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ESTIMATES OF MEAN CONSEQUENCES AND CONFIDENCE BOUNDS ON THE MEAN ASSOCIATED WITH LOW-PROBABILITY SEISMIC EVENTS IN TOTAL SYSTEM PERFORMANCE ASSESSMENTS

Osvaldo Pensado and James Mancillas
Center for Nuclear Waste Regulatory Analyses (CNWRA)
Southwest Research Institute®, San Antonio, Texas

ABSTRACT

An approach is described to estimate mean consequences and confidence bounds on the mean of seismic events with low probability of breaching components of the engineered barrier system. The approach is aimed at complementing total system performance assessment models used to understand consequences of scenarios leading to radionuclide releases in geologic nuclear waste repository systems. The objective is to develop an efficient approach to estimate mean consequences associated with seismic events of low probability, employing data from a performance assessment model with a modest number of Monte Carlo realizations. The derived equations and formulas were tested with results from a specific performance assessment model. The derived equations appear to be one method to estimate mean consequences without having to use a large number of realizations.

INTRODUCTION

Regulations for the disposal of high-level radioactive waste in the United States require that a mean dose to members of the public does not exceed a prescribed value. Nuclear waste could be isolated from the environment via natural and engineered barriers. Such barriers could include an engineered barrier system (waste forms, cladding, waste packages (WPs), drip shields, and drifts) and geologic formations [1]. A possible mechanism for early breaching of WPs intended to isolate radioactive waste from the environment is drift degradation and seismic events of high magnitude (i.e., low recurrence rate). Estimates of radionuclide release and dose consequences of WP failure can be obtained using total system performance assessments where a large number of variables affecting the potential repository performance are evaluated using a Monte Carlo approach. Due to the complexity of the overall total system model (total system models generally include engineered barrier system degradation, source term, contaminant transport, and dose models), only a few hundred to few thousand Monte Carlo realizations are usually executed. The relatively small number of realizations constrains the statistical confidence of the consequence analysis of low probability events (infrequently sampled in Monte Carlo approaches). In this paper, consequences of low recurrence rate seismic events are estimated by increasing the frequency of events causing WP failure and scaling down the mean consequences. Formulas are presented for appropriate scaling factors and to estimate a standard deviation and confidence bounds on mean consequences for particular kinds of performance assessment models (i.e., where the magnitude of the radionuclide release rate per WP is independent, or quasi-independent, of the process causing WP failure).

PROBLEM DEFINITION

Seismic events could breach components of the engineered barrier system. Breaching scenarios and mechanisms for the considered repository system in the United States have been discussed elsewhere [2–5] and will not be described in this paper. In this paper, it is assumed that the number of seismic events in a period T follows a Poisson distribution and that events of a recurrence rate higher than λ do not cause WP damage. If an event of a recurrence rate less than λ occurs, it is assumed there is a probability p_e that such an event will cause WP breaching. Typical considered values of λ are of the order of 10^{-4} 1/yr [2, 4–7]. As a first approximation, the probability p_e is assumed independent of the time of the event. In the system described by Ibarra et al. [2], assuming a constant value of p_e is equivalent to ignoring transient states of drift degradation, rubble consolidation, and accumulated WP damage. In other words, the probability p_e

that an event (of recurrence rate less or equal than λ) could cause WP breach is computed, using the Ibarra et al. conceptual model, as a function of the system stochastic parameters (e.g., material properties, contact areas between drip shield and WP structures, transferred loads, and seismic event intensity) assuming full drift degradation and rubble consolidation. Changes in the repository temperature could also introduce time dependencies in p_e as material properties, such as strength and stiffness of engineered materials, are commonly temperature dependent. However, because the interest is in long-term estimates, when repository temperatures are stable, material property changes are ignored in the analysis.

A total system performance assessment model is used to assess consequences of events leading to WP breach. A *realization* of the model is a simulation of the repository performance for a period of duration T, with a particular sample of stochastic parameters. In other words, a realization is an estimate of repository performance in time T. Monte Carlo statistics are commonly computed on multiple realization data to derive mean consequences and confidence bounds. In the performance assessment (PA) model considered in this analysis, consequences (radionuclide release rate or dose) per breached WP are assumed independent of the magnitude of the event causing the breach. In other words, the PA model disregards the severity of the WP breaching (and associated radionuclide release rates) as a function of the event intensity. In the PA model, WPs are only in two states: breached or not breached. Once a WP is breached, it is considered that all WPs in a region (referred to as *repository subarea* or simply *subarea*) are simultaneously breached. Therefore, in the PA model for this analysis, only the first event leading to WP breaching is considered. As defined in the PA model, consequences are a function of the WP breach time distribution and of stochastic parameters controlling radionuclide release rates and doses, but not directly a function of the distribution of event intensities. For example, radionuclide release rates leaving the engineered barrier system are computed as a function of water flow rates, waste form dissolution rates, solubilities of radionuclide-bearing phases, availability of colloids to carry radionuclides, and temperature.

In a set of R realizations of the PA model, only a subset (set with N_a realizations) will exhibit events leading to WP breach in a subarea (see Figure 1). For the sake of simplicity, the symbols R and N_a are used to represent sets and their number of elements, indistinctly. The subindex a in N_a highlights that the discussion applies to one subarea only. The size of the subset N_a is a monotonically increasing function of the probability p_e (the precise functional dependence is derived later). With the assumptions of the PA model, mean consequences per breached WP in the subset N_a are independent of the size of the subset, provided that the distribution of WP breach times $\{t_i, i=1,2, \ldots, N_a\}$ is a weak function of p_e (value controlling the size of the set N_a). It is straightforward to derive the statistics for the set R from the N_a subset statistics, because realizations in the complement set, R–N_a, have zero consequence as there is no WP breach in such a set. If N_a is small compared to R, it is difficult to compute converging statistics for the R set using standard Monte Carlo methods. The reason is that if N_a is small, there is no dense coverage of sampled stochastic parameters controlling release and dose consequences in this subset. Instead, statistics for the N_a subset are computed using a case with a high value of p_e (if the set N_a is large, the sampling coverage of the stochastic parameter space is enhanced) and then scaled, using the actual p_e value, to estimate statistics for the complete R set. The only requirement for the approach validity is that the distribution of WP breach times $\{t_i, i=1,2, \ldots, N_a\}$ does not significantly vary as a function of p_e.

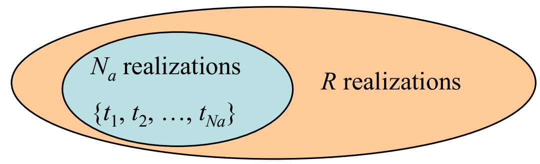


Figure 1. N_a realizations in a set of R exhibit WP breach. The set $\{t_i, i=1, 2, \ldots, N_a\}$ is the set of breach times. Only the first event leading to WP breach per realization is considered in the PA model.

The discussion that follows is aimed at proving that the distribution of WP breach times is similar for a range of p_e values provided that p_e is small and $\lambda\, T \le 1$. If the number of events in the simulation time is determined by a Poisson distribution, then the waiting time for the first event leading to WP breach follows an exponential distribution with shape parameter equal to $p_e\, \lambda$ (recall that events with a recurrence rate greater than λ are assumed not to cause WP damage); i.e.,

$$P(\text{breach time} < t) = 1 - e^{-p_e \lambda t} \qquad [1]$$

Consequently, the distribution of WP breach times in a single subarea for the set of realizations where WP breach occurs in the period T (i.e., the set of realizations in subset N_a in Figure 1) is

$$P(\text{breach time} < t \,|\, \text{WP breach occurs in } T) = \frac{1 - e^{-p_e \lambda t}}{1 - e^{-p_e \lambda T}} \qquad [2]$$

The denominator in Equation [2] equals the probability that WP breach occurs in a time less than T. Equation [2] is approximately a uniform distribution for small values of p_e if $\lambda\, T \le 1$ [Figure 2(A)]. In general, the distribution of the time of the first event leading to WP breach is a weak function of p_e. On the other hand, if $\lambda\, T > 1$, variations in p_e could cause significant changes in the shape of the distribution [Figure 2(B)]. The interest in this paper is in the case $\lambda\, T \le 1$. The case $\lambda\, T > 1$ is of no interest because it can be addressed with common Monte Carlo approaches. For example, if the event recurrence rate is $\lambda = 10^{-4}$ 1/yr and $T = 10^5$ years, then an average of 10 events occurs in every realization of the PA model. In this case, there is a significant chance that at least one event will induce WP breach in every realization; thus, WP breach occurs in a significant number of realizations; and, therefore, there is no need to apply special variance reduction techniques to estimate the distribution of breach times from Monte Carlo data. If the number of realizations is large

enough, statistically convergent consequence estimates can be derived. Figure 2(C) is an actual result from a PA model with p_e=0.5, λ = 10^{-4} 1/yr, and T=10^4 years. The stepwise lines correspond to 10 subareas considered in the PA model. The smooth thick line is the Equation [2] approximation. The actual PA distribution of WP breach times differs from the approximation at early times. The difference in the first 1,000 years is due to assumed gradual drift degradation, causing a transient temporal dependence of p_e in the PA model. After the 1,000-year transience, the Equation [2] approximation well reproduces the PA model numerical distributions.

The main conclusion from Figure 2 is that the distribution of WP breach times during T is a weak function of the probability p_e. Thus, any simulation with a high value of p_e can be used to estimate consequence statistics for the subset N_a in Figure 1. A second conclusion is that a run artificially imposing a uniform distribution of WP breach times can also be used to approximately derive statistics for the subset N_a. The following section presents the derivation of mean consequences and confidence intervals for the N_a and R sets.

MATHEMATICAL APPROACH

The PA model is based on the concept of a representative WP. If the representative WP is breached after a seismic event, all of the WPs in a subarea are assumed to be breached simultaneously. In other words, each WP is assumed to exhibit the same radionuclide release behavior governed, for example, by water flow rates, waste form dissolution, solubilities, colloidal phases, and temperatures. The number of WPs in a subarea is assumed constant for all realizations. The number of WPs in a subarea is denoted as w_a, and the number of subareas as A. In a realization, either all or none of the WPs may be breached by a seismic event in a subarea (later the approach is generalized where the fraction of WPs varies from realization to realization). The number of breached WPs in realization r in subarea a, is denoted as γ_a^r:

$$\gamma_a^r = \begin{cases} 0 \\ w_a \end{cases}, \ a = 1, 2, ..., A \qquad [3]$$

The dose per realization r, $\overline{D}^r$, includes contributions from all WPs breached in all subareas. If breaching in a realization occurs, the dose per breached WP, δ^r, is defined as

$$\delta^r = \frac{\overline{D}^r}{\sum_{a=1}^{A} \gamma_a^r} \qquad [4]$$

If there is no breaching, δ^r is defined as zero. The contribution of a subarea a to the total dose, D_a^r, is defined as

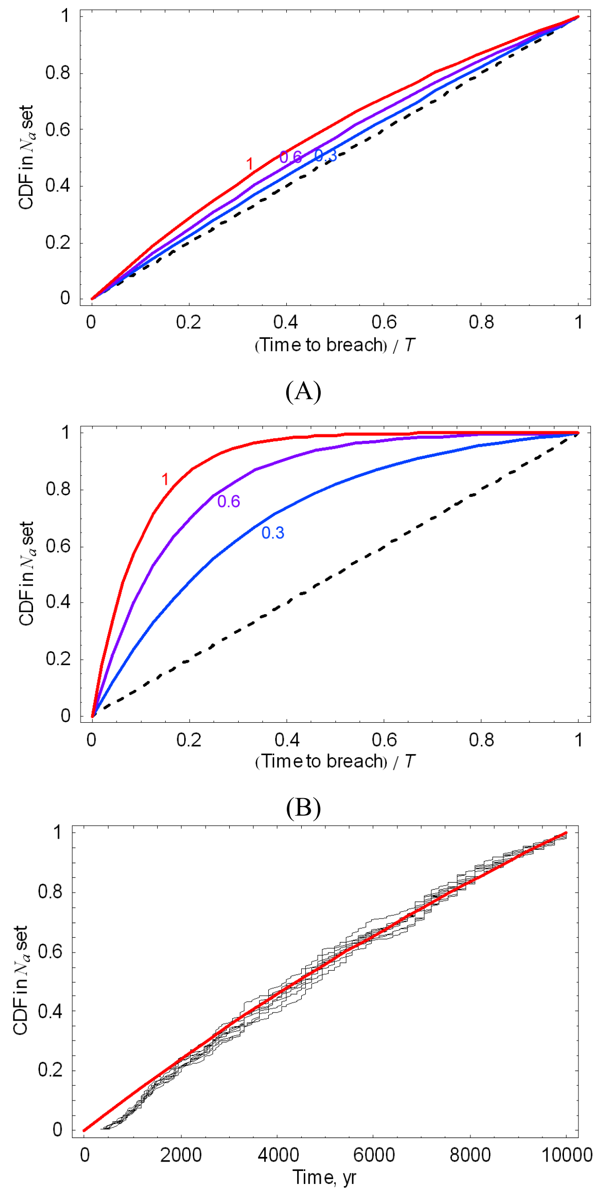
$$D_a^r = \gamma_a^r \delta^r \qquad [5]$$

Such definition of the subarea contribution is proposed so that the total dose equals the sum of the subarea contributions; i.e.,

$$\overline{D}^r = \sum_{a=1}^{A} D_a^r = \sum_{a=1}^{A} \gamma_a^r \delta^r \qquad [6]$$

The mean subarea dose is (mean value of the set R in Figure 1)

$$\mathrm{Mean}(D_a) = \langle D_a \rangle = \frac{1}{R} \sum_{r=1}^{R} \gamma_a^r \delta^r = \frac{1}{R} \sum_{\substack{r=1 \\ \gamma_a^r > 0}}^{R} \gamma_a^r \delta^r = \frac{w_a}{R} \sum_{\substack{r=1 \\ \gamma_a^r > 0}}^{R} \delta^r \qquad [7]$$



(C)

Figure 2. Plot of Equation [2] for (A) λT=1 and (B) λT=10 and different values of p_e (=1, 0.6, 0.3). The dashed line is the distribution derived in the limit $p_e \rightarrow 0$. (C) Cumulative distribution of WP breach time assuming p_e=0.5, λ =10^{-4} 1/yr, and T=10,000 yr. Ten subareas were considered in the performance assessment model. The smooth line is the Equation [2] approximation.

On the other hand, the mean subarea dose for the set of realizations with WP breach is (mean value of set N_a in Figure 1)

$$\underset{\gamma_a^r>0}{\mathrm{Mean}}(D_a)=\langle D_a\rangle_{\gamma_a^r>0}=\frac{1}{N_a}\sum_{\substack{r=1\\ \gamma_a^r>0}}^{R}\gamma_a^r\,\delta^r=\frac{w_a}{N_a}\sum_{\substack{r=1\\ \gamma_a^r>0}}^{R}\delta^r=w_a\langle\delta_a\rangle_{\gamma_a^r>0} \quad [8]$$

where

$$\underset{\gamma_a^r>0}{\mathrm{Mean}}(\delta_a)=\langle\delta_a\rangle_{\gamma_a^r>0}=\frac{1}{N_a}\sum_{\substack{r=1\\ \gamma_a^r>0}}^{A}\delta^r=\frac{\langle D_a\rangle_{\gamma_a^r>0}}{w_a} \quad [9]$$

Thus, a simple relationship exists between the means $\langle D_a\rangle$ and $\langle D_a\rangle_{\gamma_a^r>0}$:

$$\langle D_a\rangle=\frac{N_a}{R}\langle D_a\rangle_{\gamma_a^r>0} \quad [10]$$

Equation [10] is a general formula to transfer statistics in the set N_a to statistics in the set R; i.e., if X_a^r is a quantity with a value of zero in the complement set $R - N_a$, then

$$\langle X_a\rangle=\frac{N_a}{R}\langle X_a\rangle_{\gamma_a^r>0} \quad [11]$$

The subarea construction in the PA model is based on the distribution of deep water percolation and stratigraphic layers below the repository footprint in the unsaturated rock. Therefore, no relationship exists between WP breaching trends and subarea definitions. In other words, there is identical probability for each subarea to exhibit WP breach in a realization. An estimator for that probability, p, is defined as follows

$$p=\frac{1}{R\,A}\sum_{a=1}^{A}N_a \quad [12]$$

The quantity p is the probability that at least one WP breaching event occurs in a realization (i.e., in a time less than T) in one subarea. Therefore, p can also be computed by substituting the time T into Equation [1]; i.e.,

$$p=1-e^{-p_e\lambda T} \quad [13]$$

Equations [12] and [13] provide a simple approach to verify results. If the number of realizations where WP breach occurs is known, then p_e can be estimated as well as the expected distribution of WP breach times (which should be in agreement with the actual distribution from the PA model). Alternatively, if p_e is known, then the expected number of realizations with WP breach can be anticipated.

Similarly as p, the quantity $\langle\delta_a\rangle_{\gamma_a^r>0}$ (Equation [9]) should be subarea independent. The quantity $\langle\delta_a\rangle_{\gamma_a^r>0}$ is an estimator of the average dose per waste package in a subarea, computed as a function of the total dose per realization, $\overline{D}^r$. It is recognized that a PA model may independently track doses per subarea, and that accurately computed mean dose per waste package may vary from subarea to subarea due to differences in subarea features. The Equation [9] is adopted for the sake of simplicity, to facilitate total mean dose computations. Thus, an estimator of the mean dose per WP is defined as

$$\langle\delta\rangle_{\gamma^r>0}=\frac{1}{A}\sum_{a=1}^{A}\langle\delta_a\rangle_{\gamma_a^r>0} \quad [14]$$

Analogous estimators can be defined for higher moments $\langle\delta^m\rangle_{\gamma^r>0}$. Note that the subscript a in the term γ_a^r in the left-hand side of Equation [14] is dropped to highlight the subarea independence. From Equations [7]–[14], the following approximations are valid

$$\langle\delta^m\rangle=p\langle\delta^m\rangle_{\gamma^r>0},\ m=1,2,... \quad [15]$$

and

$$\langle D_a^{\,m}\rangle=p\langle D_a^{\,m}\rangle_{\gamma^r>0}=p\,w_a\,\langle\delta^m\rangle_{\gamma^r>0},\ m=1,2,... \quad [16]$$

From the definition of the total dose, Equation [6], the mean dose is estimated as

$$\langle\overline{D}\rangle=\sum_{a=1}^{A}\langle D_a\rangle=p\langle\delta\rangle_{\gamma^r>0}\sum_{a=1}^{A}w_a=p\langle\delta\rangle_{\gamma^r>0}\,W \quad [17]$$

where W is the total number of waste packages in the repository. Because WP breach is not necessarily an independent process from subarea to subarea, deriving higher moments, $\langle\overline{D}^m\rangle$, cannot be done analytically in general. However, an upper bound in the second moment in $\overline{D}$ can be derived from Equation [6]:

$$\overline{D}^r=\sum_{a=1}^{A}D_a^r=\sum_{a=1}^{A}\gamma_a^r\,\delta^r\le W\,\delta^r \quad [18]$$

Therefore,

$$\langle\overline{D}^2\rangle\le W^2\langle\delta^2\rangle=W^2p\langle\delta^2\rangle_{\gamma^r>0} \quad [19]$$

An upper bound in the variance, Var($\overline{D}$), is defined as

$$\mathrm{Var}(\overline{D})=\langle\overline{D}^2\rangle-\langle\overline{D}\rangle^2\le W^2\left(p\langle\delta^2\rangle_{\gamma^r>0}-p^2\langle\delta\rangle_{\gamma^r>0}^2\right) \quad [20]$$

On the other hand, a lower bound in the variance is estimated by assuming that subareas are independent. From Equation [6]

$$\mathrm{Var}(\overline{D})\ge\sum_{a=1}^{A}\mathrm{Var}(D_a)=\left(p\langle\delta^2\rangle_{\gamma^r>0}-p^2\langle\delta\rangle_{\gamma^r>0}^2\right)\sum_{a=1}^{A}w_a^{\,2} \quad [21]$$

Therefore, combining Equations [20] and [21]

$$\left(p\langle\delta^2\rangle_{\gamma^r>0}-p^2\langle\delta\rangle_{\gamma^r>0}^2\right)\sum_{a=1}^{A}w_a^{\,2}\le\mathrm{Var}(\overline{D})\le\left(p\langle\delta^2\rangle_{\gamma^r>0}-p^2\langle\delta\rangle_{\gamma^r>0}^2\right)W^2 \quad [22]$$

Equations [17] and [22] define mean and variances in $\overline{D}$ as functions of invariants $\langle\delta\rangle_{\gamma'>0}$ and $\langle\delta^2\rangle_{\gamma'>0}$ and of the probability p. Practically any multiple-realization simulation of the PA model can be used to estimate $\langle\delta\rangle_{\gamma'>0}$ and $\langle\delta^2\rangle_{\gamma'>0}$, provided that $\lambda\ T\leq 1$. If a simulation with R realizations is considered, then an upper bound in the standard deviation of the sample mean, SDev($\langle\overline{D}\rangle$), is estimated as [8]

$$\mathrm{SDev}\left(\langle\overline{D}\rangle\right)\leq\frac{W}{\sqrt{R}}\sqrt{p\langle\delta^2\rangle_{\gamma'>0}-p^2\langle\delta\rangle^2_{\gamma'>0}} \qquad [23]$$

If a predefined precision level on the sample mean $\langle\overline{D}\rangle$ is desired, Equation [23] defines the number of realizations, R, to be executed to achieve such a precision.

Figure 3 shows results with several 600-realization runs of the PA model considering p=0.004, 0.065, 0.15, 0.2, 0.28, and 0.39. The moments $\langle\delta\rangle_{\gamma'>0}$ [thick line in Figure 3(A)] and $\langle\delta^2\rangle_{\gamma'>0}$ [thick line in Figure 3(B)] were estimated with the case p=0.39. Note that in Figure 3(A) the moment $\langle\delta\rangle_{\gamma'>0}$ compares well to $\langle\overline{D}\rangle/(pW)$ (see Equation [17]) except in the case of p=0.004. In this case, too few realizations include WP breach in 10,000 years; therefore, the mean dose, $\langle\overline{D}\rangle$, derived with a direct Monte Carlo method, is highly uncertain. In Figure 3(B), the moment $\langle\delta^2\rangle_{\gamma'>0}$ is compared to $\frac{1}{p}\left[\mathrm{Var}(\overline{D})/W^2+p^2\langle\delta\rangle^2_{\gamma'>0}\right]$ (see Equation [20]). Figure 3(B) is consistent with the statement that Equation [20] defines an upper bound in the variance, Var($\overline{D}$). Interestingly, the upper bound is relatively close to the computed variance. This suggests that, in general, when WPs in one subarea are breached, they are also breached in multiple subareas, if not in all of them. The anomaly in Figure 3(B) also corresponds to the case p=0.004.

The moments for the case p=0.004 directly computed with the Monte Carlo data are not well converged. Equations [17] and [23] allow definition of the sample mean dose, $\langle\overline{D}\rangle$, and the standard deviation in the sample mean, SDev($\langle\overline{D}\rangle$), for the case p=0.004 as functions of moments $\langle\delta\rangle_{\gamma'>0}$ and $\langle\delta^2\rangle_{\gamma'>0}$ derived with the case p=0.39. Figure 4 shows the sample mean dose, $\langle\overline{D}\rangle$, as a continuous blue line. The continuous jagged line is the mean dose directly computed with the Monte Carlo data. The dotted lines are the 0.025, 0.05, 0.2, 0.35, 0.5, 0.65, 0.8, 0.95, and 0.975 quantiles in the sample mean dose assuming that at every timestep, the sample mean dose follows a log-normal distribution with mean as in Equation [17] and standard deviation as in Equation [23].

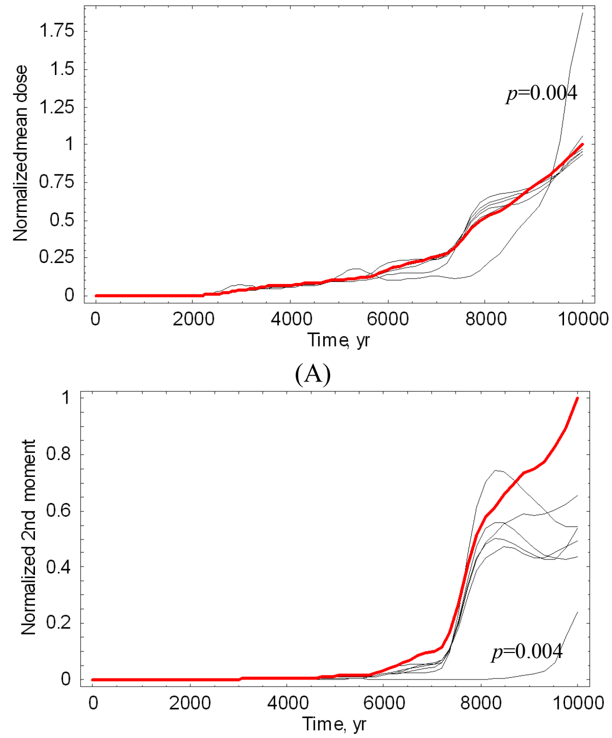


Figure 3. Moments $\langle\delta\rangle_{\gamma'>0}$ and $\langle\delta^2\rangle_{\gamma'>0}$ computed with a 600-realization run with p=0.39 [thick lines in (A) and (B)]. In (A), $\langle\delta\rangle_{\gamma'>0}$ is compared to $\langle\overline{D}\rangle/(pW)$ from 600-realization runs with p=0.004, 0.065, 0.15, 0.2, 0.28, and 0.39. In (B), $\langle\delta^2\rangle_{\gamma'>0}$ is compared to $\frac{1}{p}\left[\mathrm{Var}(\overline{D})/W^2+p^2\langle\delta\rangle^2_{\gamma'>0}\right]$ derived from 600-realization runs (p=0.004, 0.065, 0.15, 0.2, 0.28, and 0.39).

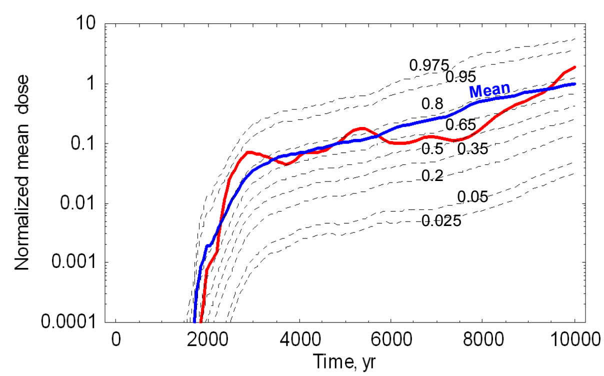


Figure 4. Sample mean versus time computed according to Equation [17] (continuous blue line) and directly from Monte Carlo data (jagged red line) for the case p=0.004. The dotted lines represent quantiles in the sample mean (quantiles in labels).

GENERALIZATION TO A VARIABLE NUMBER OF WASTE PACKAGES

A number of modifications in the approach are needed if the fraction of breached WPs varies from realization to realization. The dose per WP for realization r, δ^r, is defined as

$$\delta^r = \frac{\overline{D}^r}{\sum_{a=1}^{A} \gamma_a^r \, n^r} \qquad [24]$$

where n^r is the fraction of WPs breached in realization r. In this case, the following identities are valid:

$$\begin{aligned} \left\langle n^m \delta^m \right\rangle &= p \left\langle n^m \delta^m \right\rangle_{\gamma^r>0}, \quad m = 1,2,3,\ldots \\ \left\langle D_a^m \right\rangle_{\gamma_a^r>0} &= w_a \left\langle n^m \delta^m \right\rangle_{\gamma^r>0}, \quad m = 1,2,3,\ldots \\ \left\langle D_a^m \right\rangle &= p \left\langle D_a^m \right\rangle_{\gamma_a^r>0} = p\, w_a \left\langle n^m \delta^m \right\rangle_{\gamma^r>0}, \quad m = 1,2,3,\ldots \\ \left\langle \overline{D} \right\rangle &= p\, W \left\langle n\, \delta \right\rangle_{\gamma^r>0} \end{aligned} \qquad [25]$$

An upper bound in the second moment $\left\langle \overline{D}^2 \right\rangle$ is computed by noting that

$$\overline{D}^r = \sum_{a=1}^{A} \gamma_a^r n^r \delta^r \le n^r \delta^r \sum_{a=1}^{A} w_a = n^r \delta^r W \qquad [26]$$

Therefore,

$$\left\langle \overline{D}^2 \right\rangle \le p W^2 \left\langle n^2 \delta^2 \right\rangle_{\gamma^r>0} \qquad [27]$$

and

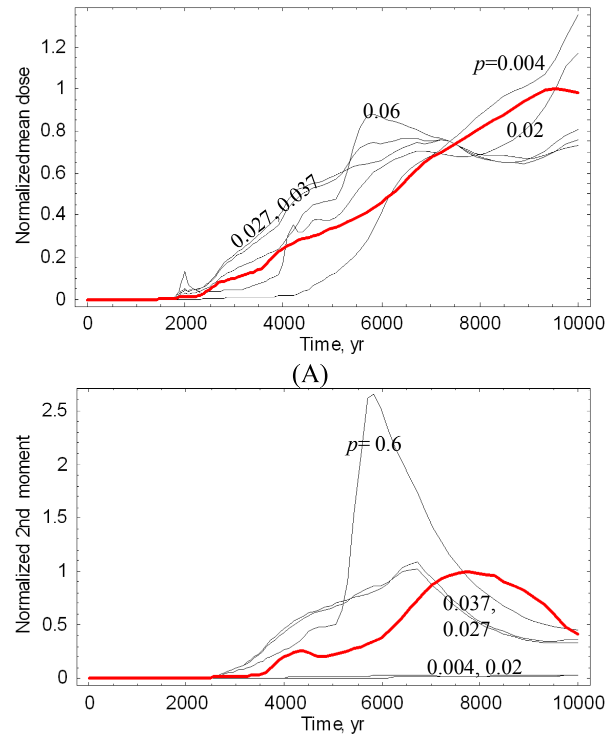
$$\mathrm{Var}\left(\overline{D}\right) = \left\langle \overline{D}^2 \right\rangle - \left\langle \overline{D} \right\rangle^2 \le W^2 \left(p \left\langle n^2 \delta^2 \right\rangle_{\gamma^r>0} - p^2 \left\langle n\, \delta \right\rangle^2_{\gamma^r>0} \right) \qquad [28]$$

The right-hand side in Equation [28] defines an upper bound in the variance in $\overline{D}$.

If the distribution for the variable δ for the set of realizations with WP breach is known as well as the distribution for the fraction n, then the moments $\left\langle n\delta \right\rangle_{\gamma^r>0}$ and $\left\langle n^2\delta^2 \right\rangle_{\gamma^r>0}$ can be numerically derived. Alternatively, these moments can be derived with a run of the PA model with high values of p_e and p, provided that the fraction n is part of the Monte Carlo sampling. An upper bound in the standard deviation in the sample mean, $\mathrm{SDev}\left(\left\langle \overline{D} \right\rangle\right)$, for sets of R realizations, can be estimated as

$$\mathrm{SDev}\left(\left\langle \overline{D} \right\rangle\right) \le \frac{W}{\sqrt{R}} \sqrt{p \left\langle n^2 \delta^2 \right\rangle_{\gamma^r>0} - p^2 \left\langle n\, \delta \right\rangle^2_{\gamma^r>0}} \qquad [29]$$

In Figure 5, results of 500-realization runs are presented with p=0.004, 0.02, 0.27, 0.37, and 0.6. The fraction n was uniformly sampled in the range 0.01 to 1. To compute the reference moments [thick lines in Figure 5(A) and (B)], a simplified case of the PA model was executed assuming that WP breach times are uniformly distributed in the simulation time of 10,000 years. The other data presented in Figure 5 were derived with regular runs of the PA model to estimate consequences of seismic events. The introduction of the fraction n introduced an extra source of variability. For example, estimates of the moment $\left\langle n\delta \right\rangle_{\gamma^r>0}$ [Figure 5(A)] show more variance than the case n=1 [Figure 3(A)]. The second moment estimator, $\left\langle n^2\delta^2 \right\rangle_{\gamma^r>0}$ [thick line in Figure 5(B)], bounds the cases p=0.004 and 0.02.



(A)

(B)

Figure 5. Moments $\left\langle n\delta \right\rangle_{\gamma^r>0}$ and $\left\langle n^2\delta^2 \right\rangle_{\gamma^r>0}$ computed with 500-realization runs with a simplified case assuming uniform breaching times during the simulation period [thick red lines in (A) and (B)]. In (A), $\left\langle n\delta \right\rangle_{\gamma^r>0}$ is compared to $\left\langle \overline{D} \right\rangle/(pW)$ for various 500-realization runs with p=0.004, 0.02, 0.27, 0.37, and 0.6. In (B), $\left\langle n^2\delta^2 \right\rangle_{\gamma^r>0}$ is compared to $\frac{1}{p}\left[\mathrm{Var}\left(\overline{D}\right)/W^2 + p^2\left\langle n\,\delta \right\rangle^2_{\gamma^r>0}\right]$ derived also with various 500-realization runs (p=0.004, 0.02, 0.27, 0.37, and 0.6). The second moment $\left\langle n^2\delta^2 \right\rangle_{\gamma^r>0}$ bounds the cases p=0.004 and 0.02.

The interest is in estimating moments for cases where WP breach infrequently occurs in Monte Carlo realizations. For example, for the case p=0.004, moments are poorly converged if directly computed from Monte Carlo data. Instead, Equations [25] and [29] can be used to estimate means and standard deviations for the case p=0.004 from estimators $\langle n\,\delta\rangle_{\gamma'>0}$ and $\langle n^2\,\delta^2\rangle_{\gamma'>0}$ displayed in Figure 5. Confidence bounds on the mean dose are displayed in Figure 6, derived in an analogous manner as the data displayed in Figure 4.

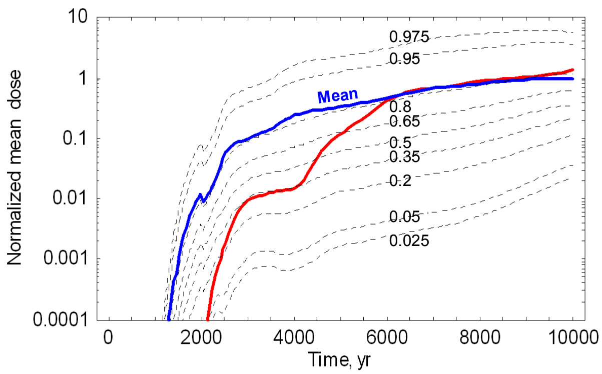


Figure 6. Sample mean versus time computed as $\langle \overline{D}\rangle = p\;W\langle n\,\delta\rangle_{\gamma'>0}$ (continuous blue line) and directly from Monte Carlo data (red line) for the case p=0.004. The dotted lines represent quantiles in the sample mean (quantiles in labels).

CONCLUSIONS

An approach was derived to estimate mean consequences of seismic events infrequently occurring in realizations of a PA model. In the PA model considered here, it is assumed that if a seismic event of sufficient intensity occurs, all of the waste packages in a *subarea* simultaneously breach. The approach was extended to account for the case where only a fraction of the waste packages may be breached, with a fraction defined by a distribution function. The equations derived were tested with data from multiple simulations of the PA model considering different probabilities for events to cause waste packages to breach. Two examples were presented using equations to estimate mean consequences of cases with low probabilities for seismic WP breaching. The derived equations can be used to estimate mean consequences without the need to execute a large number of PA model realizations.

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REFERENCES

1. Bechtel SAIC, Company, LLC. "Total System Performance Assessment- Licence Application Methods and Approach." TDR-WIS-PA-00006 Rev 00, ICN 01. Las Vegas, Nevada; Bechtel SAIC Company, LLC. 2003.
2. Ibarra, L., T. Wilt, G. Ofoegbu, R. Kazban, F. Ferrante, and A.H. Chowdhury. "Revised: Drip Shield-Waste Package Mechanical Interaction—Progress Report." San Antonio, TX: Center for Nuclear Waste Regulatory Analyses: February 2007.
3. Bechtel SAIC Company, LLC. "Drip Shield Structural Response to Rockfall." 000–00C–SSE0–00300–000–00A. Las Vegas, Nevada: Bechtel SAIC Company, LLC. 2004.
4. Bechtel SAIC Company, LLC. "Structural Calculations of Waste Package Exposed to Vibratory Ground Motion. 000–00C–WIS0–01400–000–00A. Las Vegas, Nevada: Bechtel SAIC Company, LLC. 2004.
5. Bechtel SAIC Company, LLC. "Seismic Consequence Abstraction." MDL–WIS–PA–000003. Rev. 01. Las Vegas, Nevada: Bechtel SAIC Company, LLC. 2004.
6. Bechtel SAIC Company, LLC. "Peak Ground Velocities for Seismic Events at Yucca Mountain, Nevada." ANL–MGR–GS–000004. Las Vegas, Nevada: Bechtel SAIC Company, LLC. 2005.
7. Bechtel SAIC Company, LLC. "Development of Earthquake Ground Motion Input for Preclosure Seismic and Postclosure Performance Assessment of a Geologic Repository at YM Nevada." MDL–MGR–GS–000003. Rev. 01. Las Vegas, Nevada: Bechtel SAIC Company, LLC. 2004.
8. Hald, A. *Statistical Theory with Engineering Applications*. New York City: John Wiley & Sons, Inc. 1952.