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THE FINITE ELEMENT CODE FEAT TO CALCULATE TEMPERATURES IN SOLIDS OF ARBITRARY SHAPES

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Structures often operate at elevated non-uniform temperatures, leading to thermal stresses. The computer code FEAT is used at Atomic Energy of Canada Limited to calculate temperatures in nuclear fuel elements. This paper presents an overview of the code. The code can model: conduction; internal generation of heat; prescribed convection to a heat sink; prescribed temperatures at boundaries; prescribed heat fluxes on surfaces; and temperature-dependence of materials properties like thermal conductivity. Gaps between neighbouring surfaces can also be simulated. The finite element methods is used to solve the classical multidimensional equation for conduction of heat. Iterations are used for the non-linear part of the solution; this makes the code inexpensive to run. The predictions of the code compare well (within ±5%) against independent analytical solutions and against thermocouple measurements in pressure tubes. The code predictions are also consistent with isotherms inferred from grain-growths and from voids in fuel pellets. Although the development of FEAT was driven by applications to nuclear fuel, the code is versatile. It can also be used for temperature calculations in other components of the reactor, or even for non-nuclear systems.

1. Introduction

Many engineered structures operate at non-uniform temperatures. Examples are: components of thermal power plants, engines, turbines, boilers, pumps, furnaces, even electronic circuits. Heat is also transferred in equipment in chemical process industries, and in petroleum refineries [1] *. Therefore, one important step during product design is to calculate the levels and the multi-dimensional distributions of temperatures, and to compare them against safe limits.

As a specific example, consider a nuclear fuel bundle. Fig. 1 identifies its parts. Elevated and/or non-uniform temperatures occur at several locations within a fuel element. The following are some illustrative examples:

- In normal pellets, radial gradients of temperature are evident from radially non-uniform growths of UO_2 grains [2], see for example fig. 2a.
- In grooved pellets, radial, circumferential, and longitudinal gradients of temperature are indicated by multi-dimensional profiles of grain-growth [3], see for example fig. 2b.
- In end pellets, end flux peaking [4] leads to longitudinal and to radial gradients of temperature. This is confirmed by the longitudinal and the radial variations in the shapes and sizes of voids formed near the ends of fuel elements after prolonged operation at high power, see fig. 2c.
- The appendages (bearing pads and spacer pads) can impede heat flow by insulating the sheath from the pellet. They can also increase the heat flow by providing a larger area for heat transfer (fin effect). The two mechanisms can lead to circumferential variations in sheath temperatures, see for example fig. 3.
- Voids in the braze between the sheath and the bearing pad lead to perturbations in the temperatures [5].

Excessive temperatures, and their gradients, can lead to the following mechanisms of failure:

- Reduced and non-uniform material strength as a function of changing temperature;
- Accelerated oxidation;
- Interference among neighbouring surfaces, caused by thermal expansion;
- Thermal stresses;

* [ ] Denotes the reference number.

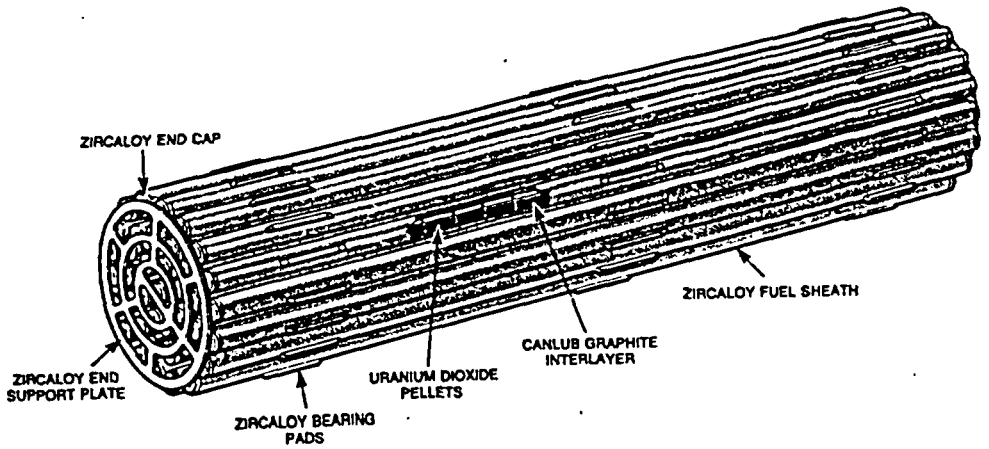


Fig. 1. Nuclear fuel bundle.

- modified material properties as a result of thermal gradients: For example, hydrogen diffuses to the colder regions, leading to precipitation of hydrides and to embrittlement;
- Fission product release in nuclear fuel, leading to stress corrosion cracking;
- Melting.

Temperature calculations are needed by the designer to avoid structural failures from the above mechanisms, by pointing the way to suitable materials, geometry, and operating conditions.

FEAT ** is a general-purpose finite-element computer-code to calculate temperatures in solids of arbitrary shapes. It solves the classical equation for steady-state non-linear conduction of heat, in two or in three dimensions.

The incentive to develop FEAT came from applications intended for nuclear fuel (for example figs. 2 and 3). Therefore, the numerical scheme in FEAT was chosen to minimize the computing time for the types of geometries, and for the non-linearities in thermal conductivities, that are often encountered in the analysis of nuclear fuel. This, however, does not affect the general applicability of FEAT. The fuel-specific features are limited to pre-coded material properties and to pre-coded flux profiles. They are coded in FEAT as options, and are activated by including extra data in the input deck. This can also be seen from the theoretical details and from the test cases reported later in this paper. Hence, the FEAT code is generic, and it can also be used outside of the nuclear fuel industry. Some potential applications outside of the nuclear fuel industry are:

- Pressure tube temperatures to help calculate diffusion of hydrogen during normal operation;
- Temperature profile in pressure tubes due to possible contact with an accidentally bowed fuel element.

In heat transfer calculations involving nuclear fuel, the material non-linearities arise mainly from the temperature-dependent thermal conductivities of Zircaloy and of UO_2, see figs. 4 and 5. Their effects on temperatures can be calculated either via an incremental approach or via an iterative one [6]. The large variations in thermal conductivites, especially of UO_2, can sometimes lead to a significant number of calculation-steps in the *incremental* approach, making the calculations expensive.

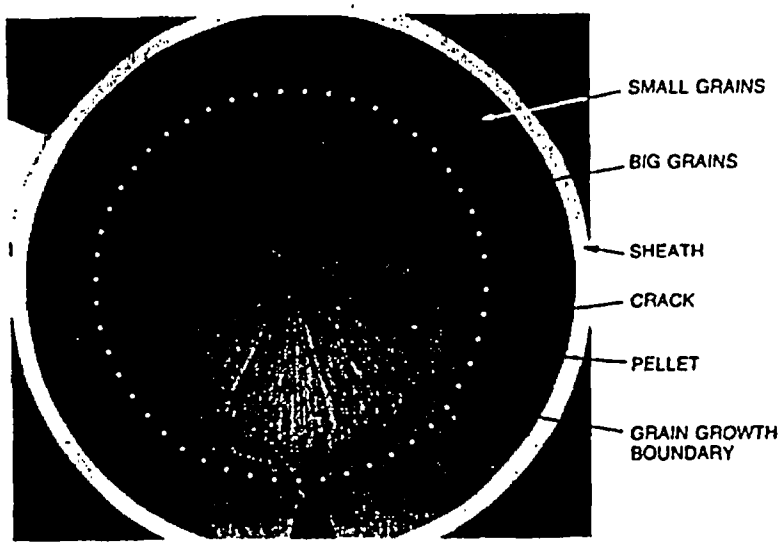
Since steady-state heat transfer is *not* path-dependent, the incremental approach is not really technically necessary for the temperature calculations. To reduce the computing cost, the non-linear calculations in FEAT are based on an *iterative formulation* [6]. The details are described later in this paper.

The present paper first summarizes the features and capabilities of FEAT, then describes their theoretical bases. Then the paper compares FEAT predictions to some analytical solutions and to experimental measurements. Illustrative examples are also given.

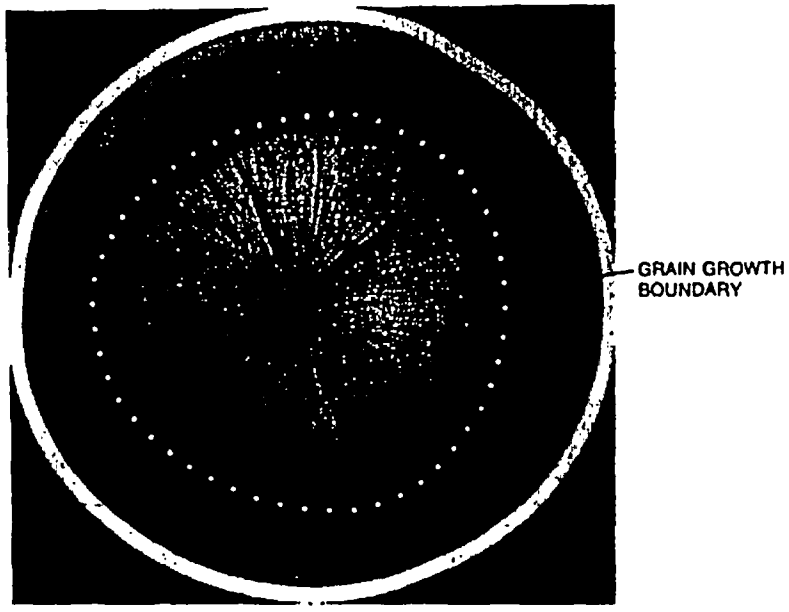
2. Mechanisms

FEAT accounts for the effects of generation and conduction of heat in solids of arbitrary shapes, by

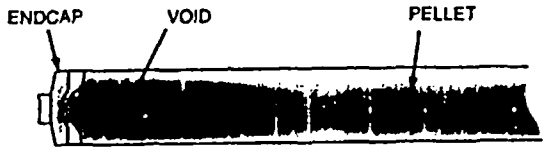
** Finite Element Analyses for Temperatures.



(a) GRAIN GROWTH: NON-UNIFORM IN THE RADIAL DIRECTION

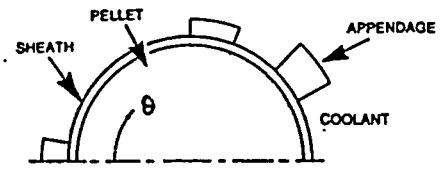


(b) GRAIN GROWTH: NON-UNIFORM RADIALLY AND CIRCUMFERENTIALLY

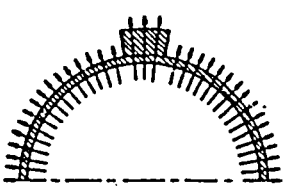


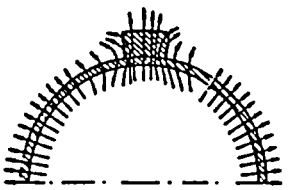
(c) TEAR-DROP VOID DUE TO END FLUX PEAKING

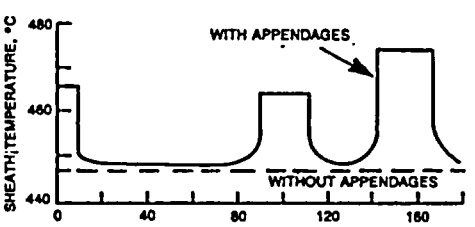
Fig. 2. Consequences of non-uniform temperatures in nuclear fuel.



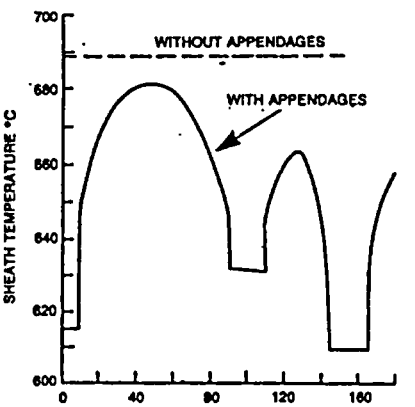
(a) ASSUMED LOCATIONS OF APPENDAGES







(b) INSULATION EFFECT : AT HIGH HEAT TRANSFER COEFFICIENT BETWEEN THE SHEATH AND THE COOLANT (30 kW/(m²K))



(c) FIN EFFECT : AT LOW HEAT TRANSFER COEFFICIENT BETWEEN THE SHEATH AND THE COOLANT (5 kW/(m²K))

Fig. 3. Effect of appendages on temperatures at the outer surface of the sheath.

solving the classical multi-dimensional equation for conduction of heat [1]. This means that the thermal energy is balanced at each point and in each direction. The heat flow, and the temperatures, at all points and in all directions, are calculated from the local gradient of temperature, and from the local thermal conductivity. At the same time, the calculated temperatures are consistent with the prescribed fluxes/heat sources/sinks/boundary-temperatures. The user can specify arbitrary distributions of: fluxes, convection to heat sinks; and prescribed temperatures at boundaries.

The finite element method [7] is used to solve the

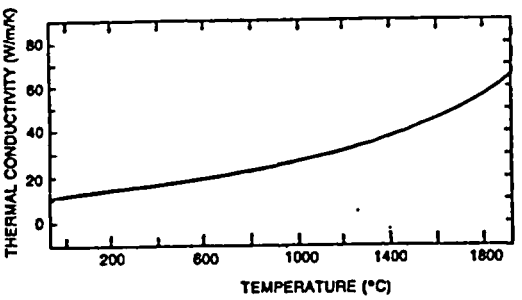


Fig. 4. Thermal conductivity of Zircaloy-4.

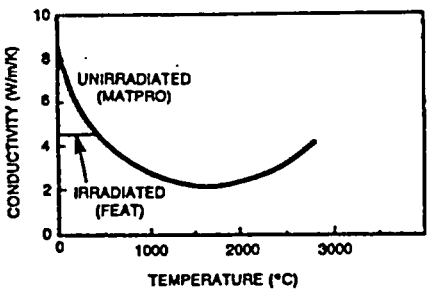


Fig. 5. Thermal conductivity of UO_2.

equation of conduction. Therefore, the code can easily and accurately accomodate [5,6]: complex geometries; curved surfaces; and unusual boundary-conditions.

3. Geometries

The finite element method [6,7] divides the analyzed region into a number of smaller, idealized, subregions called finite elements. The elements are connected to each other at the corner-points, called nodes. Fig. 6 shows some nodes and triangular elements in an illustrative finite element mesh. This mesh was used to calculate temperatures in electric heaters that simulate the critical heat flux in fuel bundles [8].

Fig. 7 shows the types of finite elements available in FEAT. For complete generality, the code permits the geometry to be built from three-dimensional, isoparametric finite elements [7]. Options are provided to select from elements with six to twenty nodes each [9], to suit the analyzed problem.

In applications typical of nuclear fuel, the majority of analyses do not require the three-dimensional capa-

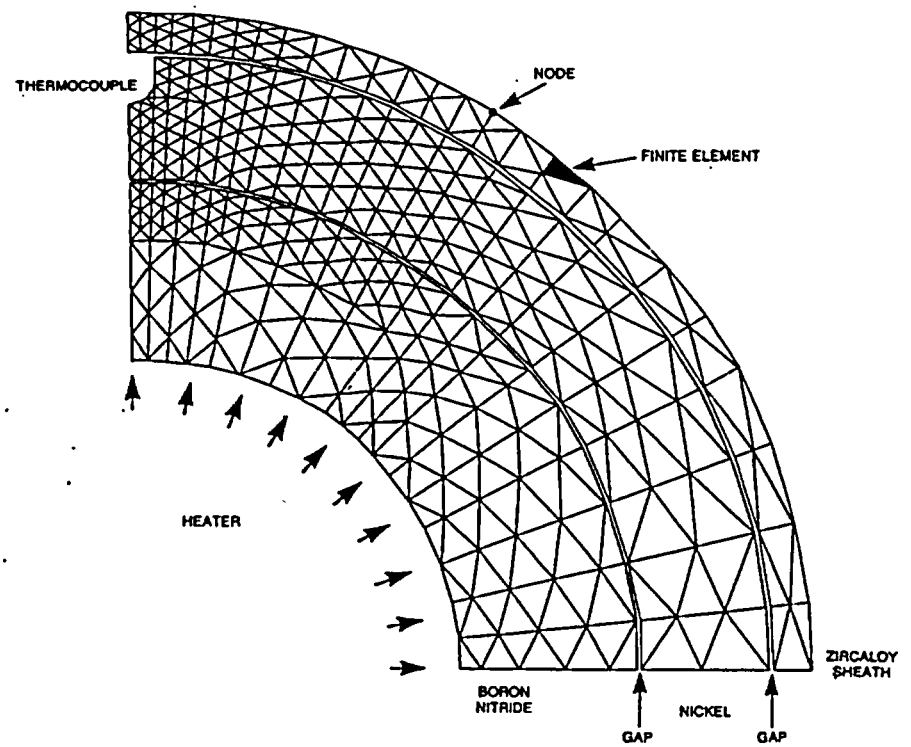


Fig. 6. Illustrative finite element mesh.

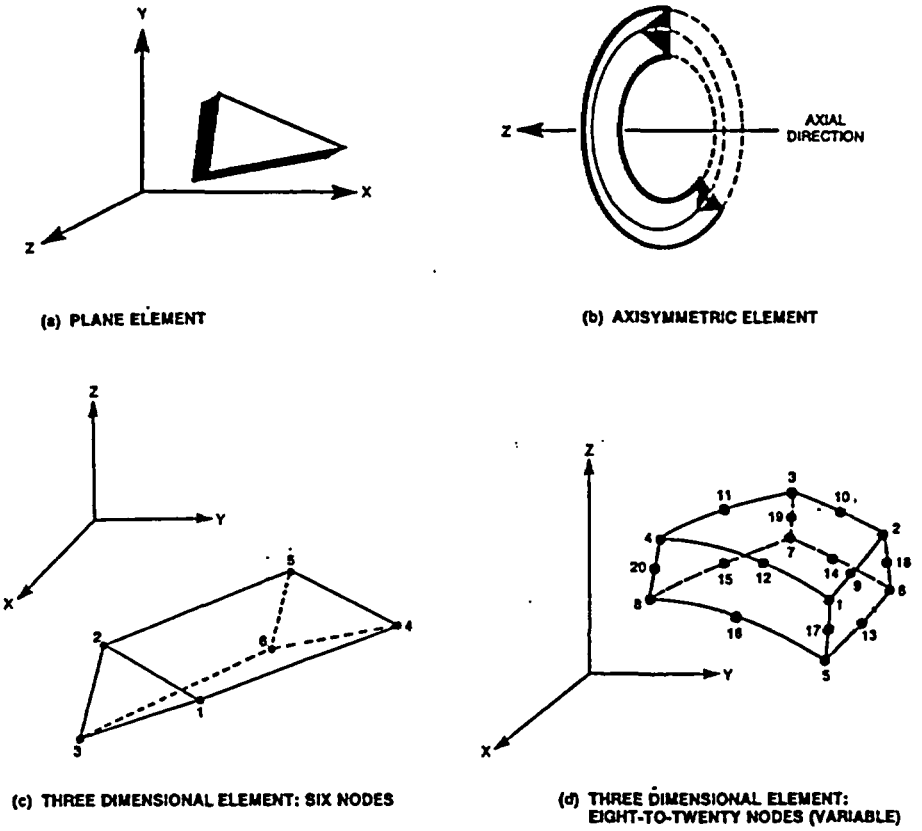


Fig. 7. Types of finite elements in FEAT.

bility of FEAT. Two-dimensional calculations are often sufficient either for reasons of symmetry (e.g. axisymmetric), or because the heat conduction is negligible along one dimension (plane conduction).

Under such conditions, two-dimensional analyses offer significant advantages [10] in accuracy and in cost, compared with three-dimensional analyses. For example:

(i) Two-dimensional meshes require about 10% of the memory for storage and for processing in the computer. Hence, for a given size of the computer, much finer meshes can be used in two-dimensional simulations, significantly improving the accuracy of temperatures in regions of sharp gradients.

(ii) Set-up of a two-dimensional mesh requires an order-of-magnitude less human effort from the analyst, lowering the cost of the analysis.

(iii) For a given number of finite elements (hence accuracy), a two-dimensional solution requires 0.2–3% of the computing time of a three-dimensional solution. The CDC/CYBER-175 computer requires 12 s to calculate non-linear temperatures in 700 two-dimensional finite elements.

To exploit the above advantages, FEAT provides options for using two types of two-dimensional elements: plane, and axisymmetric.

The analyzed system can consist of more than one component. Each component can be made of a different material. An example is a nuclear fuel element, which includes UO_2 pellets and Zircaloy sheath. Special finite elements, called gap elements, are available to simulate the interfaces between neighbouring surfaces. Figure 8 illustrates the gap element available in FEAT.

4. Material properties

The code permits the material properties to depend on the local temperature. The user has an option of specifying the detailed variation of thermal conductivity

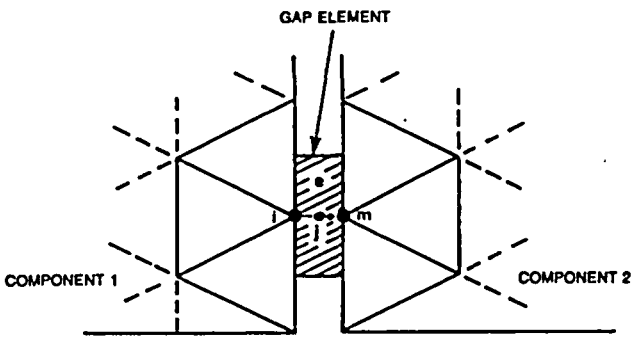


Fig. 8. Gap element(e) to model heat transfer between mating surfaces.

with temperature. Or, for convenience to the nuclear fuel industry, the user can opt for pre-coded values of thermal conductivities of UO_2 and of Zircaloy.

Fig. 4 shows the pre-coded thermal conductivity of Zircaloy, obtained from the MATPRO data base [11]. Fig. 5 shows the thermal conductivity of unirradiated UO_2, also from MATPRO. Irradiation at low temperature lowers the thermal conductivity of UO_2 [12]. Fig. 5 also includes the pre-coded thermal conductivity of irradiated UO_2, obtained from ref. [13].

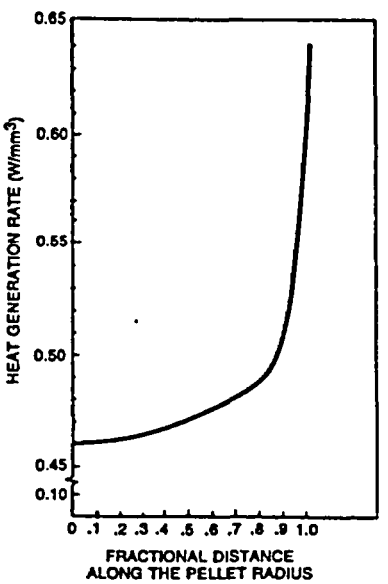


Fig. 9. Heat generation in a nuclear fuel element.

5. Conveniences

In finite element analyses, a major cost is the time required to generate the coordinates and the connectivities of hundreds, sometimes thousands, of nodes and finite elements. This cost is made negligible in FEAT, by an optional link to a digitizer. FEAT is also linked to a post-processor, which plots isotherms. FEAT has been successfully tested and run on mainframe (IBM, CDC) and on personal (IBM) computers.

The code checks for some types of possible errors in the input data. For example, a warning message is printed if a finite element contains two identical nodes. Also, if the node-numbers are found to be clockwise in a two-dimensional problem, the code rearranges them anti-clockwise, and prints a message. For convenience, the user can choose either cartesian or polar coordinate systems, to suit the analyzed geometry.

In nuclear reactors, the local rate of heat generation depends, among others, on the local rate of nuclear fission and hence on the local neutron flux. Within a fuel element, the neutron flux varies radially [13] due to absorption of neutrons, due to production of new isotopes, and due to radioactive decay. Fig. 9 shows an illustrative example. The flux also varies axially within a fuel element, due to end flux peaking [4] and due to the cosine flux shape in some reactors. Fig. 10 shows an example of end flux peaking. For further convenience to the nuclear fuel industry, FEAT also provides options to use pre-coded curves for the radial distribution of heat generation typical of nuclear reactors, and for end flux peaking.

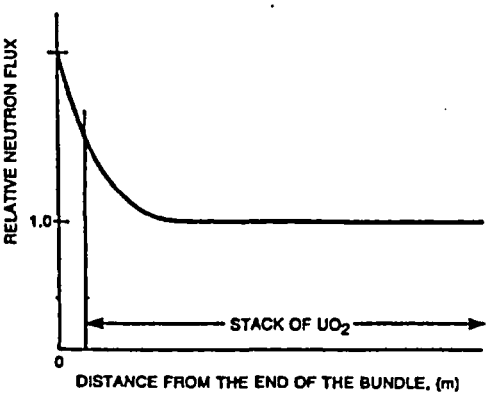


Fig. 10. End flux peaking in a nuclear fuel element.

6. Printout

FEAT prints the following results:
- printout of the input data,
- temperatures at the nodes, and
- heat fluxes at the boundaries.

7. Mathematical formulations

Conservation of thermal energy is ensured by the following classical equation for steady-state conduction of heat [1], expressed here in cartesian coordinates:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q=0. \qquad (1)$$

The symbols are explained in the nomenclature towards the end of the paper.

Many text books [6,7] describe how the finite element method solves the above equation. For the sake of completeness, the major features are summarized here. Matrix notation [7] is used.

Table 1
Element conductivities

$a_i = x_j y_m - x_m y_j,$
$a_j = x_m y_i - x_i y_m,$
$a_m = x_i y_j - x_j y_i,$

$b_i = y_j - y_m,$
$b_j = y_m - y_i,$
$b_m = y_i - y_j,$

$c_i = x_m - x_j,$
$c_j = x_i - x_m,$
$c_m = x_j - x_i.$

(a) For plane elements:

$$[h]=\frac{k}{4\Delta}\begin{bmatrix} b_ib_i & b_jb_i & b_mb_i \\ b_ib_j & b_jb_j & b_mb_j \\ b_ib_m & b_jb_m & b_mb_m \end{bmatrix}+\frac{k}{4\Delta}\begin{bmatrix} c_ic_i & c_jc_i & c_mc_i \\ c_ic_j & c_jc_j & c_mc_j \\ c_ic_m & c_jc_m & c_mc_m \end{bmatrix}$$

(b) For plane gaps:

$$[h]=\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1/\alpha A & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

and the nodal unknown is the heat flow rather than the temperature.

(c) For axisymmetric analyses, replace in the above equations, x by r, y by z, and k by rk.

First, the heat-balance is applied to individual finite elements, to calculate the thermal conductivity of each finite element. The conductivity matrix accounts for the shape, size, and location of each finite element. It also accounts for the type of heat conduction, e.g. plane, axisymmetric, three-dimensional.

Table 1 summarizes the conductivity matrices, [h], for two-dimensional elements including gaps. Ref. [9] gives the conductivity matrices for the three-dimensional elements.

Next, the conductivities of all the individual finite elements are assembled into a global conductivity matrix, which describes the thermal conductivity of the entire system. The system equations are modified to account for the boundary conditions.

This process enables one to form a set of simultaneous equations relating the known external heat fluxes ($[Q]$), to the unknown local temperatures ($[T]$), via the known global conductivity ($[H]$), as follows:

$$\{Q\}=[H]\{T\}. \qquad (2)$$

Solution of eq. (2) gives the nodal temperatures, and the heat fluxes on the boundaries.

8. Non-linear conduction

The thermal conductivity can sometimes depend on the local temperature, see figs. 4, 5. This makes eq. (1) non-linear. FEAT uses iterations to solve the non-linear equations. The first iteration uses the thermal conductivity at the room temperature. The calculated temperatures are then used to revise the thermal conductivities, which are then used to recalculate the temperatures. The iterations continue until two successive calculations show negligible changes in temperatures. For applications typical of nuclear fuel, temperature converge to $\pm 1\%$ in 3–5 iterations.

9. Accuracy

The accuracy of FEAT was checked by comparing its predictions against a number of analytical solutions. A brief summary is given below:

(i) *Case 1*: Rectangular plate generating a heat of 100 W/cm^3, see fig. 11. Its three sides are insulated, and the fourth is cooled by convection to a sink at 300°C. This was analyzed as a plane problem using cartesian coordinates. Fig. 11 shows that FEAT predictions agree with the classical solution [1], to $\pm 1\%$.

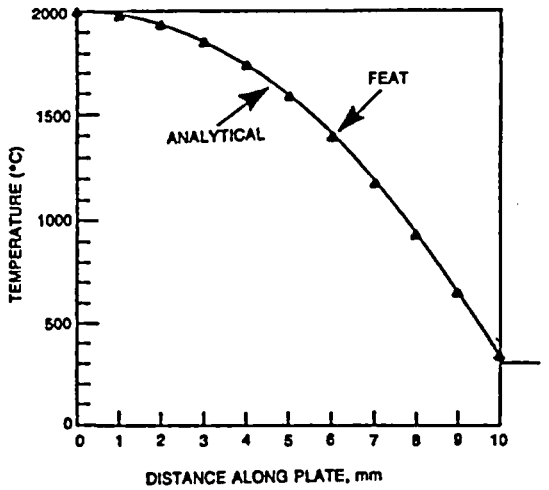


Fig. 11. Temperatures in a rectangular plate generating heat.

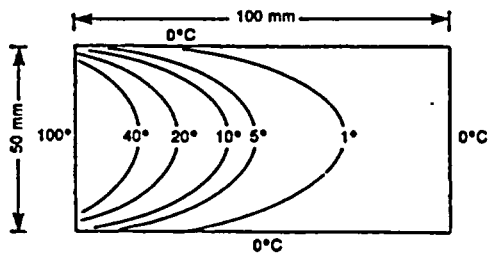


Fig. 12. Isotherms in a rectangular plate with prescribed temperatures at boundaries.

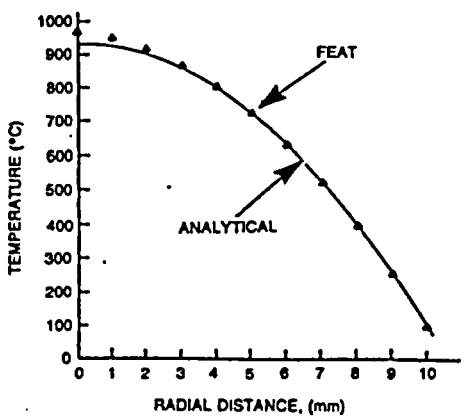


Fig. 13. Temperatures in a solid cylinder generating heat internally.

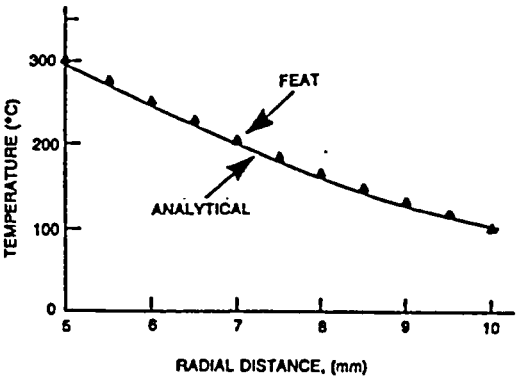
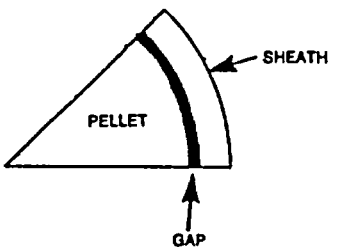


Fig. 14. Temperatures in a hollow cylinder receiving heat flux at its inner diameter.

(ii) *Case 2*: Rectangular plate, specified temperatures at the boundaries, fig. 12. The temperature profile is two-dimensional. The isotherms predicted by FEAT cannot be distinguished from those obtained by a classical solution using Fourier series [1].



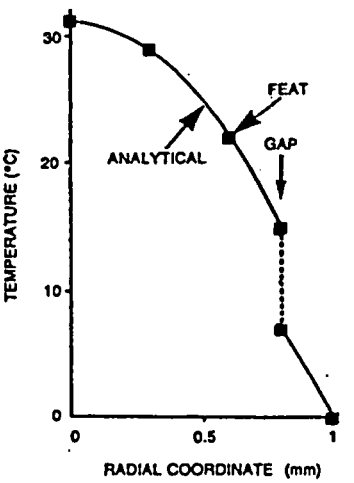


Fig. 15. Temperatures in two concentric cylinders.

(iii) *Case 3*: Solid cylinder, specified internal heat generation rate, fig. 13. The cylinder was analyzed as an axisymmetric problem. The agreement, FEAT vs classical [1], is ±4%.

(iv) *Case 4*: Hollow cylinder, receiving a specified heat flux at its inner surface, fig. 14. This was analyzed as an axisymmetric problem. The maximum difference, FEAT vs. classical [1], is ±3%.

(v) *Case 5*: Two concentric cylinders, separated by a thin gap. The inner cylinder generates heat, fig. 15. The cylinders were analyzed as a plane problem. FEAT agrees with the classical solution [1] to ±2%.

(vi) *Check against ELESIM*: The ELESIM code [13] is frequently used to calculate one-dimensional (i.e. along the radius) temperatures in nuclear fuel elements. The one-dimensional predictions of FEAT were compared to those of ELESIM for natural UO_2 fuel operating under the following conditions: linear heat generation rate of 57.2 kW/m; coolant temperature of 289°C; burnup of 50 MWh/kg U. The pellet diameter was 12 mm. FEAT and ELESIM predicted essentially the same temperatures in the pellet, see fig. 16.

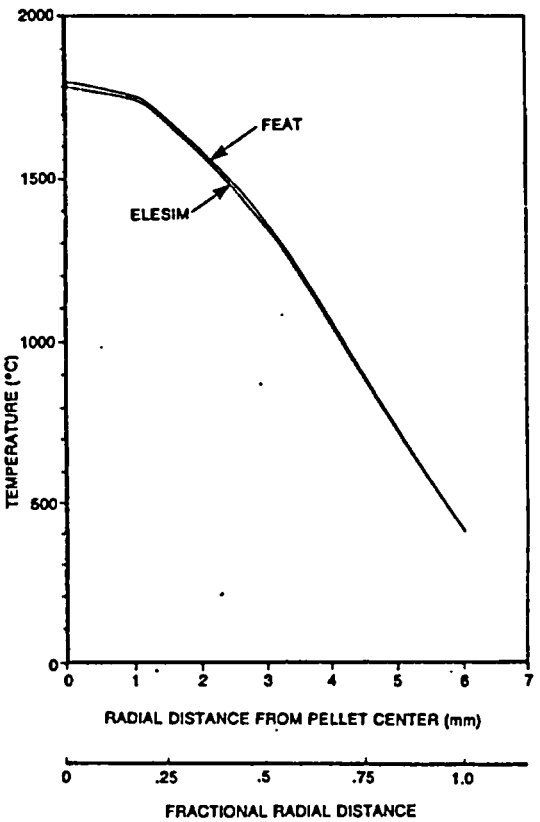


Fig. 16. Comparison of FEAT and ELESIM predictions for temperature across the pellet radius.

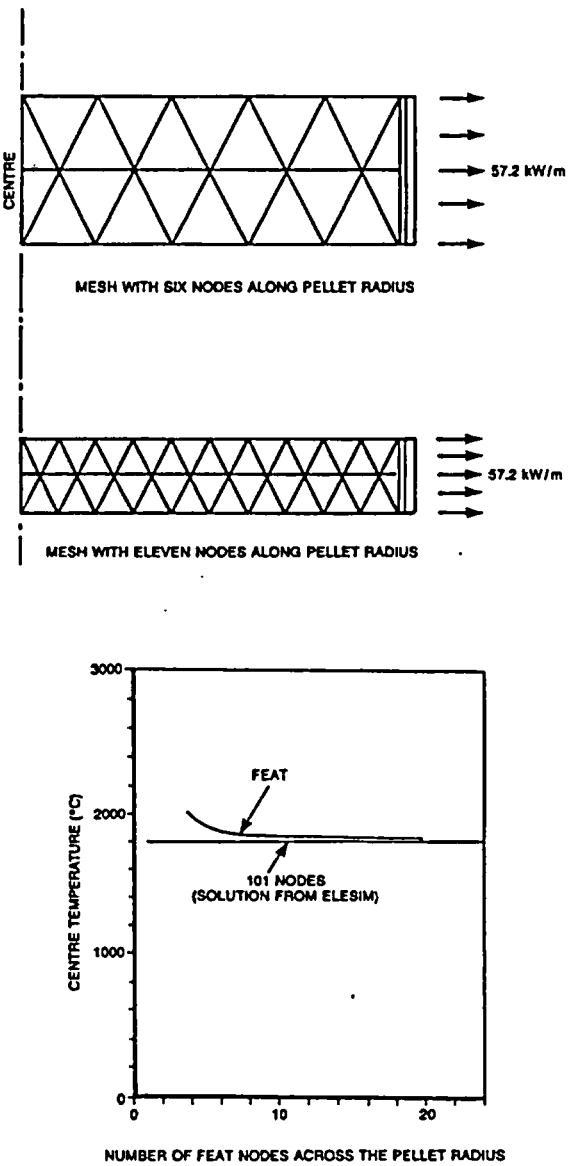


Fig. 17. Convergence of FEAT predictions for pellet center temperature.

10. Convergence

To check the convergence of FEAT, an axisymmetric cylinder made of UO_2 was simulated on FEAT using several finite element meshes representing different degrees of discretization. The cylinder, representing a fuel pellet near the midlength of a fuel element, was assumed to generate 57.2 kW/m of heat. Fig. 9 shows the relative heat generation profile along the radius of the cylinder, which is typical of a natural-UO_2 pellet at the plutonium-peak. The coolant temperature was assumed 289°C. The thermal conductivity is given in fig. 5. In a parametric study, the number of nodes across the radius of the cylinder were varied between 4 and 20. Fig. 17 shows two illustrative meshes. The figure also shows the maximum temperature in the cylinder, which occurs at the center, as a function of the discretization (i.e. the number of nodes across the radius). For reference, the figure also shows the solution obtained from the ELESIM code using 101 radial nodes. The figure shows that the FEAT solution converges monotonically as the number of nodes increases. Ten to fifteen nodes in FEAT provide a converged solution for cylinder temperatures.

11. Previous applications

Since its introduction in 1979, FEAT has been used for several applications [5,9,14] in nuclear reactors. This section summarizes five of the major uses, and the associated checks with experimental data.

11.1. Pressure tube

A check against thermocouple measurements is available, see fig. 18. It shows a section of a sheath and bearing pad in contact with the pressure tube. The heat flux at the inner surface of the sheath was 110 W/cm^2; the coolant temperature outside the pressure tube was ~ 302°C. Thermocouple measurements at two locations in the pressure tube are shown in fig. 18, and compared with temperature contours predicted by FEAT [5]. Fig. 18 shows that the prediction of FEAT for the temperature drop between the two thermocouples is consistent with measurements.

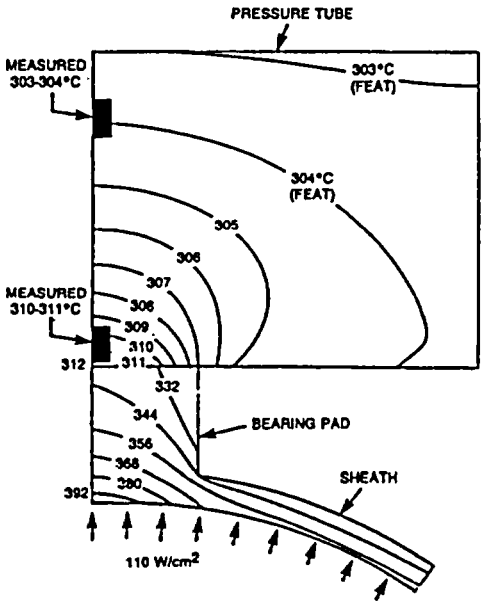


Fig. 18. Isotherms near a bearing pad: thermocouple measurements vs. FEAT predictions.

11.2. Bearing pad design

Shallow corrosion marks have been observed [14] in CANDU * pressure tubes in locations of contact with fuel bearing pads. The corrosion marks are not considered to be a determining factor in the service life of the pressure tubes or of the fuel. However, it is good engineering practice to seek ways to eliminate the cause of such marks.

Such localized corrosion is avoided if the heat flux towards the bearing surface of the pad is reduced to less than approximately 10% of the current value. Changinges in bearing pad geometry can achieve such a reduction.

FEAT simulations assisted in designing the changed geometry of the bearing pads [9]. Temperatures were calculated for a large number of modified geometries. Fig. 19 shows the isotherms in four different designs. The modified designs lower the temperature at the interface between the pad and the pressure tube. Further details are available from ref. [14].

* CANDU – CANada Deuterium Uranium – is a registered trademark of Atomic Energy of Canada Limited.

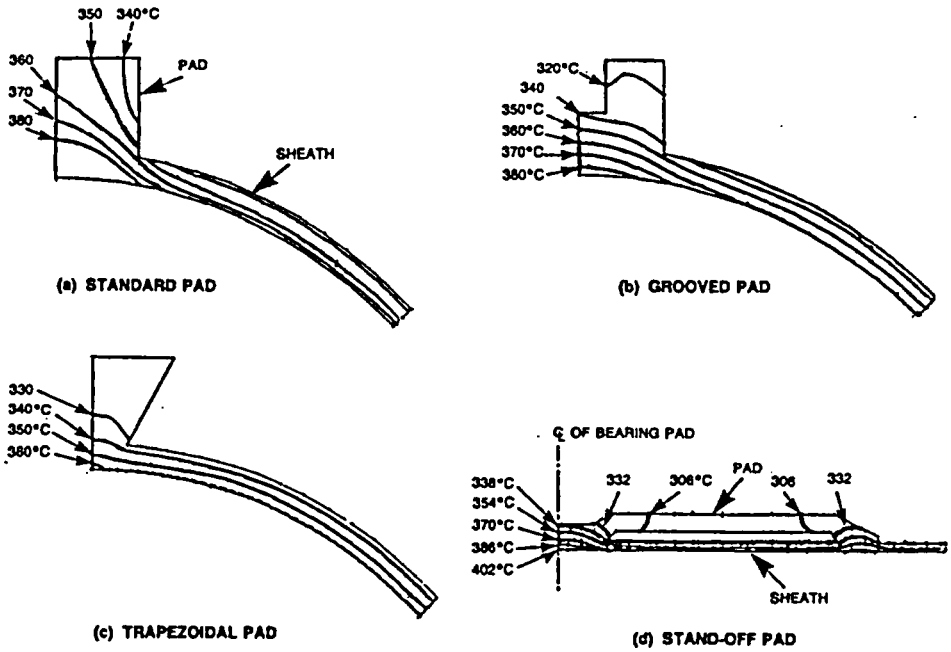


Fig. 19. Isotherms in bearing pads of nuclear fuel.

11.3. End flux peaking

Neutron flux is usually higher near the ends of the fuel element than at the centre [4], see fig. 10. This is called end flux peaking. It occurs because UO_2 is absent at the end of the bundle, and replaced by D_2O and by Zircaloy (endcap, endplate). Both of these have a low cross section for absorbing neutrons. This lack of thermal neutron absorption results in an increase in neutron flux at the end of the stack of pellets. This means that extra heat is generated near the end of the bundle. To compensate, heat conduction via endcaps provides an extra degree of cooling in that region. The end of the fuel element can therefore be either hotter or cooler than the center, depending on the degree of end flux peaking versus the degree of cooling via the endcap. The heat flow is two-dimensional (axisymmetric). Since peak pellet temperature is a consideration in fuel design, and since surface heat flux is an input to thermohydrualic calculations for safety analyses, FEAT has been used to calculate two-dimensional axisymmetric temperatures near the endcap. During these studies, some relationships were observed between FEAT predictions and structural changes in UO_2 during irradiation.

During irradiation, pores of UO_2 migrate to hotter areas, and cause structural changes like voids and grain growth [2]. Fig. 2 shows some examples of structural changes. The degree of the structural change depends on the temperature, on the gradient of temperature, and on the duration of the irradiation [2]. In one-dimensional studies such as ELESIM [2,13], the shape of the structural change is considered indicative of the shape of the isotherm [2,13]. The same concept applies to two-dimensional heat flow. Hence, as a first approximation, the isotherms predicted by FEAT were compared

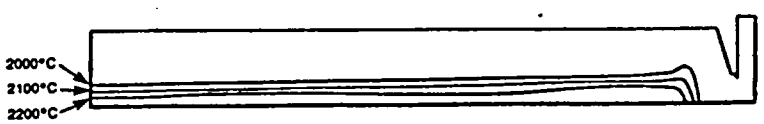


Fig. 20. Isotherms predicted in enriched experimental fuel at high power.

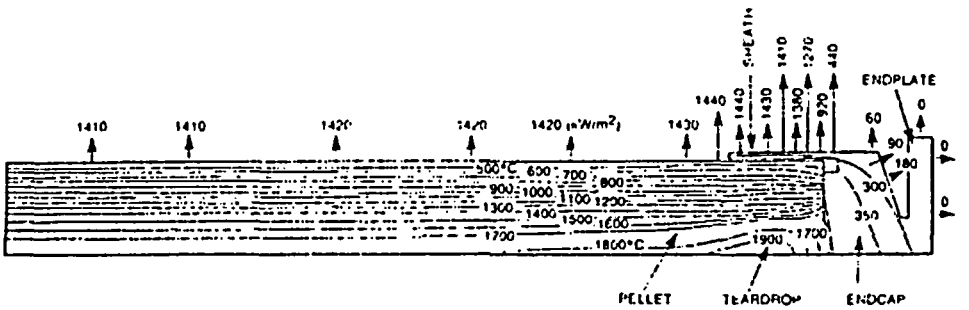
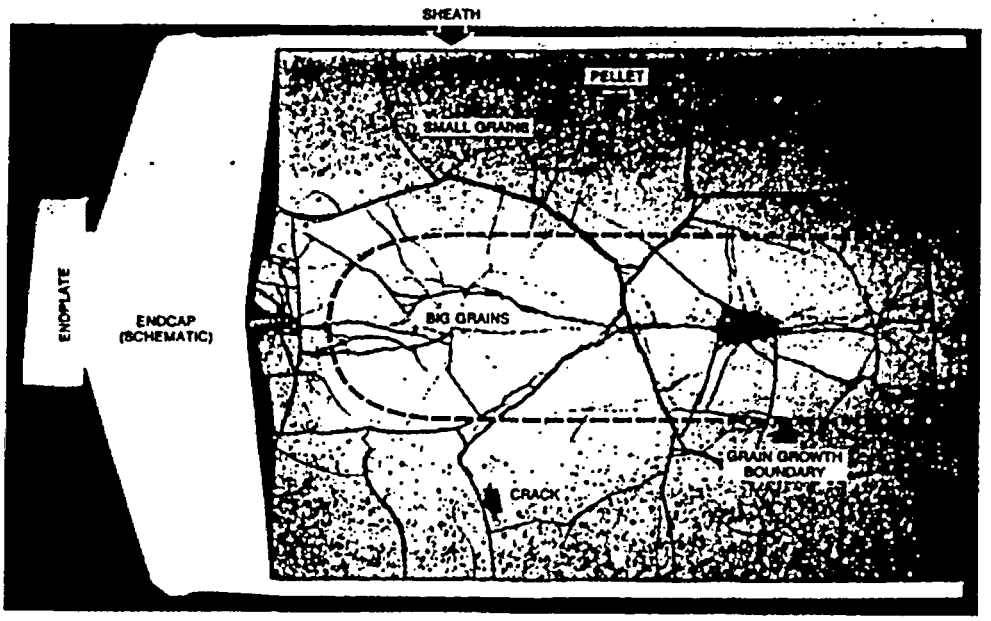
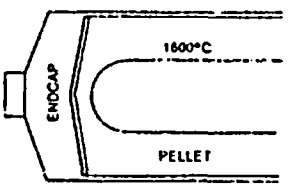


Fig. 21. Isotherms and heat fluxes near the end of a fuel stack of natural UO_2.



(a) OBSERVED GRAIN SIZES



(b) FEAT PREDICTION FOR 1600°C ISOTHERM

Fig. 22. Consequences of irradiation of natural UO_2 at 53 kW/m.

to the shapes of observed voids and grain growths. Voids are discussed in this section, grain growth in the next.

The neutron radiographs of fig. 2c show central voids near the ends of pellets after an experimental irradiation [15]. Central voids are generally believed to be associated with temperatures above 2100°C [2]. In this experiment, the pellets were made from enriched UO_2, increasing the amount of end flux peaking. Further, to be conservative, the testing was done at very high powers: the linear power started at 73 kW/m, and declined gradually to 48 kW/m. Fig. 20 shows some isotherms predicted by FEAT at 70 kW/m. The shapes and the locations of the isotherms are consistent with the shapes of the voids in the radiographs. This is encouraging, and suggests that with further development, FEAT can be extended to also predict the sizes of the voids as a function of time-at-temperature.

Fig. 21 shows an application from a power reactor. Natural UO_2 is used in a power reactor, giving a lower level of end flux peaking. Also, the element powers are lower. Here, FEAT was used to calculate the surface heat fluxes near the endcaps of a natural-UO_2 fuel element operating at 57 kW/m. Although the isotherms are still shaped like tear-drops, the temperatures are too low for central voids. This is consistent with post-irradiation examinations of fuel elements discharged from commercial CANDU reactors: they do not show central voids. Fig. 21 also shows that the peaking in surface heat flux is $\sim 2\%$ $(=(1440/1410-1)\times 100)$, even though the neutron flux peaking is $\sim 10\%$ [4]. This result permitted more realistic (less pessimistic) calculations of thermohydraulics [16] and of fuel strains [17] during accidents: local heat flux of 102% (vs 110%) gives lower coolant temperature, thus less strain in the sheath, hence lower probability of failure of the fuel element during an accident.

These analyses suggest that FEAT is a useful tool to predict pellet temperatures and surface heat fluxes in nuclear fuel.

11.4. Grain growth

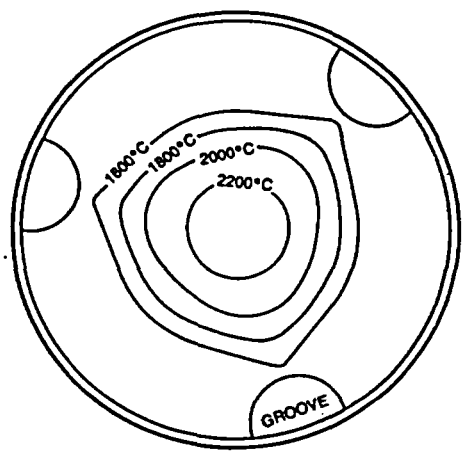
Fig. 22a shows the grains observed in pellets near an endcap of a fuel bundle discharged from a commercial CANDU reactor. This fuel operated at a peak rating of 53 kW/m. There is no evidence of central voids, but the grain sizes show axial variations near the endcap. Like central voids, grain sizes too follow isotherms [2,13]. Here, 1600°C is taken as an approximate marker for visible growth of grains [2,13]. Fig. 22b shows FEAT predictions for the 1600°C isotherm in this fuel element. The shape and the location of the isotherm are consistent with the grain growth pattern. They both show that at the comparatively low rating of this natural-UO_2 fuel element, the effect of end flux peaking is less important than the extra cooling provided by the endcap, giving a net decrease in temperature at the end.

11.5. Grooved pellets

Grooved pellets have been used in some experimental fuel element designs [3] to provide space for storing fission gases. Fig. 23 shows an illustrative prediction of FEAT for isotherms in a grooved pellet. The predicted isotherms have the same shape as the observed patterns of grain growth [3] shown in fig. 2b, with temperatures being higher near the groove.

11.6. Non-nuclear applications

The applicability of FEAT to non-nuclear systems has not yet been demonstrated via an illustrative example. However, the theoretical formulations discussed earlier do not contain limitations specific to nuclear fuel. The test cases and some of the applications confirm that with appropriate input data, FEAT can be used to simulate heat conduction in a variety of geometries and for a variety of material properties. Hence FEAT can also be applied to analyze heat conduction in non-nuclear systems.



(FIGURE COURTESY OF DR. S. GIRGIS, AECL)

Fig. 23. Isotherms predicted for a grooved pellet.

12. Conclusions

The computer code FEAT calculates temperatures in solids of arbitrary shapes. FEAT solves the classical equation for steady state conduction of heat. The solution is obtained for two-dimensional (plane or axisymmetric) or for three-dimensional problems. Gap elements are used to simulate interfaces between neighbouring surfaces.

The code can model: conduction; internal generation of heat; prescribed convection to a heat sink; prescribed temperatures at boundaries; prescribed heat fluxes on surfaces; and temperature-dependence of material properties like thermal conductivity. The user has an option of specifying the detailed variation of thermal conductivity with temperature. Or, for convenience to the nuclear fuel industry, the user can opt for pre-coded values of thermal conductivity, which are obtained from the MATPRO data base.

The predictions of FEAT have been compared against several analytical solutions. The agreement is usually better than 5%. Thermocouple measurements show that FEAT predictions are consistent with measured changes in temperatures in simulated pressure tubes. FEAT predictions were also found consistent with the radial, axial, and circumferential variations in temperatures in the pellets (UO_2) of fuel elements irradiated in experimental and in commercial CANDU reactors. Interesting relationships between two-dimensional temperatures, temperature-gradients, voids, and grain growths are also observed. A knowledge of temperatures is thus shown to be useful in analyzing complex changes in the nature of fuel following irradiation.

The finite element method makes FEAT versatile, and enables it to accurately accomodate complex geometries. The optional link to MATPRO makes it convenient for the nuclear fuel industry to use FEAT, without loss of generality. An iterative solution procedure makes the code inexpensive to run, for the type of material non-linearities often encountered in the analysis of nuclear fuel. The code, however, is general, and can be used for other components of the reactor, or even for non-nuclear systems.

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Nomenclature

Subscripts

i, j, m nodes of a finite element.

Symbols

$\{\ \}$	represents a column vector, arranged in the order of increasing node numbers,
$[\ ]$	represents a rectangular matrix,
A	surface area of the gap across which heat flow occurs,
a	function of nodal coordinates, defined in table 1,
b	function of nodal coordinates, defined in table 1,
c	function of nodal coordinates, defined in table 1,
$[h]$	element conductivity matrix,
$[H]$	system conductivity matrix,
k	local thermal conductivity,
q	heat generation rate per unit volume,
$\{Q\}$	nodal heat flux vector,
r	radial coordinate,
T	temperature,
x, y, z	cartesian coordinates,
α	heat transfer coefficient,
∂	infinitesimal increment,
Δ	area of the finite element,
θ	angular coordinate.

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