Calculation Cover Sheet

Contract No. 5057-05 Discipline UMTRA/ES&I Calc. No. 10-436-10-0 No. of Sheets 11

Project UMTRA-GRN

Feature Compaction of Contaminated Materials

Item Statistical Evaluation of degree of compaction

~~Sources of Data~~

Sources of Formulae & References

1. 'Mechanics of Particulate Media', Milton E. Harr, published by McGraw Hill, 1977.
2. "Contaminated Material Moisture Content, Density and Compaction Data - Green River, Utah", prepared by MK-ES (in progress).

Preliminary Calc. ☐ Final Calc. ☒ Supersedes Calc. No. ______

Rev No.	Revision	Calculation By	Date	Checked By	Date	Approved By	Date
C	—	Martin Goodman	25 Oct 89	Rick Stanber	10-30-89	D.K. Chen	10-31-89

Form 42-110

PURPOSE

Using construction data, evaluate the likelihood that contaminated materials were compacted to less than 90% compaction.

APPROACH

In-place compacted density (and percentage compaction) are random variables. An upper bound to the probability that these random variables will be outside a specified range can be estimated using the Chebyshev Inequality, which states:

$$P\left(|x - \bar{x}| > n\sigma_x\right) \leq 1/n^2 \qquad \text{(Ref 1, pg 121)}$$

in which:

x = a random variable

$\bar{x}$ = mean value of random variable

σ_x = standard deviation of the random variable

n = the number of σ-bounds of the random variable beyond the mean of the random variable

P = probability

$$0 \leq P \leq 1$$

The Chebyshev Inequality applies regardless of the probability density function of the random variable, however, if the probability density function is approximately symmetric and has a finite upper bound*, the probability that the value of a random variable being more than n-standard deviations beyond its mean reduces to:

$$P\left(|x-\bar{x}| > n\sigma_x\right) \leq \frac{4}{9n^2} \qquad \text{(ref 1, pg. 123)}$$

For symmetric distributions, the probability that the value of a random variable will exceed n-standard deviations beyond its mean value on one side of the mean can be taken to be one-half the probability the value of the random variable will be more than n-standard deviations beyond its mean on both sides, i.e.,

$$P\left[(x-\bar{x}) > n\sigma_x\right] \leq \frac{2}{9n^2} \qquad \text{for} \quad x \geq \bar{x}$$

$$P\left[(\bar{x}-x) > n\sigma_x\right] \leq \frac{2}{9n^2} \qquad \text{for} \quad x \leq \bar{x}$$

* Percentage Compaction is inherently has a finite upper bound determined from the zero-voids condition.

The symmetry assumption may be checked by calculating the skewness of the random variable data set. The coefficient of skewness is calculated as follows:

$$\beta_1 = \frac{(1/n)\sum_{1}^{n}(x_i-\bar{x})^3}{\sigma_x^3}$$

where
β_1 = coefficient of skewness
n = number of random variable data points
x_i = i^{th} random variable observation
$\bar{x}$ = mean value of random variable
σ_x = standard deviation of random variable

Generally, probability density functions with $-0.5 < \beta_1 < 0.5$ may be considered to be symmetric, although if the distribution is skewed towards the bound whose exceedance probability is being evaluated, the resulting estimate of exceedance probability may be somewhat underconservative.

For the case of the % compaction values of the Green River disposal cell, the random variable is the %-compaction at a random location* measured at regular volume intervals. Therefore, the Chebyshev Inequality may be used to estimate the upper bound in the probability that the %-compaction at any random location will be less than 90% compaction.

Since probability can not be less than zero, the actual probability that compaction at any random location will be less than 90% may be estimated as one-half the upper bound probability. This value is more reasonable for design purposes because the joint probability that many of the lifts will have densities less than 90% is considerably smaller than the probability that just one lift will have a density smaller than 90% compaction.

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Project	UMTRA-GJO	Contract No. 5057-05	Sheet 5/7
Feature	Compaction of Contaminated Materials	Designed MJC	File No.
Item	Statistical Evaluations	Checked RMS	Date 26 Oct 89
			Date 10/30/89

CALCULATIONS

- Tailings % Compaction

a) Demonstration of symmetry

calculate coefficient of skew $\beta_1 = \frac{(1/n)\sum_1^n (x_i - \bar{x})^3}{\sigma^3}$

$\beta_1 = -0.265^* > -0.5$ * (see appended data)

∴ distribution function of percent compaction is approximately symmetric.

b) Determination of n-σ bounds

for percent compaction:

mean = 95.65 %

σ = 2.16 %

} (see appended data)

for x = design compaction percentage, i.e., 90%

$n = (\bar{x} - x) / \sigma$ for $x < \bar{x}$

$= (95.65 - 90)/2.16$

$= 2.62$

c) Calculation of Probability of noncompliance.

$$P(x < 90\%) \leq \frac{2}{9n^2} = \frac{2}{9(2.62^2)} = \underline{0.033}$$

i.e., the theoretical upper bound probability that %-compaction will be less than 90% at any random location is about 3.3%.

– Offpile Contaminated Materials % Compaction

a) demonstration of symmetry

$\beta_1 = -0.215^* > -0.5$ * see appended data

$\therefore$ distribution function of percentage compaction ~ symmetric.

b) $n-\sigma$ bounds

for percent compaction:

mean = 94.67 %
σ = 2.71 % } see appended data

$n = (\bar{x} - x)/\sigma$ for $x < \bar{x}$

$= (94.67 - 90)/2.71$

$= 1.72$

c) Calculation of Probability of Noncompliance

$$P(x < 90\%) \leq \frac{2}{9n^2} = \frac{2}{9(1.72^2)} = \underline{0.075}$$

i.e., the theoretical upper bound probability that % compaction is less than 90% at any random location is about 7.5%.

CONCLUSIONS

The theoretical upper bound probabilities of noncompliance for percent compaction of tailings and off-pile contaminated materials at any random location in the disposal cell are 3.3% and 7.5%, respectively.

The "actual" probabilities of noncompliance for percent compaction at a random location, which are estimated to be approximately one-half of the theoretical upper bound values, agree well with the 1 in 79 noncompliance rate achieved using the Method Specification in the Subcontract Documents.

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31 Oct 89

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[illegible]

~~TABLE 9A~~

OFF-PILE CONTAMINATED MATERIAL **
(Sheet 1 of 2)

Sample No.*	Dry Density (pcf)	Percent Compaction	Moisture Content(%)	Percent Dryer than OMC
MKE-001	108.7	97.8	5.5	7.6
MKE-002	113.4	91.3	5.6	5.1
MKE-003	105.2	90.3	6.3	5.6
MKE-004	112.2	90.3	5.6	5.1
MKE-005	114.1	91.1	6.8	3.4
MKE-006	118.2	94.4	5.6	4.6
MKE-007	120.7	92.2	4.1	3.8
MKE-008	118.7	94.5	7.0	3.0
MKE-009	117.2	94.4	5.4	5.3
MKE-010-R1	118.1	95.1	5.1	5.6
MKE-011	117.2	96.3	7.5	3.3
MKE-012	115.0	94.5	5.7	5.1
MKE-013	116.8	96.0	6.3	4.5
MKE-014	115.3	94.7	4.4	6.4
MKE-015	119.5	96.8	6.0	4.6
MKE-016	119.0	96.4	5.0	5.6
MKE-017	115.6	93.7	6.6	4.0
MKE-018	111.2	96.9	7.0	5.3
MKE-019	117.0	94.8	6.8	3.8
MKE-020	118.1	95.7	6.0	4.7
MKE-021	111.7	90.5	6.6	4.0
MKE-022	119.3	96.1	7.6	3.1
MKE-023	114.2	91.9	5.8	4.9
MKE-024	117.8	96.8	4.8	6.0
MKE-025	121.3	99.7	6.4	4.4
MKE-026	114.5	98.3	5.8	6.1
MKE-027	114.6	98.4	5.7	6.2
MKE-028	119.7	96.2	4.8	6.0
MKE-029	113.1	91.1	5.3	5.9
MKE-030	113.0	97.0	5.3	6.6

*Note: 1 failed point not included.

** Data are extracted from ref. 2.

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~~TABLE 3A~~

OFF-PILE CONTAMINATED MATERIAL **
(Sheet 2 of 2)

Sample No.*	Dry Density (pcf)	Percent Compaction	Moisture Content (%)	Percent Dryer Than OMC
MKE-031	105.2	90.3	6.3	5.6
MKE-032	114.9	98.6	4.7	7.2
MKE-033	114.0	91.6	7.0	3.8
MKE-034	115.1	92.5	7.7	3.1
MKE-035	119.4	97.1	5.7	5.2
MKE-036	119.2	96.9	4.1	6.8
MKE-037	105.5	90.9	5.4	7.1
MKE-077	108.7	96.4	4.9	7.6
MKE-078	115.1	92.7	6.1	4.6
MKE-079	120.0	96.6	5.7	5.0
MEAN	115.19	94.67	5.85	5.14
STANDARD DEVIATION	4.19	2.71	0.91	1.25
Coefficient of Variation	.0364	0.0286	0.1560	0.2435
Coefficient of Skewness	--	-0.215	--	--

*Note: 1 failed point not included.

** Data are extracted from reference 2.

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~~TABLE 3B~~

TAILINGS *
(Sheet 1 of 2)

Sample No.	Dry Density (pcf)	Percent Compaction	Moisture Content(%)	Percent Dryer Than OMC
MKE-038	104.2	93.7	5.9	7.4
MKE-039	104.9	94.3	3.3	9.9
MKE-040	106.9	90.0	5.5	7.7
MKE-041	102.3	98.0	2.9	10.6
MKE-042	99.4	95.2	4.4	9.1
MKE-043	100.0	95.8	4.7	8.8
MKE-044	97.1	95.2	3.7	10.8
MKE-045	96.2	95.2	3.9	11.1
MKE-046	97.8	96.8	2.8	12.2
MKE-047	98.8	96.6	3.3	11.7
MKE-048	102.0	99.7	3.0	12.2
MKE-049	99.6	97.4	5.3	9.9
MKE-050	101.2	97.1	3.9	10.4
MKE-051	97.6	93.5	4.1	9.4
MKE-052	98.8	94.6	3.4	10.1
MKE-053	95.7	91.7	5.1	8.3
MKE-054	96.3	93.0	6.3	7.5
MKE-055	99.2	95.8	3.3	10.5
MKE-056	99.1	95.7	3.3	10.5
MKE-057	97.9	96.9	6.1	8.9
MKE-058	97.0	96.0	3.2	11.8
MKE-059	96.8	95.8	4.0	11.0
MKE-060	98.1	95.2	3.7	11.1
MKE-061	101.9	98.8	5.8	8.9
MKE-062	103.1	100.0	5.4	9.4
MKE-063	98.7	97.7	1.9	13.1
MKE-064	98.1	97.1	6.7	8.9
MKE-065	97.5	96.5	5.3	9.7
MKE-066	98.4	91.4	4.1	9.0
MKE-067	101.0	93.9	5.3	7.8

* Data are extracted from reference 2.

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~~TABLE 3B~~ 2

TAILINGS *
(Sheet 2 of 2)

Sample No.	Dry Density (pcf)	Percent Compaction	Moisture Content(%)	Percent Dryer Than OMC
MKE-068	104.4	95.0	6.7	6.6
MKE-069	102.9	99.4	4.8	9.0
MKE-070	99.5	96.1	3.9	10.2
MKE-071	96.8	93.5	5.0	8.8
MKE-072	109.9	95.1	3.9	8.8
MKE-073	97.9	96.9	3.9	11.1
MKE-074	95.2	94.3	4.2	10.8
MKE-075	98.7	95.7	5.9	8.9
MKE-076	98.7	95.7	5.2	9.6
MEAN	99.73	95.65	4.44	9.78
STANDARD DEVIATION	3.21	2.16	1.18	1.46
COEFFICIENT OF VARIATION	0.0321	0.0226	0.2654	0.1494
Coefficient of Skewness	--	-0.265	--	--

* Data are extracted from reference 2.