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HOLTEC INTERNATIONAL  
CALCULATION SHEETOFFICE OF SECRETARY  
RULEMAKINGS AND  
ADJUDICATIONS STAFFPage F-1 Rev \_\_\_\_\_

Project No

Alou Soler

Report No

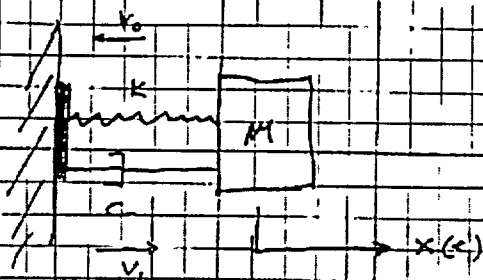
Other

Prepared By

Date

Reviewed By

Date

COEFFICIENT OF RESTITUTION AND  
LINEAR VISCOUS DAMPINGConsider a 1-DOF mass-spring-damper  
after impact with a fixed target

The equation of motion is:

$$M \ddot{x} + c \dot{x} + K x = 0$$

$$x(0) = 0 \quad \dot{x}(0) = -v_0$$

If we define  $\omega_0$ ,  $\beta$  by the relations

$$\omega_0 = \sqrt{K/M} \quad 2\beta\omega_0 = \frac{c}{M}$$

then the equation of motion is

$$\ddot{x} + 2\beta\omega_0 \dot{x} + \omega_0^2 x = 0 \quad (1)$$

Per C. Smith, Applied Mechanics - Kinso Dynamics,  
John Wiley, 1976, pp 187-189, the solution  
to eq (1), subject to the prescribed initial conditions  
is:

DECLARATION OF COMMISSION

Bucket No. \_\_\_\_\_ Official Exh. No. TT  
In the matter of PFS  
Staff \_\_\_\_\_ IDENTIFIED ✓  
Applicant ✓ RECEIVED ✓  
Intervenor \_\_\_\_\_ REJECTED \_\_\_\_\_  
Other \_\_\_\_\_ WITHDRAWN \_\_\_\_\_  
DATE 4-30-02 Witness \_\_\_\_\_  
Clerk emp

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Project No. \_\_\_\_\_ Report No. \_\_\_\_\_ Other \_\_\_\_\_  
Prepared By: Alan Soler Date \_\_\_\_\_ Reviewed By \_\_\_\_\_ Date \_\_\_\_\_

$$\dot{x}(t) = -\frac{V_0}{\omega} e^{-\beta \omega_0 t} \sin \omega t \quad (2)$$

The mass leaves the target at time,  $t_f$ , when

$$\dot{x}(t_f) = 0 = -\frac{V_0}{\omega} e^{-\beta \omega_0 t_f} \sin \omega t_f$$

or, when

$$\omega t_f = \pi = \omega_0 (1 - \beta^2)^{1/2} t_f \quad (3)$$

During the time  $0 \leq t \leq t_f$  when the mass impacts the target, compresses the spring and damper, and reverses direction to return to the starting point, the velocity of the mass is

$$\dot{x}(t) = -\frac{V_0}{\omega} \left[ -\beta \omega_0 e^{-\beta \omega_0 t} \sin \omega t + \omega e^{-\beta \omega_0 t} \cos \omega t \right]$$

so that at time,  $t_f$ , when  $\omega t_f = \pi$

$$\dot{x}(t_f) = V_1 = V_0 e^{-\beta \omega_0 t_f} \quad (4)$$

Since  $\frac{V_1}{V_0} = e^{-\beta \omega_0 t_f} =$  coefficient of restitution, and

$$\omega_0 = \frac{\omega}{\sqrt{1 - \beta^2}}$$

Then

$$e = e^{-\frac{3\pi}{(1 - \beta^2)^{1/2}}} \quad (5)$$

The above equation is plotted in the following

$i = 1 \dots 40$

$$z_i = \frac{(i-1)}{40}$$

$$r_i = \frac{z_i}{\left[1 + (z_i)^2\right]^{1/2}} \pi$$

$$\text{cor}_i = e^{r_i}$$

i	cor <sub>i</sub>	z <sub>i</sub>
1	1	0
2	0.924	0.025
3	0.854	0.05
4	0.79	0.075
5	0.729	0.1
6	0.673	0.125
7	0.621	0.15
8	0.572	0.175
9	0.527	0.2
10	0.484	0.225
11	0.444	0.25
12	0.407	0.275
13	0.372	0.3
14	0.34	0.325
15	0.309	0.35
16	0.281	0.375
17	0.254	0.4
18	0.229	0.425
19	0.205	0.45
20	0.183	0.475
21	0.163	0.5

