Scientific Notebook #326, continuation from notebook #285

LABORATORY NOTEBOOK

CNWRA/SwRI

NOTEBOOK NO. ______________

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ON ______________________________ **19** ______

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CNWRA
CONTROLLED
COPY 326

—SCIENTIFIC NOTEBOOK CO.—
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INSTRUCTIONS

1. **The primary purpose of this notebook is to protect your and the Company's Patent-Rights by keeping records of all original work in a form acceptable as evidence if any legal conflict arises.**

2. • When starting a page, enter the title, project number, and book number.
   • Use <u>ink</u> for permanence -- avoid pencil.
   • Record your work as you progress, including any spur-of-the-moment ideas which may be developed later.
   • Avoid making notes on loose paper to be recopied.
   • Record your work in such a manner that a co-worker can continue from where you stop. You might be ill and to protect your priority it could be urgent that the work continue while you are absent.

3. • Give a complete account of your experiments and the results, both positive and negative, including your observations.
   • Record all diagrams, layouts, plans, procedures, new ideas, or anything pertinent to your work including the details of any discussions with suppliers, or other people outside the Company.
   • Do not try to erase any incorrect entries; draw lines deleting them, note the corrections, sign and date the changes. This extra care is worthwhile because of the necessity of original data to prove priority of new discoveries.

4. • After entering your data, sign and date the entries.
   • Explain your work to at least two witnesses who are not co-inventors, and have them sign and date the pages in the place provided.
   • Record the names of operators and witnesses present during any demonstration and have at least two witnesses sign the page. If no witnesses are present during an experiment of importance, repeat it in the presence of two witnesses.

5. Since computer programs can be patented these instructions apply to the development of computer software. In this case a description of the structure and operation of the program should be recorded in the notebook, together with a basic flow diagram which illustrates the essential features of the program. In the course of developing the code, the number of lines of code written each day should be recorded in the notebook, together with a statement of the portion of the flow diagram to which the section of code is directed.

6. This notebook and its contents are the exclusive property of the Company. It is confidential and the contents are not to be disclosed to anyone unless authorized by the Company. You must return it when completed, upon request, or upon termination of employment. It should be kept in a protected place. **If loss occurs, notify your supervisor immediately, and make a written report describing the circumstances of the loss.**

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manuscript submitted to SME 2000	88-91

Debra L. Hughson

Project No.

TITLE

Book No.

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This Notebook is a continuation of CNWRA Controlled copy 285

modeling of seepage into drifts under isothermal conditions

11/3/95

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Book No. TITLE

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		Recorded by	

TITLE USFIC Project No. 20-1402-861
Book No. 326

From Page No.

-r2

Flux	Pc	Se
8.0e-5 kg/m²/s	154.2	.992
8.6e-5 kg/m²/s	4.08	.998
8.62e-5 kg/m²/s	0	1
8.61e-5 kg/m²/s = 2720 mm/yr	1.69	.9999

-r1

Flux	Pc	Se
8.7e-5 kg/m²/s	20.24	.999
8.75e-5 kg/m²/s	8.468	.9996
8.77e-5 kg/m²/s	3.28	.9998
8.78e-5 kg/m²/s = 2774 mm/yr	1.44	.9999

-r0

Flux	Pc	Se
1.0785e-5 kg/m²/s	3.27	.9998
1.0787e-5 kg/m²/s	2.89	.9999
1.0789e-5 kg/m²/s	2.506	.9999
1.0791e-5 kg/m²/s	2.126	.9999
1.07950-5 kg/m²/s	0	1.0
1.07930-5 kg/m²/s = 3410 mm/yr	1.74	.9999

Effect of grid refinement on threshold flux obtained from unstructured grid multiflo

Grid DH 4/14/99

Grid	
-r0	#285 page 91
-r1	#285 page 95
-r2	this notebook following page

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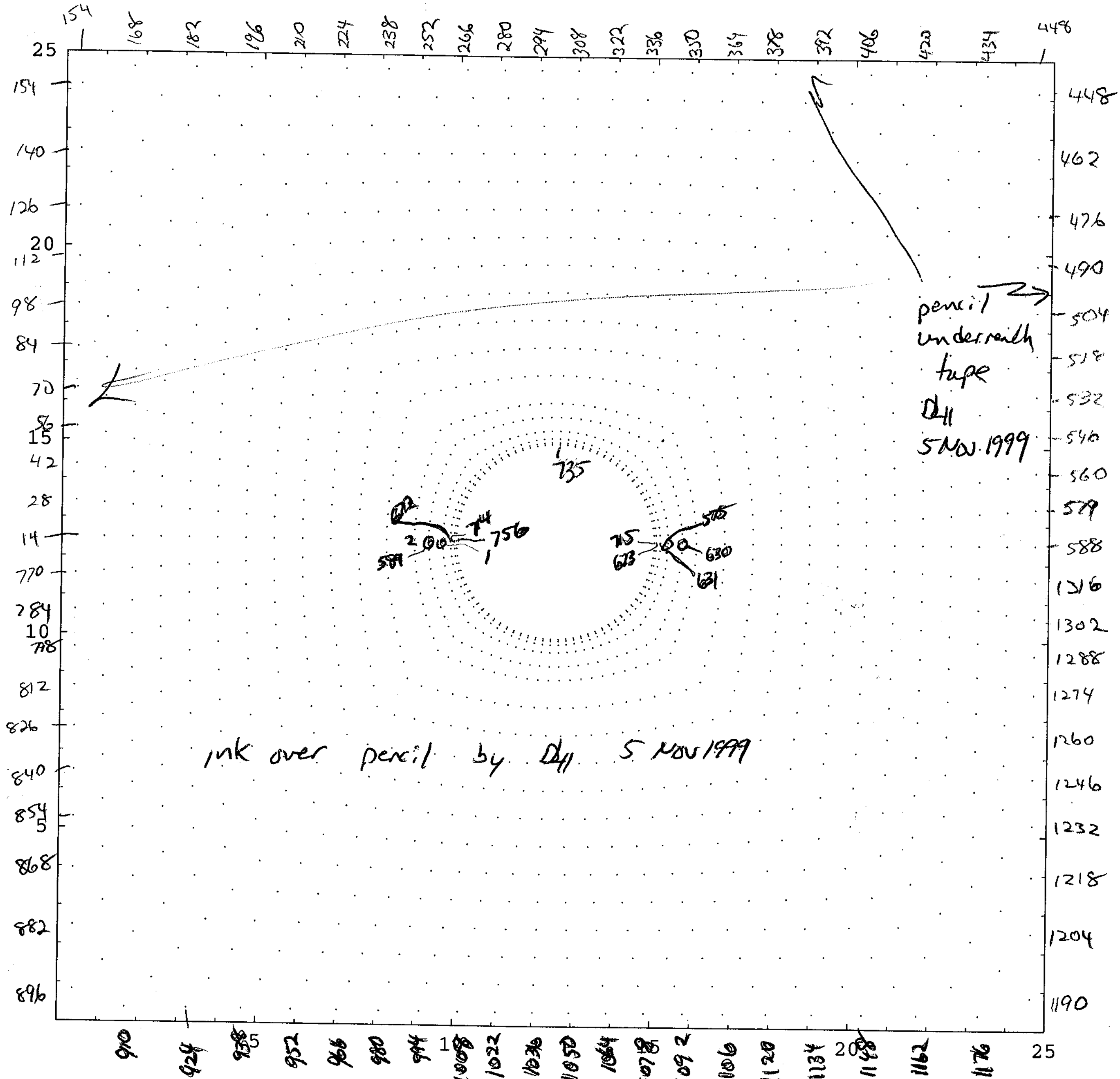

Grid – r2

1476

nextkey
dw2
dwt
dw

DH

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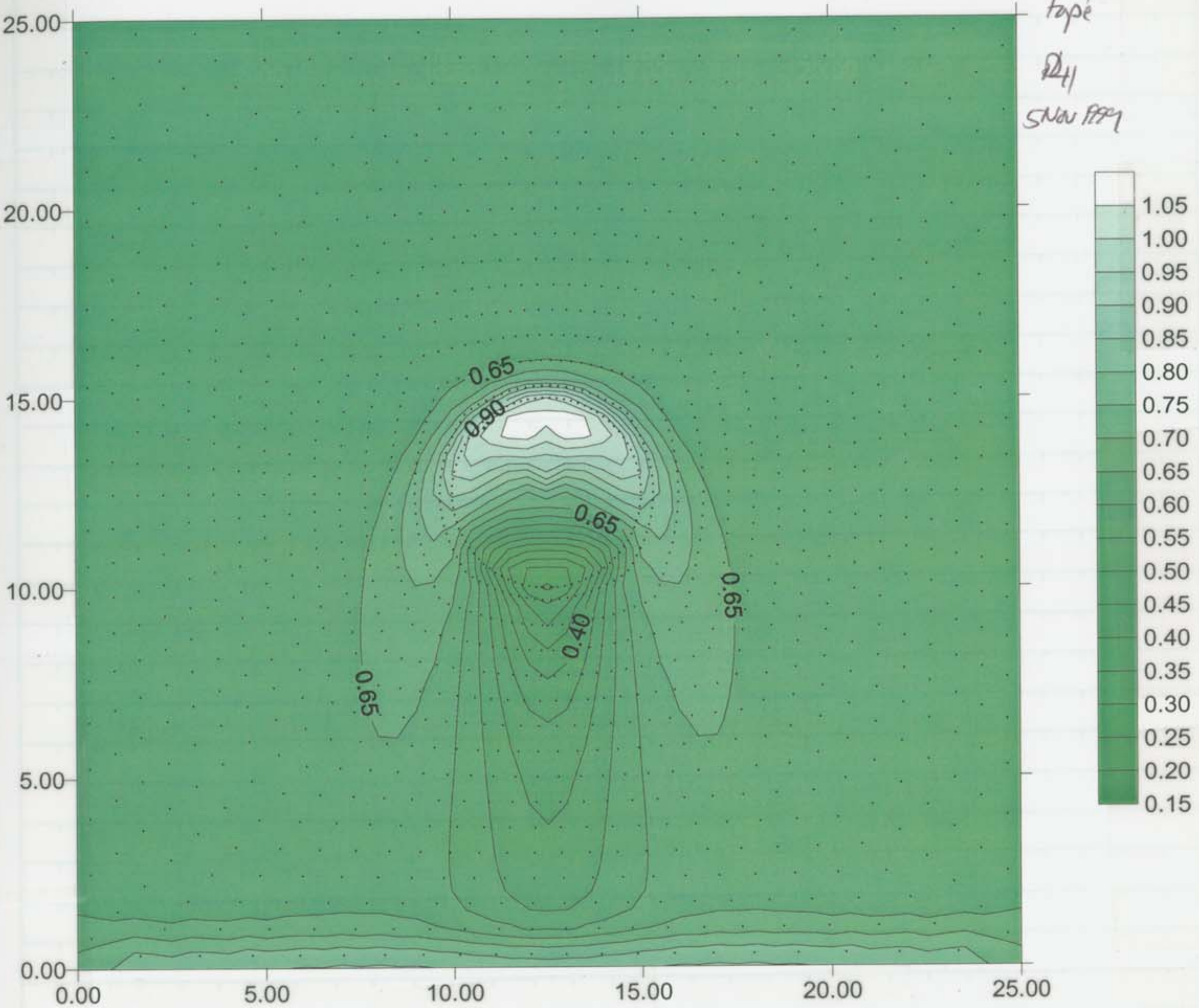
threshold flux 1.0793e-5 kg/m²/s = 3410 mm/yr
Philip solution = 2260 mm/yr ← pencil underneath tape

_r0

DH 5 Nov 1999



~dhughson/multiflo/inputbak/multi.r0
nodes.r0

Ink over pencil by DH 5 Nov 1999

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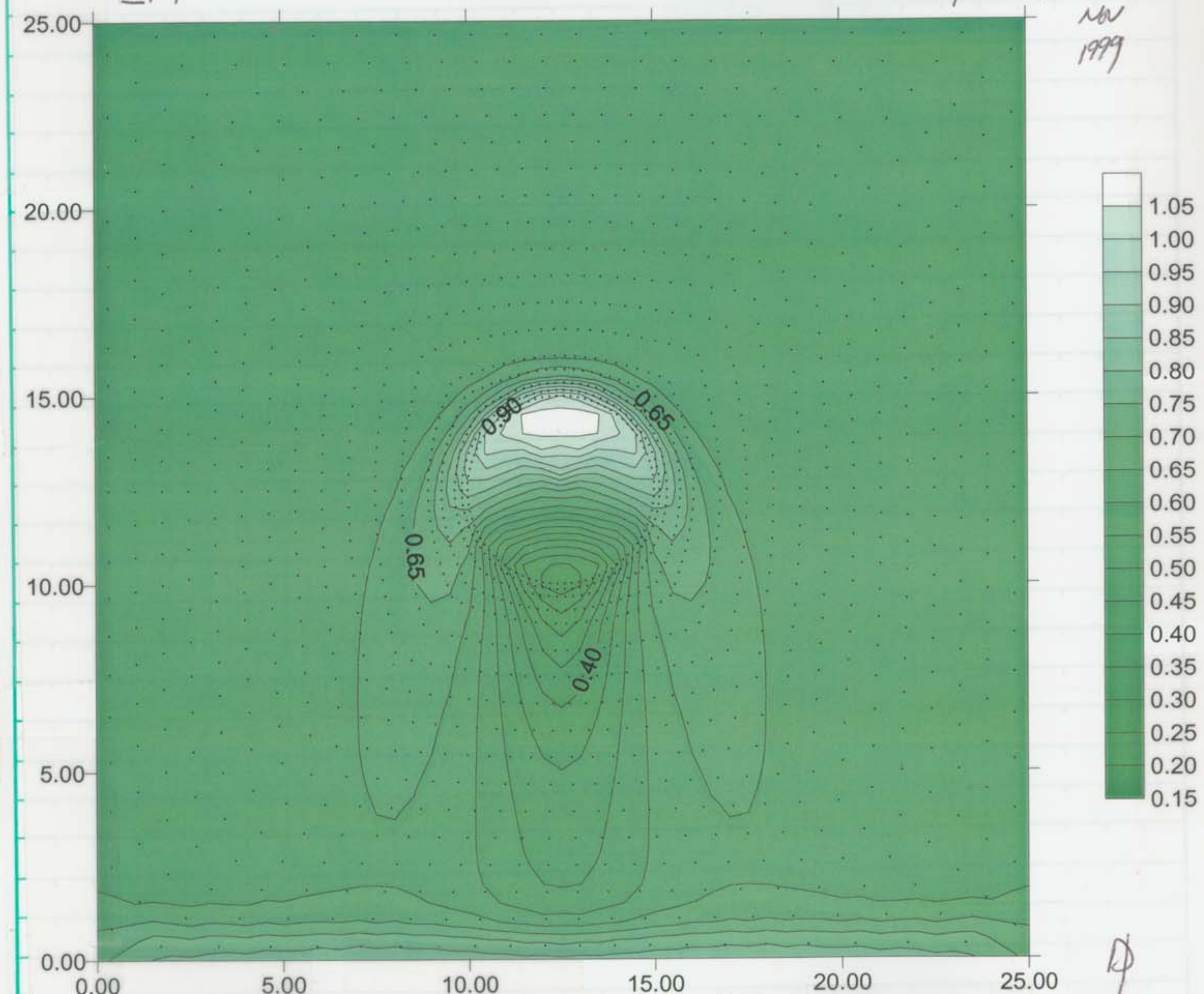
threshold flux 8.78e-5 kg/m²/s = 2774 mm/yr

_r1

Philip solution = 2260 mm/yr

ink over pencil except under tape DH 5 Nov 1999



DH

~dhughson/multiflo/inputbak/multi.r1
nodes.r1

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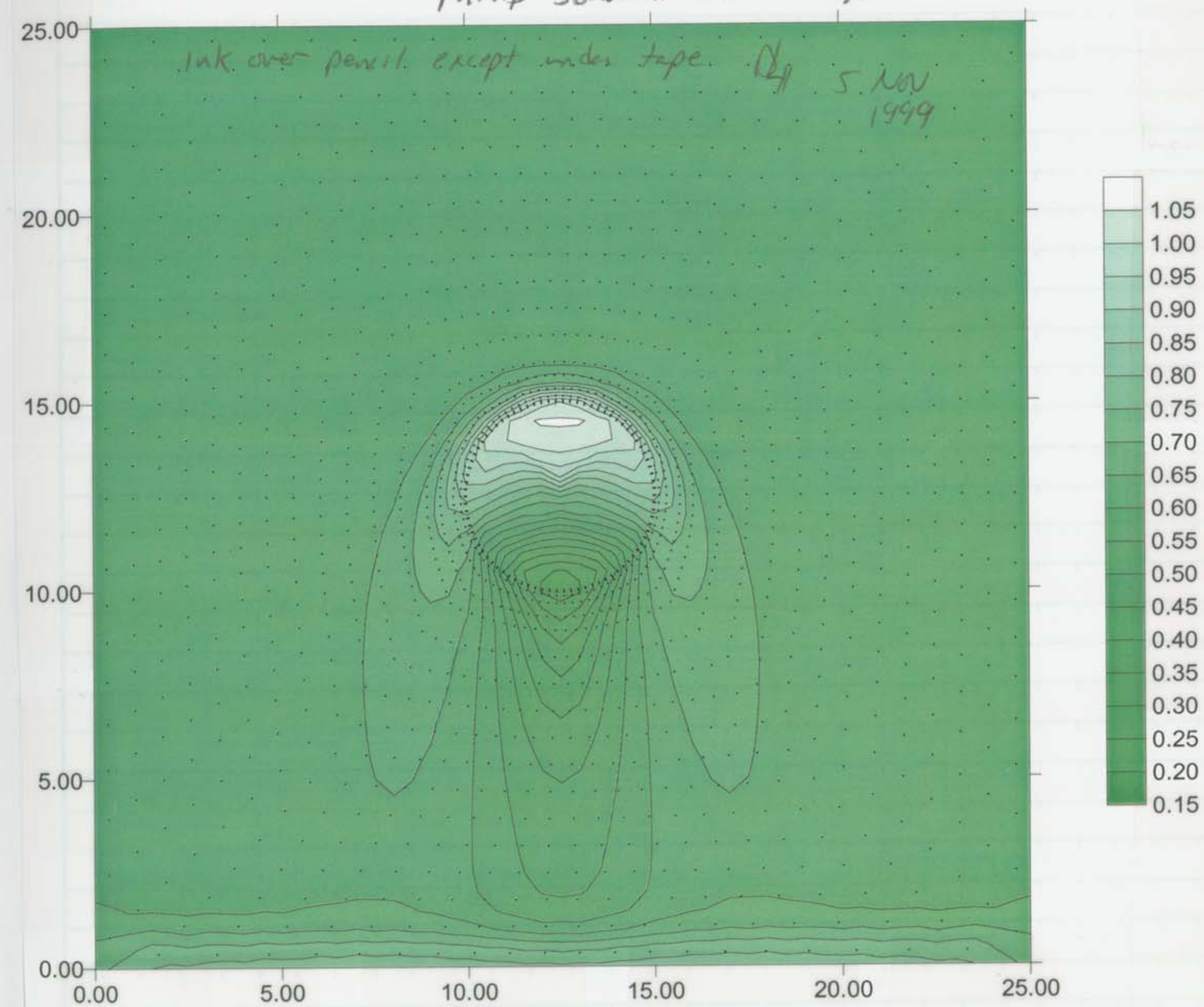
threshold flux 8.61×10⁻⁵ kg/m²/s = 2720 mm/yr

_r2

Philip solution 2260 mm/yr

ink over pencil except under tape. DH 5 Nov 1999



~dhughson/multiflo/inputbak/multi.r2
nodes.r2

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Discrepancy between analytical solution q_{max} = 2257.6 mm/yr and grid - r2 ~ 2720 mm/yr (notebook 285 page 90) Calls for another grid refinement.

First ring of nodes at r = 2.501 located 1mm = 0.1cm = .001 m from drift wall

~~sec~~ second ring at r = 2.51 m
third ring at r = 2.52m
4th ring at r = 2.55m
5th ring at r = 2.6m
6th ring at r = 2.7m

GRID - r3

Single line of nodes direction vertical from drift crown ~ node 882 top dead center and 1mm from drift wall boundary.

flux	P_c (PA)	Se
8.61e-5 kg/m²/s	0	1.0
8.0e-5 kg/m²/s	24.14	.9988
8.1e-5 kg/m²/s	0	1.0
8.05e-5 kg/m²/s	11.15	.9995
8.06e-5 kg/m²/s	8.58	.9996
8.07e-5 kg/m²/s	5.98	.9997
~~8.08e-5 kg/m²/s~~		DH
8.09e-5	0.86	1.0

= 2556 mm/yr

Philip (2260 mm/yr) % error $\frac{2556-2260}{2260}$ = 13.1%

α_G = 4.85e-4 Pa^{-1}

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For Gardner alpha 1.5e-4 Pa^{-1}

flux (kg/m²/s)	P_c (Pa)	Se
8.09e-5	6859	.8344
1.0e-4	5267	.8803
5.0e-4	0	1.0
DH ~~1.1e-4~~ 2.0e-4	97.75	.9985
2.1e-4	0	1
2.05e-4	0	1
2.02e-4	24.1	.9996
2.03e-4	0	1
2.025e-4 = 6397 mm/yr	5.9	.9999
2.026e-4 = 6400 mm/yr	0	1.0

Philip sol = 5910 mm/yr % error = $\frac{6400-5910}{5910}$ = 8.3%

For Gardner alpha 1.0e-5 Pa^{-1}

Philip = 23854

flux (kg/m²/s)	P_c (Pa)	Se
2.026e-4	9.67e+4	.8466
3.0e-4	7.092e+4	.8952
4.0e-4	5.072e+4	.9304
5.0e-4	3.367e+4	.9572
6.0e-4	1.971e+4	.9771
7.0e-4	6694	.993
7.5e-4	956.5	.9990
7.6e-4	0	1.0
7.58e-4	75.14	.9999
7.581e-4	49.45	1.0
7.582e-4	73.51	.9999 ?
7.583e-4	133.7	.9999 ?
7.585e-4	17.66	1.0
7.587e-4	65.85	.9999
7.590e-4 ≈ 23977 mm/yr	37.4	1.0
7.593e-4	0	1.0
7.592e-4	0	1.0 (1.0)
7.591e-4 (7.5905e-4) = 23978 mm/yr	0 (0)	1.0

% error $\frac{23977-23854}{23854}$ 0.5%

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_r3

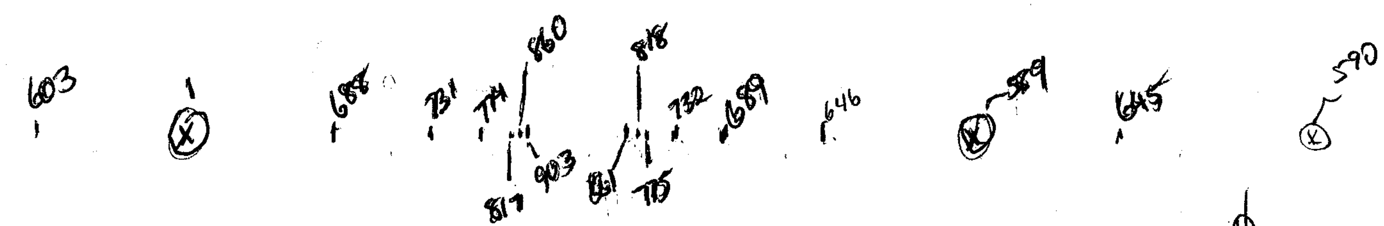
ink over pencil except under tape

DH 5 Nov 1999

882

2 0 1

589 0 0590



1764

Witnessed & Understood by me, | Date | Invented by | Date

Recorded by DH Apr 21 1999

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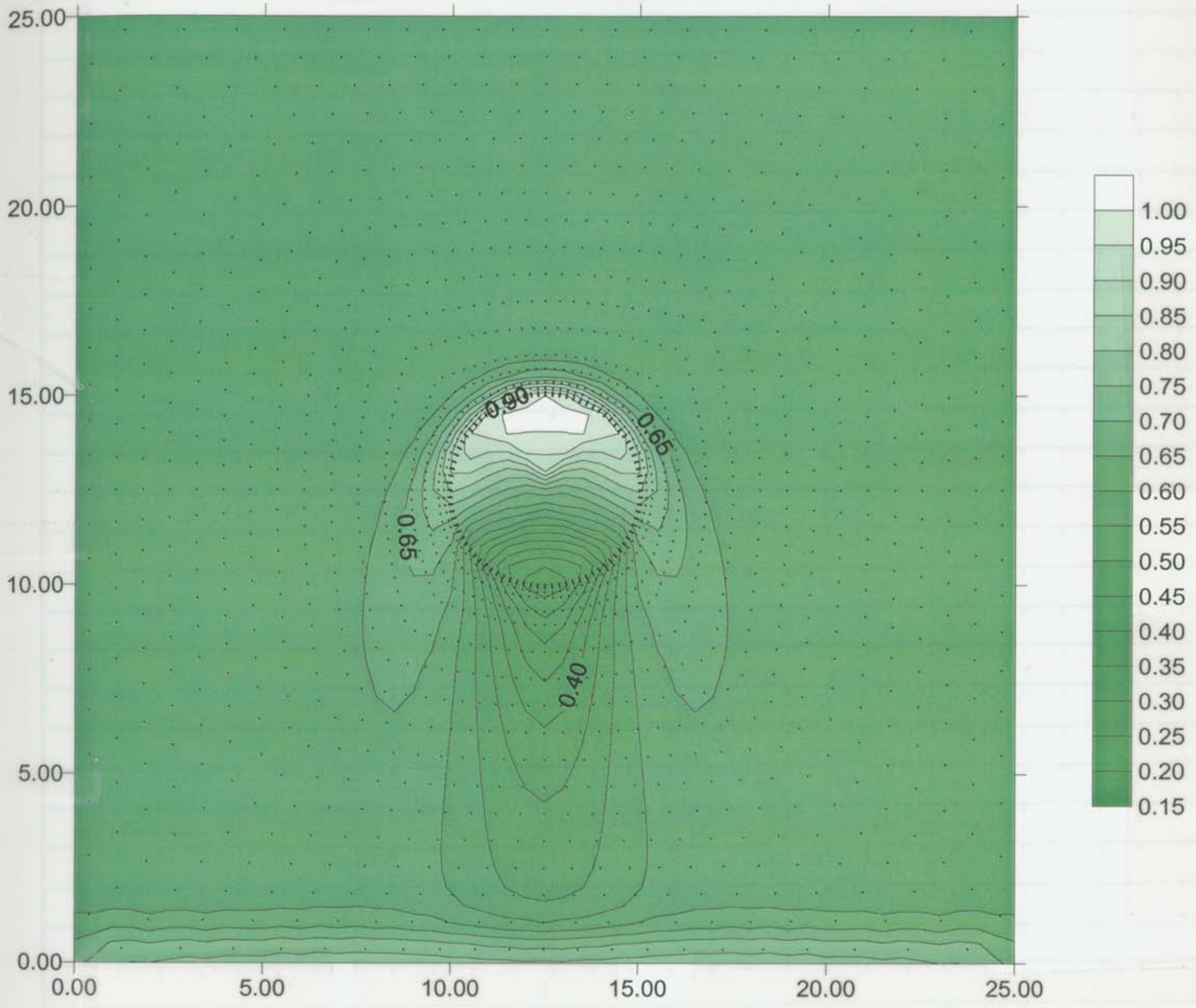

This is the result with $\alpha_G = 4.85e\text{-}4\ Pa^{-1}$ from page 8 with grid refinement – r3 (page 10)

~ dhughson/multiflo/inputbak/multi.r3
nodes.r3

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		Recorded by DH	Apr 21 1999

From Page No. ___ April 30, 1999

Return from trip to ESF, Thermal Test Workshop, Appendix 7 and tour of EBSO. Learned that repository drifts are to be 5.5 m diameter instead of the 5.0 m I've been using. Therefore, the next grid design -r4 will be -r3 with r = 2.75 m

The first perturbed driftwall input file I created with r = 2.5 will be saved in case I need it later as ~dhughson/multiflo/inputbak/multi.r3p

Node numbering for -r4 is identical to -r3 on page 10, the only difference is r = 2.75 m instead of r = 2.5 m

Using table lookup Gardner alpha 1.0e-5 Pa^{-1} at node 882

Flux	Pc	Se
7.5905e-4	0	1.0
8.0e-5	1.407e+5	.76
1.0e-4	1.320e5	.771
5.0e-4	3.118e4	.977
6.0e-4	1.672e4	.981
7.0e-4	3.785e3	.9961
7.3e-4	3.35e2	.9997
7.4e-4	0	1.0
7.35e-4	0	1.0
7.32e-4	131.9	.9999
7.325e-4 = 23146.4	0	1.0
7.324e-4 = 23143.3	26.2	1.0

Philip 23291.1 $\frac{23144-23291}{23291} = -0.63\%$ error

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For Gardner alpha 4.85e-4 Pa^{-1}
Philip = 2076.6 mm/yr

Flux	Pc	Se
6.57e-5	257	.9862
6.6e-5	247.7	.9868
6.7e-5	216.8	.9886
6.9e-5	156.1	.9919
7.1e-5	96.3	.9952
7.2e-5	66.9	.9967
7.4e-5	7.006	.9996
7.5e-5	0	1.0
7.45e-5	0	1.0
7.42e-5	3.352	.9998
7.43e-5 = 2347.8	0.514	1.0

$\frac{2348-2077}{2077} = 13\%$

archive ~dhughson/multiflo/inputbak/multi.r4

extensive grid refinement to capture detail of wavy wall boundary

created in D:\work\grid\MeshGrid6.nb

$r_0 = 2.75$ m d = amplitude = 0.1 n = 20 waves

(DH 5 Nov 1999 ~~Ext~~) First perturbed boundary result for grid -r5p

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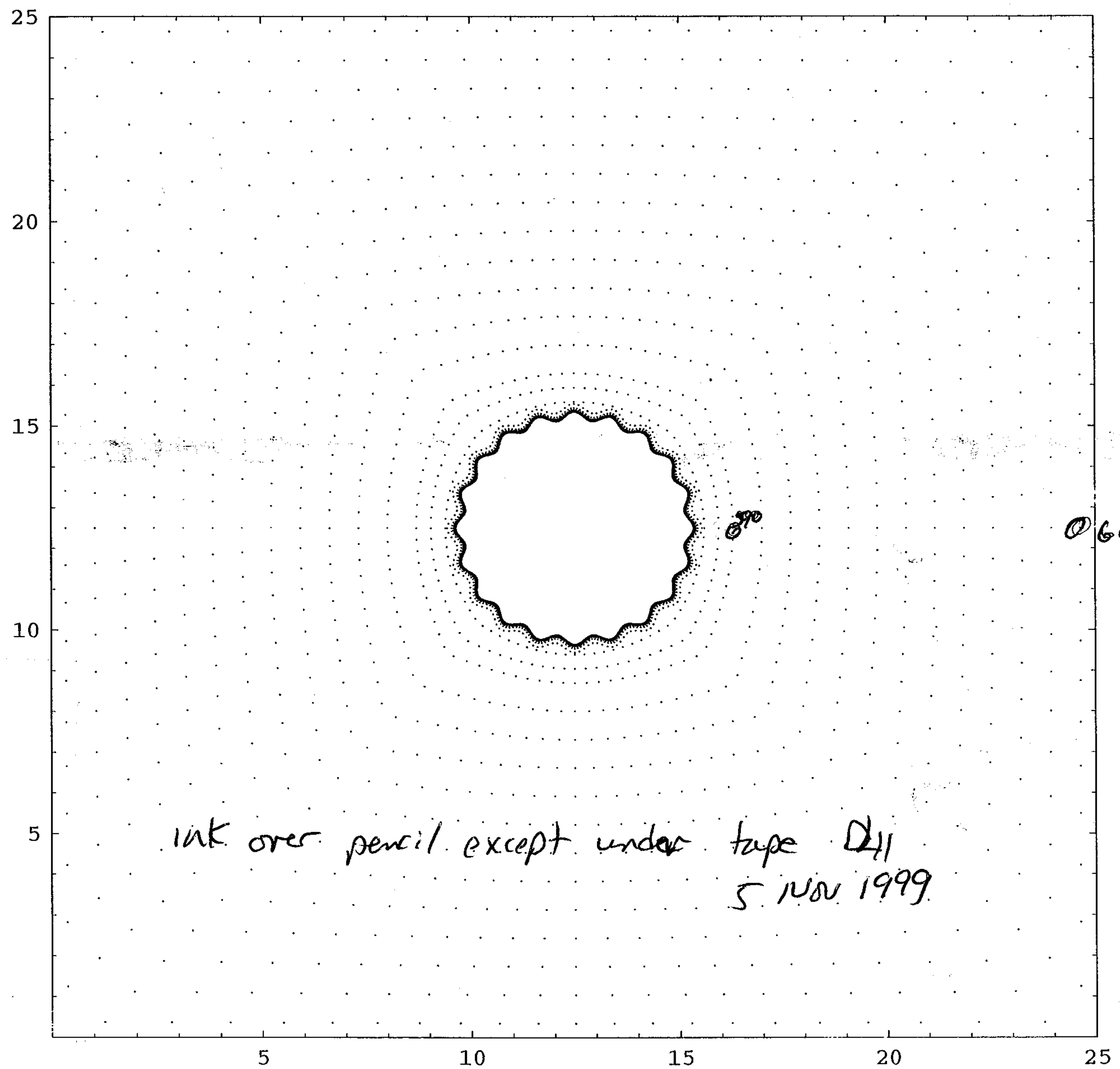
MeshGrid6.nb 1



3696

rSP

603 1 1730 857 1026 1237 1532 1869 1533 1238 1027 858 731 646 589 645 590

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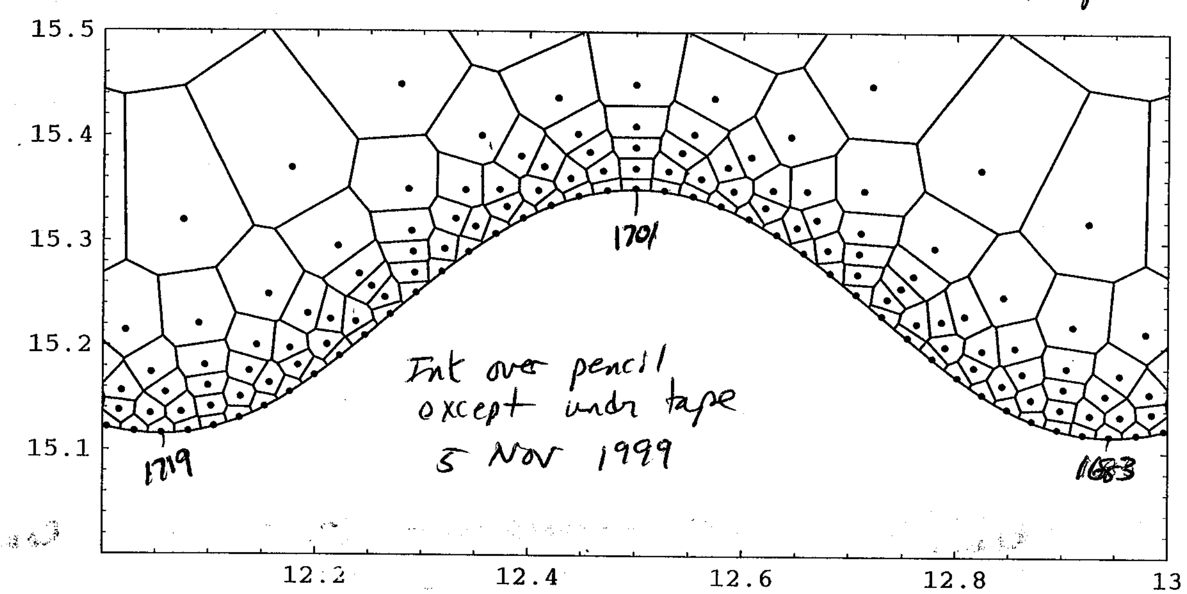
From Page No. Detail of grid in top boundary layer

MeshGrid6.nb

rSP



Flux	Pc	Se	node
7.43e-5	635.9	.9616	1701
7.43e-5	0	1.0	1683
~~7.43e-5~~	~~0~~	~~1.0~~	~~1719~~
6.0e-5	0	1.0	1719
1.0e-5 does not converge			
5.0e-5	0	1.0	
4.0e-5	0	1.0	
~~4.0e-5~~ 3.0e-5 (DH 5 Nov 1999)	0	1.0	
2.0e-5	0	1.0	
~~1.0e-5~~ 1.0e-6	0	1.0	
1.0e-7 does not converge	0	1.0	
6.0e-7	0	1.0	
5.0e-7	0	1.0	
4.0e-7	0	1.0	
3.0e-7 does not converge			
~~3.8e-7~~ 4.0e-7 — DH 5 Nov 1999			

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Preliminary results from -r5p and Frankl.nb

For α_g = 4.85e-4 Pa^{-1} (998.3 kg/m^3)(9.81 m/s^2) = 4.75 m

Capillary rise ≈ 1/α = 0.21m

-r5p grid converges ok for q_{in} > 4.0e-7 kg/m^2/s

4.0e-7 kg/m^2/s = 12.6 mm/yr refer to diagram pg 15

node 1701 S_e = ~~.86~~ .1394 (DH 5 Nov 1999) P_c = 1.519e4
node 1719 S_e = 0 P_c = 0
For nodes above 1719

node	x	z	S_e	P_c
1719	12.055	15.12	1.0	0
1401	12.045	15.135	1.0	0
1143	12.059	15.155	1.0	0
951	12.046	15.174	1.0	0
800	12.09	15.214	.1646	1.394e4

Sum of perched water in trough

z coord of containing peak 15.221

From this simulation 12.6 mm/yr is leaking.
Frankl with number of terms = 20 says (wrong DH)
threshold = 35.3 mm/yr for trough
= 846.8 mm/yr for top central peak

check
2.67e-5 kg/m^2/s S_e = .7921 P_c = 2570 node 1701
3.0e-5 kg/m^2/s = 948 .8118 2364

Running a check on Frankl.nb with r0 = 1

Correction on above ——— next page

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From Page No. Frankl.nb r0 must = 1

Threshold for top central peak = 2746.91 mm/yr
trough adj = 880.586 mm/yr

Try 2747 mm/yr = 8.693e-5 kg/m^2/s S_e = .9853 P_c = 273.7

-r5p & Frankl agree reasonably well at the top central peak but not for the trough. Suspect gridding problems with -r5p. Need to try a smoother (lower amplitude) wave for driftwall

---

New Grid -r6p 16 waves
radius = 2.75m
wave amplitude = .06875m

Flux	S_e	P_c	node
3.17e-7 kg/m^2/s			

(Run DH 5 Nov 1999) Nodes above driftwall in troughs are all S_e=1 P_c=0 and nodes above or immediately adjacent are S_e = 0.13. Some kind of gridding problem.

Create new grid with 1st set of nodes .005m away instead of .001m away. New grid is called -r7p. This time

nodes 1676-1680 and 1721-1726 are S_e=1 P_c=0 at 10 mm/yr infiltration.

~~Try looking at~~ Try ignoring these nodes (DH 5 Nov 1999)

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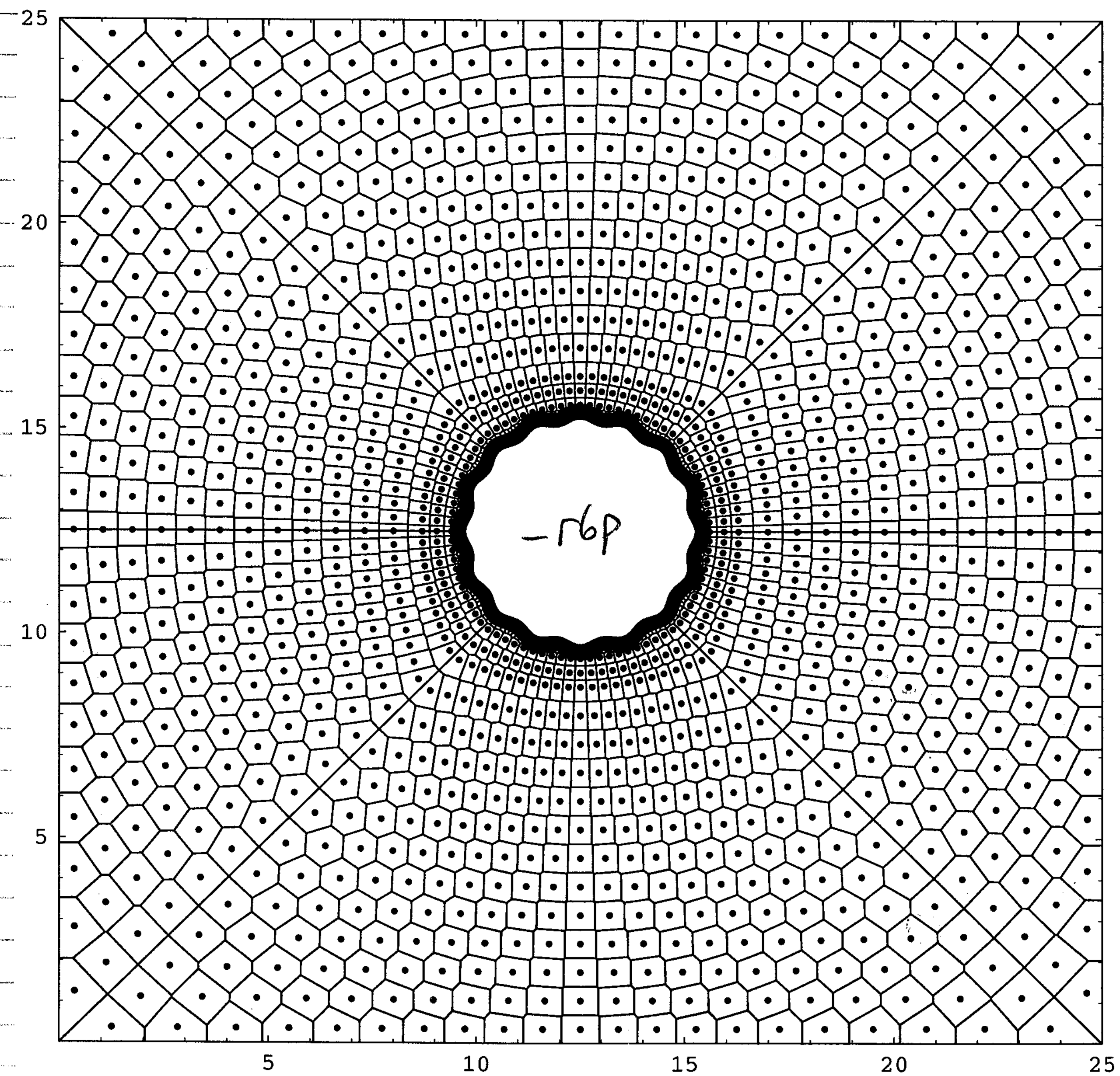

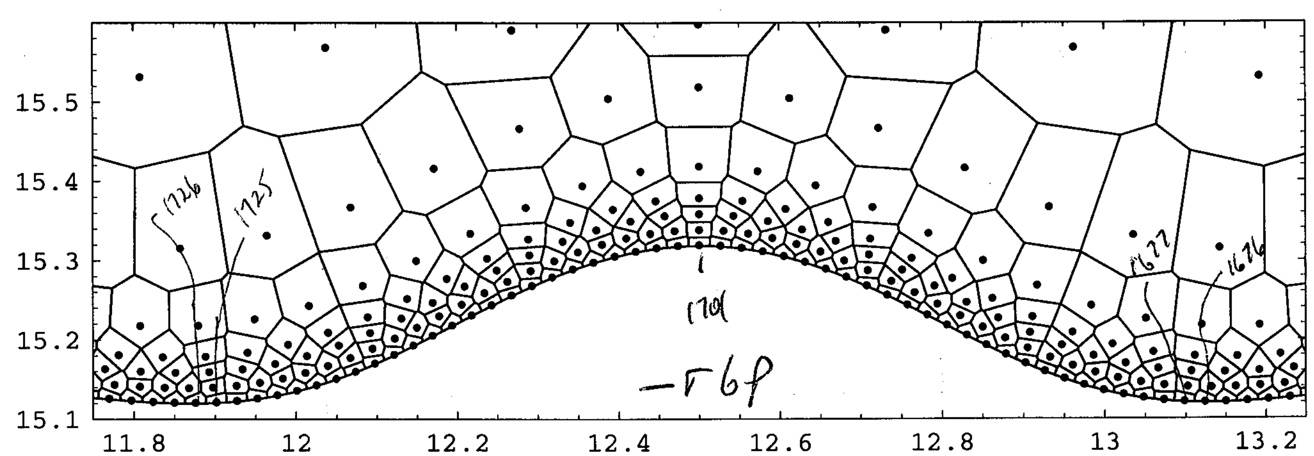

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From Page No. For grid -r7p

flux	Se	Pc	nodes
3.17e-7	.1318	1.556e4	1675, 1727
3.17e-6	.3963	2.465e3	" "
3.17e-5	.8563	1.893e3	
should be looking at		1672, 1730	
	.8818	1.614e3	
2.0e-5	1.0	0	1672, 1730
5.0e-5	.977	408.2	1674, 1728
5.5e-5	.9927	142	1674, 1728
5.7e-5 = 1801 mm/yr			

Grids -r5p -r6p -r7p (DH 5 NOV 1999) all have $S_e = 1.0$ $P_c = 0$ in driftwall boundary nodes ~~for~~ for small ~ 10 mm/yr infiltration. Problem reduced with -r7p by moving nodes from .001m to .005m away from dw bc. Still a problem

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mathematica notebook Frank2.nb checked against Frank's mathcad calculation for

S=4 waves = 16 ~~tip~~ amplitude = .025/1
number of terms = 80

Next run Frank2.nb with $\alpha_g = 4.85e-4$ Pa^{-1}

so $s = \alpha_g \theta / 2 = 6.531$ to get results for numerical grid comparison.

~~start 8:40 AM~~ DH
DH 5 Nov 1999

From Page No. Taylor results for DOE/NRC Tech Exchange scheduled for Wed, May 26, 1999 ~ prepare 15 min
Include Ron Green's stuff from App 7 meeting.

Generally this starts material creation and organization for AGU poster with possible inclusion in Tech Exchange.
FROM THE BASE CASE
TSPA seepage parameters 3.3e-4 9.7e-4 3.3e-3 α
10^{-14} 10^{-13} 10^{-12} K
grid r4.wpg gridr4zoom.wpg
Grid -r4 radius = 2.75 m rings .001 2.751 m
.01 2.76
.02 2.77
.05 2.8
.1 2.85
.2 2.9

For $\alpha_g = 1.0e-5$ Pa^{-1} $K = 10^{-14}$ m^2

$q^* =$ threshold ~ 7.324e-5 = 2314.3 mm/yr
tested – yes exactly r4benc1.dat

Philip solution = 2329.1 %error = (~~2329.1 – 2314.3~~) DH 5 Nov 1999
(2314.3 – 2329.1) / 2329.1 × 100 = –0.63%
r4bencl.dat r4bengdat
This is close to analytical but the graph looks wierd

What does $\alpha_g = 3.3e-3$ Pa give for q^* with $K = 10^{-14}$ m^2

DH 5 Nov 1999

Flux	Se	Pc
7.43e-6 ok	1.0	0
~~7.e-7~~ = 22.1 mm/yr	1.0	0
7.0e-8	1.0	0
1.0e-10 = .003 mm/yr	1.0	0

something is wierd here

Using $K = 10^{-13}$ m^2 $\alpha_v = 3.3e-3$ Pa^{-1}

5.0e-7	.648	303.8
7.0e-7 = 22.1 mm/yr	.703	274.0
1.0e-6 = 31.6 mm/yr	.762	242.2
5.0e-6 = 158 mm/yr	.985	76.78
6.0e-6 = 189.6 mm/yr	.995	51.8
6.5e-6	.9974	39.87
7.0e-6 = 221.2 mm/yr	.999	28.22
7.5e-6	.9998	16.7

From Page No. ___ Grid – r4 smooth wall r = 2.75 m
properties $K = 10^{-13}$ m^2 $\alpha_{G} = 3.3e-3$ Pa^{-1} (cont.)

~~5.0e-6~~ DH 5 NOV 1999	1.0	0
~~7.8e-6~~ 7.85e-6	1.0	8.6

= 2.48 mm/hr

r4benvq.dat

From Philip's solution this is equivalent to
$\alpha_G = 4.54e-3$ Pa^{-1} which gives

$q^* = 248.26$ mm/hr

Numerical $\alpha_G = 4.54e-3$ $K = 10^{-13}$ m^2 Tab in multiflo

2.85e-6	1.0	0	
6.0e-6	1.0	0	
5.0e-6	1.0	0	
5.0e-7	1.0	0	Haywire again
5.0e-9	1.0	0	Definitely haywire

It's the damned initial condition that's throwing it off

~~5.0e-9~~	~~.84~~	~~1.71e3~~	wrong
5.0e-8	.2016	1.185e3	
5.0e-6	.89	162.1	
9.0e-6	.982	34.77	
1.0e-5	.995	9.7	
~~2.0e-5~~ 1.5e-5 (DH 5 NOV 1999)	1.0	0	
1.1e-5	1.0	0	
1.05e-5	1.0	0	
1.02e-5	.997	5.6	
1.03e-5	.998	3.59	
1.04e-5	.9993	1.45	
1.045e-5 = 330.2	.9999	.26	✓

$$\text{Error} = \frac{330.2 - 248.26}{248.26} \times 100 = 33\%$$

r4 beng.dat

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Invented by ___ Date May 2, 1999
Recorded by DH

From Page No. ___ Result from pg 13 – r4 with
$\alpha_q = 4.85e-4$ Pa^{-1} $K = 10^{-13}$ m^2 saved as
r4benc2.dat

try – r7p with $\alpha_q = 4.54e-3$ $K = 10^{-13}$ m^2
1.0e-6

So far not too much success with numerical results for irregular boundary – may or may not continue this line

The following section records some work conducted jointly by Frank Dodge and myself on ~~an~~ (DH NOV 5 1999) a semi-analytical approach to irregular cavity walls. Reference all of ~~JR~~ JR Philip's papers in WRR around 1989 – 1990

Presented as a poster session at Spring AGU 1999 in Boston

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Witnessed & Understood by me, ___ Date ___
Invented by ___ Date May 24 1999
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pg 1

Fracture Continuum in the Dual Continuum model Approach ~~provides~~ (5 NOV 1999 DH)

Boundary condition on vertical seepage flux
q_{inf} = uniform & constant

↓ ↓ ↓ ↓ ↓

Continuum (DH 5 NOV 1999) ~~Discrete~~ Fracture Model

Ignore matrix flux

~~Ignore fracture intersections~~ (DH 5 NOV 1999)

seepage = $f(q_{inf}$, fracture distribution, fracture properties)

Drift

Assume: Flow ~~in each fracture~~ can be described by a 2D form of Richard's Eq. (steady state) (DH 5 Nov 1999)

~~For a fracture parallel to gravity~~: (DH 5 NOV 1999)

Isotropic

$$\nabla\cdot\left(\underline{K}\,\nabla(\psi - z)\right) = 0$$

$$\nabla\cdot\left(\underline{\underline{K}}\left[\frac{\partial\psi}{\partial x},\ \frac{\partial\psi}{\partial z} - 1\right]^T\right) = 0 \qquad K = \begin{bmatrix} K_x & 0 \\ 0 & K_z \end{bmatrix}$$

$$\frac{\partial}{\partial x}\left(K\frac{\partial\psi}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial\psi}{\partial z}\right) - \frac{\partial K}{\partial z} = 0$$

$$\nabla\cdot(K\nabla\psi) = \frac{\partial K}{\partial z}$$

Kirchoff transformation:

$$\Theta = \int_{-\infty}^{\psi} K(\psi)\,d\psi$$

using the Gardner model

$$K(\psi) = K_s e^{\alpha\psi}$$

Leibnitz Rule:

$$\frac{\partial}{\partial x}\int_{a(x)}^{b(x)} f(x,y)\,dy = \int_{a(x)}^{b(x)} \frac{\partial f(x,y)}{\partial x}\,dy + f(b,y)\frac{\partial b}{\partial x} - f(a,y)\frac{\partial a}{\partial x}$$

From

pg 2

$$\frac{\partial\Theta}{\partial x} = \frac{\partial}{\partial x}\int_{-\infty}^{\psi} e^{\alpha\psi}\,d\psi = \int_{-\infty}^{\psi} 0\,d\psi + e^{\alpha\psi}\frac{\partial\psi}{\partial x} - 0 = K\frac{\partial\psi}{\partial x}$$

$$\frac{\partial^2\Theta}{\partial x^2} = \frac{\partial}{\partial x}\left(K\frac{\partial\psi}{\partial x}\right) \qquad \text{likewise} \qquad \frac{\partial^2\Theta}{\partial z^2} = \frac{\partial}{\partial z}\left(K\frac{\partial\psi}{\partial z}\right)$$

so $\nabla\cdot(K\nabla\psi) = \frac{\partial K}{\partial z}$ becomes $\nabla^2\Theta = \frac{\partial K}{\partial z}$

$$\frac{\partial K}{\partial z} = \frac{\partial(e^{\alpha\psi})}{\partial z} \qquad \Theta = \int_{-\infty}^{\psi} e^{\alpha\psi}\,d\psi = \left.\frac{e^{\alpha\psi}}{\alpha}\right|_{-\infty}^{\psi} = \frac{e^{\alpha\psi}}{\alpha}$$

$$\alpha\Theta = e^{\alpha\psi}$$

α = constant so $\frac{\partial K}{\partial z} = \alpha\frac{\partial\Theta}{\partial z}$

$$\nabla^2\Theta = \alpha\frac{\partial\Theta}{\partial z}$$

Far from the drift seepage is uniform $K_o = q_{inf}$

so $\lim_{x^2+y^2\to\infty} \Theta = \Theta_o$

normalize $\vartheta = \frac{\Theta}{\Theta_o}$ $\quad \varsigma = \frac{x}{l}$ $\quad \eta = \frac{z}{l}$

$$\frac{\partial}{\partial x}\left(\frac{\partial\vartheta}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial\vartheta}{\partial z}\right) = \alpha\frac{\partial\vartheta}{\partial z}$$

$$\frac{\partial}{\partial(l\varsigma)}\left(\frac{\partial\vartheta}{\partial(l\varsigma)}\right) + \frac{\partial}{\partial(l\eta)}\left(\frac{\partial\vartheta}{\partial(l\eta)}\right) = \alpha\frac{\partial\vartheta}{\partial(l\eta)}$$

$$\nabla^2\vartheta = \alpha l\frac{\partial\vartheta}{\partial\eta}$$

where ∇ is on the transformed coordinates

let $s = \frac{\alpha l}{2}$ then $\nabla^2\vartheta = 2s\frac{\partial\vartheta}{\partial\eta}$

pg 3

Let $H = (\vartheta - 1)e^{-s\eta} = \vartheta e^{-s\eta} - e^{-s\eta}$

$\frac{\partial H}{\partial \zeta} = \frac{\partial \vartheta}{\partial \zeta} e^{-s\eta}$ $\qquad \frac{\partial^2 H}{\partial \zeta^2} = \frac{\partial^2 \vartheta}{\partial \zeta^2} e^{-s\eta}$

$\frac{\partial H}{\partial \eta} = \frac{\partial \vartheta}{\partial \eta} e^{-s\eta} - \vartheta s e^{-s\eta} + s e^{-s\eta}$

$\frac{\partial^2 H}{\partial \eta^2} = \frac{\partial^2 \vartheta}{\partial \eta^2} e^{-s\eta} - \frac{\partial \vartheta}{\partial \eta} s e^{-s\eta} - \frac{\partial \vartheta}{\partial \eta} s e^{-s\eta} + \vartheta s^2 e^{-s\eta} - s^2 e^{-s\eta}$

substitute in $\frac{\partial^2 \vartheta}{\partial \zeta^2} + \frac{\partial^2 \vartheta}{\partial \eta^2} = 2s \frac{\partial \vartheta}{\partial \eta}$

$\frac{\partial^2 \vartheta}{\partial \zeta^2} = \frac{\partial^2 H}{\partial \zeta^2} e^{s\eta}$ $\qquad \frac{\partial \vartheta}{\partial \eta} = \frac{\partial H}{\partial \eta} e^{s\eta} + \vartheta s + s$

$\frac{\partial^2 \vartheta}{\partial \eta^2} = \frac{\partial^2 H}{\partial \eta^2} e^{s\eta} + 2 \frac{\partial \vartheta}{\partial \eta} s - \vartheta s^2 + s^2$

$\frac{\partial^2 H}{\partial \zeta^2} e^{s\eta} + \frac{\partial^2 H}{\partial \eta^2} e^{s\eta} + 2 \frac{\partial \vartheta}{\partial \eta} s - \vartheta s^2 + s^2 = 2s\left(\frac{\partial H}{\partial \eta} e^{s\eta} + \vartheta s + s\right)$

$= 2s \frac{\partial H}{\partial \eta} e^{s\eta} + 2\vartheta s^2 + 2s^2$

$\frac{\partial^2 H}{\partial \zeta^2} + \frac{\partial^2 H}{\partial \eta^2} = 2s \frac{\partial H}{\partial \eta} - 2s \frac{\partial \vartheta}{\partial \eta} e^{-s\eta} + 3s^2 \vartheta e^{-s\eta} + s^2 e^{-s\eta}$

$= 2s \frac{\partial H}{\partial \eta} - 2s e^{-s\eta}\left(\frac{\partial H}{\partial \eta} e^{s\eta} + \vartheta s + s\right) + 3s^2 \vartheta e^{-s\eta} + s^2 e^{-s\eta}$

$= 2s \frac{\partial H}{\partial \eta} - 2s \frac{\partial H}{\partial \eta} - 2s^2 \vartheta e^{-s\eta} - 2s^2 e^{-s\eta} + 3s^2 \vartheta e^{-s\eta} + s^2 e^{-s\eta}$

$= s^2 \vartheta e^{-s\eta} - s^2 e^{-s\eta}$

$= s^2 e^{-s\eta}(\vartheta - 1) = s^2 H$

then $\nabla^2 H = s^2 H$ Helmholtz Eq.

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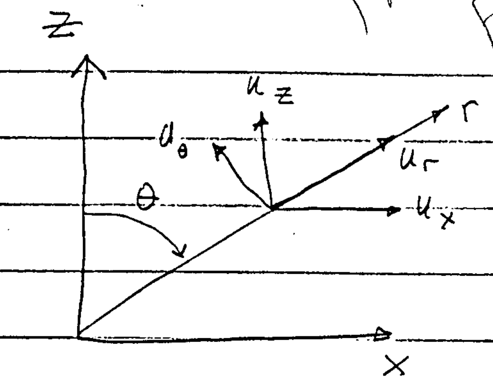
Witn

Recorded by DH Aug 16 1999

From Frank Dodge

1 of 2

circa March 19



$z = r\cos\theta$

$x = r\sin\theta$

$r^2 = x^2 + z^2$

$\tan\theta = \frac{x}{z}$

$u_r = u_z\cos\theta + u_x\sin\theta \qquad u_\theta = u_z\sin\theta - u_x\cos\theta$

θ = const

$\frac{\partial z}{\partial r} = \cos\theta \qquad \frac{\partial z}{\partial \theta} = -r\sin\theta \qquad \frac{\partial x}{\partial r} = \sin\theta \qquad \frac{\partial x}{\partial \theta} = r\cos\theta$ r = const

$\frac{\partial r}{\partial x} = \frac{x}{r} = \sin\theta \qquad \frac{\partial r}{\partial z} = \frac{z}{r} = \cos\theta$ x = const

z = const $\frac{\partial \theta}{\partial x} = \frac{\cos^2\theta}{z} = \frac{\cos\theta}{r} \qquad \frac{\partial \theta}{\partial z} = \frac{-x\cos^2\theta}{z^2} = -\frac{\sin\theta}{r}$

For any function $f(x,z) = F(r,\theta)$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} = \sin\theta\frac{\partial f}{\partial r} + \frac{\cos\theta}{r}\frac{\partial f}{\partial \theta}$ z = const

$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial f}{\partial \theta}\cdot\frac{\partial \theta}{\partial z} = \cos\theta\frac{\partial f}{\partial r} - \frac{\sin\theta}{r}\frac{\partial f}{\partial \theta}$ x = const

$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial r} = \sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial z}$ θ = const

$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial \theta} = r\cos\theta\frac{\partial f}{\partial x} - r\sin\theta\frac{\partial f}{\partial z}$ r = const

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CHECK BY A SIMPLE EXAMPLE

$f = z^2 + 3xz$ or $f = r^2(\cos^2\theta + 3\sin\theta\cos\theta)$

$\frac{\partial f}{\partial z} = 2z + 3x = r(2\cos\theta + 3\sin\theta)$ $\frac{\partial f}{\partial r} = 2r(\cos^2\theta + 3\sin\theta\cos\theta)$

$\frac{\partial f}{\partial x} = 3z = 3r\cos\theta$ $\frac{\partial f}{\partial \theta} = r^2[-2\cos\theta\sin\theta + 3\cos^2\theta - 3\sin^2\theta]$

BY THE FORMULAS

$$\frac{\partial f}{\partial r} = \sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial z} = \sin\theta[3r\cos\theta] + r\cos\theta[2\cos\theta + 3\sin\theta]$$
$$= r(2\cos^2\theta + 6\sin\theta\cos\theta) \checkmark$$

$$\frac{\partial f}{\partial \theta} = r\cos\theta\frac{\partial f}{\partial x} - r\sin\theta\frac{\partial f}{\partial z} = r\cos\theta(3r\cos\theta) - r\sin\theta[r(2\cos\theta + 3\sin\theta)]$$
$$= r^2[3\cos^2\theta - 2\sin\theta\cos\theta - 3\sin^2\theta] \checkmark$$

$$\frac{\partial f}{\partial x} = \sin\theta\frac{\partial f}{\partial r} + \frac{\cos\theta}{r}\frac{\partial f}{\partial \theta} = \sin\theta[2r(\cos^2\theta + 3\sin\theta\cos\theta)]$$
$$+ \frac{\cos\theta}{r}[r^2][-2\cos\theta\sin\theta + 3\cos^2\theta - 3\sin^2\theta]$$
$$= r\{2\sin\theta\cos^2\theta + 6\sin^2\theta\cos\theta - 2\sin\theta\cos^2\theta + 3\cos^3\theta - 3\sin^2\theta\cos\theta\}$$
$$= r\{3\sin^2\theta\cos\theta + 3\cos^3\theta\} = 3r\cos\theta \checkmark$$

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$$\frac{\partial f}{\partial z} = \cos\theta\frac{\partial f}{\partial r} - \frac{\sin\theta}{r}\frac{\partial f}{\partial r} = 2r(\cos^3\theta + 3\sin\theta\cos^2\theta)$$

~~$- r[-2\cos\theta\sin^2\theta + 3\sin\theta\cos^2\theta - 3\sin^3\theta]$~~

$$= r\{2\cos^3\theta + 6\sin\theta\cos^2\theta + 2\cos\theta\sin^2\theta - 3\sin\theta\cos^3\theta + 3\sin^3\theta\}$$
$$= r\{2\cos^3\theta + 2\cos\theta\sin^2\theta + 3\sin\theta\cos^2\theta + 3\sin^3\theta\} = r(2\cos\theta + 3\sin\theta) \checkmark$$

Wi

TITLE

From Pa

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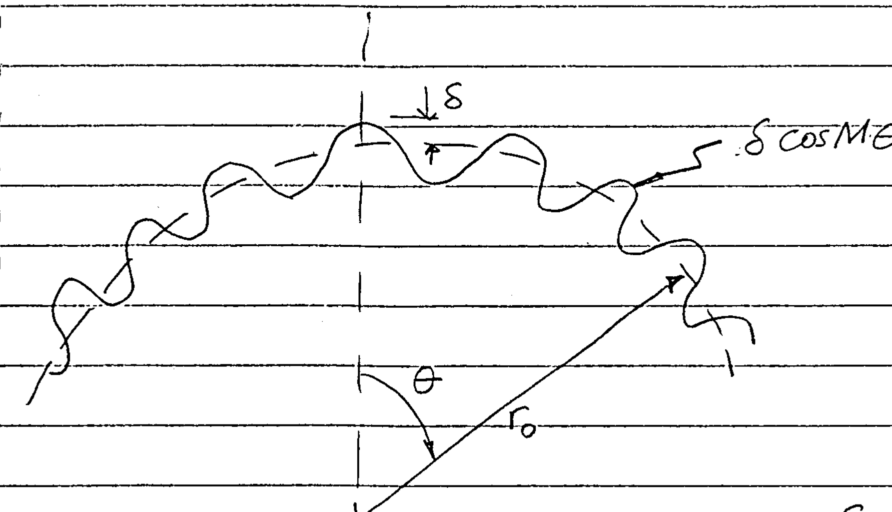
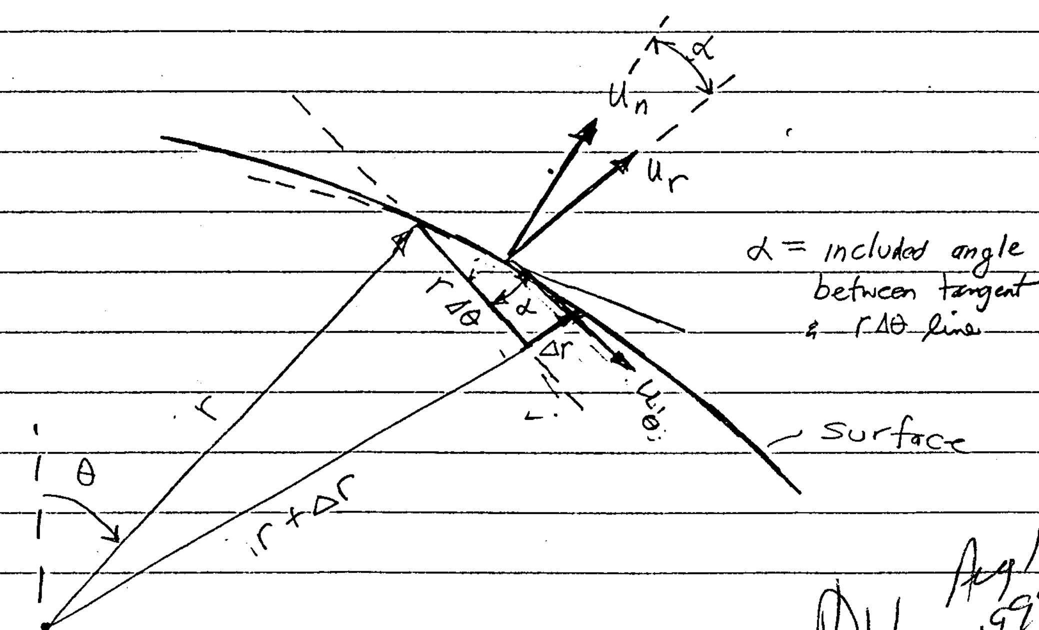
April 13, 1999



$r = r_o + \delta\cos M\theta$

Derivation of normal velocity = 0 on surface ($u_n = 0$)



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$u_n = u_r\cos\alpha - u_\theta\sin\alpha$ (1)

$\tan\alpha = \frac{\Delta r}{r\Delta\theta}$ (2)

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So the normal velocity is

$$U_n = \left(U_r - U_\theta \frac{\sin\alpha}{\cos\alpha}\right)\cos\alpha = \left(U_r - U_\theta \frac{\Delta r}{r\Delta\theta}\right)\cos\alpha$$

In the limit $\Delta\theta \to 0$, the normal velocity is zero when

$$\boxed{U_r - U_\theta\left(\frac{dr}{rd\theta}\right) = 0} \qquad (3)$$

where $r = r_0 + \delta\cos M\theta$

From Phillip's papers, we know

$$U_r = \psi\cos\theta - \frac{1}{2s}\frac{\partial\psi}{\partial r} \quad (\text{I use } \psi \text{ instead of } \varphi) \qquad (4)$$

$$U_\theta = \psi\sin\theta - \frac{1}{2sr}\frac{\partial\psi}{\partial\theta} \qquad (5)$$

$$\psi = He^{sz} + 1 = He^{sr\cos\theta} + 1 \qquad (6)$$

From (6)

$$\frac{\partial\psi}{\partial r} = e^{sr\cos\theta}\frac{\partial H}{\partial r} + Hs\cos\theta\, e^{sr\cos\theta} \qquad (7)$$

$$\frac{\partial\psi}{\partial\theta} = e^{sr\cos\theta}\frac{\partial H}{\partial\theta} - Hsr\sin\theta\, e^{sr\cos\theta} \qquad (8)$$

Put (4), (5), (7), and (8) into (3) :

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$$\left[e^{sr\cos\theta}H + 1\right]\cos\theta - \frac{1}{2s}\left[e^{sr\cos\theta}\frac{\partial H}{\partial r} + Hs\cos\theta\, e^{sr\cos\theta}\right]$$

$$-\left\{\left[e^{sr\cos\theta}H + 1\right]\sin\theta\left(\frac{dr}{rd\theta}\right) - \frac{1}{2sr}\left[e^{sr\cos\theta}\frac{\partial H}{\partial\theta} - Hsr\sin\theta\, e^{sr\cos\theta}\right]\right\}\frac{dr}{rd\theta} = 0$$

Multiply through by $2se^{-sr\cos\theta}$ and collect terms

$$\boxed{\frac{\partial H}{\partial r} - \frac{1}{r}\frac{\partial H}{\partial\theta}\left(\frac{dr}{rd\theta}\right) = s\cos\theta\left[H + 2e^{-sr\cos\theta}\right] - s\sin\theta\left[H + 2e^{-sr\cos\theta}\right]\left(\frac{dr}{rd\theta}\right) = 0} \qquad (9)$$

Eq. (9) is Phillip's eq. (32) when $\frac{dr}{rd\theta} = 0$ and $r = 1$

Multiply by r^2 to get r's out of the denominator and evaluate for $r = r_0 + \delta\cos M\theta = \bar{r}$ for short

$$\boxed{\bar{r}^2\frac{\partial H}{\partial r} - \frac{\partial H}{\partial\theta}\left(\frac{dr}{d\theta}\right) = \bar{r}^2 s\cos\theta\left[H + 2e^{-s\bar{r}\cos\theta}\right] - \bar{r}s\sin\theta\left[H + 2e^{-s\bar{r}\cos\theta}\right]\left(\frac{dr}{d\theta}\right) = 0} \qquad (10)$$

This is the boundary condition at $r = \bar{r}$

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SOLUTION

$$H(r,\theta) = \sum_{n=0}^{\infty} a_n K_n(sr)\cos n\theta + \sum_{n=1}^{\infty} b_n K_n(sr)\sin n\theta \qquad (11)$$

satisfies $\nabla^2 H = s^2 H$ and $H \to 0$ as $r \to \infty$.

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How do we make (11) satisfy (10). Substitute (11) into (10) and use the relation

$$\frac{d}{dr} K_n(sr) = -\frac{s}{2}\left[K_{n-1}(sr) + K_{n+1}(sr)\right]$$

After some rearranging we get

$$\frac{s}{2}\bar{r}^{-2}\left\{\sum_{n=0}^{\infty}(a_n\cos n\theta + b_n\sin n\theta)\left[K_{n-1}(s\bar{r}) + K_{n+1}(s\bar{r})\right]\right\}$$

$$+\left(\frac{d\bar{r}}{d\theta}\right)\left\{\sum_{n=1}^{\infty}\left(-na_n\sin n\theta + nb_n\cos n\theta\right)K_n(s\bar{r})\right\}$$

$$+ s\left[\bar{r}^2\cos\theta - \bar{r}\left(\frac{d\bar{r}}{d\theta}\right)\sin\theta\right]\sum_{n=0}^{\infty}(a_n\cos n\theta + b_n\sin n\theta)K_n(s\bar{r}) \quad (12)$$

$$= -4s\left(e^{-s\bar{r}\cos\theta}\right)\left[\bar{r}^2 - \bar{r}\frac{dr}{d\theta}\right] = 0$$

For any r_0, δ, s, and M, both sides of this equation can be expanded into a Fourier series:

$$\text{Left-Hand Side} = \sum_{n=0}^{\infty} A_n \cos n\theta + \sum_{n=1}^{\infty} B_n \sin n\theta \quad (13)$$

where

A_n, B_n are functions of $a_0, a_1, a_2, a_3, \ldots, b_1, b_2, b_3, \ldots$

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For example, with $s=4$, $r_0=1$, $\delta=0.1$, and $M=8$, we find the expansion for

$$\frac{s}{2}\bar{r}^{-2}\left\{\sum_{n=0}^{\infty}(a_n\cos n\theta + b_n\sin n\theta)\left[K_{n-1}(s\bar{r}) + K_{n+1}(s\bar{r})\right]\right]$$

to be

$$\frac{s}{2}\Big\{0.025a_0 + 0.029a_1\cos\theta + 0.029b_1\sin\theta$$
$$+ 0.043a_2\cos 2\theta + 0.043b_2\cos 2\theta$$
$$+ 0.083a_3\cos 3\theta + 0.083b_3\sin 3\theta$$
$$+ \ldots$$
$$+ 797.45a_{10}\cos 10\theta + 797.45b_{10}\sin 10\theta$$
$$+ \ldots\Big\}$$

This is particularly simple because each A_n and B_n depends only on the corresponding a_n and b_n. Note also that a_n and b_n should $\to 0$ as $n \to \infty$, otherwise A_n and B_n get bigger rather than smaller as $n \to \infty$.

Likewise, the RHS is

$$RHS = \sum_{n=0}^{\infty} C_n\cos n\theta + \sum_{n=1}^{\infty} D_n \sin n\theta \quad (14)$$

where C_n and D_n are numbers

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Since LHS = RHS term-by-term:

$A_n \equiv C_n$

$B_n \equiv D_n$

In general, $A_m = \sum_{n=0}^{\infty} \alpha_{mn} a_n + \sum_{n=1}^{\infty} \beta_{mn} b_n$

$B_m = \sum_{n=0}^{\infty} \gamma_{mn} a_n + \sum_{n=1}^{\infty} \mu_{mn} b_n$

where $\alpha_{mn}, \beta_{mn}, \gamma_{mn}, \mu_{mn}$ are numbers determined from the Fourier expansions.

Thus

(15) $\sum_{n=1}^{\infty} \alpha_{mn} a_n + \sum_{n=1}^{\infty} \beta_{mn} b_n = C_m \qquad m = 1, 2, 3, \ldots, K, \ldots$

(16) $\sum_{n=0}^{\infty} \gamma_{mn} a_n + \sum_{n=1}^{\infty} \mu_{mn} b_n = D_m \qquad m = 1, 2, 3, \ldots, K, \ldots$

Truncate after K terms (K is a big number ≈ 10) and solve (15) and (16) [2K equations] for the 2K unknowns: $a_0, a_1, a_2, \ldots, a_K, b_1, b_2, \ldots, b_K$

Then use these in (6) to solve for potential φ

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$$\varphi = e^{sr\cos\theta} \sum_{n=0}^{\infty} \left[a_n \cos n\theta + b_n \sin n\theta \right] K_n(sr) + 1$$

for $r = r_0 + \delta \cos M\theta$, θ = various values to evaluate φ on the boundary!

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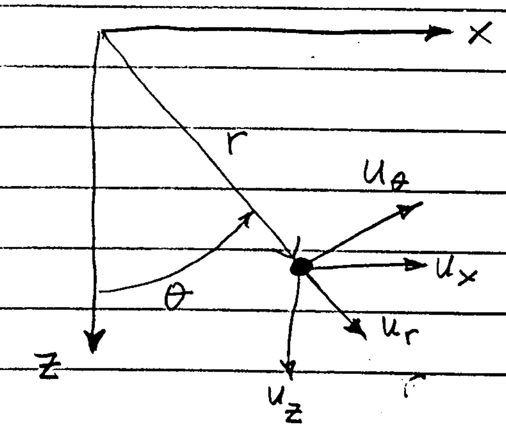
TRANSFORMATIONS & VELOCITIES



$z = r\cos\theta$

$x = r\sin\theta$

$r^2 = x^2 + z^2$

$\tan\theta = \frac{x}{z}$

$U_r = U_x \sin\theta + U_z \cos\theta$

$U_\theta = U_x \cos\theta - U_z \sin\theta$

CHECK

$\theta = 90°$ U_θ should be $-U_z$ ✓

$\theta = 0°$ U_θ should be $+U_x$ ✓

~~$\theta = 0°$ U_r should be $+U_z$~~ ✓

$\theta = 90°$ U_r should be $+U_x$ ✓

$\frac{\partial z}{\partial r} = \cos\theta \quad \frac{\partial x}{\partial r} = \sin\theta \quad \frac{\partial z}{\partial \theta} = -r\sin\theta \quad \frac{\partial x}{\partial \theta} = r\cos\theta$

$\frac{\partial r}{\partial z} = \frac{z}{r} = \cos\theta \quad \frac{\partial r}{\partial x} = \frac{x}{r} = \sin\theta$

$\frac{\partial \theta}{\partial z} = -\frac{x\cos^2\theta}{z^2} = -\frac{\sin\theta}{r} \quad \frac{\partial \theta}{\partial x} = \frac{\cos^2\theta}{z} = \frac{\cos\theta}{r}$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} = \sin\theta \frac{\partial f}{\partial r} + \frac{\cos\theta}{r}\cdot\frac{\partial f}{\partial \theta}$

$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial z} = \cos\theta\frac{\partial f}{\partial r} - \frac{\sin\theta}{r}\frac{\partial f}{\partial \theta}$

$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial r} = \sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial z}$

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Witness $\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \theta} = r\cos\theta\frac{\partial f}{\partial x} - r\sin\theta\frac{\partial f}{\partial z}$

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Check

let $\frac{\partial f}{\partial r} = u_r \quad \frac{\partial f}{r\partial\theta} = u_\theta \quad \frac{\partial f}{\partial x} = u_x \quad \frac{\partial f}{\partial z} = u_z$

$$\frac{\partial f}{\partial r} = u_r = \sin\theta \frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial z} = \sin\theta\, u_x + \cos\theta\, u_z \quad ✓$$

$$\frac{1}{r}\frac{\partial f}{\partial\theta} = u_\theta = \cos\theta\frac{\partial f}{\partial x} - \sin\theta\frac{\partial f}{\partial z} = \cos\theta\, u_x - \sin\theta\, u_z \quad ✓$$

Now $u_x = -\frac{1}{2s}\frac{\partial\psi}{\partial x} \qquad u_z = \psi - \frac{1}{2s}\frac{\partial\psi}{\partial z}$

Note $\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0 \Rightarrow$ conservation of mass

so $-\frac{1}{2s}\frac{\partial^2\psi}{\partial x^2} + \frac{\partial\psi}{\partial z} - \frac{1}{2s}\frac{\partial^2\psi}{\partial z^2} = 0$

or $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2} = 2s\frac{\partial\psi}{\partial z}$ ← Phillip's Eq. 13 ✓

$$u_r = u_x\sin\theta + u_z\cos\theta = -\frac{1}{2s}\sin\theta\frac{\partial\psi}{\partial x} + \cos\theta\left[\psi - \frac{1}{2s}\frac{\partial\psi}{\partial z}\right]$$

$$= -\frac{\sin\theta}{2s}\left[\sin\theta\frac{\partial\psi}{\partial r} + \frac{\cos\theta}{r}\frac{\partial\psi}{\partial\theta}\right] + \psi\cos\theta - \frac{\cos\theta}{2s}\left[\cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right]$$

$$= -\frac{(\sin^2\theta+\cos^2\theta)}{2s}\frac{\partial\psi}{\partial r} + \psi\cos\theta - \frac{\sin\theta\cos\theta}{2sr}\frac{\partial\psi}{\partial\theta} + \frac{\sin\theta\cos\theta}{2sr}\frac{\partial\psi}{\partial\theta}$$

$$\boxed{u_r = \psi\cos\theta - \frac{1}{2s}\frac{\partial\psi}{\partial r}}$$

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$$u_\theta = u_x\cos\theta - u_z\sin\theta = \cos\theta\left[-\frac{1}{2s}\frac{\partial\psi}{\partial x}\right] - \sin\theta\left[\psi - \frac{1}{2s}\frac{\partial\psi}{\partial z}\right]$$

$$= -\frac{\cos\theta}{2s}\left[\sin\theta\frac{\partial\psi}{\partial r} + \frac{\cos\theta}{r}\frac{\partial\psi}{\partial\theta}\right] - \psi\sin\theta + \frac{\sin\theta}{2s}\left[\cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right]$$

$$\boxed{u_\theta = -\psi\sin\theta - \frac{1}{2sr}\frac{\partial\psi}{\partial\theta}}$$

From Page N

CHECK

$\theta = 90°$ $\quad u_\theta = -u_z$ and $\frac{\partial}{r\partial\theta} = -\frac{\partial}{\partial z}$

so $u_\theta = -\psi + \frac{1}{2s}\frac{\partial\psi}{\partial z} = -\left[\psi - \frac{1}{2s}\frac{\partial\psi}{\partial z}\right] = -u_z$ ✓

$u_r = u_x$ and $\frac{\partial}{\partial r} = \frac{\partial}{\partial x}$

so $u_r = \psi\cdot 0 - \frac{1}{2s}\frac{\partial\psi}{\partial x} = -\frac{1}{2s}\frac{\partial\psi}{\partial x} = u_x$ ✓

$\theta = 0°$ $\quad u_\theta = u_x$ and $\frac{\partial}{r\partial\theta} = \frac{\partial}{\partial x}$

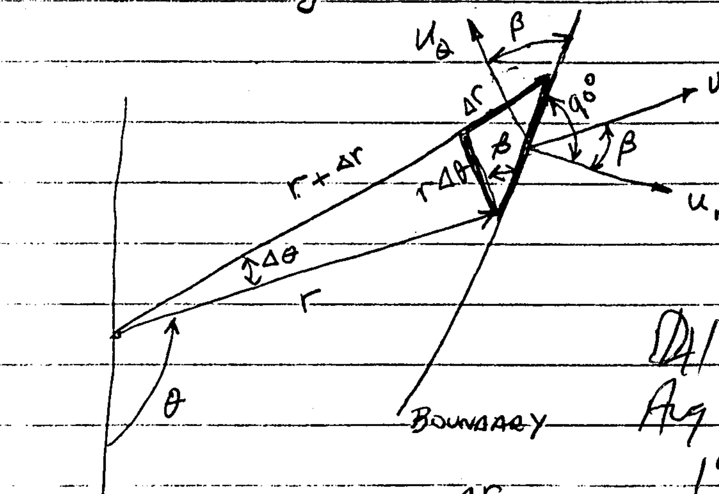
so $u_\theta = -\psi\cdot 0 - \frac{1}{2s}\frac{\partial\psi}{\partial x} = -\frac{1}{2s}\frac{\partial\psi}{\partial x} = u_x$ ✓

$u_r = u_z$ and $\frac{\partial}{\partial r} = \frac{\partial}{\partial z}$

so $u_r = \psi - \frac{1}{2s}\frac{\partial\psi}{\partial z} = u_z$ ✓

SO FAR, SO GOOD!

Normal velocity = 0 condition $\quad u_n = 0$



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$$\tan\beta = \frac{\Delta r}{r\Delta\theta}$$

$$u_n = u_r \cos\beta - u_\theta \sin\beta = (u_r - u_\theta \tan\beta)\cos\beta$$

So $u_r - u_\theta \tan\beta = 0$

or

$$u_r - \left(\frac{dr}{r\,d\theta}\right) u_\theta = 0$$ is $u_n = 0$ condition

---

BACK UP A MINUTE & MAKE A CHECK

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_\theta}{r\,\partial\theta} = 0$$ conservation of mass

So

$$\left[\cos\theta \frac{\partial\psi}{\partial r} - \frac{1}{2s}\frac{\partial^2\psi}{\partial r^2}\right] + \left[\frac{\cos\theta}{r}\psi - \frac{1}{2sr}\frac{\partial\psi}{\partial r}\right]$$

$$+ \left[-\frac{\psi}{r}\cos\theta - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta} - \frac{1}{2sr^2}\frac{\partial^2\psi}{\partial\theta^2}\right]$$

$$= -\frac{1}{2s}\frac{\partial^2\psi}{\partial r^2} - \frac{1}{2sr}\frac{\partial\psi}{\partial r} - \frac{1}{2sr^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\cos\theta}{r}\psi - \frac{\cos\theta}{r}\psi$$

$$+ \cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta} = 0$$

or

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{r^2\partial\theta^2} = 2s\left[\cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right]$$

$$= 2s\left[\frac{\partial\psi}{\partial z}\right]$$

= Phillip's Eq. (13) in cylindrical coordinates ✓

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Now back to $u_n = 0$ condition

$$u_r - \left(\frac{dr}{r\,d\theta}\right) u_\theta = 0$$

or

$$\left[\psi\cos\theta - \frac{1}{2s}\frac{\partial\psi}{\partial r}\right] - \left(\frac{dr}{r\,d\theta}\right)\left[-\psi\sin\theta - \frac{1}{2sr}\frac{\partial\psi}{\partial\theta}\right] = 0$$

or

$$-\frac{\partial\psi}{\partial r} + \frac{1}{r}\left(\frac{dr}{r\,d\theta}\right)\frac{\partial\psi}{\partial\theta} + 2s\left[\psi\cos\theta + \left(\frac{dr}{r\,d\theta}\right)\psi\sin\theta\right] = 0$$

or

$$\frac{\partial\psi}{\partial r} - \left(\frac{dr}{r\,d\theta}\right)\frac{\partial\psi}{r\,d\theta} - 2s\psi\left[\cos\theta + \left(\frac{dr}{r\,d\theta}\right)\sin\theta\right] = 0$$

But $\psi = H e^{sr\cos\theta} + 1$

$$\frac{\partial\psi}{\partial r} = \left[(s\cos\theta)H + \frac{\partial H}{\partial r}\right] e^{sr\cos\theta}$$

$$\frac{\partial\psi}{\partial\theta} = \left[-(sr\sin\theta)H + \frac{\partial H}{\partial\theta}\right] e^{sr\cos\theta}$$

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so

$$\left[(s\cos\theta)H + \frac{\partial H}{\partial r}\right] e^{sr\cos\theta} + \left(\frac{dr}{r^2 d\theta}\right)\left[(sr\sin\theta)H - \frac{\partial H}{\partial\theta}\right] e^{sr\cos\theta}$$

$$- 2s\left[H e^{sr\cos\theta} + 1\right]\left[\cos\theta + \left(\frac{dr}{r\,d\theta}\right)\sin\theta\right] = 0$$

or

$$\frac{\partial H}{\partial r} - \left(\frac{dr}{r\,d\theta}\right)\left(\frac{\partial H}{r\,d\theta}\right) + sH\left[\cos\theta + \left(\frac{dr}{r\,d\theta}\right)\sin\theta\right]$$

$$- 2sH\left[\cos\theta + \left(\frac{dr}{r\,d\theta}\right)\sin\theta\right] = 2s e^{-sr\cos\theta}\left[\cos\theta + \left(\frac{dr}{r\,d\theta}\right)\sin\theta\right]$$

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or

$$\frac{\partial H}{\partial r} - \left(\frac{dr}{r d\theta}\right)\frac{\partial H}{r\partial\theta} - sH\left[\cos\theta + \left(\frac{dr}{r d\theta}\right)\sin\theta\right]$$

$$= 2s e^{-sr\cos\theta}\left[\cos\theta + \left(\frac{dr}{r d\theta}\right)\sin\theta\right]$$

[This is Phillip's Eq. (32) when $\frac{dr}{d\theta} = 0$] !

Now let

$$H = \sum_{n=0}^{\infty} a_n \frac{K_n(sr)}{K_n(sR_0)} \cos n\theta$$

$$\frac{\partial H}{\partial r} = -s \sum_{n=0}^{\infty} \left[\frac{K_{n-1}(sr) + K_{n+1}(sr)}{2K_n(sR_0)}\right] a_n \cos n\theta$$

$$\frac{\partial H}{\partial \theta} = \sum_{n=0}^{\infty} -n a_n \sin n\theta \frac{K_n(sr)}{K_n(sR_0)}$$

so

$$-s \sum_{n=0}^{\infty} a_n \cos n\theta \left[\frac{K_{n+1}(sr) + K_{n-1}(sr)}{2K_n(sR_0)}\right]$$

$$+ \left(\frac{dr}{r d\theta}\right) \sum_{n=0}^{\infty} \frac{n a_n \sin n\theta}{r} \left[\frac{K_n(sr)}{K_n(sR_0)}\right]$$

$$- s\left[\cos\theta + \left(\frac{dr}{r d\theta}\right)\sin\theta\right] \sum_{n=0}^{\infty} a_n \cos n\theta \left[\frac{K_n(sr)}{K_n(sR_0)}\right]$$

$$= 2s e^{-sr\cos\theta}\left[\cos\theta + \left(\frac{dr}{r d\theta}\right)\sin\theta\right]$$

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or

$$\sum_{n=0}^{\infty} a_n \cos(n\theta) \left[\frac{K_{n+1}(sr) + K_{n-1}(sr)}{2K_n(sR_0)}\right]$$

$$+ \sum_{n=0}^{\infty} a_n \left[\cos\theta + \left(\frac{dr}{r d\theta}\right)\sin\theta\right] \cos n\theta \left[\frac{K_n(sr)}{K_n(sR_0)}\right]$$

$$- \frac{1}{s} \sum_{n=0}^{\infty} \frac{a_n}{r} \left(\frac{dr}{r d\theta}\right) \sin n\theta \left[\frac{K_n(sr)}{K_n(sR_0)}\right]$$

$$= -2e^{-sr\cos\theta}\left[\cos\theta + \left(\frac{dr}{r d\theta}\right)\sin\theta\right]$$

This is equivalent to Phillip's Eq. (42)

with $R_0 = r = 1$; $e^{-sr\cos\theta} = \sum (-1)^n I_n(s) \cos n\theta$

$\frac{dr}{d\theta} = 0$

and Phillip's $\bar{a}_n = a_n K_n(s)$

Now let $r = R_0 + \delta \cos M\theta = R(\theta)$

$$\frac{dr}{d\theta} = -M\delta \sin M\theta$$

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As

$$\sum_{n=0}^{\infty} a_n R(\theta)^2 \cos n\theta \left\{ \frac{K_{n+1}[sR(\theta)] + K_{n-1}[sR(\theta)]}{2K_n(sR_0)} \right\}$$

$$+ \sum_{n=0}^{\infty} a_n \left[R(\theta)^2 \cos\theta - R(\theta) M\delta \sin M\theta \sin\theta \right] \cos n\theta \left\{ \frac{K_n[sR(\theta)]}{K_n(sR_0)} \right\}$$

$$+ \frac{M\delta}{s} \sum_{n=0}^{\infty} a_n \sin M\theta \sin n\theta \left\{ \frac{K_n[sR(\theta)]}{K_n[sR_0]} \right\}$$

$$= -2e^{-sR(\theta)\cos\theta} \left[R(\theta)^2 \cos\theta - R(\theta) M\delta \sin M\theta \sin\theta \right]$$

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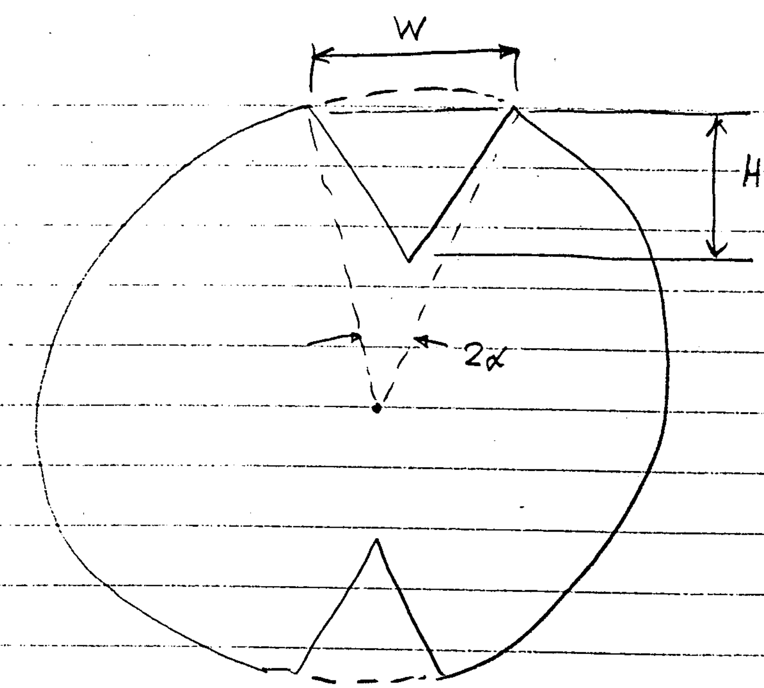

Cases Considered

$W = 0.2R_0$ $H = 0.05R_0$ $s = 4, 8, 16$

$W = 0.4R_0$ $H = 0.05R_0$ $s = 4$

$W = 0.2R_0$ $H = 0.1R_0$ $s = 4$

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s	Phillips ϕ_{max}	W/R_0	H/R_0	ϕ_{max}	% increase
4	9.7088	0.2	0.05	16.353	68.4
8	17.82	↓	↓	49.379	177.1
16	33.9			243.15 (?)	617.3 (?)
4	9.7088	0.4		21.127	117.6
4	↓	0.2	0.1	24.460	151.9

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FROM: FRANK DODGE

- $I_n(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{x\cos\theta}\cos(n\theta)\,d\theta$
- $I_{-n}(x) = I_n(x)$
- $I_n(-x) = I_n(x)$ for $n = 0, 2, 4, 6, 8, \ldots$ even
  $= -I_n(x)$ for $n = 1, 3, 5, 7, 9, \ldots$ odd

So, Fourier cosine series expansion

$$e^{-s\cos\phi} = \sum_{n=0}^{n=\infty} a_n \cos(n\phi)$$

where $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-s\cos\phi}\,d\phi = I_0(-s) = I_0(s)$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-s\cos\phi}\cos n\phi\,d\phi$$
$$= 2I_n(-s)$$
$$= 2I_n(s) \text{ for } n \text{ even}$$
$$= -2I_n(s) \text{ for } n \text{ odd}$$
$$= 2(-1)^n I_n(s)$$

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But $I_n(s) = I_{-n}(s)$ and $\cos(n\phi) = \cos(-n\phi)$

and $a_n = \frac{1}{2}a_n + \frac{1}{2}a_n$

$$= (-1)^n I_n(s) + (-1)^{-n} I_{-n}(s)$$

So

$$\sum_{n=0}^{n=\infty} a_n\cos(n\phi) = a_0 + \sum_{n=1}^{n=\infty}\frac{1}{2}a_n\cos(n\phi) + \sum_{n=-\infty}^{n=1}\frac{1}{2}a_n\cos(n\phi)$$
$$= \sum_{n=-\infty}^{\infty}\frac{1}{2}a_n\cos(n\phi)$$
$$= \sum_{n=-\infty}^{\infty}(-1)^n I_n(s)\cos(n\phi)$$

So

$$e^{-s\cos\phi} = \sum_{n=-\infty}^{\infty}(-1)^n I_n(s)\cos(n\phi)$$

= Philip's Eq. (42)

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The redacted material is from the following reference:

Hughson, D. and F.T. Dodge. “The Effect of Cavity Wall Irregularities on Seepage Exclusion from Horizontal Underground Openings.” Journal of Hydrology. Vol. 228. pp. 206-214. 2000.

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$\delta = -.01$

S	M	ϕ_{max}		ϕ/ϕ
4	4	10.47 (10.494)		1.0784
	8	10.723		1.1045
9.7088	16	10.615		1.093
	32	~~10.494~~ (DH Nov 5 1999)	doesn't converge N=80	
8	4	19.8		1.111
17.820	8	21.866	good plot	1.227
	16	21.473		1.205
	~~32~~	~~21.522~~	~~good plot~~	~~1.208~~
16	4	38.579		1.138
33.9	8	48.24		1.423
	16	51.421		1.517
	32	49.852	— crazy plot (ok)	1.4706

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$\delta = -0.025$

S	Phillip's ϕ^P_{max}	M	ϕ^W_{max}	θ_{max}	% increase	$\phi^W_{max}/\phi^P_{max}$
4	9.7088	4	11.822	π	21.8	1.2176
	↓	8	12.609	π	29.9	1.30
		16	12.450	π	28.2	1.282
4		32	12.839			1.3224
8	17.820	4	23.627	π	32.6	1.326
	↓	8	30.598	π	71.7	1.717
		16	29.964	π	68.2	1.681
8		32	30.638			1.7193
16	33.9	4	44.856	π	32.3	1.323
	↓	8	89.358	π	163.6	2.636
		16	106.306	π	213.6	3.136
16		32	103.94			3.066

$\delta = + 0.025$ .06875 m

S	Phillip's ϕ_{max}	M	ϕ_{max}	θ_{max}	% increase
4	9.7088	4	8.196	π	-15.58
6.531	↓	8	10.943	$\pi \pm 0.147\pi$	12.7
		16	21.571		
		16	12.147	$\pi \pm 0.07\pi$	25.1
8	17.820	4	14.249	π	-20.0
	↓	8	21.774	$\pi \pm 0.153\pi$	22.2
		16	27.387	$\pi \pm 0.077\pi$	56.5
16	33.9	4	28.741	π	-15.2
	↓	8	44.883	$\pi \pm 0.147\pi$	32.4
		16	80.397	$\pi \pm 0.077\pi$	137.1

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$\delta = -0.05$

S	Phillip's ϕ_{max}	M	ϕ_{max}	θ_{max}		% increase
4	9.7088	4	14.788	π	1.523	52.3
8	17.820	4	33.404	π	1.8745	87.5
	↓	8	57.066	π	3.202	220.2
		16	60.046	π	3.370	239.0

$\delta = +0.05$

S	Phillip's ϕ_{max}	M	ϕ_{max}	θ_{max}	% increase
4					
8	17.820	4	13.95	π ± 0.30π	-21.6
	↓	8	30.991	π ± 0.15π	74.0
		16	53.719	π ± 0.064π	201.5

$\delta = -.05$

	S	M	ϕ_{max}		
	4	8	17.028		1.754
9.7088	4	16	does not converge		
	4	32	18.798 (lots of tiny noise) not converged		1.936
17.820	8	32	65.886	pretty wild	3.697
	16	4	~~79.~~ 79.117		2.33
33.9	16	8	299.783		8.843
	16	16	447.571		13.203
	16	32	did not converge		

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S	w	H	ϕ_{max}	
4	.2	.05	16.353	1.684
8	.2	.05	49.379	2.771
16	.2	.05	243.15	7.173
4	.2	.025	13.48	1.388
8	.2	.025	33.643 (N=40) .458 (N=50)	1.8776
16	.2	.025	111.388	3.286
4	.2	.01	12.025	1.2386
8	.2	.01	26.871	1.508
16	.2	.01	72.642	2.143
4	.1	.01	10.883	1.121
8	.1	.01	22.369	1.255
16	.1	.01	53.002	1.563
4	.05	.01	~~48.502~~ 10.618	1.094
8	.05	.01	21.315	1.196
16	.05	.01	48.502	1.431
4	.1	.025	12.222	1.259
8	.1	.025	28.162	1.580
16	.1	.025	83.661	2.468
4	.05	.025	11.977	1.234
8	.05	.025	27.137	1.523
16	.05	.025	79.027	2.331

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S	w	H	ϕ_{max}	
4	.1	.05	14.808	1.525
8	.1	.05	41.312	2.318
16	.1	.05	177.495	5.236
4	.05	.05	14.634	1.50
8	.05	.05	40.621	2.281
16	.05	.05	181.572	5.35

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Wed May 26 8:30-10:00 Interaction 5th A EBS

9.7088
17.82
33.9

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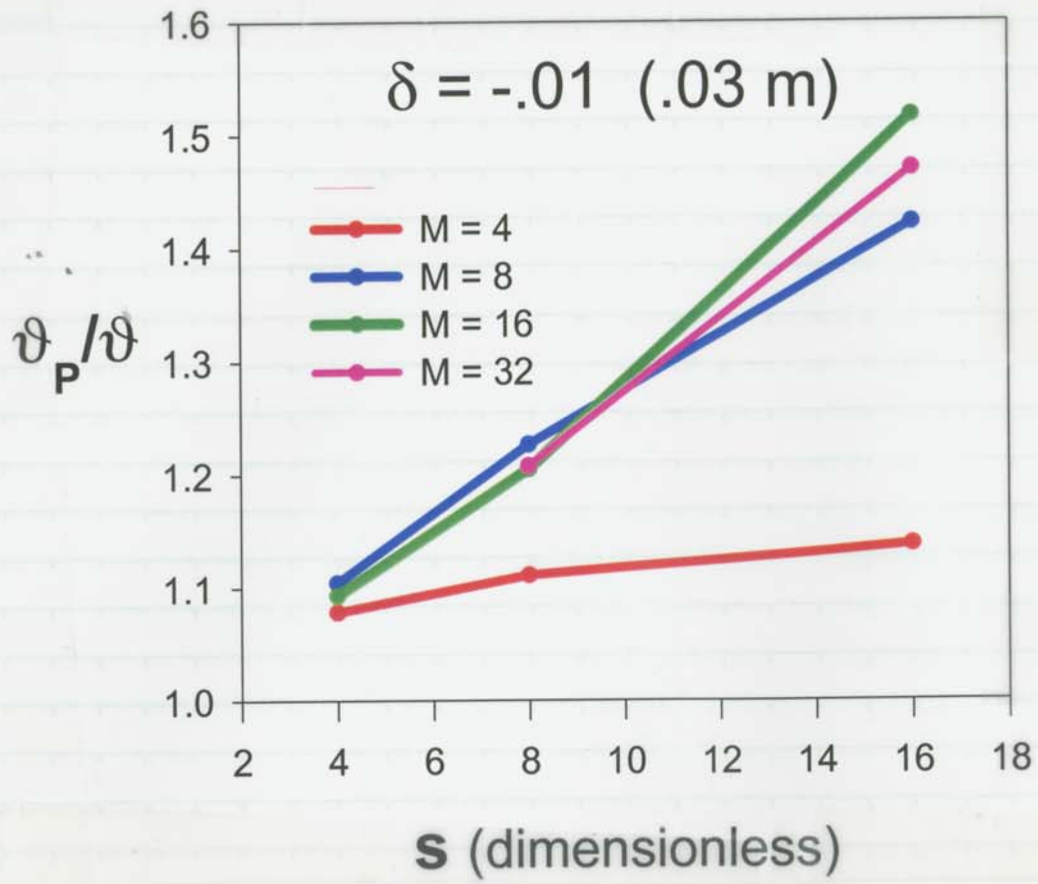

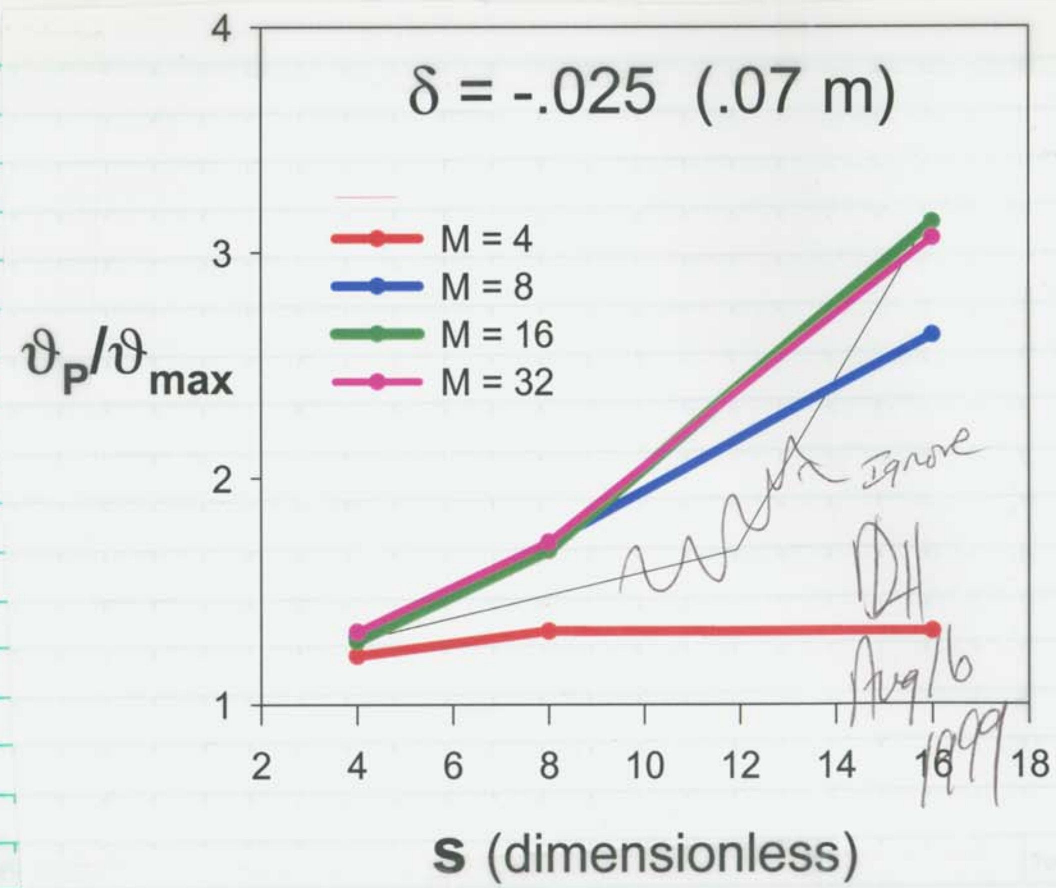

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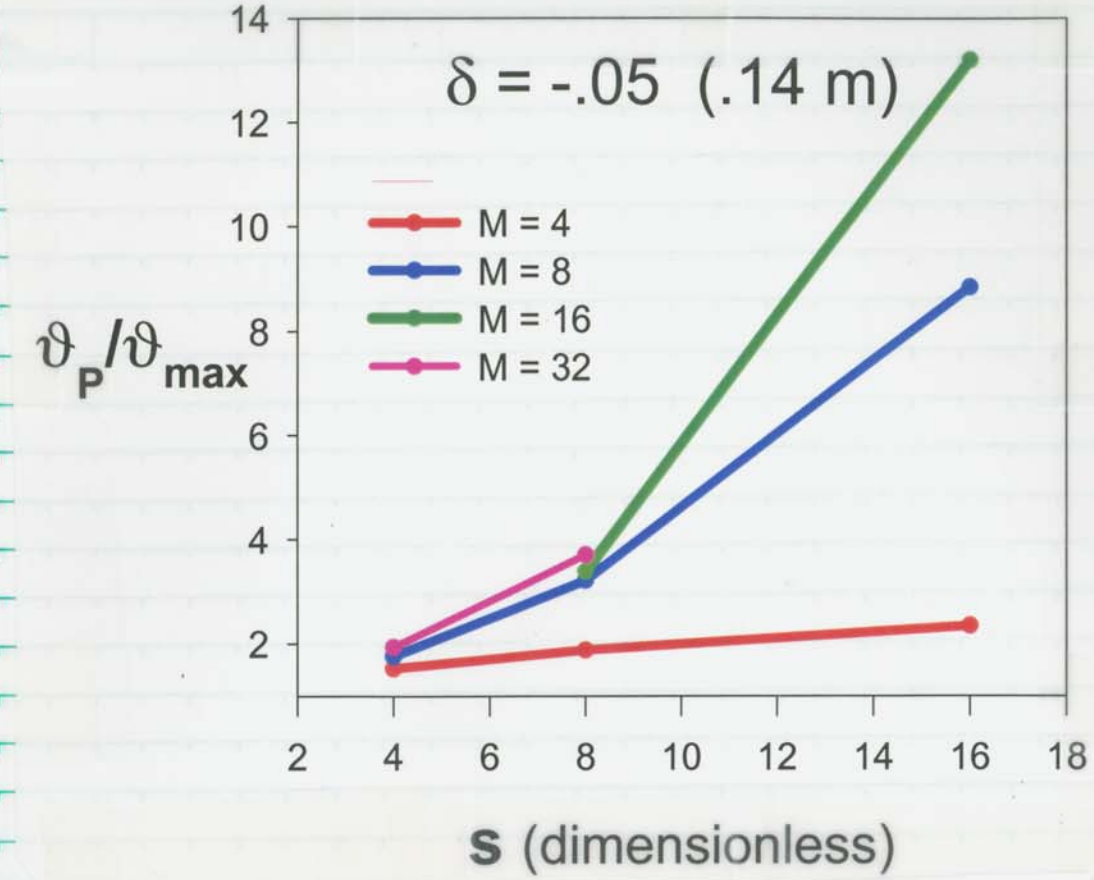

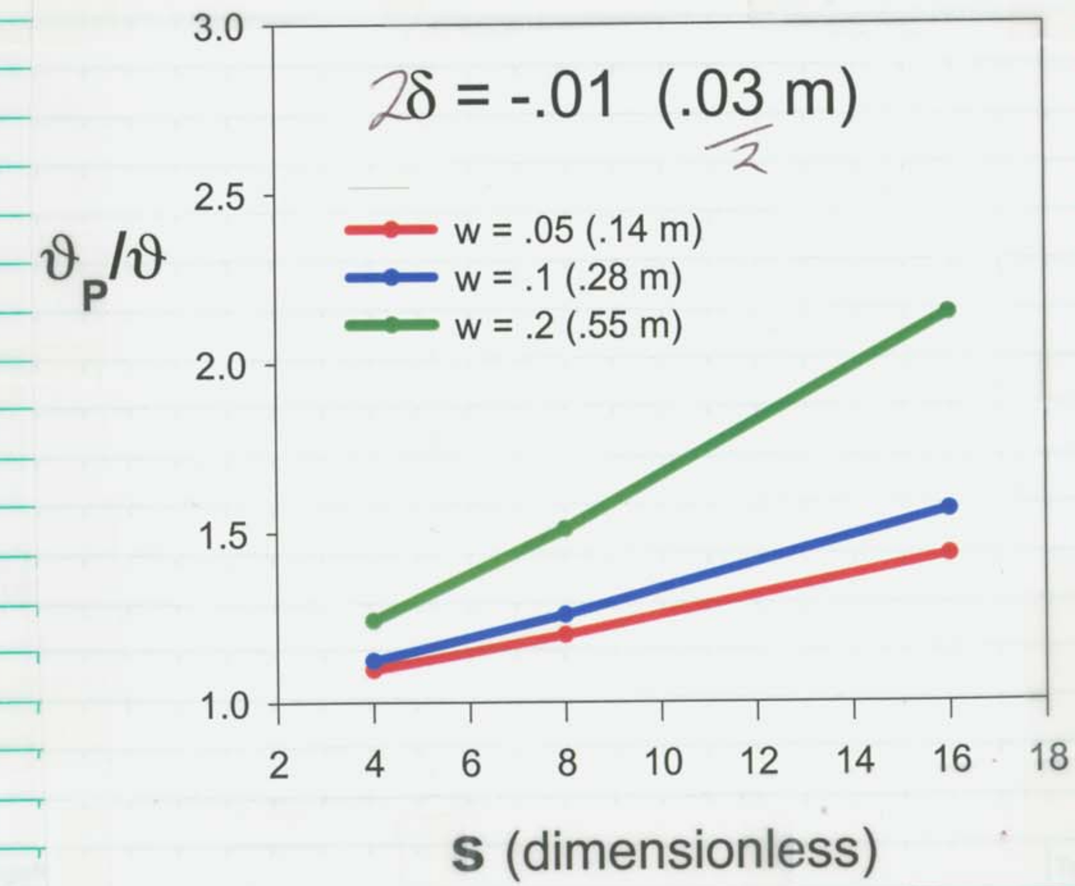

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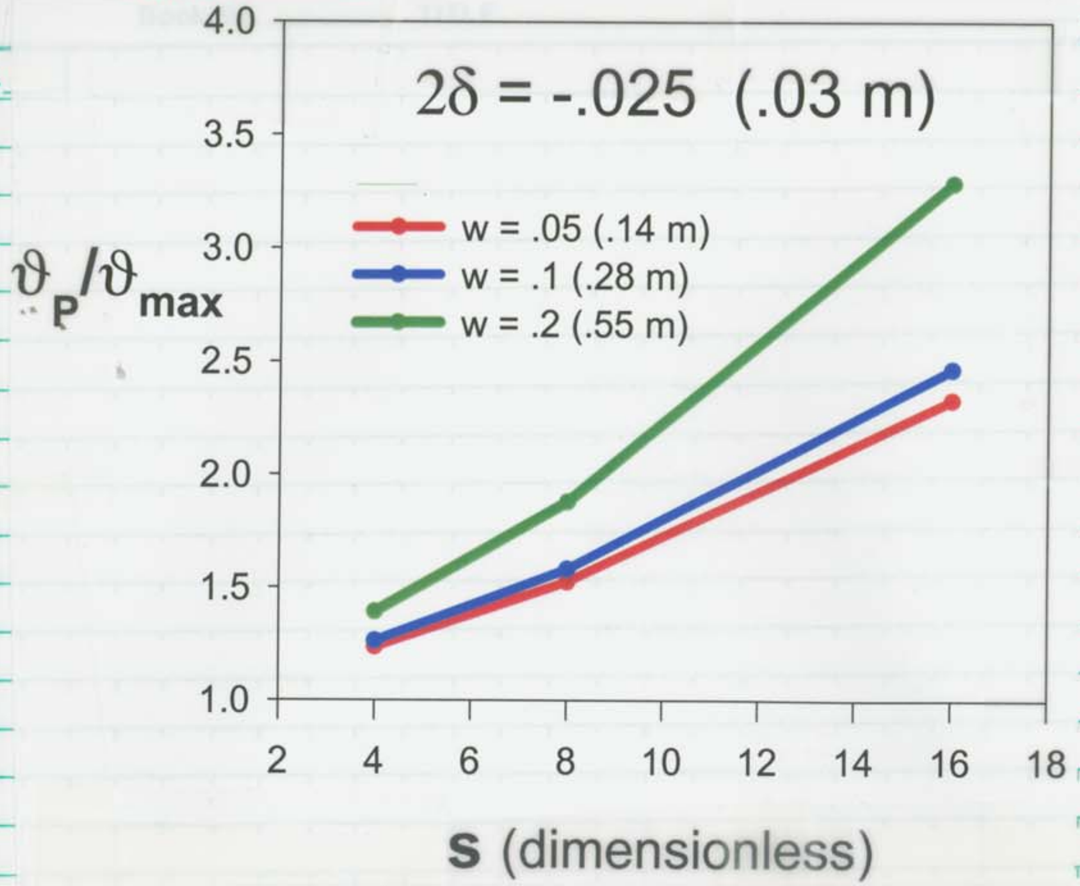

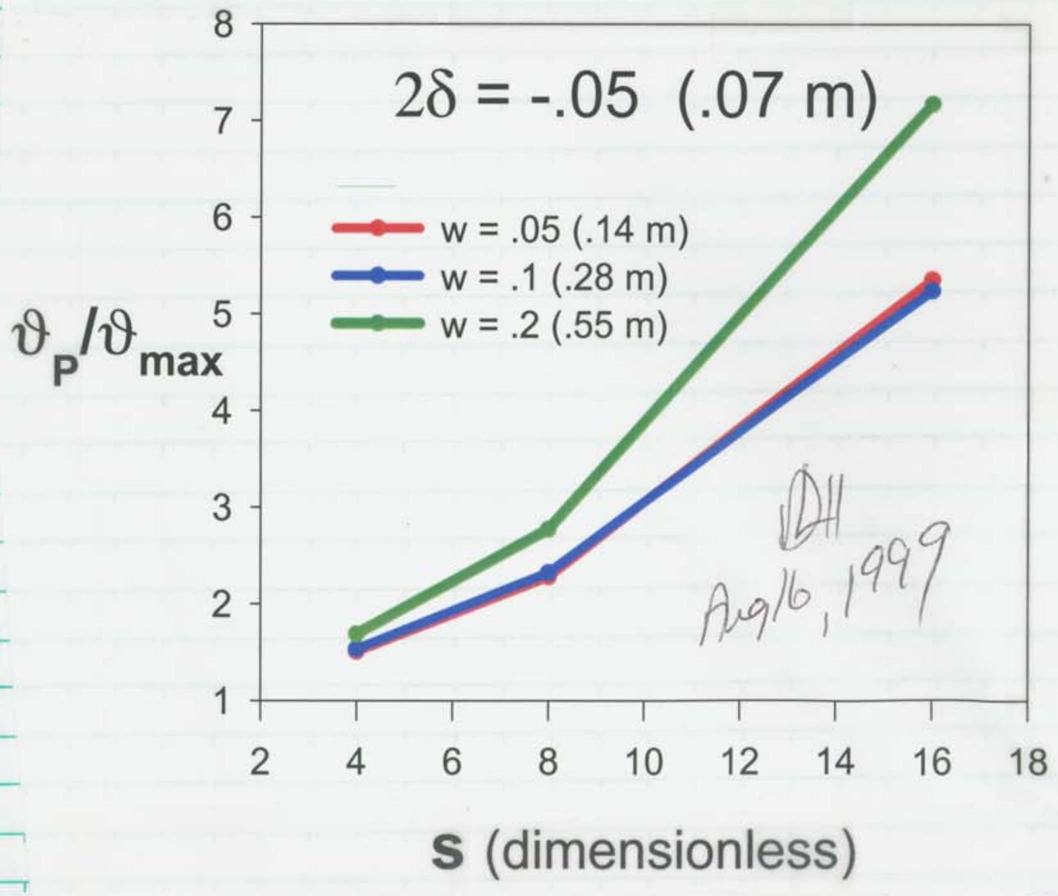

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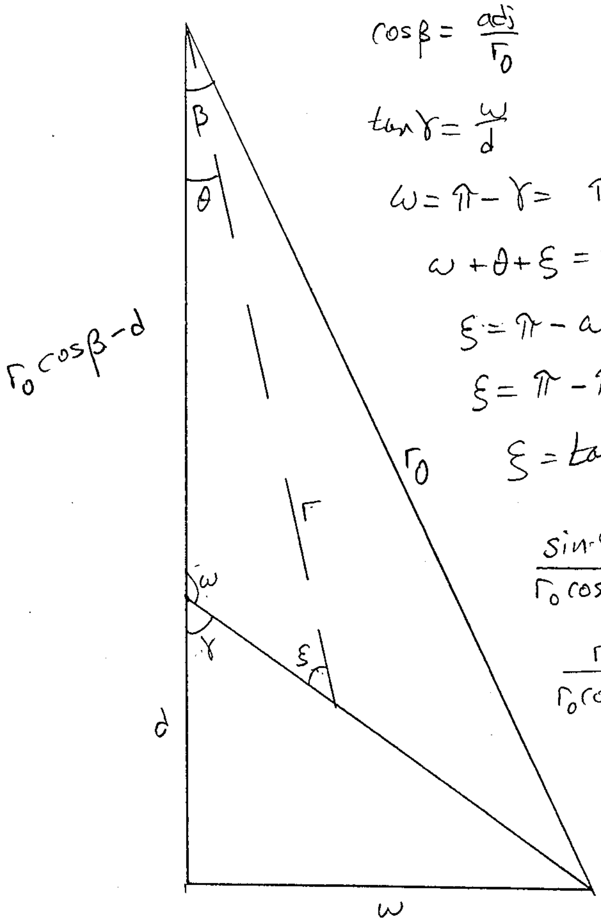

$$\sin\beta = \frac{w}{r_0}$$

$$\cos\beta = \frac{adj}{r_0}$$

$$\tan\gamma = \frac{w}{d}$$

$$\omega = \pi - \gamma = \pi - \tan^{-1}\frac{w}{d}$$

$$\omega + \theta + \xi = \pi$$

$$\xi = \pi - \omega - \theta$$

$$\xi = \pi - \pi + \tan^{-1}\frac{w}{d} - \theta$$

$$\xi = \tan^{-1}\frac{w}{d} - \theta$$

$$\frac{\sin\xi}{r_0\cos\beta - d} = \frac{\sin\omega}{r}$$

$$\frac{r}{r_0\cos\beta - d} = \frac{\sin\omega}{\sin\xi}$$

$$r = \left(r_0\cos\beta - d\right)\frac{\sin\left(\pi - \tan^{-1}\frac{w}{d}\right)}{\sin\left(\tan^{-1}\frac{w}{d} - \theta\right)}$$

Partial Derivation of the radius function for the inverted peak at the drift crown

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Total System Performance Assessment Viability Assessment Technical Basis Report Chapter 2 - Unsaturated Zone Hydrology Model

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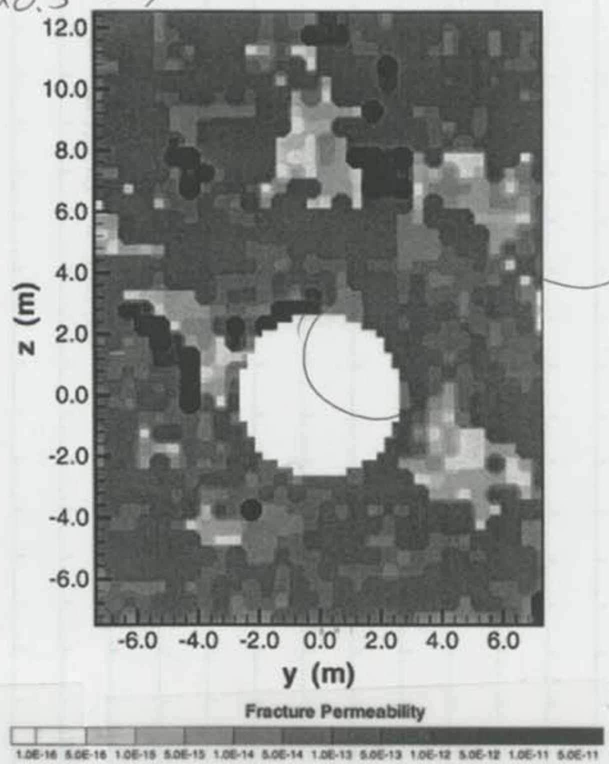
Figure 2-58a. Simulated Heterogeneous-Permeability Field for Vertical Slice 1 of the 3-D Block

From VA
blocks (0.5 x 0.5 x 0.5 m)
dim x = 15m
z = 20m
gives nx = 30
nz = 40
drift r = 2.5m



jagged edge not drawn on block bndry but must divide blocks

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Try to compare VA seepage simulation grid to Philip's sol.

No.
Date Aug 17, 1999
Invented by
Recorded by
Date
Witnessed & Understood by me,

TITLE USFIC

Project No. 20-1402-861

Book No. 326

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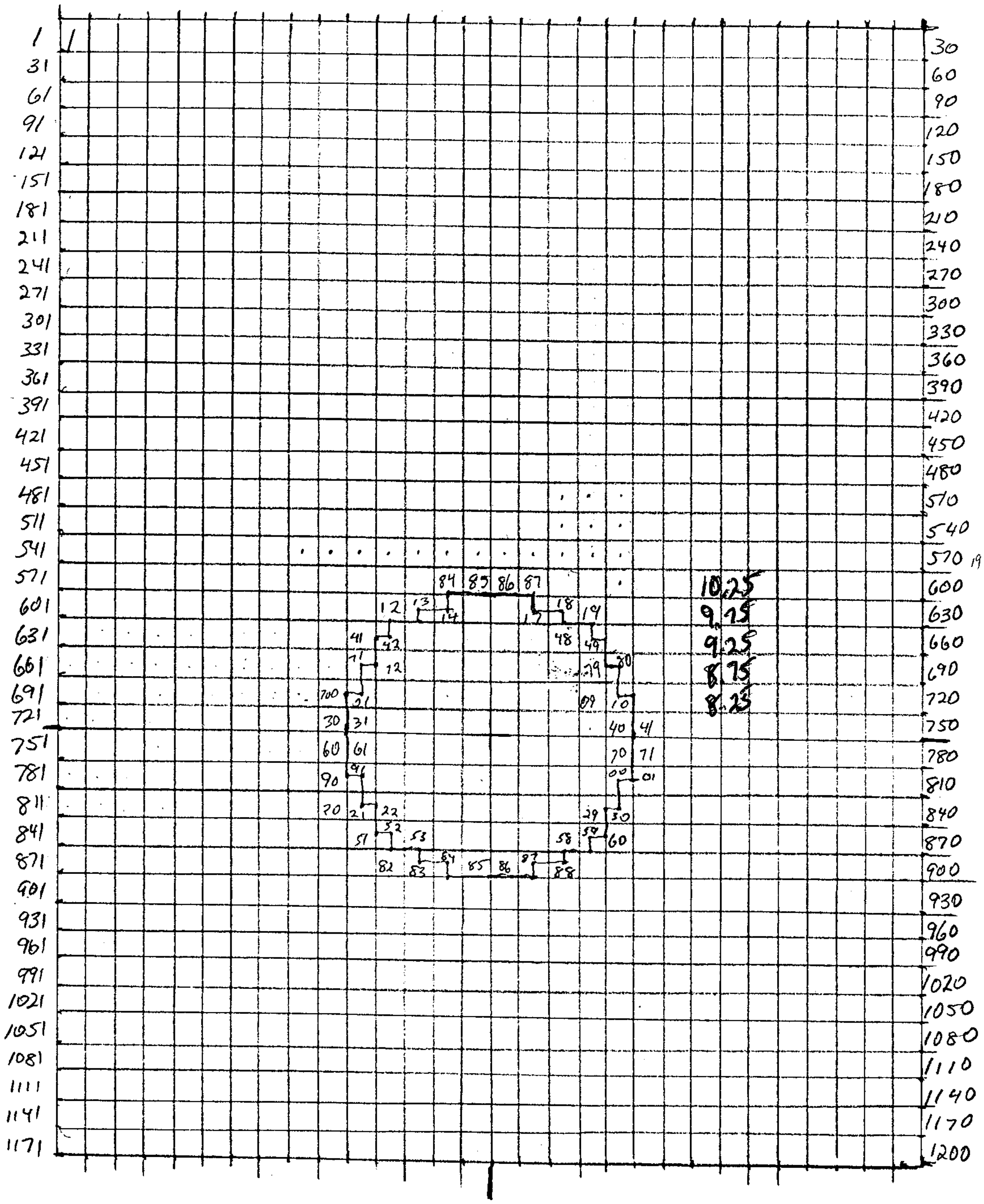

sketch of how drift outline intersects blocks

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From Page No. ___ Saturation

q ~~584 614~~ mm/yr	584	585	612	613	614	615	642	643	644
6.57e-7	.133	.147	.133	.163	.175	0	.155	0	0
6.57e-6	.4	.439	.408	.489	.53	0	.47	0	0
1.0e-5	.466	.514	.476	.573	.617	0	.548	0	0
3.0e-5	.661	.722	.678	.782	.819	0	.773	0	.322 (circled)

Increase bk vol 614, 617 ~~fr~~ from .125/4 to .125

"	.661	.722	.678	.782	.819	0	.773	0	.322

made no difference / change back / reverse left/right connections

"	.660	.721	.683	.79	.815	0	.778	0	.292

zero 644-614 and 647-617 connections - div by zero

" increase 644-614 and 647-617 conn to .25 + .25/2

"	.635	.702	.658	.752	.760	0	.751	0	.41

need to decrease this connection, try .01

"	.669	.729	.695	.804	.834	0	.789	.06 (circled)	0

worked - now reduce 643-613 648-618 conn to .01

"	.669	.729	.695	.804	.834	0	.789	0	0
4.0e-5	.727	.787	.746	.858	.888	0	.825	0	0
5.0e-5	.758	.812	.778	.889	.918	.308 (circled)	.845	0	.01 (circled)

Reduce conn

615-585	.01
616-586	.01
644-614	.005
647-617	.005

(~~644-614~~ 0 DJ 5 Nov 1999)

5.0e-5	.771	.832	.785	.897	.926	0	.848	0	0
6.0e-5	.806	.866	.815	.926	.953	0	.867	0	0

Philip's soln for $\mu = 1002e\text{-}6$ $\rho = 998.3\ kg/m^3$ $g = 9.81\ m/s^2$

$\alpha = 4.85e\text{-}4\ Pa^{-1} = 4.75\ m^{-1}$ $k = 1e\text{-}13\ m^2$

$q^* = 2076.6\ mm/yr = 6.5737e\text{-}5\ kg/m^2/s$

.823	.883	.830	.939	.965	0	.876	0	0

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Need to reduce conn

672-642	.01
679-649	.01
731-701	.01
740-710	.01

q	584	585	612	613	614	615	642	643	644
6.5737e-5 (X)	.888	.856	.953	.975	0	.941	0	0	

~~Att~~ At the theoretical q^* no node around drift is saturated, max sat occurs in corners away from crown, nodes 614 & 617

(DJ 5 Nov 1999)

~~6.7e-5~~	585-86	614-17	P_c
6.7e-5	.8917	.977	
6.8e-5	.894	.979	
7.0e-5	.8996	.9828	
7.2e-5	.9047	.9862	
7.4e-5	.9097	.9895	
7.6e-5	.9144	.9925	145.1
7.8e-5	.919	.9955	90.7
8.0e-5	.9234	.9982	36.9
8.2e-5	.9269	.9997	6.8

Reduce connections 644-614, 647-617 .001

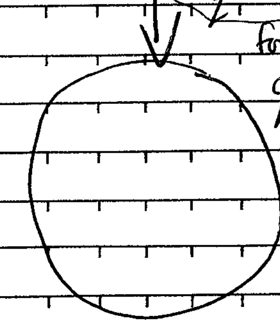
← leaking despite reducing 644-614 647-617 connections to .0005

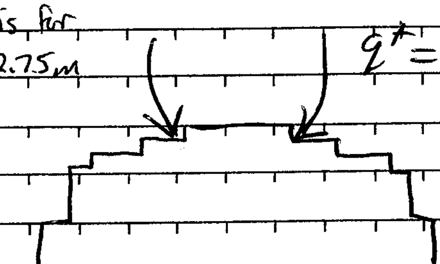
Summary $\alpha = 4.85e\text{-}4\ Pa^{-1} = 4.75\ m^{-1}$

~~$q^* = 2076.6\ mm/yr$~~ this is for r = 2.75 m (DJ 5 Nov 1999)

for r = 2.5 m
$q^* = 2259.5\ mm/y$

$q^* = 2590.4\ mm/yr$





14.6%

Numerical overpredicts q^* by ~~24.7%~~ DJ 5 Nov 1999

theoretical (Philip)

Numerical also predicts seeps at sides & not crown due to discretization

INPUT FILE ~ dhughson/multiflo/seep/ustrO/inputfiles/multi_q

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Increase α by X2 to 9.7e-4 Pa^{-1}

SAME GRID

q	Nodes 585-86	Nodes 614-17	S_l & P_c
1.0e-7	.109	.15	
3.48e-5	.811	.963	
4.0e-5	.8385	.9816	
4.5e-5	.861	.9957	42.9
4.6e-5	.8652	.998	20.4
4.7e-5	.8691	.9994	← leaking
4.65e-5		.9991	
4.66e-5		.9993	
4.67e-5		.9996	
4.68e-5	.8689	.9998	← threshold

DH 5NOV1999

theoretical ~~1101.5 mm/yr~~ [for r=2.75m] Numerical 1478.4 mm/yr

Philip for r=2.5m q^* = 1204.1 mm/y

DH ~~34.2%~~ error = 22.8%

For α = 9.7e-4 Pa^{-1} = 9.5 m^{-1} 1/α = 0.105 m

DH 5NOV1999

Increase α by X3.4 to 3.3e-3 Pa^{-1}

q	585-86	614-17	
1.0e-5	.6577	.9177	Reduce 731-701
1.075e-5		.930	740-710
1.2e-5		.9475	to .001
1.4e-5		.9709	672-642
1.6e-5		.9889	679-649
1.7e-5		.9959	
1.75e-5		.9989	
1.76e-5		.9995	← threshold
1.77e-5		leaks	

→ Plot on pg 75 →

DH 5NOV1999

Theoretical ~~339.57 mm/y~~ [for r=2.75m] Numerical 556 mm/yr

for r=2.5m q^* = 372.7 mm/yr

DH 5NOV1999 ~~63.7%~~ error = 49.2%

For α = 3.3e-3 Pa^{-1} = 32.318 m^{-1} 1/α = .031 m

~dhughson/multflo/seep/ustr0/inpfiles/multi.2 and multi.3

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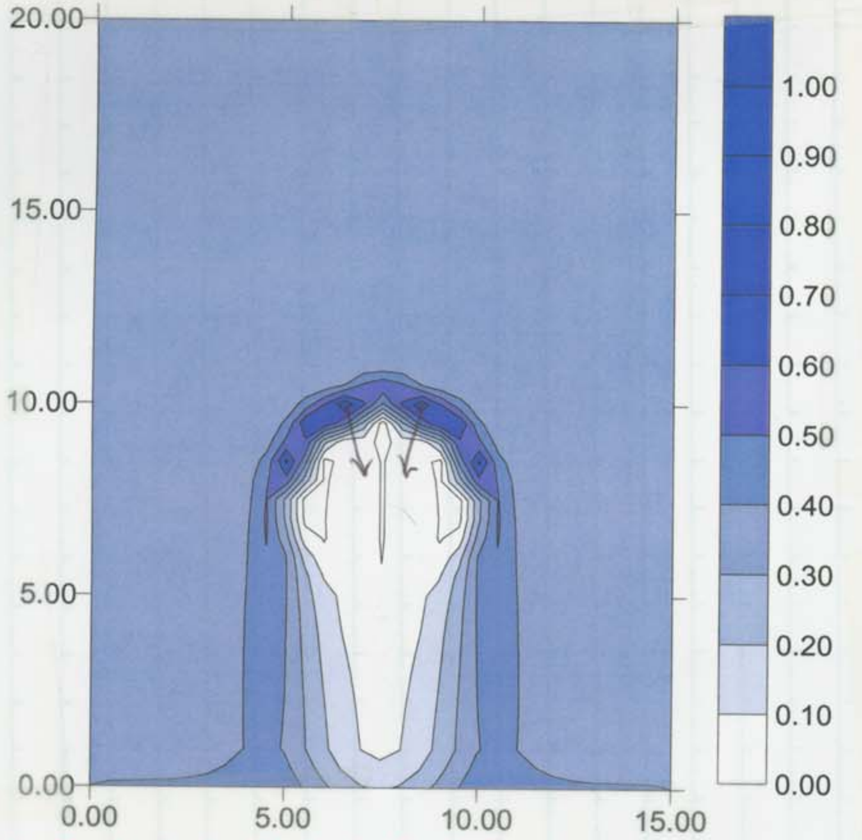

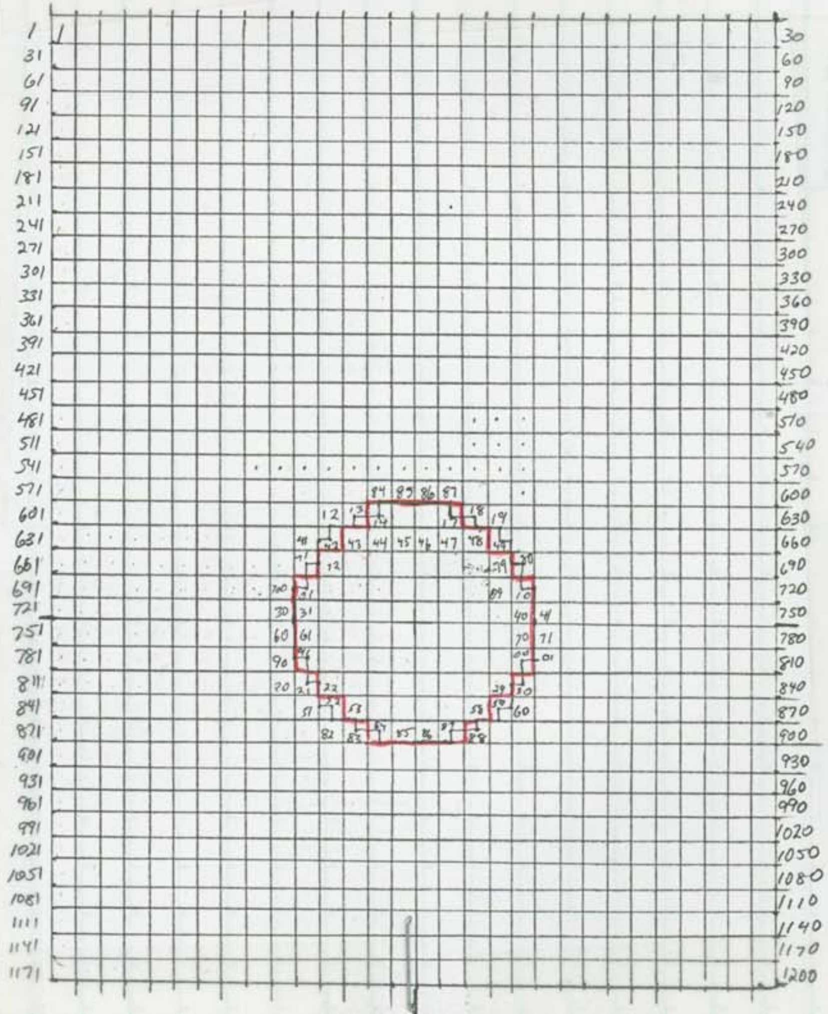

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$a = 3.3e{-}3\ Pa^{-1}$
$K = 1.0\ e{-}13\ m^2$
$q = 559\ \frac{mm}{yr}$
dripping at 614
and 617

Redo grid so connections do not divide up blocks ~ as shown in red above

Redo thres hold for Some alphas on following pages

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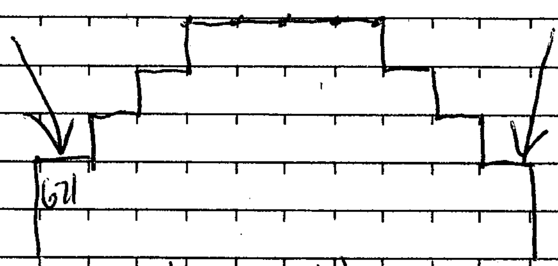
2nd cubic unstr. grid with constant uniform block size and shape

~dhughson/multiflo/seep/ustro/inputfiles/multi.4

$\alpha = 4.85e\text{-}4\ Pa^{-1}$	621
4.0e-5	.8429
7.0e-5	.9462
8.0e-5	.9664
9.0e-5	.9822
1.0e-4	.9946
1.1e-4	leaks
1.09e-4	leaks
1.05e-4	leaks
1.03e-4	.9978
1.04e-4	.9988
1.048e-4	leaks
1.045e-4	.9993
1.046e-4	.9994 ← threshold
1.047e-4	leaks



theoretical = ~~2076.6~~ [for r = 2.75m] mm/yr = 3304.3 mm/yr

for r = 2.5m $q^* = 2259.5$ DH 5 NOV 1999

~~59.1%~~ DH 5 NOV 1999 Error = 46.2%

$\alpha = 9.7e\text{-}4\ Pa^{-1}$

q	621
1.0e-5	.6725
4.0e-5	.9376
5.0e-5	.9717
5.5e-5	.9844
6.0e-5	.9947
6.2e-5	.9982
6.4e-5	leaks
6.3e-5	leaks
6.25e-5	leaks
6.24e-5	.9989 ← q^*

theoretical = [for r = 2.75m] 1101.5 mm/yr

for r = 2.5m $q^* = 1204.1$

Numerical = 1971.2

~~78.96%~~ DH

ERROR DH 5 NOV 1999

= 63.7%

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$\alpha = 3.3e-3\ Pa^{-1}$

	671
1.0e-5	.8488
3.0e-5	leaks
2.0e-5	.968
2.1e-5	.9752
2.2e-5	.9818
2.3e-5	.9876
2.4e-5	.9931
2.5e-5	leaks
2.45e-5	.9955
2.46e-5	.9960
2.47e-5	.9964 ← threshold
2.48e-5	leaked

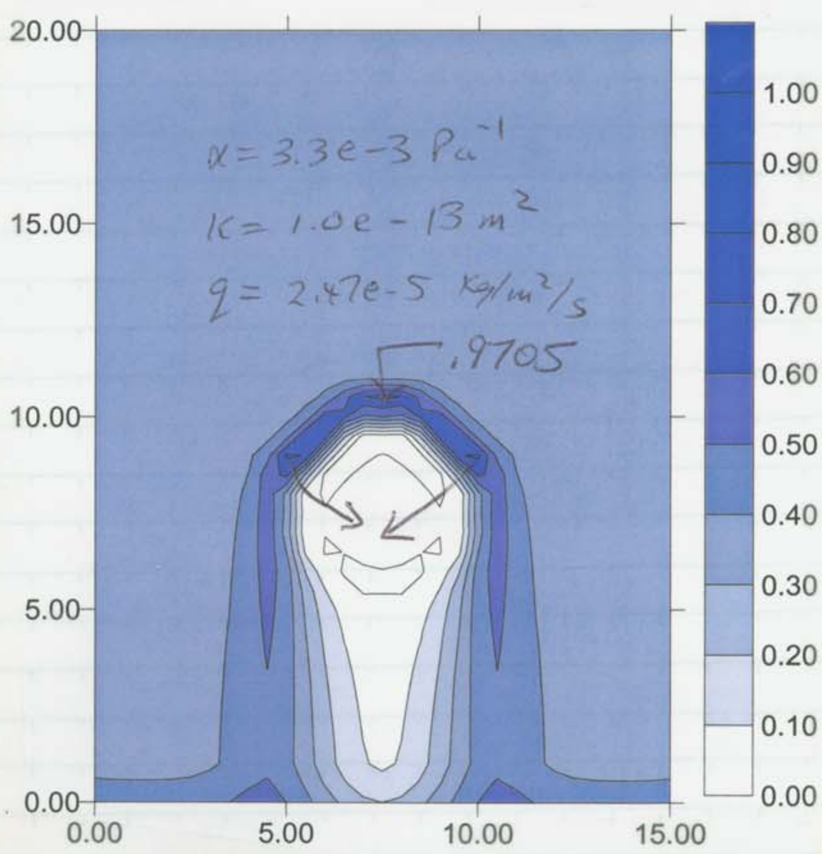
Theoretical = ~~339.57~~ for r=2.75 DH 5 Nov 1999

for r=2.5m $q^* = 372.7$

Numerical = 780.3

~~129.8% Error~~ DH 5 Nov 1999

= 109.4%



~dhughson/multiflo/seep/usfic/input files/multi.6

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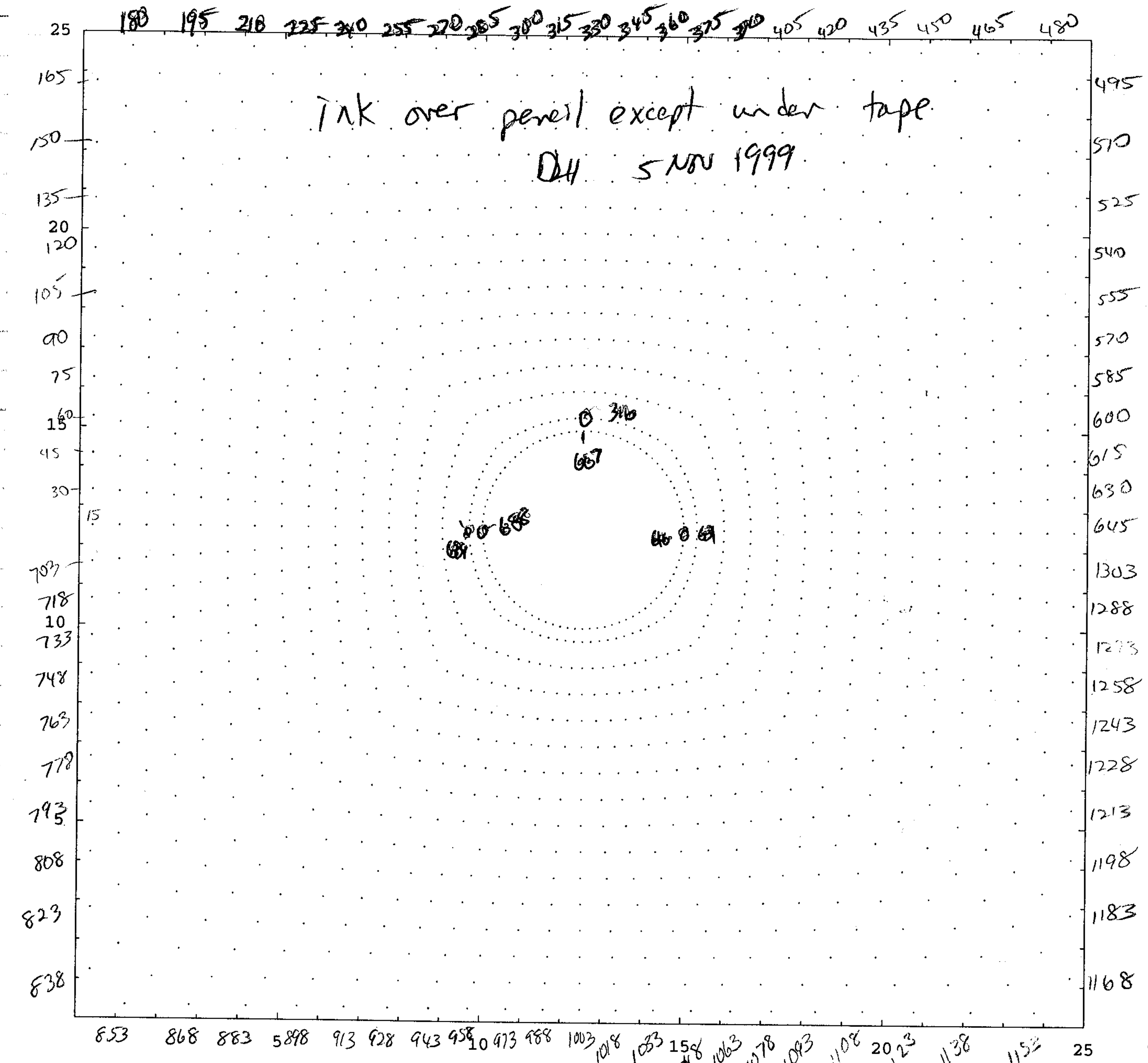

~dhughson/seep/r0

Inner most ring of nodes is .001 m from the drift wall boundary

node 316 x = 12.5 y = 15 1/3
node 667 x = 12.5 y = 15.001

From Page No.

For Gardner $\alpha = 1.0e-5\ Pa^{-1}$ $K = 1.0e-13\ m^2$

q	node 667
7.85e-6	.6828
7.55e-4	.9987
7.56e-4	.9989
7.58e-4	.9991
7.6e-4	.9994
7.62e-4	.9996
7.64e-4	.9998 ← leak
7.65e-4	1.0
7.645e-4	1.0
7.642e-4	.9998

theoretical = 23854.3 mm/yr
Numerical = 24140

1.2% Error

For Gardner $\alpha = 4.85e-4\ Pa^{-1}$

q	node 667
7.2e-5	.9505
7.3e-5	.9526
8.0e-5	.966
8.5e-5	.9745
9.0e-5	.9821
9.5e-5	.9887
1.0e-4	.9945
1.1e-4	1.0
1.05e-4	.9996 ←
1.06e-4	1.0

Theoretical = 2259.5 mm/yr
Numerical = 3317 mm/y

46.8% error

For Gardner $\alpha = 9.7e-4\ Pa^{-1}$

q	node 667
3.8e-5	.8902
4.5e-5	.9207
5.0e-5	.9371
6.0e-5	.9655
7.0e-5	.9863
7.6e-5	.9960
7.7e-5	.9974
8.0e-5	1.0
7.9e-5	1.0
7.85e-5	.9994
7.87e-5	.9997
7.88e-5	.9998
7.89e-5	.9999 ← q*

theoretical = 1204.1 mm/yr
numerical = 2492.4 mm/yr

107% error

From Page No.

For Gardner α = 3.3e-3 Pa⁻¹ r0

q	667 Se
~~6.0e-5~~ 1.7e-5	.7704
3.0e-5	.8816
6.0e-5	.9905
6.1e-5	.9924
6.3e-5	.9959
6.5e-5	.9991
6.6e-5	1.0
6.55e-5	1.0
6.54e-5	1.0
6.53e-5	1.0
6.52e-5	.9994 ←

DH 5 Nov 1999

theoretical 372.7 mm/yr

numerical = 2059.6

error = 452.6%

GRID REFINEMENT
r1

Place ring of nodes .03m from driftwall bndry

For Gardner α = 3.3e-3 Pa⁻¹

q	node 710
1.0e-5	.8034
3.0e-5	

DH 5 Nov 1999

← ~~forgot~~ Grid is screwed up because I forgot to add extra connections correctly. Needs to be redone

Sept 7, 1999 Re did the r1 unstructured grid with extra ring of nodes .03m from drift wall boundary

Results for Gardner α 3.3e-3 Pa⁻¹ on following page

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From Page No. GRID r2 - extra ring of nodes at 0.1m from drift wall

For Gardner α = 3.3e-3 Pa⁻¹

q	node 710
1.0e-5	.8586
3.0e-5	1.0
2.0e-5	.9754
2.1e-5	.9817
2.2e-5	.9871
2.3e-5	.9921
2.4e-5	.9688
2.5e-5	1.0
2.45e-5	.9988
2.46e-5	.9992
2.47e-5	.9996 ← numerical = 780.3
2.48e-5	1.0

theoretical 372.7 mm/yr

109.4% error

Grid r1 - extra ring of nodes at .03m from drift wall

q	node 710
1.0e-5	.8034
2.0e-5	.9355
3.0e-5	.9934

obviously more error than r2

Grid r3 - extra ring at 0.167m from drift wall

q	node 710
1.0e-5	.7893
2.0e-5	.9209
3.0e-5	.9825

obviously more error than either r1 or r2

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GRID r4 1 extra ring at .05 m
2 extra ring at 0.1 m

q	node 753
1.0e-5	.9206
2.0e-5	1.0
1.5e-5	.982
1.7e-5	.9966
1.75e-5	.9997
1.76e-5	1.0
1.755e-5	.9999 ← leak

Nearly ~~identical~~ identical (DH 5 Nov 1999) error to Mickey Mouse Grid pg 74

analytical = 372.7 mm/yr
numerical = 554.4 mm/yr
error 48.7%

directory
usfr0 renamed sgblkuns

Need to fill in $q^*(\alpha)$ for Grid r4

Grid r4 α = 9.7e-4 Pa^{-1}

q	node 753
~~1.8e-5~~ 1.755e-5	.8231
3.0e-5	.9229
4.0e-5	.9679
5.0e-5	.9960
5.5e-5	1.0
5.3e-5	1.0
5.1e-5	.9980
5.2e-5	.9999 ←

(DH 5 Nov 1999)

Analytical 1204.1 mm/yr
Numerical 1643 mm/yr
error 36.4%

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Grid r4 α = 4.85e-4 Pa^{-1}

q	node 753
5.2e-5	.9206
7.0e-5	.9673
8.0e-5	.9850
9.0e-5	.9984
9.5e-5	1.0
9.1e-5	.9995
9.2e-5	1.0
9.15e-5	1.0
9.14e-5	1.0
9.13e-5	.9999 ←

analytical = 2259.5 mm/yr
numerical = 2884 mm/yr
error = 27.6 %

Grid r4 α = 1.0e-5 Pa^{-1}

q	node 753
9.13e-5	.7686
2.0e-4	.8438
4.0e-4	.9292
5.0e-4	.9555
6.0e-4	.9763
7.0e-4	.9919
7.5e-4	.9982
7.9e-4	1.0
7.6e-4	.9995
7.7e-4	1.0
7.61e-4	.9996
7.62e-4	.9997
7.63e-4	.9999 ←

analytical = 23854.3 mm/yr
numerical = 24103 mm/yr
error = 1.04 %

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Grid 1 ~~pg 75~~ pg 77 ~~78~~ α = 1.0e-5 Pa⁻¹ page 77
DH 5 Nov 1999

q		node 585	
7.e-4	leaks		
6.e-4	no leak		analytical 23854.3 mm/yr
6.5e-4	leaks	1.0	
6.1e-4	leaks	1.0	numerical 19206.5 mm/yr
6.0e-4		.999	
6.01e-4		.9992	error = − 19.5%
6.03e-4		.9993	
6.05e-4		.9995	
6.07e-4		.9997	
6.08e-4		.9999 ← q*	
6.09e-4	leaks		

Grid 2 ~~pg 75~~ ~~77~~ α = 1.0e-5 Pa⁻¹ page 75 (dark ink & contours)
DH 5 Nov 1999

q		node 585	614	
6.0e-4		.9987	.9994	analytical 23854.3
6.05e-4	leaks			numerical
6.01e-4			.9996	19080.2
6.02e-4			.9997	
6.03e-4			.9998	
6.04e-4			.9999 ← q*	

error = −20.0%

Grid 1 drift wall goes between blocks as shown in red ink on page 75

Grid 2 drift wall divides up blocks as shown on page 70 & 71

Named Grid 1 & Grid 2 for convenience in SME paper

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Attempt to improve error for α = 3.3e-3 /Pa over Grid r4 and Grid 2 (pg 70, 71, 77)

3 extra rings plus one at r = 2.500 m
these are 2.53, 2.57, and 2.62 m
.03, .07, .12 from driftwall

Grid r5

q	node 796	α = 3.3e-3 Pa⁻¹
1.5e-5	.8158	
1.6e-5	.8208	
1.7e-5	.8393	
1.8e-5	.8498	Much worse, why?

Looks OK except for irregular contours in boundary layer

Try rings at .05 m 0.1 m and .15 m
Grid r6 at α = 3.3e-3 Pa⁻¹

q	node 796	
1.5e-5	.8695	
1.6e-5	.8823	
1.7e-5	.8924	
1.8e-5	.902	Very Odd

ONE MORE TEST OF TREND

	First ring					
r0	.001				.333	452.6%
r2	.001			.01	.333	109.4%
r4	.001		.05	.01	.333	48.7%
r7	.001	.025	.05	.01	.333	? 439%

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From Page No. ___ Grid r7 rings @ .001 .025 .05 .1

q	node 796
1.4e-5	.7345
1.8e-5	.7831
3.0e-5	.8843
5.0e-5	.9713
6.0e-5	.9934
6.5e-5	1.0
6.4e-5	1.0
6.35e-5	.9994
6.39e-5	1.0
6.38e-5	.9999

Analytical 372.7 mm/yr

Numerical = 2008.6 mm/yr

Error = 439%

Grid r6 .05 .1 .15 m α = 3.3e-3 / Pa

q	node 796
2.0e-5	.9197
3.0e-5	.9841
3.5e-5	1.0
3.49e-5	1.0
3.48e-5	1.0
3.46e-5	.9995
3.47e-5	.9998

Analytical 372.7 mm/yr

Numerical 1097.8

Error = 194.5%

Problem with all 1.0's for Se in lower portion grids r7 and r8 when tolr set low to 1e-15 & boundary at top itypebc = 2

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From Page No. ___ Error checking Grid r4 with itypebc = 5

q = 1.0e-5 node 753 Se = .9819 Bal 1.001213

itypebc = 2 q = 1.0e-5 kg/m²/s node 753 Se = .9823

itypebc = 2 q = 1.0e-5 set tolr 1e-15 same as problem runs with grids r7 and r8 Se = .9823 Bal 1.000003

Problem apparently does not occur in grid r4

DH 10/28/99

Grid r7 ~ itypebc = 5 q = 5.0e-5 low tolr tolr = 1e-15 mass bal = 1.000000 node 796 Se = .9713 positive air flux in no problem with Se=1 in lower part

Grid r7 use more detailed TAB for α = 3.3e-3/Pa, the rest as above mass bal = 1.000000 node 796 Se = .9713

Oct 6, 1999 I just created the 1.0 Se problem in grid r4. the change I made was to set the bottom boundary Sg from .05 to .0 and mole fraction from 0.9 to 1.0

Paper submitted to SME Annual Meeting & Exhibit 2000 Society for Mining, Metallurgy, and Exploration, Inc PO BOX 625002 Littleton CO 80162-5002

Information potentially subject to coyright protection was redacted from pages 87 through 96 of this scientific notebook. The redacted material is from the following reference:

Hughson, D.L. and R. Codell. "Comparison of Numerical and Analytical Models of Unsaturated Flow Around Underground Openings, Considerations for Performance Assessments of Yucca Mountain, Nevada, as a High-Level Nuclear Waste Repository." Society for Mining, Metallurgy, and Exploration Annual Meeting. Salt Lake City, Utah. Preprint 00-84. March 2000.

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From Page No.

Not enough space to include figures

This notebook appears to conform to QA requirements.

E. C. [signature] 11/5/99

See reference information on page 87 of this scientific notebook

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		Recorded by [signature]	Oct 28 1999

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