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Static and Sliding Friction in Feedback Systems

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One of the most common nonlinearities encountered in servomechanisms design is the friction phenomenon in electromechanical systems. Conventional linear theory fails to predict its effect upon system performance. This paper extends familiar techniques to the treatment of friction nonlinearity in servosystems. Frequency-response methods are employed throughout and the theoretical results are verified by means of an analog computer. Sliding friction and static friction are represented by describing functions which form the critical factors in determining system stability. The analysis indicates that certain series equalizers designed from linear theory may fail to achieve effective compensation in the presence of sliding and static friction. On the other hand, a subsidiary loop may avoid the stability problem while still realizing an essentially equivalent loop gain function.

I. INTRODUCTION

WHILE basic analysis and synthesis procedures for linear feedback systems have become well established during the last decade, there is no correspondingly broad approach to nonlinear problems. Except in very simple cases, no general solutions are possible, and the designer must rely either on machine computation or on various linear or quasi-linear approximations. A variety of such approximations has been developed to fit numerous types of systems and successful design procedures have been discovered for a great many practical problems. It is the purpose of this paper to extend one of these techniques so as to make it applicable to the analysis of feedback systems involving sliding and static friction. Particular attention will be paid to certain loop gain functions which appear to be quite satisfactory on the basis of linear analysis but are found to be unstable in practice as a result of friction phenomena. Methods of predicting, and hence presumably preventing, such behavior will be outlined.

II. REVIEW OF BASIC PROCEDURES

The technique to be employed was first devised by Kochenburger[1] for the analysis of contactor servomechanisms and subsequently adapted for use with other nonlinear devices.[2,3] The basic procedure has been described extensively in the literature and will therefore be outlined only briefly. It is unique in that it permits use of the frequency domain in an approach to problems involving certain types of nonlinear elements. If a sinusoidal voltage is applied to a nonlinear device, the output is generally not sinusoidal. However, under rather general conditions the fundamental component of the output will be greater than any harmonic, a difference which will be further emphasized by effective low-pass filters such as servomotors. Adequate accuracy can therefore be obtained in many cases by considering only the fundamental component of the output.

Since the amplitude and phase of the fundamental component varies with the amplitude of the applied sinusoid, the approximate characteristics of the nonlinear device are represented by an "amplitude-describing function" $H_d(x) = f(x)e^{j\theta(x)}$ (see Fig. 1). If x represents the amplitude of a sinusoidal input signal, $f(x)$ is the ratio of the fundamental output to the input amplitude and $\theta(x)$ is the phase shift of the output fundamental relative to the input signal. Note that $H_d(x)$ is frequency invariant, it depends only on the input amplitude.

Once $H_d(x)$ is known, a stability analysis can proceed essentially as in the linear case. Consider the simple system shown in Fig. 2. System stability is governed by the roots of the equation

$$1 + H_d H(s) = 0 \qquad (1)$$

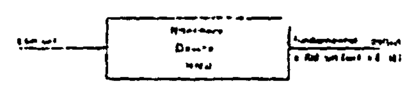

Fig. 1. Describing function of a nonlinear device.

or

$$H(s) = -1/H_d(x) \qquad (2)$$

In the linear case $H_d(x) = 1$ and the stability problem reduces to the conventional one, solved easily by means of a Nyquist plot. The only modification required for the nonlinear case under the assumptions stated is a change in the critical point which now becomes $-1/H_d(x)$ instead of -1. Thus the critical point changes with the signal amplitude, and it becomes necessary to plot an amplitude locus $-1/H_d(x)$ in addition to the frequency locus $H(s)$. If the amplitude locus lies completely outside the frequency locus, the system is stable under all conditions of operations.[4] Figure 3 shows intersecting loci. Here the system is unstable for small disturbances, but stable for large disturbances so

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[1] R. J. Kochenburger, Elec. Eng. 69, 687 (1950). See also Trans. Am. Inst. Elec. Engrs. 69, 270 (1950).

[2] E. C. Johnson, dissertation, Massachusetts Institute of Technology, 1951; Trans. Am. Inst. Elec. Engrs. 71II, 170 (1952).

[3] E. S. Sherrard, Trans. Am. Inst. Elec. Engrs. 71II, 112 (1952).

[4] Frequently the inverse loci, $1/H(s)$ and $-H_d(x)$, are plotted. The choice is governed simply by computational convenience in particular instances.

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that oscillations will tend to stabilize near the intersection point P which thus specifies the steady-state conditions, at least to a first approximation.[1]

In summary, analysis of the stability problem will require the following steps.

(a) Determination of the wave form at the output of the nonlinear device resulting from a sinusoidal input.

(b) Calculation of the describing function $N_s(x)$ from the wave shape obtained in (a).

(c) Plot and interpretation of the amplitude and frequency loci for the system under consideration. For the cases of particular interest here this step requires rearrangement of the conventional block diagram in order to secure effective separation of all transfer functions into two classes: The class of all linear but frequency sensitive components and that of all nonlinear but frequency insensitive elements.

The following definitions will be used throughout this paper. Static friction is the torque required to initiate rotation. Sliding friction is the velocity-independent component of the torque necessary to maintain such motion once started. Viscous friction is that component of the torque which is linearly proportional to the angular velocity of the rotating member.

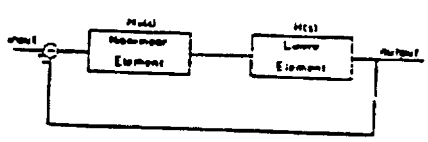

FIG. 2. Feedback loop with nonlinear element.

III. SLIDING FRICTION IN SERVOSYSTEMS

A. Wave Form Resulting from a Sinusoidal Input Torque to a System with Sliding Friction

If only sliding friction is considered the entire friction phenomenon can be represented by the characteristic curve of Fig. 4.

Consider a rotating member with moment of inertia J and angular acceleration $\ddot{\theta}$. Because of sliding friction the effective accelerating or decelerating torque τ_e is related to the applied torque τ_a through the equation

$$\tau_e = \tau_a \pm T_s. \tag{3}$$

where T_s is defined by Fig. 4. From Newton's law of motion,

$$\tau_a = T_s + J\ddot{\theta} \text{ for angular velocity } \dot{\theta} > 0 \tag{4}$$

$$\tau_a = -T_s + J\ddot{\theta} \text{ for } \dot{\theta} < 0. \tag{5}$$

From Eqs. (3), (4), and (5), the angular acceleration of the rotating member is given by

$$\ddot{\theta}(t) = \tau_e(t)/J. \tag{6}$$

Hence $\ddot{\theta}(t)$ has the same wave form as $\tau_e(t)$.

If the applied torque τ_a is sinusoidal,

$$\tau_a(t) = \Gamma_a \sin\omega t. \tag{7}$$

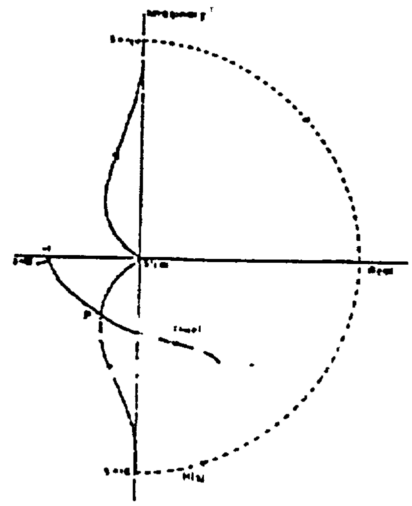

FIG. 3. Amplitude and frequency loci

The corresponding steady-state wave forms are sketched in Figs. 5 and 6. The effective torque wave derived from Eq. (4) is shown in dotted lines. The discontinuities of the τ_e wave correspond to zeros of the $\dot{\theta}$ wave because the frictional torque T_s changes sign at those instants. On the θ curve, point P is the point of inflection, corresponding to maximum acceleration. Since the steady state is of primary interest, the reference time is chosen after the oscillation has reached its steady-state value $\dot{\theta}(t)$ passes through zero at

$$\omega t = n\pi - \alpha, \quad n = 0, 1, 2, 3 \cdots,$$

while

$$\tau_a = T_s \quad \text{at} \quad \omega t = \alpha.$$

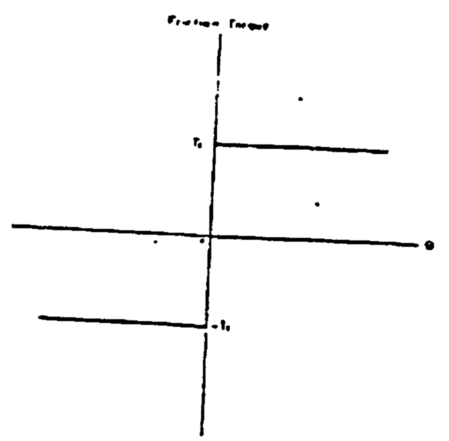

FIG. 4. Sliding friction characteristic.

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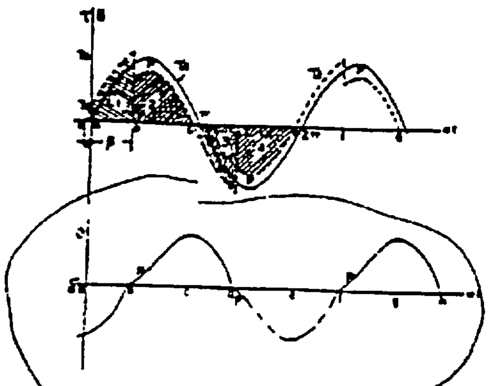

FIG. 5. Steady-state wave forms without dead zones.

It follows that

$$\alpha = \sin^{-1}\lambda, \tag{8}$$

where

$$\lambda = T_s/T_m. \tag{9}$$

Only the angle β corresponding to the first discontinuity point remains unknown. Once it has been evaluated in terms of λ, the wave form is completely determined. There are two possibilities: If $\alpha \leq \beta$, there is no dead zone in the τ_e wave (Fig. 5). If $\alpha > \beta$, there are dead zones as indicated in Fig. 6. These two cases will be considered separately.[3]

Mathematical Representation of the Steady-State Wave Forms

Case (1).—No dead zone. $\alpha \leq \beta$.

Refer to Fig. 5. In the absence of viscous friction, the following steady-state conditions exist.

Shaded area No. 1 = shaded area No. 2
= shaded area No. 3
= shaded area No. 4, etc.[4]

But between a and b,

$$\tau_e = T_m \sin\omega t + T_s$$
$$= T_m(\sin\omega t + \sin\alpha) = T_m(\sin\omega t + \lambda); \tag{10}$$

and between b and c,

$$\tau_e = T_m \sin\omega t - T_s$$
$$= T_m(\sin\omega t - \sin\alpha) = T_m(\sin\omega t - \lambda). \tag{11}$$

Then

$$\text{area No. 1} = \int_{-\alpha}^{\beta} T_m(\sin\omega t + \sin\alpha)d(\omega t)$$
$$= T_m(-\cos\beta + \beta \sin\alpha + \cos\alpha + \alpha \sin\alpha), \tag{12}$$

and

$$\text{area No. 2} = \int_{\beta}^{\pi-\alpha} T_m(\sin\omega t - \sin\alpha)d(\omega t)$$
$$= T_m[\cos\alpha - (\pi - \alpha)\sin\alpha + \cos\beta + \beta \sin\alpha]. \tag{13}$$

If Eqs. (12) and (13) are set equal and simplified, the result is

$$\cos\beta = \pi \sin\alpha / 2$$

or

$$\beta = \cos^{-1}(\pi\lambda/2). \tag{14}$$

For the extreme case, $\beta = \alpha$, Eq. (14) becomes

$$\sin^{-1}\lambda = \cos^{-1}(\pi\lambda/2)$$

or

$$\lambda^2 + (\pi\lambda/2)^2 = 1.$$

A solution for λ yields

$$\lambda = \lambda_c = 0.536 \tag{15}$$

λ_c is the critical value of the quantity T_s/T_m. There is no dead zone for $\lambda \leq \lambda_c$, and there are dead zones for $\lambda > \lambda_c$.

Case (2).—With dead zones, $\alpha > \beta$.

In like manner, one obtains from Fig. 6:

shaded area No. 1 = shaded area No. 2
= shaded area No. 3
= shaded area No. 4, etc.

But, between a and b,

$$\tau_e = T_m(\sin\omega t + \lambda), \tag{16}$$

between b and c,

$$\tau_e = 0, \tag{17}$$

and between c and d,

$$\tau_e = T_m(\sin\omega t - \lambda). \tag{18}$$

Hence,

$$\text{area No. 1} = \int_{-\alpha}^{\beta} T_m(\sin\omega t + \lambda)d(\omega t)$$
$$= T_m(-\cos\beta + \beta\lambda + \cos\alpha + \alpha\lambda), \tag{19}$$

$$\text{area No. 2} = \int_{\alpha}^{\pi-\alpha} T_m(\sin\omega t - \lambda)d(\omega t)$$
$$= T_m[\cos\alpha - (\pi - \alpha)\lambda + \cos\alpha + \alpha\lambda]. \tag{20}$$

Equating (19) and (20) and simplifying

$$\lambda\beta - \cos\beta = (1-\lambda^2)^{\frac{1}{2}} - (\pi - \sin^{-1}\lambda)\lambda. \tag{21}$$

For the extreme case $\alpha = \beta = \sin^{-1}\lambda$,

$$\lambda \sin^{-1}\lambda - (1-\lambda^2)^{\frac{1}{2}} = (1-\lambda^2)^{\frac{1}{2}} - (\pi - \sin^{-1}\lambda)\lambda,$$

or

$$\lambda = \lambda_c = 0.536 \text{ as before.} \tag{15}$$

[3] Note that a dead zone or region of zero effective torque and velocity such as bc on Fig. 6 occurs whenever the applied torque is smaller in magnitude than T_s at the instant when the velocity reaches zero.

[4] Velocity is proportional to the integral of torque in the absence of viscous friction.

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B. Calculation of the Describing Function

Since $\tau_e(t)$ is a periodic function of time, it can be expressed in terms of a Fourier series:

$$\tau_e(t)=\frac{b_0}{2}+\sum_{n=1}^{\infty}(a_n\sin n\omega t+b_n\cos n\omega t). \tag{22}$$

It has been pointed out that, to a first approximation, $\tau_e(t)$ can be represented by the fundamental component of its Fourier series. From symmetry considerations, $b_0=0$. This implies oscillation about the rest position which is a condition of primary interest in stability analyses.[7] Then

$$\tau_e(t)=a_1\sin\omega t+b_1\cos\omega t, \tag{23}$$

where

$$a_1=\frac{1}{\pi}\int_\theta^{2\pi+\theta}\tau_e(t)\sin(\omega t)d(\omega t) \tag{24}$$

and

$$b_1=\frac{1}{\pi}\int_\theta^{2\pi+\theta}\tau_e(t)\cos(\omega t)d(\omega t). \tag{25}$$

Evaluation of the Fourier Coefficients

Case (1).—No dead zone, $\lambda\leq\lambda_c$ or $\alpha\leq\beta$

$$a_1=\frac{2}{\pi}\int_{-\alpha}^{\beta}T_s(\sin\omega t+\lambda)\sin(\omega t)d(\omega t)$$

$$+\frac{2}{\pi}\int_{\beta}^{\pi-\alpha}T_s(\sin\omega t-\lambda)\sin(\omega t)d(\omega t)$$

$$=T_s(1-2\lambda^2) \tag{26}$$

Similarly

$$b_1=2T_s\lambda[(2/\pi)^2-\lambda^2]^{\frac{1}{2}}. \tag{27}$$

Equation (23) may also be written in the form

$$\tau_e(t)=C_1\sin(\omega t+\delta), \tag{28}$$

where

$$C_1=(a_1^2+b_1^2)^{\frac{1}{2}}=T_s\left[1-4\left(1-\frac{4}{\pi^2}\right)\lambda^2\right]^{\frac{1}{2}} \tag{29}$$

and

$$\delta=\tan^{-1}\frac{b_1}{a_1}=\tan^{-1}\frac{2\lambda[(2/\pi)^2-\lambda^2]^{\frac{1}{2}}}{1-2\lambda^2}. \tag{30}$$

Hence, with an applied torque $\tau_a(t)=T_a\sin\omega t$, the effective torque is

$$\tau_e(t)=T_s\left[1-4\left(1-\frac{4}{\pi^2}\right)\lambda^2\right]^{\frac{1}{2}}$$

$$\times\sin\left[\omega t+\tan^{-1}\frac{2\lambda[(2/\pi)^2-\lambda^2]^{\frac{1}{2}}}{1-2\lambda^2}\right] \tag{31}$$

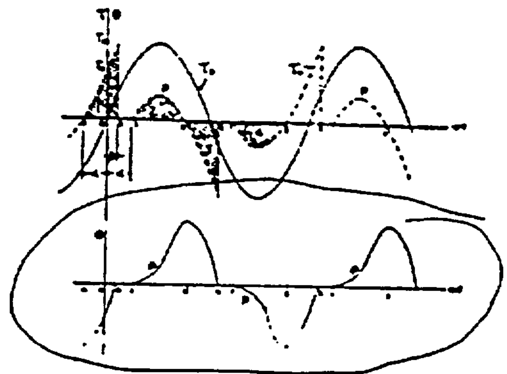

FIG. 6. Steady-state wave forms with dead zones

In accordance with the definition of Sec. II, the describing function for the sliding-friction element is

$$H_s(\lambda)=f(\lambda)\angle\delta(\lambda)$$

$$=\left[1-4\left(1-\frac{4}{\pi^2}\right)\lambda^2\right]^{\frac{1}{2}}\angle\tan^{-1}\frac{2\lambda[(2/\pi)^2-\lambda^2]^{\frac{1}{2}}}{1-2\lambda^2}. \tag{32}$$

Case (2).—With dead zones, $\lambda>\lambda_c$ or $\alpha>\beta$

$$a_1=\frac{1}{\pi}\int_\alpha^{\pi+\beta}T_s(\sin\omega t-\lambda)\sin(\omega t)d(\omega t)$$

$$+\frac{1}{\pi}\int_{\pi+\alpha}^{2\pi+\beta}T_s(\sin\omega t+\lambda)\sin(\omega t)d(\omega t)$$

$$=(T_s/\pi)[\pi-(\alpha-\beta)-\sin\alpha(\cos\alpha+\cos\beta)$$

$$-\cos\beta(\sin\alpha-\sin\beta)] \tag{33}$$

Similarly

$$b_1=(T_s/\pi)(\sin\alpha+\sin\beta)^2. \tag{34}$$

Hence in complete analogy with case (1) the friction-describing function is given by the expression

$$H_s(\lambda)=f(\lambda)\angle\delta(\lambda), \tag{35}$$

where

$$f(\lambda)=\frac{1}{\pi}\{[\pi-(\alpha-\beta)-\sin\alpha(\cos\alpha+\cos\beta)-\cos\beta(\sin\alpha+\sin\beta)]^2+[(\sin\alpha+\sin\beta)^2]^2\}^{\frac{1}{2}}$$

and

$$\delta(\lambda)=\tan^{-1}\frac{(\sin\alpha+\sin\beta)^2}{\pi-(\alpha-\beta)-\sin\alpha(\cos\alpha+\cos\beta)-\cos\beta(\sin\alpha+\sin\beta)}.$$

[7] Extensions to nonzero means are possible but complicate the analysis appreciably. See reference 1.