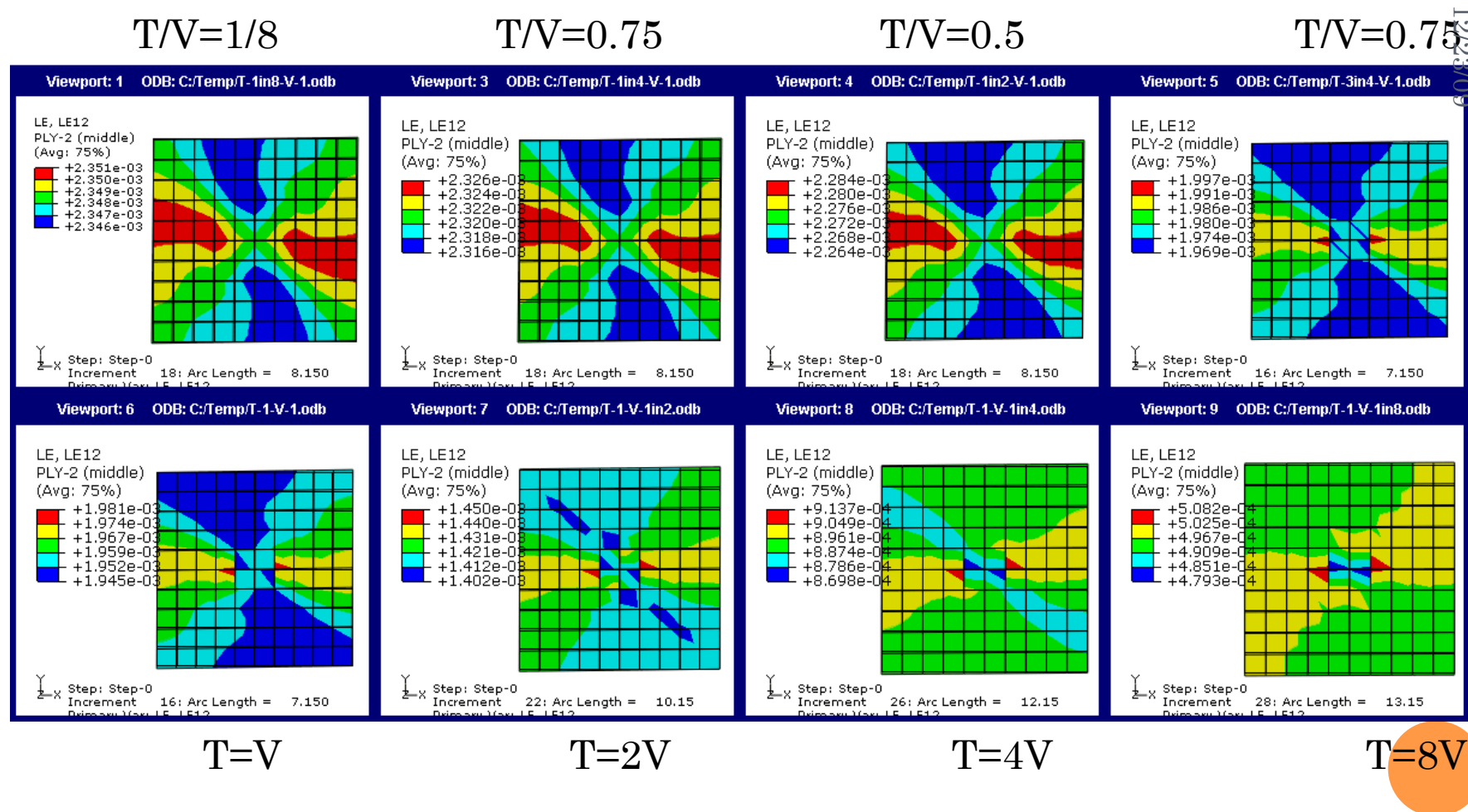


Concrete contribution (min. princ. stress) decreases as the  $T/V$  increases.

Concrete contribution is highest for the pure shear case.

# RESULTS FROM TENSION + SHEAR ANALYSIS

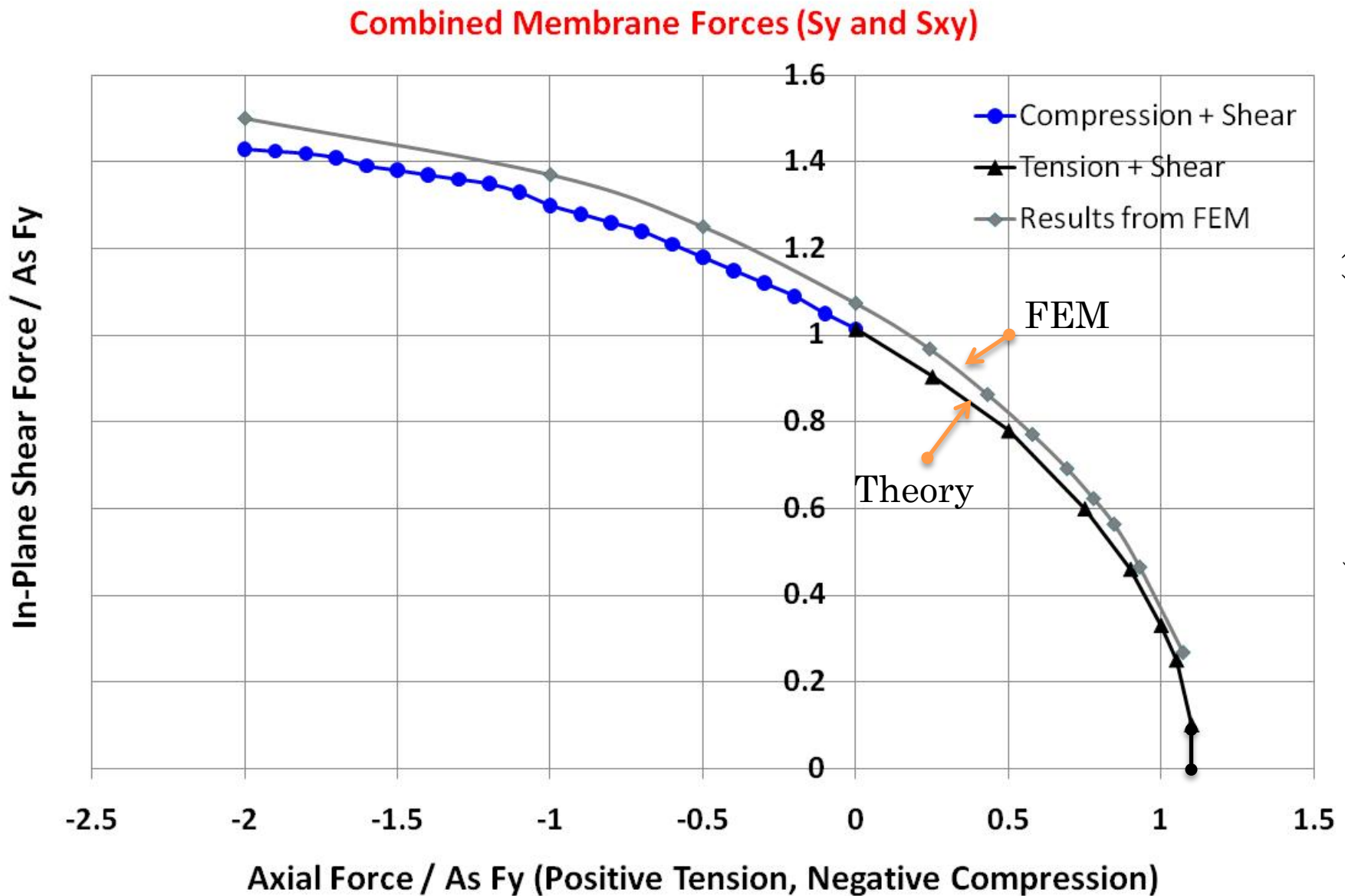
Shear strain at onset of yielding decreases as the T/V increases.  
Largest shear strain at yielding occurs for the pure shear case.



## SUMMARY FROM FEM ANALYSIS RESULTS

- The steel plates govern the behavior even more for tension + shear loading cases.
- The cracking direction does not change until there is significant tension ( $T/V$  greater than or equal to 1)
- The concrete contribution to the shear behavior decreases with increasing tension. It is maximum for the pure shear case (as compared to tension + shear).
- Compression + shear results in significant increase in the in-plane shear strength of the composite design.

# ANALYSIS FOR IN-PLANE FORCE



## RESULTS IN PRINCIPAL FORCE SPACE

- Take the LCS model with smeared cracking concrete model and fracture energy.
- Analyze for combinations of axial and shear force
- Take the results for  $S_y$  and  $S_{xy}$ , and convert into or calculate  $S_{p1}$  and  $S_{p2}$ .

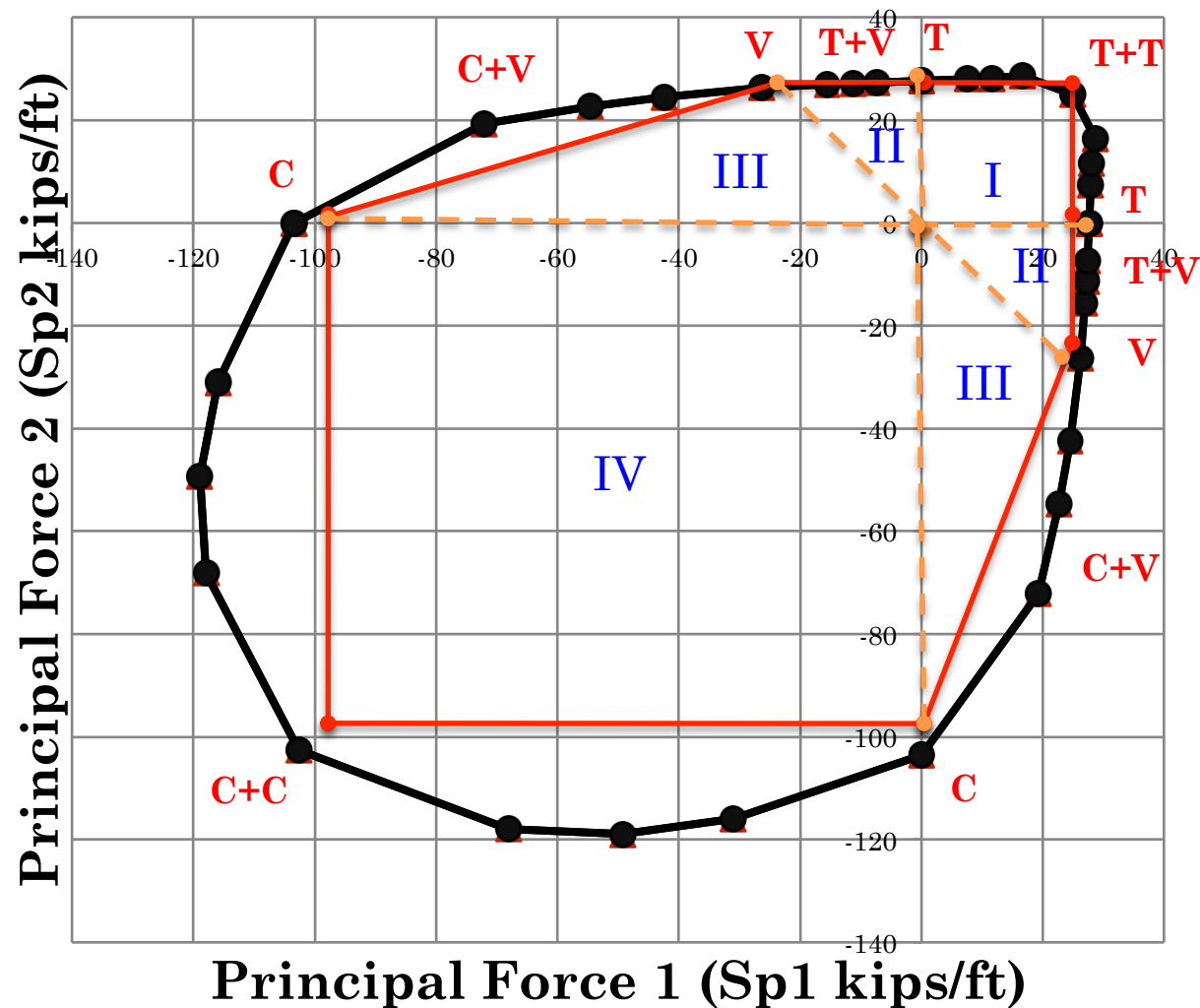
$$S_{p1,p2} = \frac{S_x + S_y}{2} \pm \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + (S_{xy})^2}$$

- The principal forces were plotted from the results.



# FAILURE SURFACE IN SP1-SP2 SPACE

## Failure Surface for In-Plane Forces in Principal Forces

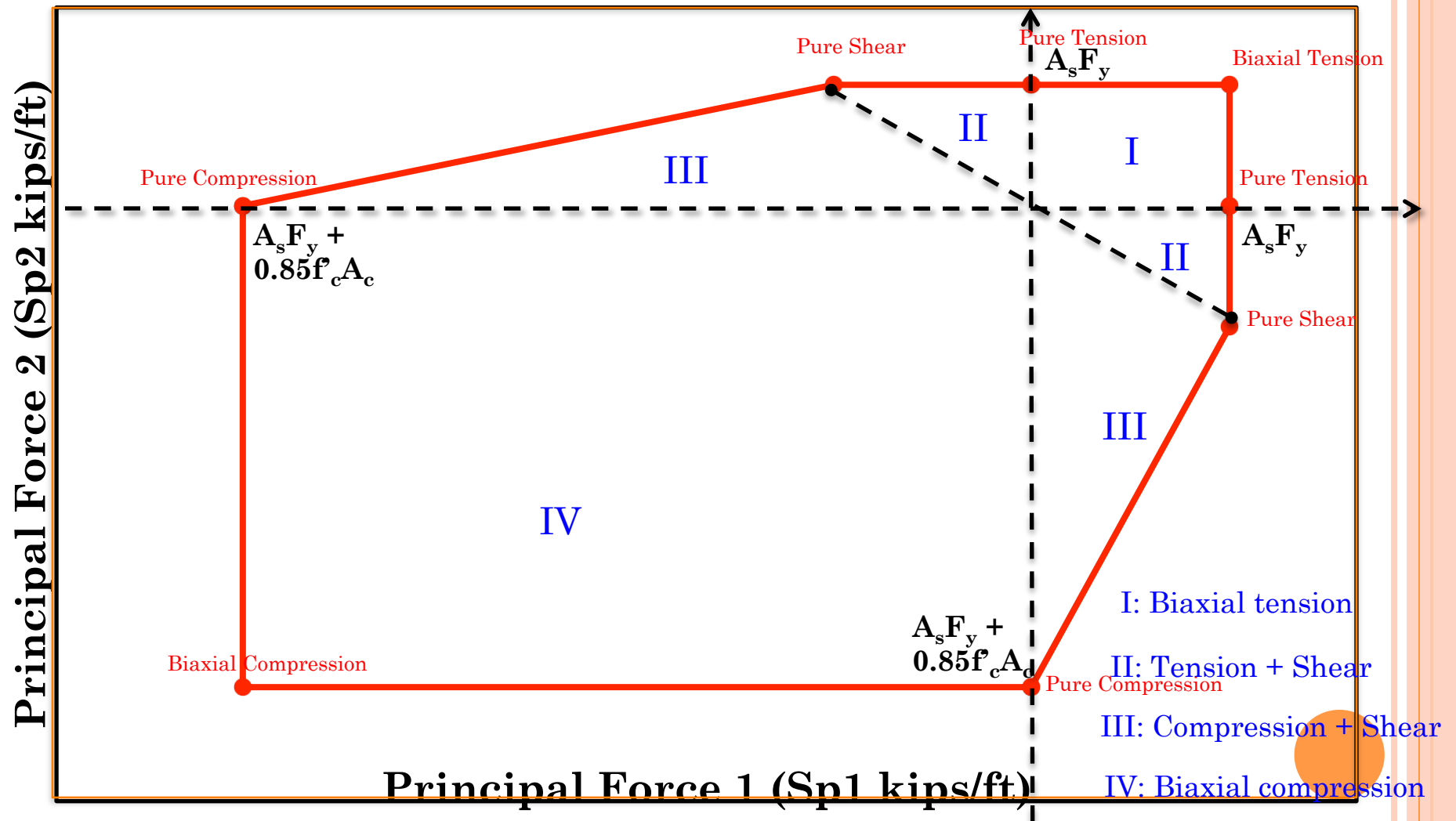


- I: Biaxial tension
- II: Tension + Shear
- III: Compression + Shear
- IV: Biaxial compression



# CAPACITY SURFACE IN PRINCIPAL FORCE SPACE

## Failure Surface for In-Plane Forces in Principal Forces



## DESIGN CAPACITIES

- The phi-factors should be representative of the failure mode (brittle and ductile)
- Creep and shrinkage effects can be included by using  $F_R$  (residual stress).
- The compression capacity can be reduced to include the effects of creep, shrinkage, local buckling (if any), and locked in stresses
- For example,  $A_s (F_y - F_r) + \beta f'_c A_c$



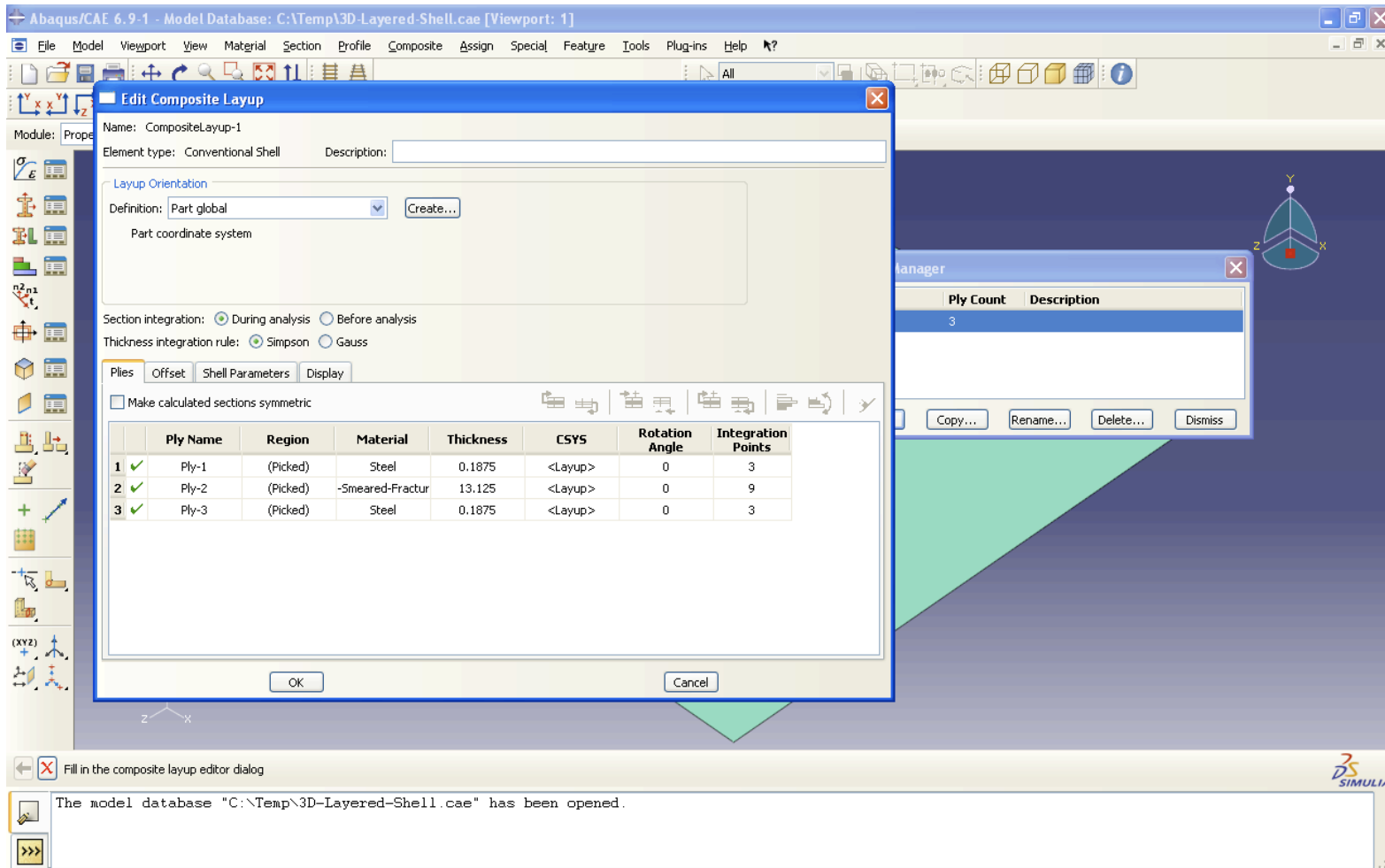
# **M- $V_{IN}$ INTERACTION**

## **DETAILS OF FEM MODELING**

**Out-of-plane Moment – In-Plane Shear Interaction**

## MODEL DETAILS

- Layered Composite Shell (LCS) model with nonlinear material models
- Model the steel-concrete composite plate with layered composite shell (LCS) elements
  - There are three plies (two steel plies at the top and bottom, and one concrete ply in between)
  - The steel plies have three integration points through the thickness
  - The concrete ply has nine integration points through the thickness

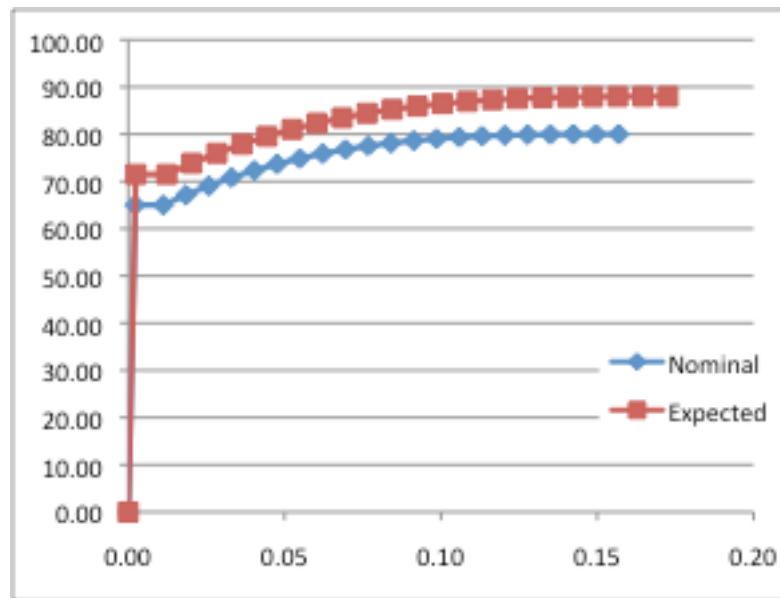


## MODEL DETAILS

- The model is for an SC structure with 3/8 in. (0.1875 in.) thick steel plates and 13.125 thick concrete infill.
- This is just a placeholder for now. The results will be no different for the full scale model.
- We have studied the relative thickness of the steel and concrete later.

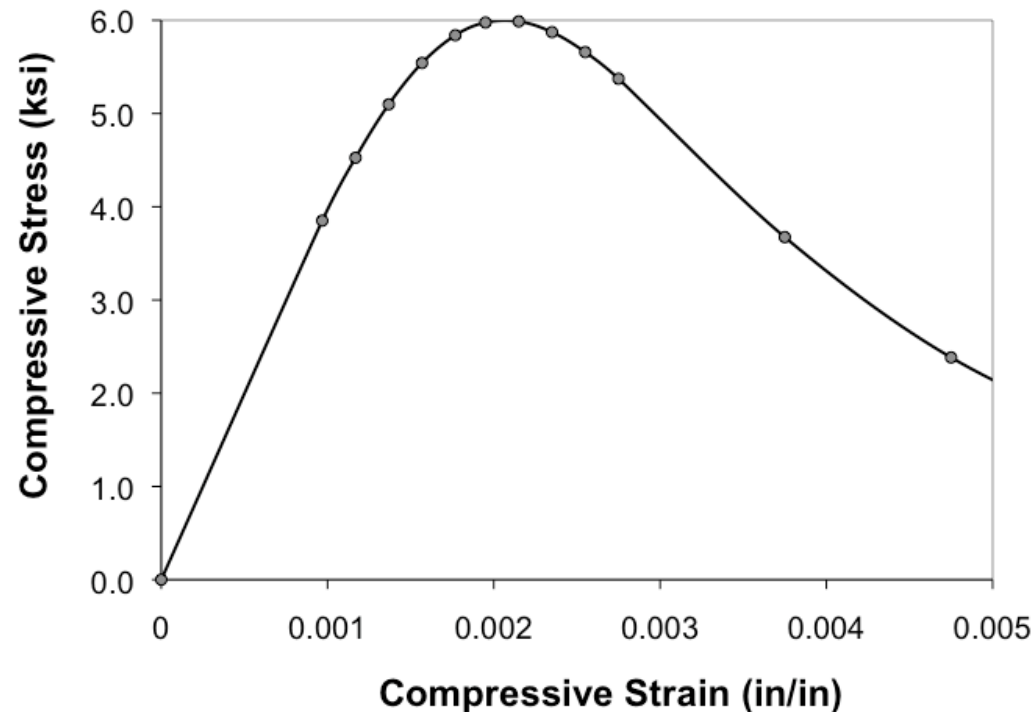
## MODEL DETAILS

- The steel plates are made from 65 ksi steel. The steel material model is shown below. Nominal properties were used.
- Von Mises yield with kinematic hardening, and associated flow.



## MODEL DETAILS

- The concrete was assumed to be 6 ksi nominal strength.
- The uniaxial compressive stress-strain is shown below.





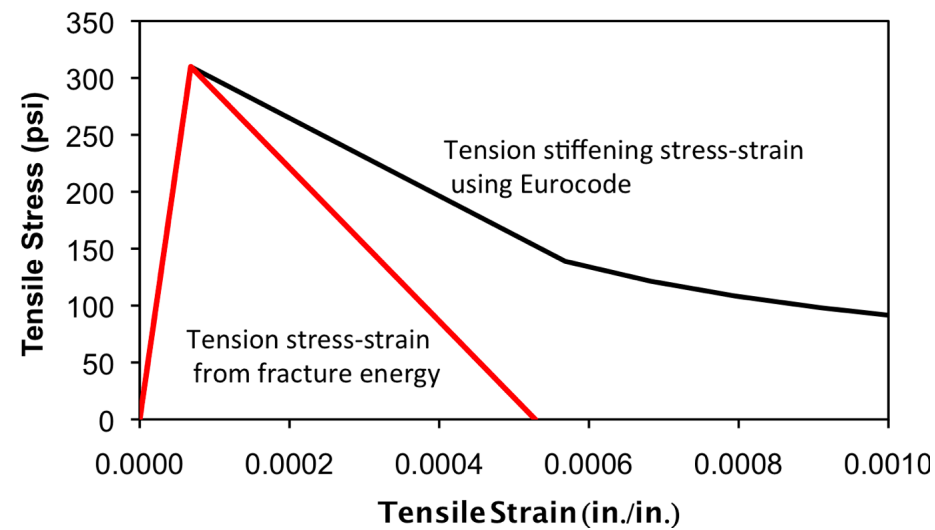
## MODEL DETAILS

- The failure ratios are as follows:
  - Ratio of biaxial compressive strength to uniaxial strength = 1.16 (Kupfer and Gerstle 1971)
  - Ratio of strain at biaxial strength to strain at uniaxial strength = 1.28 (Kupfer and Gerstle 1971)
  - Ratio of tension strength to compressive strength = 0.05 (based on  $4 \sqrt{f'_c}$ )
  - Ratio of tension strength when other principal stress is compressive (at  $f'_c$ ) to uniaxial tension strength = 0.333

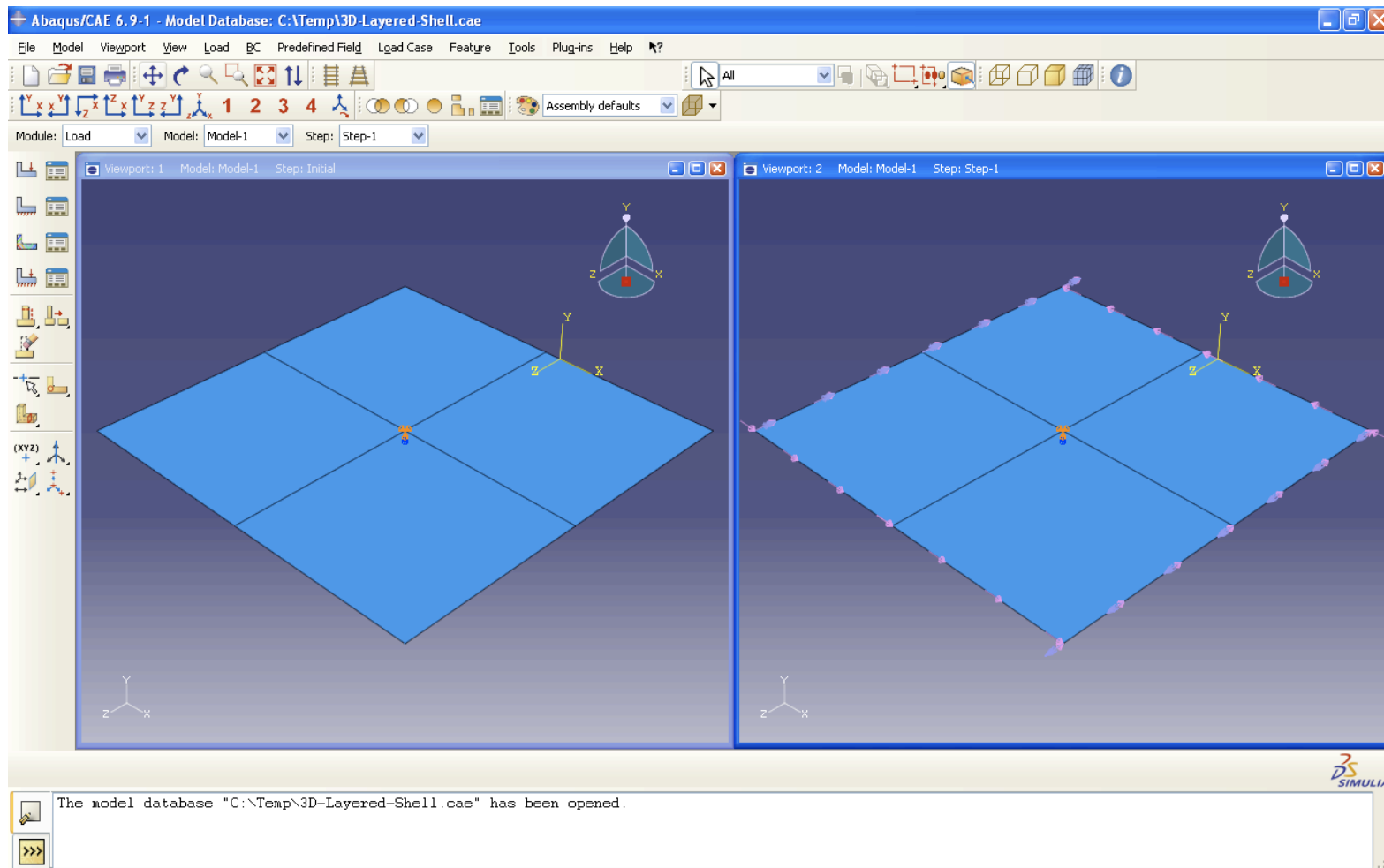
## MODEL DETAILS

- The tension smeared cracking model uses fracture energy principles
- The concrete fracture energy is estimated as = 0.717 lb/in.
- The related displacement corresponding to fracture energy =  $2 \times \text{Energy} / f'_t = 0.0046$
- The shear retention data are assumed to be as follows:
  - $\rho_{\text{close}} = 0.8$
  - strain corresponding to zero transfer ( $\rho_{\text{open}}$ ) = 0.01

# CONCRETE TENSION BEHAVIOR



# BOUNDARY CONDITIONS AND LOADING

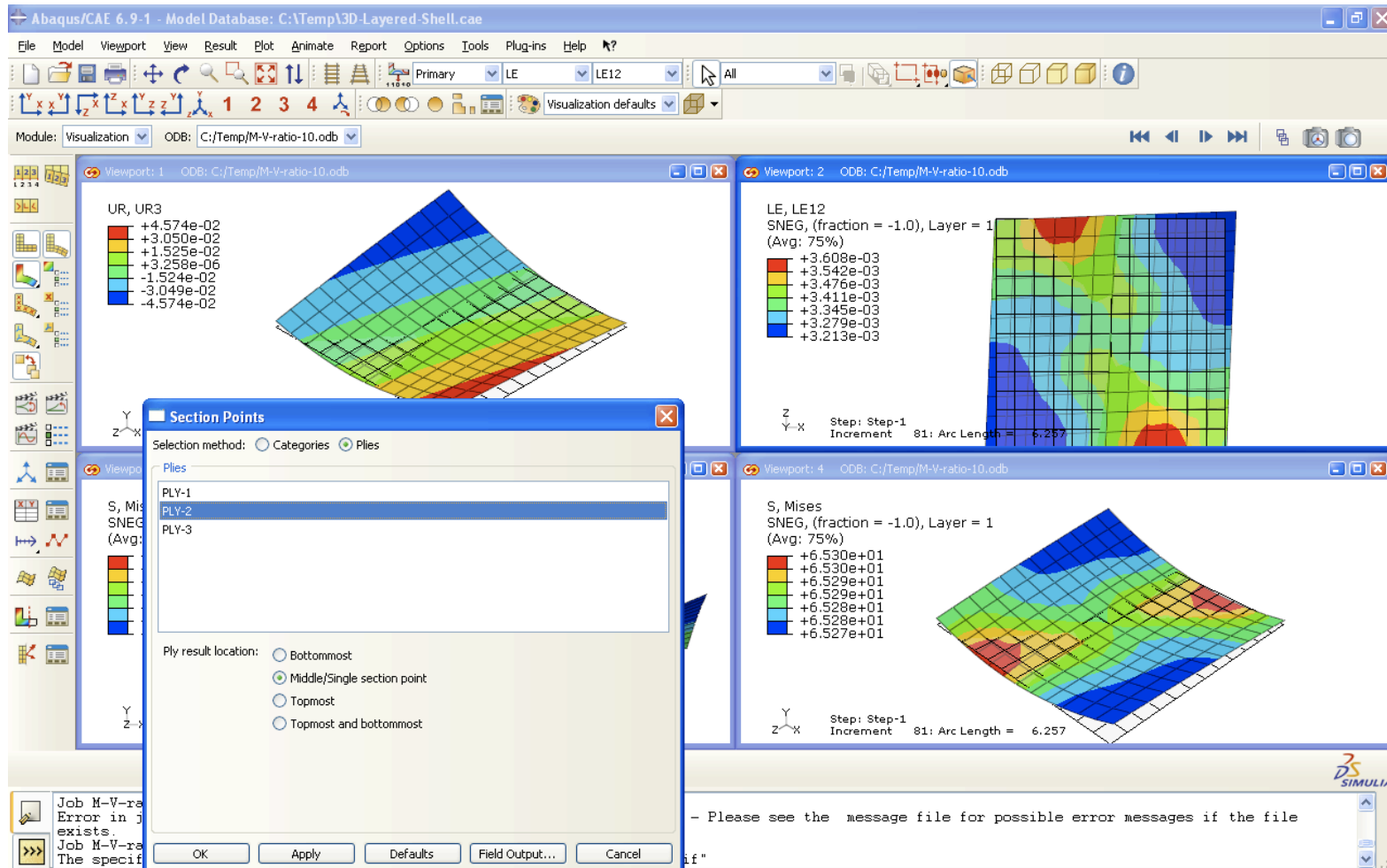


## BOUNDARY CONDITIONS AND LOADING

- The center node of the panel is restrained for U1, U2, U3, UR2. UR1 and UR3 are free
- The loads are applied as tractions to the shell edges.
  - In-plane shear forces are applied as 24.375 kips/in.
  - This value is equal to  $A_s F_y$
  - The out-of-plane moment is applied as 161.246 kip-in/in.
  - This value is equal to  $A_s F_y (T-t)$
- The analysis is done using Modified Riks. So the result is in the form of a load proportionality factor (LPF) that when applied by the multiplied the loads gives the load capacity.

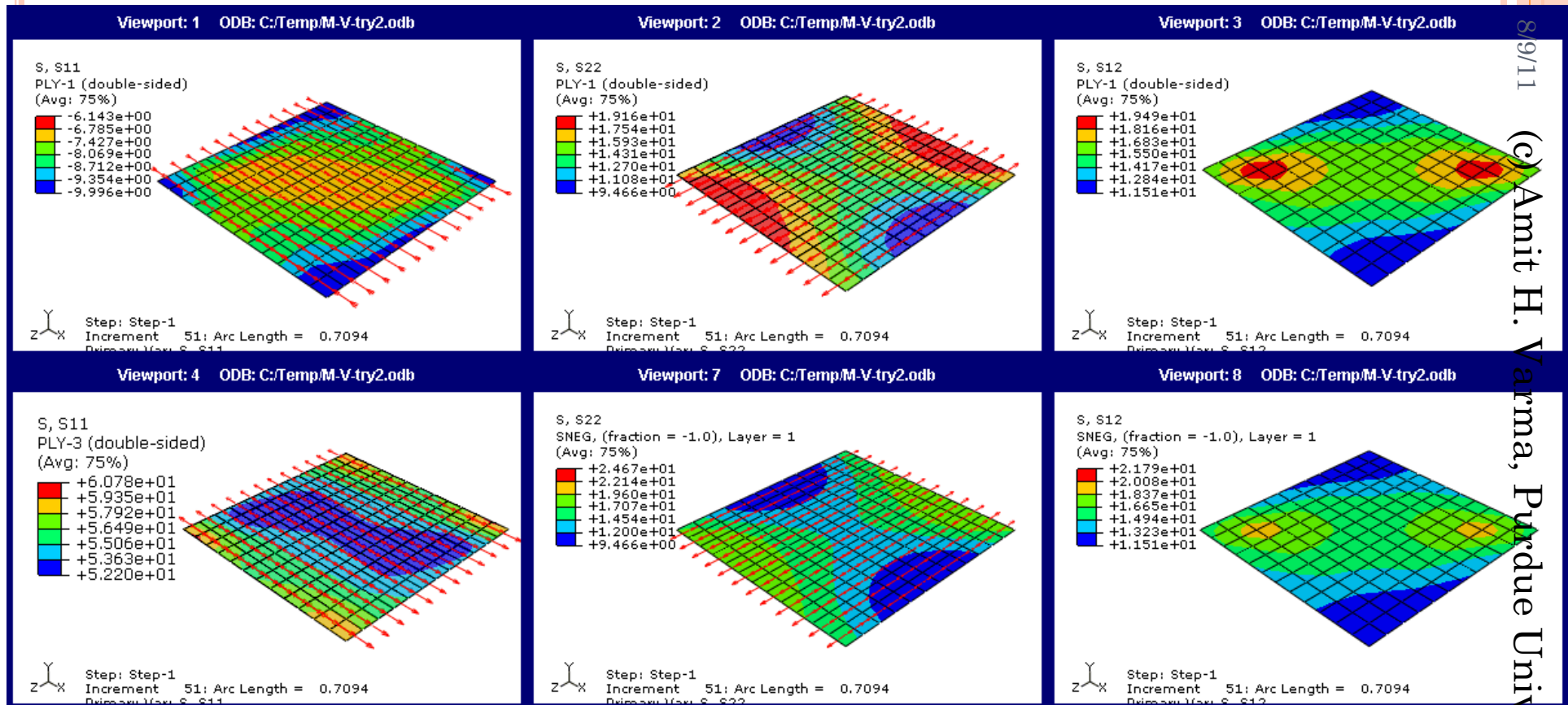
# DEFORMED SHAPES

## NOTE CURVATURE AND SHEAR STRAINS

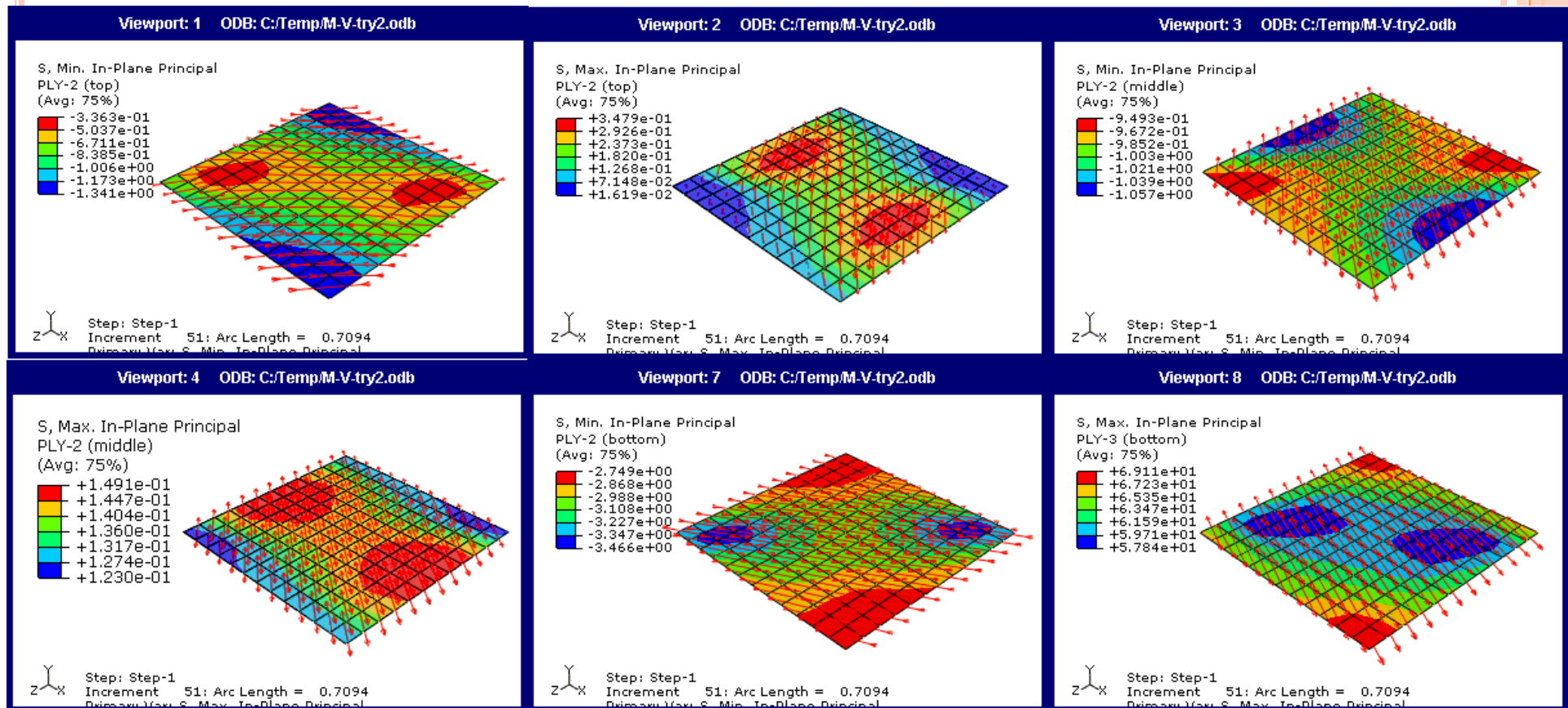


# RESPONSES FROM FEM ANALYSES

## STRESSES IN TOP AND BOTTOM STEEL PLATES



# RESULTS FROM FEM ANALYSES CONCRETE PRINCIPAL STRESSES (TOP AND BOT.) TO UNDERSTAND CRACK ORIENTATIONS ETC.





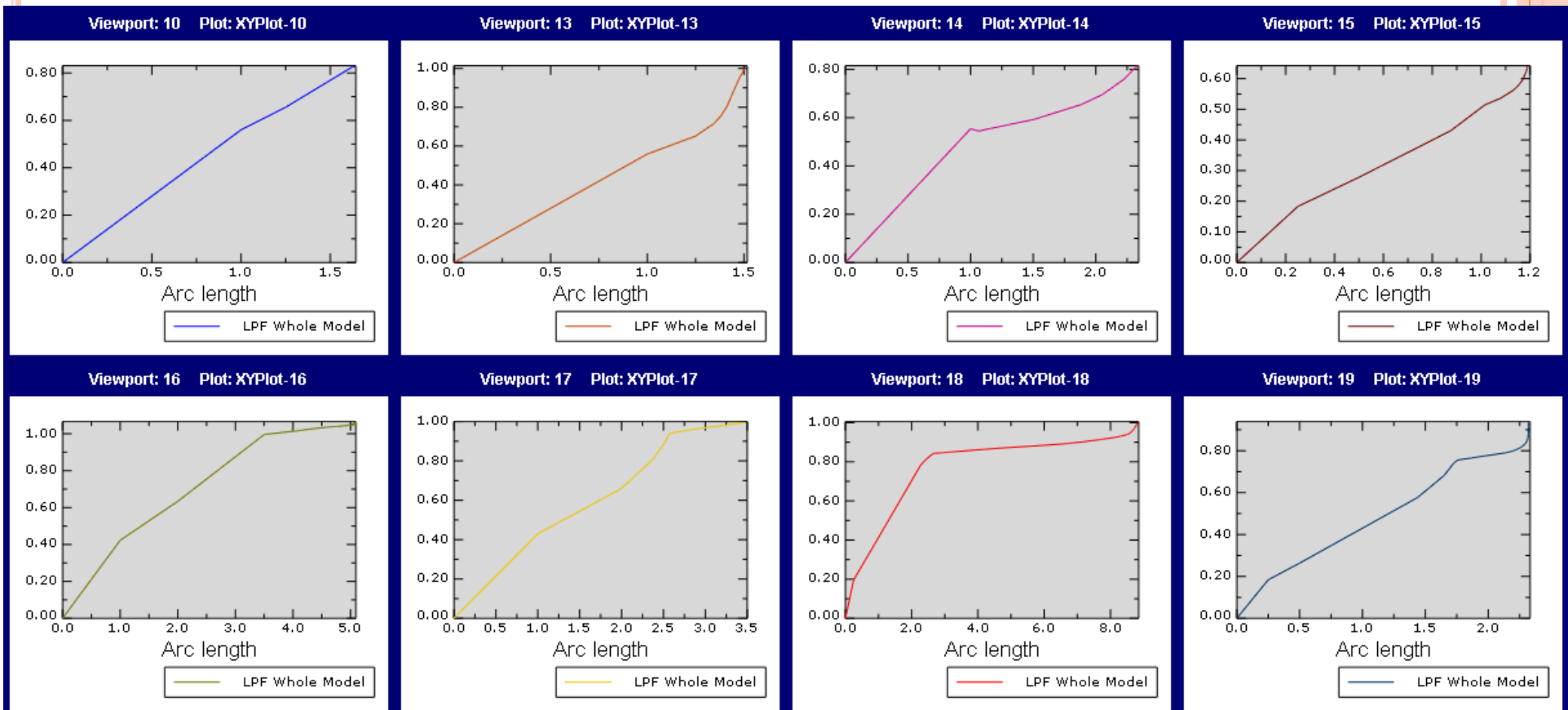
# RESULTS FROM FEM ANALYSES

$V=AsF_y/8$ ,  $M= M_p$

$V=AsF_y/4$ ,  $M= M_p$

$V=AsF_y/2$ ,  $M= M_p$

$V=3AsF_y/4$ ,  $M= M_p$



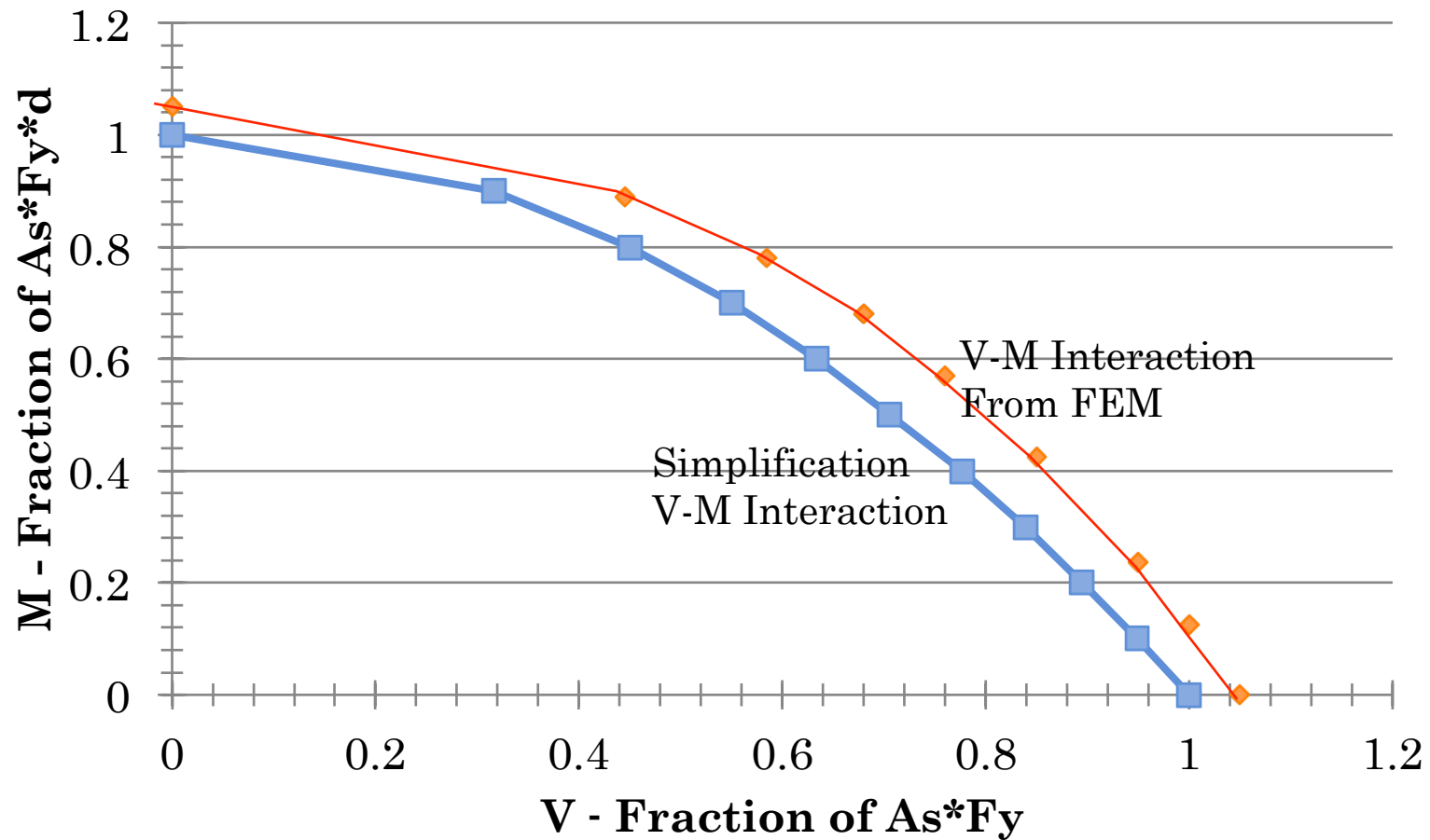
$V=AsF_y$ ,  $M= M_p/8$

$V=AsF_y$ ,  $M= M_p/4$

$V=AsF_y$ ,  $M= M_p/2$

$V=AsF_y$ ,  $M=3M_p/4$

# M-V INTERACTION



# SIMPLIFIED APPROACH

## COMBINED MEMBRANE AND FLEXURAL REINFORCEMENT IN PLATES AND SHELLS

By Ajaya K. Gupta,<sup>1</sup> M. ASCE

**ABSTRACT:** A plate or shell element subjected to membrane forces  $N_x$ ,  $N_y$ ,  $N_{xy}$  and bending moments  $M_x$ ,  $M_y$ ,  $M_{xy}$  is considered. Based on equilibrium considerations, equations for capacities of top and bottom reinforcements in two orthogonal directions have been derived. An iterative method is suggested for calculating the design capacities. The proposed equations are more general and rigorous than those derived for membrane reinforcement alone and those for

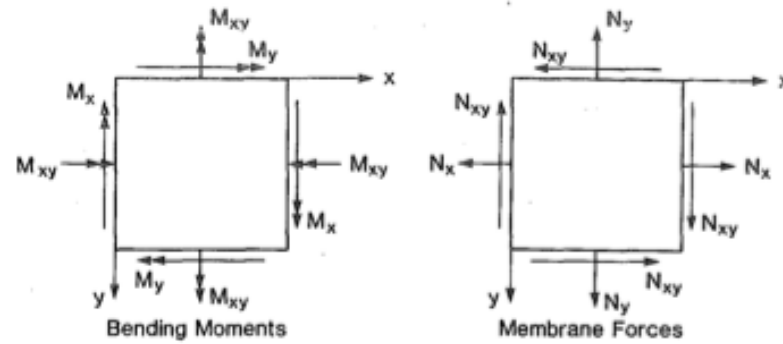
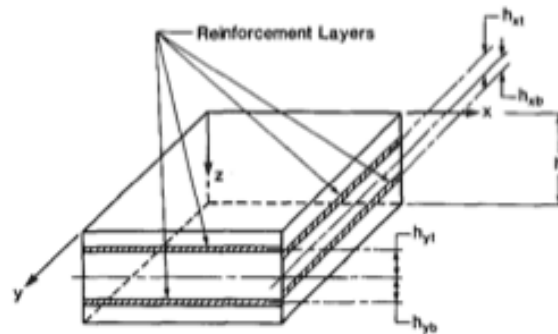
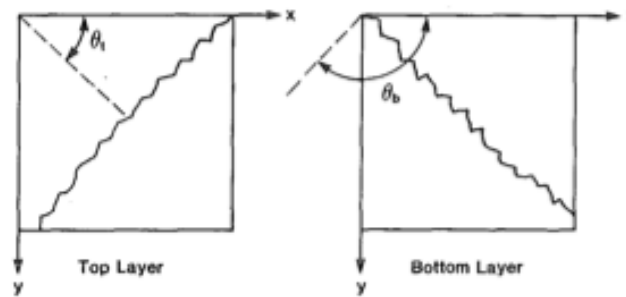


FIG. 1.—Applied Forces and Moments on an Element

and  $b$  stand for the top and bottom layers, respectively. A vertical plane of crack, whose normal makes an angle  $\theta_i$  with the  $x$ -axis in the  $xy$  plane, penetrates through the top surface. The concrete is under compression parallel to this crack; it is assumed that the depth of Whitney's stress block is  $a_i$ . The corresponding crack direction for the bottom surface is



(a) Shell Element Showing the Reinforcement Layers



(b) Crack Directions

FIG. 2.—Reinforcement Layers and Crack Directions

# SIMPLIFIED APPROACH

$$N_x^* = N_{xt}^* + N_{xb}^*; \quad N_y^* = N_{yt}^* + N_{yb}^* \dots\dots\dots (1)$$

$$M_x^* = -N_{xt}^* h_{xt} + N_{xb}^* h_{xb}; \quad M_y^* = -N_{yt}^* h_{yt} + N_{yb}^* h_{yb} \dots\dots\dots (2)$$

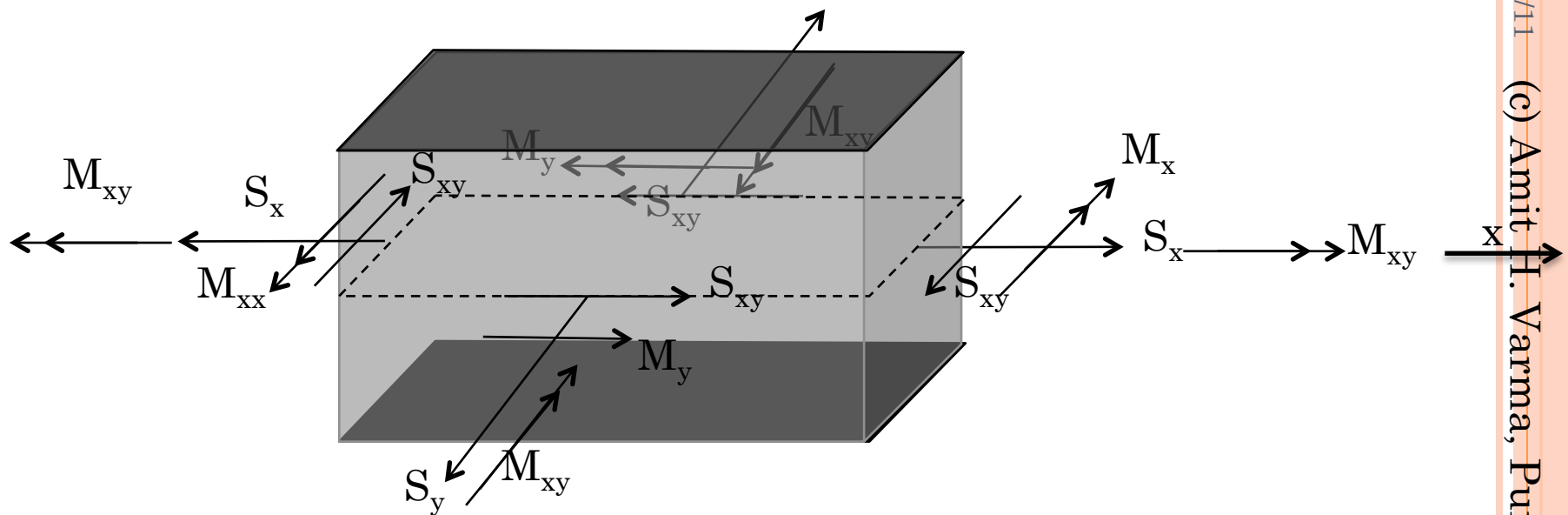
If the average compressive stress in concrete is  $f^c$ , the force and moment resultants of the top concrete block are

$$N_t^c = -a_t f_t^c; \quad M_t^c = -\frac{1}{2} (h - a_t) N_t^c \dots\dots\dots (3)$$

and for the bottom concrete block

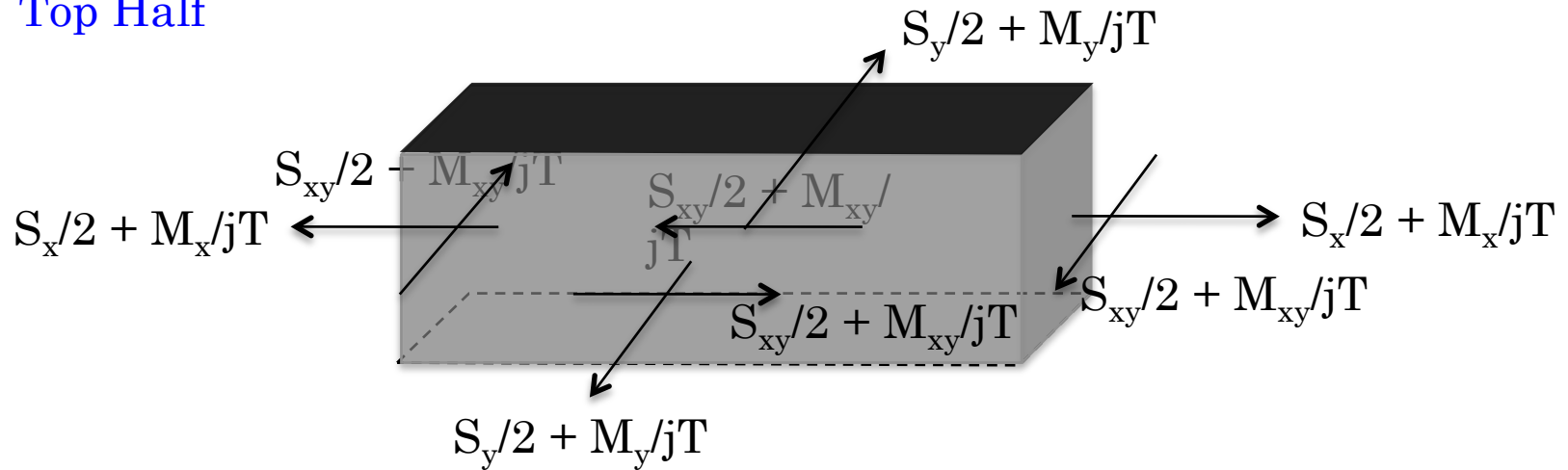
$$N_b^c = -a_b f_b^c; \quad M_b^c = \frac{1}{2} (h - a_b) N_b^c \dots\dots\dots (4)$$

# SIMPLIFIED APPROACH

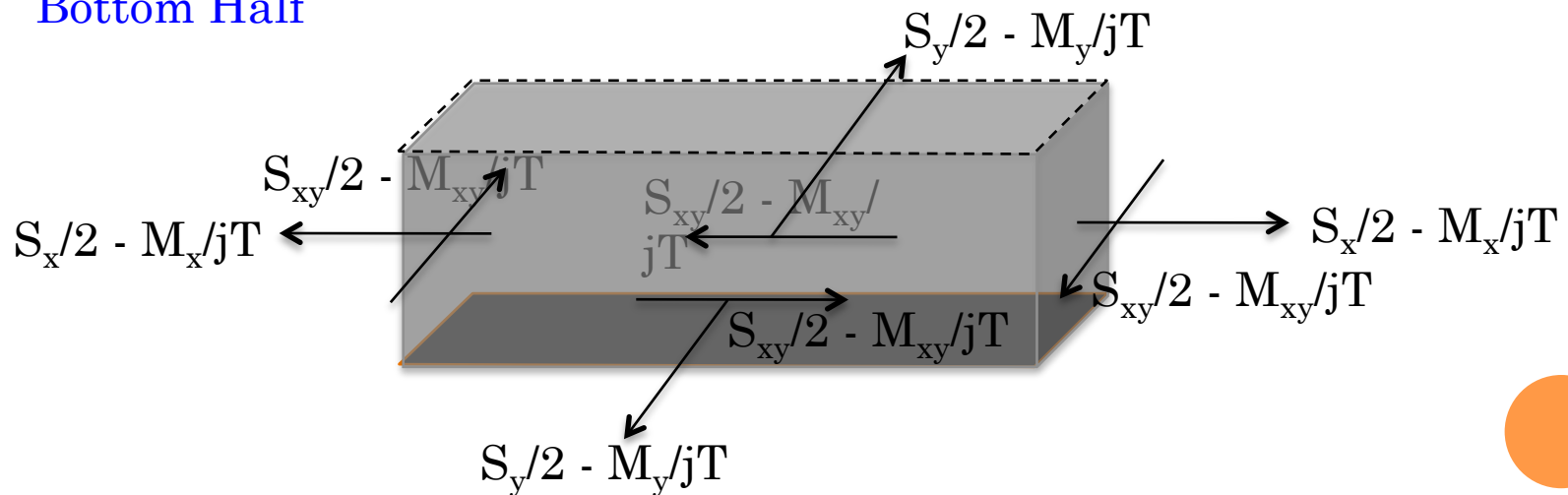


# TWO NOTIONAL HALVES

## Top Half

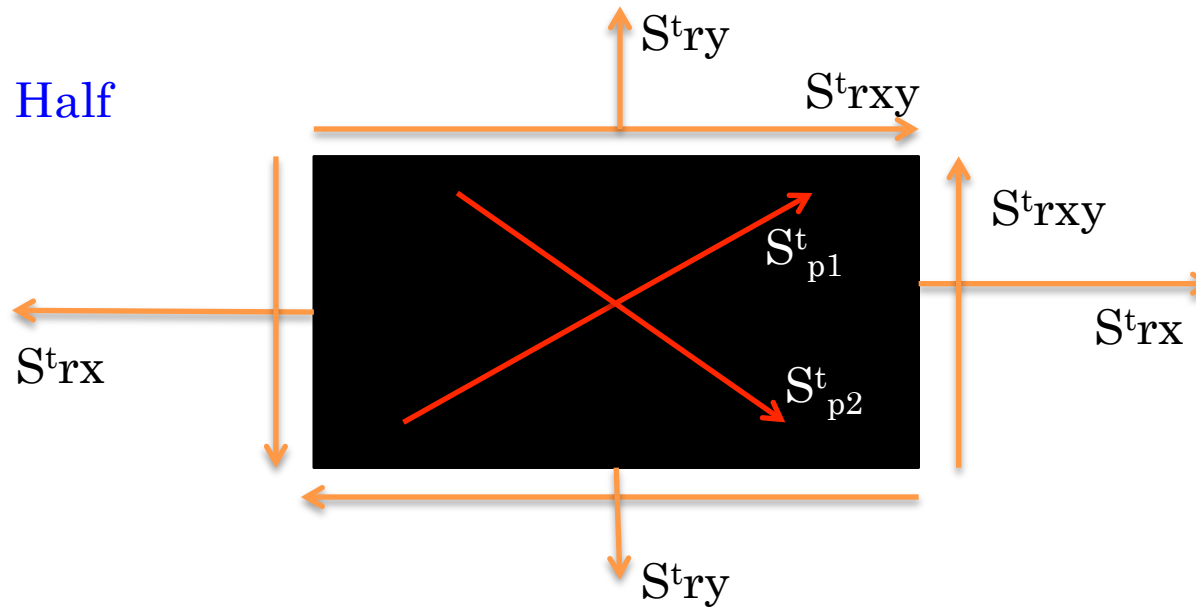


## Bottom Half

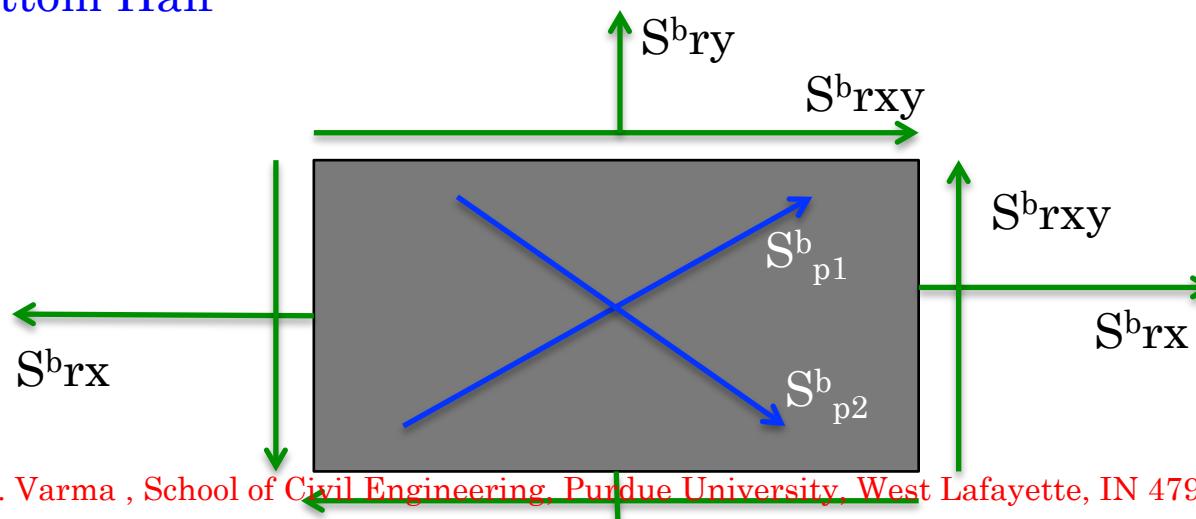


# TWO NOTIONAL HALVES (CHECK ON EACH)

Top Half



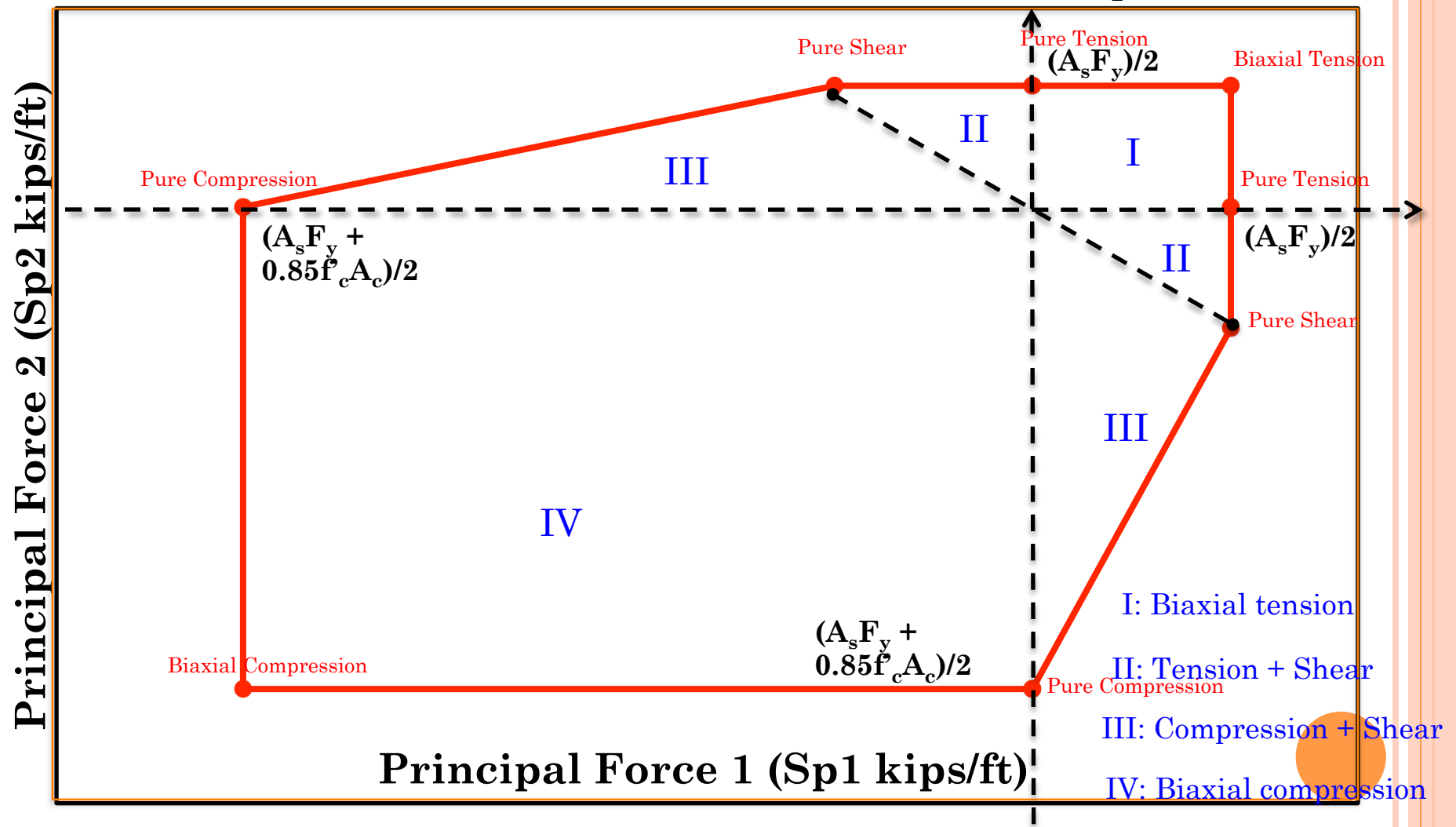
Bottom Half



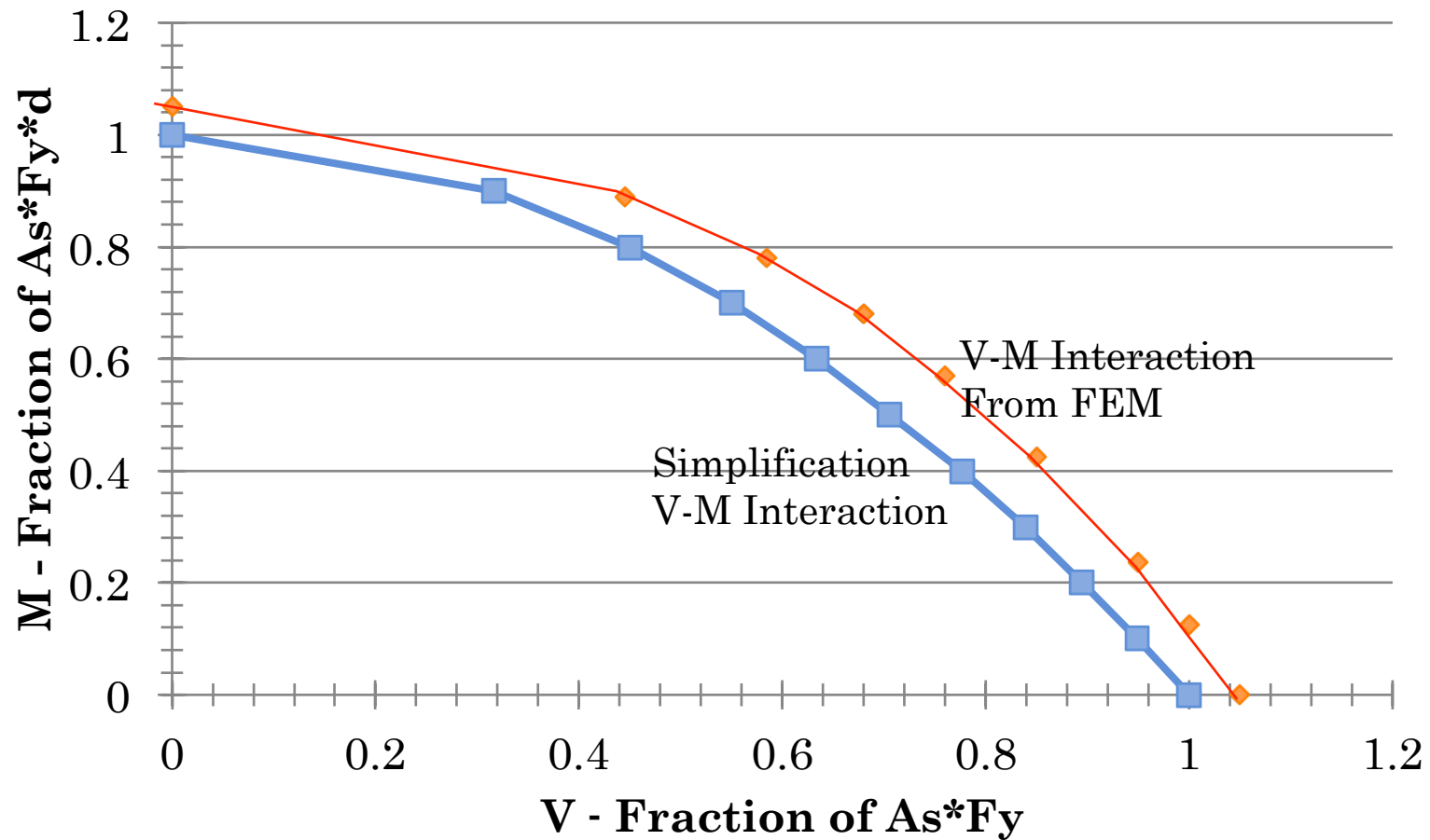


# INTERACTION SURFACE FOR EACH NOTIONAL HALF

## Failure Surface for In-Plane Forces in Principal Forces



# M-V INTERACTION



# INTERACTION EQUATIONS

$S'_{rx}$ ,  $S'_{ry}$ , and  $S'_{rxy}$  are the required in-plane membrane strengths per unit width calculated for each *notional half of the SC section* using Equations N9-H1-6 to N9-H1-8.

$$S'_{rx} = \frac{S_{rx}}{2} \pm \frac{M_{rx}}{\beta_x T} \quad (\text{N9-H1-6})$$

$$S'_{ry} = \frac{S_{ry}}{2} \pm \frac{M_{ry}}{\beta_y T} \quad (\text{N9-H1-7})$$

$$S'_{rxy} = \frac{S_{rxy}}{2} \pm \frac{M_{rxy}}{\beta_{xy} T} \quad (\text{N9-H1-8})$$

$S'_{rx}$  = *required membrane axial strength* in direction x for each notional half of SC section, kips/ft

$S'_{ry}$  = *required membrane axial strength* in direction y for each notional half of SC section, kips/ft

$S'_{rxy}$  = *required membrane in-plane shear strength* for each notional half of SC section, kips/ft

$\beta_i$  = parameter for distributing required flexural strengths ( $M_{rx}$ ,  $M_{ry}$ ,  $M_{rxy}$ ) into the corresponding membrane force couples acting on each notional half of SC section

$\beta_x$  = 0.9 if  $S_{rx} > -0.6P_{co}$       and       $\beta_x = 0.67$  if  $S_{rx} < -0.6P_{co}$

$\beta_y$  = 0.9 if  $S_{ry} > -0.6P_{co}$       and       $\beta_y = 0.67$  if  $S_{ry} < -0.6P_{co}$

$\beta_{xy}$  = 0.67

# INTERACTION EQUATIONS

$$S_{rp1,2} = \frac{S'_{rx} + S'_{ry}}{2} \pm \sqrt{\left(\frac{S'_{rx} - S'_{ry}}{2}\right)^2 + (S'_{rxy})^2} \quad (\text{N9-H1-5})$$

Range of $S_{\max}$ and $S_{\min}$	Interaction Equation	Equation Number
$S_{r,\max} \geq 0$ and $S_{r,\min} \geq 0$	$\frac{S_{r,\max}}{T_{ci}} \leq 1.0$	N9-H1-1
$S_{r,\min} < 0$ and $S_{r,\max} + S_{r,\min} \geq 0$	$\frac{S_{r,\max} + S_{r,\min}}{T_{ci}} - \frac{S_{r,\min}}{V_{ci}} \leq 1.0$	N9-H1-2
$S_{r,\max} > 0$ and $S_{r,\max} + S_{r,\min} < 0$	$\frac{S_{r,\max}}{V_{ci}} - \frac{S_{r,\max} + S_{r,\min}}{P_{ci}} \leq 1.0$	N9-H1-3
$S_{r,\max} \leq 0$ and $S_{r,\min} \leq 0$	$\frac{-S_{r,\min}}{P_{ci}} \leq 1.0$	N9-H1-4

Where,  $S_{r,\max}$  and  $S_{r,\min}$  are the larger and smaller of the two *required principal in-plane membrane strengths* ( $S_{rp1,2}$ ) per unit width for each *notional half of the SC section* calculated using Equation N9-H1-5.

# INTERACTION EQUATIONS

## **Additionally, In Equations N9-H1-1 to N9-H1-4:**

$T_{ci}$  = *available tensile strength* per unit width for each notional half of SC section, kips/ft

$V_{ci}$  = *available in-plane shear strength* per unit width for each notional half of SC section, kips/ft

$P_{ci}$  = *available compressive strength* per unit width for each notional half of SC section, kips/ft

## **For design according to N9-B.3 (LRFD)**

$T_{ci}$  =  $\phi_{ti} T_{ni}/2$ , where  $T_{ni}$  is calculated using the nominal tensile strength according to Section N9-D and  $\phi_{ti} = 0.95$

$V_{ci}$  =  $\phi_{vs} V_{ni}/2$ , where  $V_{ni}$  is calculated using the nominal in-plane shear strength according to Section N9-G and  $\phi_{ti} = 0.95$

$P_{ci}$  =  $\phi_{ci} P_{no}/2$ , where  $P_{no}$  is calculated using the nominal section compressive strength according to N9-E and  $\phi_{ci} = 0.80$



## Failure Surface for In-Plane Forces in Principal Forces

